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# Phenomenological description of spin dynamics in antiferromagnets: short history and modern development

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A brief review of alternative phenomenological approaches to the spin dynamics of antiferromagnets are discussed in virtue of modern interest to ultrafast spin dynamics and it application. Specific properties of antiferromagnets, first of all, the possibility of spin dynamics faster then for ferromagnets are described. Novel types of solitons for anisotropic antiferromagnets are discussed.

Keywords: antiferromagnet, soliton, ultrafast spin dynamics.

## 1. Introduction

Research on antiferromagnets began almost a century ago, in the 1930s, shortly after the creation of consistent quantum-mechanical theory of magnetic order due to the exchange interaction of spins [1, 2]. All these times, studies of antiferromagnetism constitute a significant part of the fundamental physics of magnetism. A detailed description of the physics of antiferromagnets can be found in the monograph by E. A. Turov with coauthors [3]. However, in the past century, there were practically no examples of the technical application of antiferromagnets. This situation is in a strict contrast with the tremendous successes achieved in the use of ferromagnets (or, equivalently, common ferrimagnets like Yttrium Iron Garnet, which are far from the compensation point) over the same period in microwave technology, magnetic sensors, and information storage and processing systems. Note especially spintronics, in which the main role is played not by the electron charge but by the electron spin, and not by the electric current, but by the spin current. The concept of spintronics appears in 90th mostly for ferromagnets, see for review [4–6]. The effects of spin current allow for the creation of various magneto-electronic devices with submicron sizes, e.g., information storages and solid-state nanogenerator, so-called spin-torque oscillator working in microwave region (up to tens of gigahertz).

Investigations of spin dynamics of antiferromagnets began in the 1950s. First, the analysis was carried out on the basis of various quantum-mechanical approaches [7–10]. Here, however, a problem arising from the fact that the exact quantum-mechanical ground state is unknown for the antiferromagnets. A little bit later, a phenomenological approach, based on the application of the system of famous Landau–Lifshitz equations for the magnetizations of each of the sublattices, $M_\alpha$, was proposed [11, 12]. All theoretical results were in good agreement with contemporary experimental data of linear spin resonance [13]. The main characteristic features of antiferromagnetic spin dynamics, in particular, much higher values of frequencies, were known to the 60th years of the past century. The results of these investigations of spin dynamics for ferromagnets and antiferromagnets were summarized in famous review articles of Akhiezer, Bar'yakhtar, and Kaganov, published in 1960 [14, 15].

For a long time, the higher frequencies of antiferromagnetic resonance were not considered as some advantage for practice, mostly as a great challenge for experimentalists, and only a few laboratories were able to work with antiferromagnetic resonance measurements, see [13, 16, 17]. The situation has changed significantly in the current century. New promising areas of applied physics of magnetism have emerged, which are based solely on the specific features of antiferromagnets; first of all, on their dynamic properties.

The fundamental point is that not only linear resonance but all types of the spin dynamics for antiferromagnets are much faster than for ferromagnets with the same values of microscopic parameters (spin values, exchange integrals, constants of spin-orbit interaction). The characteristic enhancement parameter is determined by the quantity $\sqrt{H_{\rm ex}/H_a} \gg 1$, where $H_{\rm ex}$ is the antiferromagnetic exchange field and $H_a$ is the anisotropy field. The quantity $H_a$ for common antiferromagnets is determined by relativistic interactions and, as well as for ferromagnets, very rarely exceed 10 kOe [3]. Contrary, the uniform exchange field is unique for antiferromagnets; its value is determined by the exchange integral between the spins of different sublattices. Being exchange in its nature, the value of $H_{\rm ex}$ is huge, up to tens of megaOersteds [3]. Due to this circumstance, the frequencies of magnetic resonance in antiferromagnets lie in the range from hundreds of gigahertz to several terahertz, i.e., in the terahertz range. In recent years, the interest to this range increasing sharply, see [18–20] and the recent collective review [21], but there is a clear deficit of compact devices (generators, detectors, amplifiers, ect.) working in this range. It is a common belief that antiferromagnets are the best candidates for usage in terahertz devices, playing a role similar to that of famous Yttrium Iron Garnet for microwave (gigahertz) devices [22, 23].

The employing of antiferromagnets in the terahertz technique is bases on various physical effects, for example, magneto-optical and galvanomagnetic effects. The usage of femtosecond lasers (with pulses shorter than 100 fs) makes it possible to realize non-thermal excitation of spin oscillations in transparent antiferromagnet [24–26]. The observed frequencies were as high as a few terahertz [27, 28], up to 9 terahertz [28]. These effects can be used to create generators of terahertz electromagnetic waves with the optical pump and control [29–31]. Effects of ultrafast (in picosecond time scale) switching the directions of spins of antiferromagnets under the action of light pulses [32, 33] or spin current pulses [34, 35], were also observed. This opportunity opens up an avenue for ultrafast recording and processing of information using arrays of antiferromagnetic particles [35–37].

Great expectations are associated with antiferromagnetic spintronics, a branch of magnetic electronics based on the usage of spin current in antiferromagnet, see Refs. 38–43. For this area, it is fundamentally important that efficiency of the action of spin current on the spin system of antiferromagnets, as well as efficiency of the conversion of the energy of antiferromagnetic spin oscillations into a useful ac signal, are comparable for ferromagnets and antiferromagnets [44]. To realize that for standard dielectric antiferromagnets, familiar spin Hall effect and inverse spin Hall effect can be used [39]. Resonant spin pumping at frequencies up to hundreds of gigahertz was already realized for antiferromagnet/heavy metal bilayer, with antiferromagnet as an active element with resonant frequencies in the subterahertz region [45, 46]. Several schemes of spin-torque antiferromagnetic generators, which can efficiently operate in the subterahertz and terahertz ranges, have been proposed [41, 42, 47–51]. For antiferromagnets, the effect of amplification of the spin current (to the best of our knowledge, it is not known for ferromagnets) was recently predicted [52] and observed experimentally [53].

Concept of antiferromagnetic spin dynamics can be applied to some interesting magnetic materials, having properties of antiferromagnets. Long ago it was found that some magnets (for example, FeRh, $Mn_2GaC$, and $Mn_{2-x}Cr_xSb$ with $0.025 < x < 0.2$) can demonstrate phase transition between ferromagnetic and antiferromagnetic states at some temperature [54]. Antiferromagnetic-to-ferromagnetic phase transition for iron-rhodium FeRh under the action of femtosecond laser pulses was realized on subpicosecond time scale at the very beginning of the femtomagnetic research [55, 56]. Recently strong and fast spin-pumping effects were found for this material that making it very promising candidate for perspective optical spintronics [57, 58]. FeRh in its antiferromagnetic phase is a typical two-sublattice antiferromagnet with the structure of CsCl, with large magnetic moments (3.3 Bohr magnetons) of the iron spins and zero magnetic moment of rhodium [54]. Thus two-sublattice picture, typical for standard antiferromagnets, is adequate for this material, but with significantly (till the values of anisotropy field) reduced exchange field between sublattices in the vicinity of this transition.

These modern successes, experimental and technological, are based on the theoretical description of spin dynamics of antiferromagnets. The problems of interest are mostly nonlinear, and phenomenological approach has no alternative for their analysis. As it is frequently happened, the approach developed long ago becomes in a very high demand nowadays. When writing this article, the authors took the opportunity of briefly reviewing the history of the development of the theory of antiferromagnetic spin dynamics, which was already understood in the 50s of the last century, with the invaluable contribution made by M. I. Kaganov.

## 2. Phenomenological approach to the spin dynamics of an antiferromagnet: the development of a concept

The antiferromagnetic magnetic order is commonly described on the basis of the idea of magnetic sublattices, which have nonzero magnetizations, but the antiferromagnet as a whole without magnetic field is characterized zero total magnetization, at least in the purely exchange approximation. In the simplest case, there are two such sublattices; they corresponds to equivalent crystal positions and contain identical magnetic ions. The sublattice magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ are both equal in length, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$, but they are oriented antiparallel, with the zero total magnetization $\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{M} = 0$. Antiferromagnetic order is determined by the so-called antiferromagnetic vector $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$. The condition $\mathbf{M} = 0$ can be fulfilled in a wide range of external conditions (temperature, pressure, etc.) if and only if the sublattices are equivalent. This means that at least one element of the antiferromagnetic crystal group should produce permutation for atoms belonging to different sublattices (following Turov, call it as an odd element). The importance of this condition was understood already at the end of the 50s; it is the basis of common modern definition of antiferromagnets [3]. In particular, this condition allows one to distinguish between antiferromagnets and ferrimagnets at the compensation point, see, for example, the recent review [59].

As we already mentioned, first results in the field of antiferromagnetic spin dynamics were obtained on the ground of various quantum-mechanical approaches [7–10]. Here, however, a problem, connected to the fact that the exact quantum-mechanical ground state is unknown for antiferromagnets, arose. As an alternative, a phenomenological approach, based on the application of the system of famous Landau–Lifshitz equations for the magnetizations of each of the sublattices, $\mathbf{M}_\alpha$, was proposed [11, 12]. For two-sublattice antiferromagnets $\alpha$ = 1, 2 these equations reads, see, e.g., [14–16]

$$\frac{\partial \mathbf{M}_\alpha}{\partial t} = -\gamma[\mathbf{M}_\alpha \times \mathbf{H}_\alpha^{(\mathrm{eff})}] + \mathbf{R},\ \mathbf{H}_\alpha^{(\mathrm{eff})} = -\frac{\delta W}{\delta \mathbf{M}_\alpha}, \qquad (1)$$

where $\gamma = g\mu_B/\hbar$, $g$ is the Lande factor ($g$-factor), $\mu_B$ is the Bohr magneton modulus, $\gamma \approx 2.8$ MHz/Oe at $g = 2$, $\mathbf{R}$ describes the energy dissipation. The effective magnetic fields $\mathbf{H}_\alpha^{(\mathrm{eff})}$ are defined through the variational derivatives of the energy (more precisely, of the nonequilibrium thermodynamic potential) of the two-sublattice antiferromagnet, $W = W[\mathbf{M}_1, \mathbf{M}_2]$, written as a functional of the sublattice magnetization densities $\mathbf{M}_{1,2} = \mathbf{M}_{1,2}(\mathbf{r}, t)$. This functional has the standard form $W = \int w d\mathbf{r}$, with the density $w$ depending on both magnetizations $\mathbf{M}_{1,2}$ and their gradients. Additionally to standard terms with nonuniform exchange, magnetic anisotropy, ect., common to that for ferromagnets, this energy contains uniform exchange interaction of sublattices, proportional to scalar product of their magnetizations, $w_{\mathrm{ex}} = (H_{\mathrm{ex}}/2M_0)(\mathbf{M}_1 \cdot \mathbf{M}_2)$. Here the exchange field of antiferromagnet, $H_{\mathrm{ex}}$, chosen such that in exchange approximation the antiferromagnetic order is destroyed (sublattices become parallel and $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2 = 0$) at $H \geq H_{\mathrm{ex}}$, see more about this parameter in the monograph [3] The total energy of antiferromagnet coincides with $W = W[\mathbf{M}_1, \mathbf{M}_2]$. For $\mathbf{R} = 0$, the energy is an integral of motion for antiferromagnetic dynamics.

It is convenient to rewrites the equation (1) through normalized vectors $\mathbf{M}$ and $\mathbf{L}$, instead of the magnetization vectors of the sublattices $\mathbf{M}_{1,2}$:

$$\mathbf{l} = \mathbf{L}/2\mathbf{M}_0,\ \ \mathbf{m} = \mathbf{M}/2\mathbf{M}_0. \qquad (2)$$

These vectors are constrained as

$$\mathbf{l}^2 + \mathbf{m}^2 = 1, \quad \mathbf{ml} = 0. \qquad (3)$$

The equations of motion for the vectors $\mathbf{l}$ and $\mathbf{m}$ can be easily obtained from the set of Landau–Lifshitz equations (1). These equations (here and below the relaxation processes are be neglected) take the form

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma[\mathbf{m} \times \mathbf{H}_m] + \gamma[\mathbf{l} \times \mathbf{H}_l], \qquad \frac{\partial \mathbf{l}}{\partial t} = \gamma[\mathbf{h}_m \times \mathbf{l}] + \gamma[\mathbf{H}_l \times \mathbf{m}], \qquad (4)$$

where we introduce effective fields,

$$\mathbf{H}_m = -\frac{1}{2M_0}\frac{\delta W}{\delta \mathbf{m}},\ \ \mathbf{H}_l = -\frac{\mathbf{l}}{2M_0}\frac{\delta W}{\delta \mathbf{l}}, \qquad (5)$$

and $W = W[\mathbf{m}, \mathbf{l}]$ is the energy functional of antiferromagnet rewritten through the new variables, $\mathbf{m}$ and $\mathbf{l}$. In particular, the density of the uniform exchange interaction, with accounting for constraint (3), takes the form $w_{\mathrm{ex}} = H_{\mathrm{ex}} M_0 \mathbf{m}^2$. It is clear that two sets of equation, Eq. (1) with $\mathbf{R} = 0$ and Eqs. (4) are completely equivalent, and the choice of this or that depends on the convenience only.

The great advantage of the above phenomenological approach is the possibility of the description of nonlinear dynamics. Nonlinear dynamical states, antiferromagnetic solitons, were considered by many authors on the equivalent footing of Eq. (1) or Eqs. (4) [62–66]. It is interesting to note that the authors of Ref. 11 saw the main motivation for using the phenomenological approach in the possibility, at least in words, to overcome the problem of the ground state of the antiferromagnets. Indeed, the knowledge of the quantum ground state is not necessary within this approach. This approach is based only on the assumption of the existence of sublattices, not necessary with saturated magnetizations, which could be considered an experimental fact. The authors of the phenomenological approach themselves understood that their results are not rigorous proofs, but only "arguments in favor" of sublattice picture of antiferromagnets. M. I. Kaganov recalls the discussion of the work Ref. 11 with L. D. Landau "Getting acquainted with our work, Landau "threatened" that he would show us how to construct a theory of weakly excited states of antiferro-magnets" [67]. M. I. Kaganov also discussed the article of Andreev and Marchenko [68]; he had estimated this work as a step forward to the Landau's idea about construction of fully-phenomenological theory of AFM [67].

Indeed, the work Ref. 68 is based on the natural idea, general for various area of macroscopic theory of electrodynamics of continuous media: if the microscopic charge density, averaged over some media, equals to zero, this media must be described in terms of moments of this density; dipole, quadruple, etc. As applied to magnets, the role of scalar quantity, charge density is played by the vector quantity, microscopic spin density. Following [68], within macroscopic approach spin state of all antiferromagnets can be described by a few unit vectors (maximum three), which are orthogonal to each other. For standard antiferromagnets with two sublattices, the corresponding "dipolar moment of the magnetization vector" is one normalized (unit) vector $\mathbf{l}$, $\mathbf{l}^2 = 1$, with transformation properties completely analogues of that for the antiferromagnetic vector $\mathbf{L}$ introduced above. (Despite of some differences in the definition of the unit vector $\mathbf{l}$ here and normalized antiferromagnetic vector $\mathbf{L}$ above, we are using the same notations for both them, for the reason discussed below.) It turns out that a closed effective equation for the vector $\mathbf{l}$, which describes the dynamics of an antiferromagnet, can be obtained by consideration of symmetry [68]; now this equation is commonly called as the sigma model equation. The sigma model equation for antiferromagnet can be written by the variation of the Lagrangian $\mathcal{L} = \int L d\mathbf{r}$ as the following

$$[\mathbf{l} \times (\delta\mathcal{L} / \delta\mathbf{l})] = 0,$$

where density $L$ has the form [68]:

$$L = \frac{\chi_\perp}{2\gamma^2}\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2 + \frac{\chi_\perp}{\gamma}\left(\mathbf{H}\cdot\left[\mathbf{l}\times\frac{\partial \mathbf{l}}{\partial t}\right]\right) - w(\mathbf{l}, \nabla\mathbf{l}), \qquad (6)$$

$\chi_\perp = 2M_0 / H_{\text{ex}}$ is the transversal susceptibility of antiferromagnet, $H_{\text{ex}}$ is the aforementioned exchange field of antiferromagnet, $\mathbf{H}$ is the magnetic field, and the density of static energy $w(\mathbf{l}, \nabla\mathbf{l})$ can be taken of the form

$$w(\mathbf{l}, \nabla\mathbf{l}) = \frac{A}{2}\left(\frac{\partial \mathbf{l}}{\partial x_i}\right)^2 + w_a(\mathbf{l}) + \frac{\chi_\perp}{2}(\mathbf{H}\cdot\mathbf{l})^2, \qquad (7)$$

where the first term is the energy of non-uniform exchange (written here in the simplest, isotropic over space coordinates form), second term describes the magnetic anisotropy and the last term determines specific contribution of the magnetic field, which can be treated as a renormalization of the anisotropy energy. The second term in the Lagrangian (6) determines the gyroscopic dynamics, which for antiferromagnets occurs because of the magnetic field only. Note the absence of uniform exchange interaction in this energy at zero magnetic field, instead exchange field enters inertial term with $(\partial\mathbf{l} / \partial t)^2$.

Within the sigma model approach, the only vector $\mathbf{l}$ plays the role of dynamical variable; the magnetization $\mathbf{M}$ is a "slave" variable and it is fully determined by $\mathbf{l}$ and its time derivative [68],

$$\mathbf{M} = \chi_\perp\left[\mathbf{H} - \mathbf{l}(\mathbf{H}\cdot\mathbf{l}) + \frac{1}{\gamma}\left(\frac{\partial \mathbf{l}}{\partial t}\times\mathbf{l}\right)\right]. \qquad (8)$$

Generally speaking, here and above in the Lagrangian (6), the effective magnetic field

$$\mathbf{H} \to \mathbf{H}_{\text{eff}} = \mathbf{H}_0 + \mathbf{H}_D$$

should be used. Here $\mathbf{H}_0$ is the external magnetic field and $\mathbf{H}_D$ is so-called Dzyaloshinskii field, intrinsic symmetry-determined vector characteristics of AFM, responsible for canting of sublattices (weak ferromagnetism), see [3, 68]. It is interesting to note that Dzyaloshinskii field in the simplest antisymmetric form, $\mathbf{H}_D \propto (\mathbf{d}\times\mathbf{l})$, where the vector $\mathbf{d}$ is determined by the symmetry of the magnet [3], contribute to the weak magnetic moment, see (8) but not contribute to the gyroscopic term in Eq. (6) [68]. Note that some more general forms of Dzyaloshinskii field leads to specific gyroscopic dynamics, see for more details the recent review [69], but we will not discuss their effects here.

The Lagrangian of the sigma model (6) rewritten as $\mathcal{L} = T + G - U$ has a clear mechanical analogy: the first two terms in (6) are "kinetic energy" $T$ and gyroscopic term $G$ and the last term has the sense of "potential energy". The energy of the antiferromagnet $W$ within the sigma model approach contains two terms $W = W[\mathbf{l}] = T + U$,

$$T = \frac{\chi_\perp}{2\gamma^2}\int\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2 d\mathbf{r}, \quad U = \int w(\mathbf{l}, \nabla\mathbf{l}) d\mathbf{r}. \qquad (9)$$

Two aforementioned phenomenological approaches, the first one, based on the set of Landau–Lifshitz equations, and the second one, based on the sigma model, look quite different. Even orders of time derivatives are different for these equations. Dynamical variables are also formally different: two constrained vectors $\mathbf{m}$ and $\mathbf{l}$, $\mathbf{l}^2 \le 1$ for (4) and unit vector $\mathbf{l}$ for sigma model (7). But in fact these two approaches are quite common, and it is possible to find an important link between them. For this reason we are keeping the same designation for both vectors, it will not lead to confusing below. Using the condition that the exchange interaction is much stronger that all other interactions in the system, equations for sublattice magnetizations $\mathbf{M}_1$, $\mathbf{M}_2$ can be reduced to the sigma model equation [69, 70]. Let us explain this briefly, more detailed description can be found in original articles [70, 71], and also in the books and reviews [69, 72, 73].

First note that all terms in the equation for $d\mathbf{l} / dt$ in the system (4) are bilinear on the components of the vectors $\mathbf{l}$ and $\mathbf{m}$ (this general feature is dictated by the different transformation properties of these vectors and, correspondingly, effective fields $\mathbf{H}_m$ and $\mathbf{H}_l$ under the action of an odd element of the crystal symmetry group). In the presence of the exchange term $\mathbf{H}_{\text{ex}} = H_{\text{ex}}\mathbf{m}$, all other small relativistic terms with the common structure can be neglected in this equation. If we omit these small terms, the equation simplifies to the form

$$(1/\gamma)\partial\mathbf{l} / \partial t = H_{\text{ex}}(\mathbf{l}\times\mathbf{m}) + (\mathbf{H}_{\text{eff}}\times\mathbf{l}),$$

that directly leads to the equation for magnetization (8), characteristic for the sigma model. Then we can use (8), exclude magnetization from the equation for $d\mathbf{m}/dt$ and obtain the equation for the vector $\mathbf{l}$ alone, which coincides with the sigma model equation. If the dynamics is slow enough ($|d\mathbf{l}/dt| \ll \gamma H_{\rm ex}$) and the magnetic field is small comparing with the exchange field, the value of the normalized magnetization $\mathbf{m}$ is small, $\mathbf{m}^2 \ll \mathbf{l}^2$, and the vector $\mathbf{l}$ can be treated as unit vector. Note that the typical values of the resonance frequency for antiferromagnets, $\omega_{AFM} \simeq \simeq \gamma\sqrt{H_{\rm ex}H_a} \ll \gamma H_{\rm ex}$ and typical characteristic fields, like spin-flop field $H_{SF} \simeq \sqrt{H_{\rm ex}H_a} \ll H_{\rm ex}$, satisfy the above inequalities. Thus the sigma model equation is equivalent to the set of Landau–Lifshitz equations of the form of (1) or (4); at least, in the first approximation over small parameter $\sqrt{H_a/H_{\rm ex}}$.

It is instructive to compare these two approaches to antiferromagnetic spin dynamics. Sigma model equation is quite convenient, especially having in mind the formal Lorentz-invariance of this equation in many interesting systems and consequent simplification of the description of common moving magnetic solitons [69]. For standard antiferromagnets, the value of the parameter $\sqrt{H_a/H_{\rm ex}}$ is actually small, but some important exception can be mentioned also. Note the description of ultrafast phase transition between ferromagnetic and antiferromagnetic states in iron-rhodium FeRh [55, 56], which is of large interest for modern spintronic applications [57, 58]. This transition is caused by the change of the sign of the uniform exchange interaction between sublattices, and the value of exchange field significantly reduced in the vicinity of the transition [54]. Of course, this transition is of the first order, and exchange field is not exactly zero, but the values comparable with relativistic(anisotropy) fields are expected. For description of spin dynamics for such materials the analysis of full set of two equations sublattice magnetizations have no alternative. It is worth to note that the constant of non-uniform exchange $A$ is determined by both inter-sublattice and intra-sublattice exchanges; it is not, in general, small near this transition. For this reason, the characteristic length scale $\Delta = \sqrt{A/(M_0 H_a)}$ exceed the lattice constant $a$ and macroscopic approximation is still valid.

The situation is not completely clear even for standard antiferromagnets with $\sqrt{H_a/H_{\rm ex}} \sim 10^{-2} \ll 1$. The presence of a small parameters and expansion over them is an unavoidable procedure for solution of any physical problem. But the theorist should be careful when using this procedure, especially looking for some delicate problems like soliton theory or nonlinear oscillations in systems with a few degrees of freedom, see, e.g., [74–79]. In particular, for non-integrable dynamical systems omitting of some small parameters could restore integrability and thus change the principal properties of the dynamics of this system [74]. For the complete analysis it is useful to control the small corrections to the standard sigma model (6) that can be done by the analysis of the full set of Eqs. (4).

Note one interesting example, nonlinear waves in isotropic antiferromagnet, when the sigma model in its simplest form is not valid. For such antiferromagnets, sigma model allows nonlocalized nonlinear wave of the form of nonuniform in space precession of the vector $\mathbf{l}$,

$$l_x + il_y = l_\perp \exp(i\mathbf{kr} - i\omega_k t), \quad l_z = \text{const} = \pm\sqrt{1-l_\perp^2},$$

where the frequency $\omega_k$ depends on the "wave vector" $\mathbf{k}$ in the same way as for linear wave, but this frequency is independent of the wave amplitude $l_\perp$ for any allowed values $l_\perp \le 1$. Of course, this property witnesses the strong degeneration of this version of the sigma model as a nonlinear system. As well, in this system traveling-wave localized solutions, which are most typical nonlinear excitations (solitons), are absent. Contrary such solitons, common to well-known Lieb states, appears in isotropic antiferromagnet being described beyond the sigma model [80]. On the other hand, sigma model approach is quite effective for many nonlinear effects, like domain wall dynamics in biaxial antiferromagnets [73], inertial spin switching under an action of laser pulses [24, 32], spin currents effects [52] and many others. Thus the answer on the question, whether or not these "abnormal" solitonic states absent for the sigma model for isotropic antiferromagnets survives for the general models of antiferromagnets, is of large interest.

### 3. Lagrange and Hamilton approach to dynamics of sublattice magnetizations

Equation of motion for the vectors $\mathbf{l}$ and $\mathbf{m}$ are determined by the density of energy. General form of this energy can be written as follows

$$w = H_{\rm ex} M_0 \mathbf{m}^2 + \frac{A}{2}(\nabla \mathbf{l})^2 + \frac{\bar{A}}{2}(\nabla \mathbf{m})^2 + w_a(\mathbf{m},\mathbf{l}). \quad (10)$$

Here $H_{\rm ex}$ is the exchange field of antiferromagnet, $A$ and $\bar{A}$ are the constant of nonuniform exchange, and $w_a(\mathbf{m},\mathbf{l})$ is anisotropy energy, its form will be specified for the concrete models discussed below. In the following, we will not consider the effects of external magnetic field and Dzyaloshinskii–Moria interaction; thus all the contributions to the anisotropy energy are some functions of invariants, bilinear over components either $\mathbf{l}$ or $\mathbf{m}$. Note that contrary to sigma model approximation, two independent constants of nonuniform exchange are present here.

Application of Lagrange formalism is quite convenient for the description of nonlinear dynamics, either for uniform oscillations or for solitons. It is enough to mention that the knowledge of Lagrangian allows building energy-momentum tensor for the system and writing basic integrals of motion. In principle, dynamic part of Lagrangian for the above set of Landau–Lifshitz equations has the form of the sum of two independent contributions for $\mathbf{M}_1$ and $\mathbf{M}_2$, their form is well known [77, 78], but these contributions are singular and inconvenient for analysis. Equations (4) for the irreducible vectors $\mathbf{l}$ and $\mathbf{m}$ keep all the properties of the set of Landau–Lifshitz equations, and they allow direct comparison with the sigma model results. For the case of interest, highly-nonlinear spin dynamics with accounting for constraint (3), the following parametriczation through angular variables is convenient:

$$\mathbf{l} = \mathbf{e}_3 \cos\mu, \mathbf{m} = \sin\mu(\mathbf{e}_1 \cos\psi + \mathbf{e}_2 \sin\psi), \quad (11)$$

where the auxiliary set of orthogonal unit vectors is introduced

$$\begin{aligned} \mathbf{e}_3 &= \mathbf{e}_z \cos\theta + \sin\theta(\mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi), \\ \mathbf{e}_1 &= -\mathbf{e}_z \sin\theta + \cos\theta(\mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi), \quad (12) \\ \mathbf{e}_2 &= -\mathbf{e}_x \sin\varphi + \mathbf{e}_y \cos\varphi, \end{aligned}$$

$\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ determine Cartesian coordinate system, which can be chosen along the crystalline axis.

The physical sense of these variables is quite clear: the angles $\theta$ and $\varphi$ determine the direction of the vector $\mathbf{l}$, and the angle $\mu$ determines the modules of the vectors $\mathbf{m}$ and $\mathbf{l}$. It is easy to see that the equations for $\mathbf{m}$ and $\mathbf{l}$ can be obtained by the variation of Lagrangian with the kinetic part of the form of $(2M_0/\gamma)(\mathbf{m}\cdot[\mathbf{l}\times\partial\mathbf{l}/\partial t])/|\mathbf{l}|^2$. The density of Lagrangian for these four angular variables can be written as $\mathcal{L}[\theta, \varphi, \mu, \psi] = \int L d\mathbf{r}$. Lagrangian density is as the following

$$L = \frac{2M_0}{\gamma}\left(\frac{\partial\theta}{\partial t}\sin\psi - \frac{\partial\varphi}{\partial t}\sin\theta\cos\psi\right)\sin\mu - w, \quad (13)$$

where $w$ is the density of energy of antiferromagnet written through angular variables and their gradients. The energy of the antiferromagnet is determined by the standard equation

$$E = \int w d\mathbf{r}. \quad (14)$$

In particular case $\theta = \pi/2$ and $\psi = 0$, this Lagrangian transforms to the Lagrangian describing simple particular case of planar spin dynamics [80]. Contrary to that for the sigma model (6), this Lagrangian contains singular terms of the form common to that for ferromagnets. This singularity is reflected in the formula for the linear momentum of spin field $\mathbf{P}$,

$$\mathbf{P} = \frac{2M_0}{\gamma}\int \sin\mu(\nabla\varphi\sin\theta\cos\psi - \nabla\theta\sin\psi)d\mathbf{r}. \quad (15)$$

In particular, this singularity appears at the value $\mu = \pi/2$, when sublattices are parallel and $\mathbf{l} = 0$. Such singular terms leads to some nontrivial dynamical properties of magnetic solitons, for example, soliton energy becomes a periodic function of the linear momentum [80].

Within Hamilton approach, angular variables $\theta$ and $\varphi$ are generalized coordinates, with the conjugated canonical momenta $p_\theta$ and $p_\varphi$,

$$p_\theta = \frac{2M_0}{\gamma}\sin\mu\sin\psi, \quad p_\varphi = -\frac{2M_0}{\gamma}\sin\mu\sin\theta\cos\psi. \quad (16)$$

The canonical momentum $p_\varphi$ is proportional to $z$ projection of the magnetization $M_z = 2M_0 m_z$, it is equal to the density of $z$ projection of the spin angular momentum, $\hbar s_z = -M_z / g\mu_B$. If $z$ axis is chosen along the principal axis of uniaxial antiferromagnet, this quantity is the density of an integral of motion, $z$ projection of the total spin.

## 4. Solitons in anisotropic antiferromagnets beyond the sigma model

Now we are in the position to apply this general formalism to the analysis of concrete models of antiferromagnets. First note that the wide class of dynamical states, describing so-called planar solution and discussed for isotropic antiferromagnets [80], is present for wide class of anisotropic antiferromagnets, including uniaxial and biaxial (with rhombic symmetry) magnets. For this class, $\mathbf{m}$ is parallel to one of the crystal axis (say, $z$ axis) and $\mathbf{l}$ is laying in the orthogonal plane ($xy$ plane); thus, two variables are fixed, $\theta = \pi/2$ and $\psi = 0$. To proof this nontrivial property (by the way, such planar solutions are absent for ferromagnets) consider the general form of magnetic anisotropy for biaxial magnet (with rhombic symmetry), which can be written as an expansion over squares of Cartesian components of vectors $\mathbf{m}$ and $\mathbf{l}$. All terms like $l_x^2$, $l_y^2$ or $l_z^2$ are independent of $\psi$ and they are proportional to $\sin^2\theta$ or $\cos^2\theta$, thus, their contribution to both $\partial w_a/\partial\theta$ and $\partial w_a/\partial\psi$ vanish on the planar solution with $\theta = \pi/2$. It is straightforward to show that on the planar solution the values of $\partial m_\alpha^2/\partial\theta = 0$ and $\partial m_\alpha^2/\partial\psi = 0$, where index $\alpha$ take the values $x$, $y$, and $z$. After long but simple algebra the same conditions can be proofed for the contributions of inhomogeneous exchange interactions of both types, $(\nabla\mathbf{l})^2$ and $(\nabla\mathbf{m})^2$. Thus the variations of the energy over these two variables, $\theta$ and $\psi$, vanish on the planar solution. Two equations obtained by variation of $\mathcal{L}$ over $\theta$ and $\psi$ with substituting $\cos\theta = 0$ and $\sin\psi = 0$ to their right-hand sides acquire the form

$$\delta w/\delta\theta = -(\partial\psi/\partial t)\sin\mu, \quad \delta w/\delta\psi = (\partial\theta/\partial t)\sin\mu,$$

and at $\partial\theta/\partial t = 0$ and $\partial\psi/\partial t = 0$ they become identities. Thus, the planar solution exists for quite general form of the energy of antiferromagnets, including various contributions like $l_z^2$, $l_x^2$, $m_z^2$, ect.

For isotropic antiferromagnet, the class of planar solutions includes various magnetic soliton states, one-dimensional moving solitons, common to Lieb states known for non-ideal Bose gas and two-dimensional vortex-like solitons [80]. Let us discuss now solutions for real models of antiferromagnets, which contain magnetic anisotropy.

The form of planar solutions is governed by the simple Lagrangian

$$\mathcal{L} = \int d\mathbf{r}\left(-\frac{2M_0}{\gamma}\frac{\partial\varphi}{\partial t}\sin\mu - w\right), \qquad (17)$$

where $w$ is the energy density (10), presented through angular variables $\mu$ and $\varphi$, with the substitution $\cos\theta = 0$ and $\sin\psi = 0$

$$w = \frac{E}{2}\sin^2\mu + \frac{A}{2}\cos^2\mu(\nabla\varphi)^2 + $$
$$+\frac{1}{2}\left[A\sin^2\mu + \bar{A}\cos^2\mu\right](\nabla\mu)^2 + w_a, \qquad (18)$$

where $w_a = w_a(\mu,\varphi)$ is the anisotropy energy, which can depend on both angular variables. This system has two integrals of motion, energy and linear momentum $\mathbf{P}$. The linear momentum is determined as the total field momentum of two-component field, $\mu$, $\varphi$,

$$\mathbf{P} = \frac{2M_0}{\gamma}\int d\mathbf{r}(\nabla\varphi)\sin\mu\,. \qquad (19)$$

The dynamical part of the Lagrangian (17) and the expression for momentum (19) contain singularities common to that for ferromagnets and connected with non differentiability of the azimuthal angle $\varphi$. Note that this planar model looks similar to the Landau–Lifshitz equation for ferro-magnet in angular variables $\Theta$ and $\Phi$, with the correspondence $\sin\mu \Leftrightarrow \cos\Theta$ and $\varphi \Leftrightarrow \Phi$. The difference is in the form of non-uniform terms, for ferromagnet the contribution of $(\nabla\Theta)^2$ [i.e., $(\nabla\mu)^2$ in (18)] is independent of $\Theta$. This feature produces great difference in mathematical properties of these two model: Landau–Lifshitz equation for ferromagnet with biaxial anisotropy of the simplest type is exactly integrable by inverse scattering method [75], and allow construction of multi-soliton solutions [76], see also [77–79], whereas it is probably not the case for the model (18) with $A \neq \bar{A}$. On the other hand, at $A = \bar{A}$ the results known for Landau–Lifshitz equation can be directly applied to the planar model of antiferromagnet.

The analysis shows that these solitons are present for anisotropic antiferromagnet with pure uniaxial anisotropy of the easy-plane type. For such magnets, anisotropy energy is a function of $l_z^2$ and $m_z^2$ only, and for antiferromagnetic state the minimum of the energy corresponds to $\mathbf{m} = 0$ and the vector $\mathbf{l}$, perpendicular to the hard axis $z$. In this case, the anisotropy energy within the class of planar solutions depends on $\mu$ only, $w_a = w_a(\mu)$, and one more integral of motion, namely, the conservation law of the $z$ projection of the total spin, is present

$$2M_0\frac{\partial}{\partial t}\sin\mu = -\gamma A\,\mathrm{div}\left[(\nabla\varphi)\cos^2\mu\right]. \qquad (20)$$

The presence of this integral of motion allows construction of one-dimensional soliton solution of the type of traveling waves with one parameter, velocity $v$. For such solitons, $\mu = \mu(\xi)$, $\varphi = \varphi(\xi)$, $\xi = x - vt$, and far from the soliton, at $\xi \to \pm\infty$, $\mu(\xi) \to 0$, while $\varphi(\xi)$ has constant values, $\varphi_\pm$. Equation (20) together with the aforementioned boundary conditions gives an explicit formula for $d\varphi/d\xi$ in the following form:

$$\frac{d\varphi}{d\xi} = v\frac{2M_0}{\gamma A}\frac{\sin\mu}{\cos^2\mu}. \qquad (21)$$

Then this expression can be substituted to the Lagrange equation obtained by variation $\delta L/\delta\mu = 0$ that gives ordinary second-order equation for $\mu$. It can be integrated ones, and then the equation for $\mu(\xi)$ in soliton solution acquires the following form:

$$(\bar{A}\cos^2\mu + A\sin^2\mu)\left(\frac{\partial\mu}{\partial\xi}\right)^2 =$$
$$= \sin^2\mu\left[E - \left(\frac{2M_0}{\gamma}\right)^2\frac{v^2}{A\cos^2\mu}\right] + 2w_a(\mu)\,. \qquad (22)$$

For the simplest case of uniaxial anisotropy of the form of $2w_a(\mu) = \bar{K}m_z^2 = \bar{K}\sin^2\mu$, Eq. (22) has the same form as for the isotropic case, with the trivial renormalization of the exchange constants, $E \to E + \bar{K}$. Even for more general form of anisotropy $w_a(\mu)$ [such that $w_a(\mu) > 0$ at $\sin^2\mu \neq 0$], the soliton solution of this equation can be written in quadratures. But unfortunately, even for the simplest form of the anisotropy, the functions $\mu(\xi)$ cannot be written in the explicit form through some elementary functions. Anyway, some important properties of these solitons can be found by qualitative analysis.

Note that these soliton states exist only at $\bar{A} > 0$; it is a formal confirmation of the fact that for their analysis one should go beyond the sigma model, where this constant is not presented at all. It is easy to see that the soliton velocity cannot exceed some limit value. For any anisotropy energy with the asymptotic behavior $w_a(\mu) \to \bar{K}\mu^2/2$, this value is $c = (\gamma/2M_0)\sqrt{(J+\bar{K})A}$. This limit velocity coincides with phase velocity of linear excitations (magnons) for this model of antiferromagnet. The structure of the planar solitons in anisotropic antiferromagnets, as well as the energy dependence on the soliton velocity and linear momentum, is common to that for solitons in isotropic antiferromagnet [80]. Hence, we will not discuss it in details and we limit ourselves with description of the results obtained.

First of all, the form of the solution depends strongly on the value of soliton velocity $v$. For any nonzero velocity the function $\mu(\xi)$ has standard bell-like shape with the maximal value $\mu_{\max} < \pi/2$. If the velocity $v$ is nearly $c$, the soliton amplitude $\mu_{\max}$ is small, proportional to $\sqrt{c^2 - v^2}$. The maximal value of $\mu_{\max}$ is reached at zero velocity of the soliton, it equals to $\pi/2$. The values of $\varphi$ at the right and left of the soliton differ by a certain value $\Delta\varphi$. For any values of $A \neq \bar{A}$, the limit value $\Delta\varphi = \pi$ appears at $v = 0$, but $\Delta\varphi < \pi$ for $v \neq 0$; $\Delta\varphi$ vanishes at $v \to c$. In principle, all these features are common to that for many solitons in the media with spontaneous break of continuous symmetry, like rotary waves for easy plane ferromagnets [81, 82] dark solitons in nonlinear optics [83, 84], or Lieb states in one-dimensional non-ideal Bose gas with repulsive interaction [85]. The discussion of common properties of all these solitons can be found in review article [77].

The energy of a soliton and its dependence on the soliton linear momentum is one of most important soliton characteristics. Using Eqs. (21) and (22), the soliton energy $E$ can be written as a definite integral over $\mu$ from $\mu = 0$ till its maximal value $\mu_{\max}$. The explicit value of this integral cannot be present through analytical function. The exception is again the special case $A = \bar{A}$ and $2w_a(\mu) = \bar{K}\sin^2\mu$, for which the explicit form for soliton energy as a function of its velocity can be written as a simple square root dependence,

$$E = E_0\sqrt{1 - \frac{v^2}{c^2}}, \qquad (23)$$

where $c$ is the spin wave speed, $E_0 = 2\sqrt{(E + \bar{K})A}$ is the maximal soliton energy, corresponding to $v = 0$. For a general model, it is possible to show that this square-root dependence is hold at $c^2 - v^2 \ll c^2$ and the energy is maximal at zero velocity $v = 0$.

The calculation of the linear momentum meets some principal difficulties, due to the singular character of the Lagrangian. This problem can be analyzed with usage topological properties of this Lagrangian represented through Dirac monopole field on the manifold of field variables of the planar solution, which is two-dimensional sphere $l_x^2 + l_y^2 + m_z^2 = 1$ in three-dimensional spin space $\{l_x, l_y, m_z\}$ [80]. Note that the same method was used for domain walls in general model of ferromagnets [86, 87]. This kind of calculation of the linear momentum for solitons in planar model of antiferromagnet is model-free. It is practically the same as for isotropic antiferromagnet; and we will not discuss details and just present the results.

The dispersion law for one-dimensional planar solitons in antiferromagnets $E(P)$ is periodic, $E(P + P_0) = E(P)$, where the value of this period $P_0 = 4\pi M_0/\gamma$. For the model of literally one-dimensional antiferromagnet, spin chain with antiferomagnetic interaction of spins $S$ and the period $a$, the value $P_0 = 2\pi\hbar S/a$; thus, this period is commensurate with the size of Brillouin zone, $P_B = 2\pi\hbar/a$. Such periodic dependence is characteristic for various solitons in ferromagnets, for example, domain walls and spin complexes [77, 78], but never appears for one-dimensional antiferromagnetic solitons within the sigma model approach, even with Lorentz-invariance broken by Dzyaloshinskii field or external magnetic field [69].

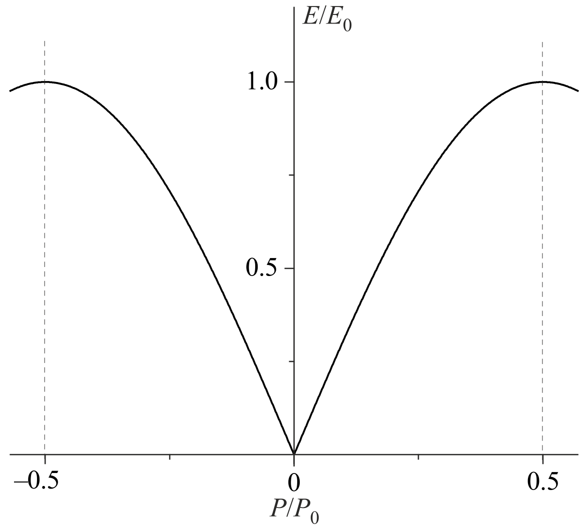


*Fig. 1.* Schematic view of the dispersion relation for one-dimensional planar soliton. Here $E_0$ and $P_0$ are maximal soliton energy and the value of period, correspondingly.

One more interesting type of solitons, planar vortices with ferromagnetic core, has been found for two-dimensional isotropic antiferromagnet [80]. It is straightforward to generalize these solutions to the case of easy-plane antiferromagnets, and we will not discuss their properties in many details. Note only that dynamic properties of these planar vortex are quite unusual for antiferromagnetic vortices. Moving planar antiferromagnetic vortex is subjected to the gyroscopic force $G[\mathbf{e}_z \times \mathbf{v}]$, equivalent to the Lorentz force for a charged particle in the uniform magnetic field. This property is in a strong contrast to standard antiferromagnetic vortices with three-dimensional distribution of the vector $\mathbf{l}$, see for review [69], but it is common to that for ferromagnetic vortices. Again, this property is model-independent, and for antiferromagnet with square lattice of spins $S$ the value of gyroconstant $G = 2\pi\hbar S/a^2$; it depends on sublattice spin density $\hbar S/a^2$ only (here $S$ is the atomic spin, $a$ is the lattice constant).

## 5. Conclusion

The phenomenological concept of spin dynamics of antiferromagnets, the foundations of which were laid more than 60 years ago, now has turned into a theoretical basis for an interesting and important area of fundamental and applied physics. Despite the successes of the sigma model, the initial form of this theory, based on the system of equations for the sublattice magnetizations, is also in great demand now. We have already mentioned practically important materials demonstrating the transition from antiferromagnetic to ferromagnetic states, where exchange field is abnormally weak and sigma model is hardly applicable. Even for standard antiferromagnets with large parameter $\sqrt{H_{\text{ex}}/H_a}$, this version of the phenomenological theory is also of a large practical interest. As we have shown in this article, for some model of antiferromagnets important nonlinear features are lost within sigma model description. It is also important that the initial version of the phenomenological approach can be easily used for description of antiferromagnet with standard programs of micromagnetic simulation code like MuMax3 [88], which are quite effective but written for ferromagnets. To do this, antiferromagnet is modeled by some system of parallel plates of ferromagnets coupled antiferromagnetically, see, e.g., [89, 90] This trick is working pretty well, whereas direct simulation of the sigma model needs is principally different software; on the best of our knowledge, such software is not developed yet. To resume, the first phenomenological approach for spin dynamics of antiferro-magnet, developed more then a half of century ago with an important contribution of M. I. Kaganov, still to be an important tool in modern physics of antiferromagnets and their applications.

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## Феноменологічний опис спінової динаміки в антиферомагнетиках: коротка історія та сучасний розвиток

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Короткий огляд альтернативних феноменологічних підходів до спінової динаміки антиферомагнетиків обговорюється в зв'язку з сучасним інтересом до надшвидкої спінової динаміки та її застосування. Описано специфічні властивості антиферомагнетиків, в першу чергу можливість спінової динаміки, більш швидкої, ніж у феромагнетиків. Обговорюються нові типи солітонів у анізотропних антиферомагнетиках.

Ключові слова: антиферомагнетик, солітон, надшвидка спінова динаміка.