USE OF THE DETERMINED PROPERTIES OF A ROUGHNESS OF A SURFACE IN MEASUREMENT OF A GEOMETRICAL FORM OF OBJECT

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Classification of surfaces as roughness is carried out. The roughness can have the determined, stochastic or chaotic character, depending on a method of their forming. In the work the problem of finding of normal vector to a surface and coordinates of a point of lighting by a laser beam for the case of approximation of a roughness by a flat reflective diffraction grating is solved.
Keywords: surface, roughness, diffraction grating, coherent radiation.

ИСПОЛЬЗОВАНИЕ ДЕТЕРМИНИРОВАННЫХ СВОЙСТВ ШЕРОХОВАТОСТИ ПОВЕРХНОСТИ В ИЗМЕРЕНИИ ГЕОМЕТРИЧЕСКОЙ ФОРМЫ ОБЪЕКТА

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Проведена классификация поверхностей по типу шероховатостей. Шероховатость в зависимости от метода возникновения может иметь детерминированный, стохастический или хаотический характер. В работе решена задача нахождения вектора нормали к поверхности и координаты точки освещения лучом лазера для случая аппроксимации шероховатости плоской отражательной дифракционной решёткой.
Ключевые слова: поверхность, шероховатость, дифракционная решетка, когерентное излучение.

ВИКОРИСТАННЯ ДЕТЕРМІНОВАНИХ ВЛАСТИВОСТЕЙ ШОРСТКОСТІ ПОВЕРХНІ У ВИМІРІ ГЕОМЕТРИЧНОЇ ФОРМИ ОБ'ЄКТУ

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Проведена класифікація поверхонь за типом шорсткостей. Шорсткість залежно від методу виникнення може мати детермінований, стохастичний або хаотичний характер. У роботі вирішено завдання знаходження вектора нормалі до поверхні і координати точки освітлення променем лазера для випадку апроксимації шорсткості плоскими відбивними дифракційними гратками.
Ключові слова: поверхня, шорсткість, дифракційні грати, когерентне випромінювання.

Surface of objects can have various character of a roughness (fig. 1). The roughness, as a rule, is formed as a result of external mechanical (or another) impacts on object, for example, polishing. The surface with the determined roughness turns out by impact on object of the determined process. For example, diffraction gratings possess such surface. The surface with a stochastic roughness is formed at action on object of several processes some of which are determined, and others are casual. The surface with a chaotic roughness turns out at impact on object of chaotic process, for example, a polishing by a sand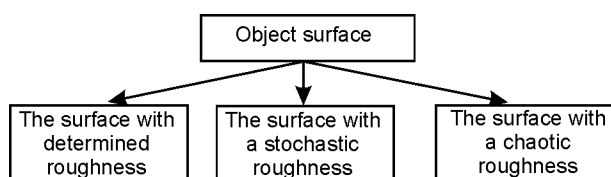 stream. In many cases it is possible to present a roughness of surfaces of the first two types as a combination of several diffraction gratings with various options of their relative positioning (fig. 2).
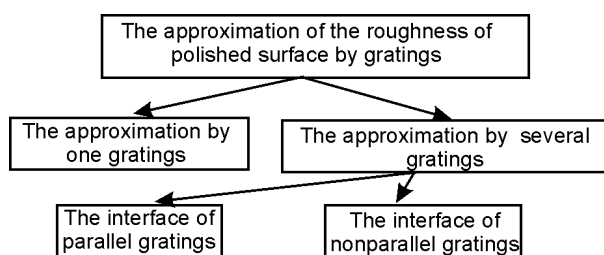
Fig. 2. The approximation of the roughness of polished surfaces by gratings.

The simplest case – one diffraction grating use. At approximation by several gratings there are options of their relative positions, there is need to enter concept of conjugation of gratings. Conjugation defines mechanical ways of their imposing at each

Fig. 1. Object surfaces classification.

other. So, the diffraction pattern, received at interaction of coherent radiation with similar surfaces, represents quite determined structure (fig. 3) [1].
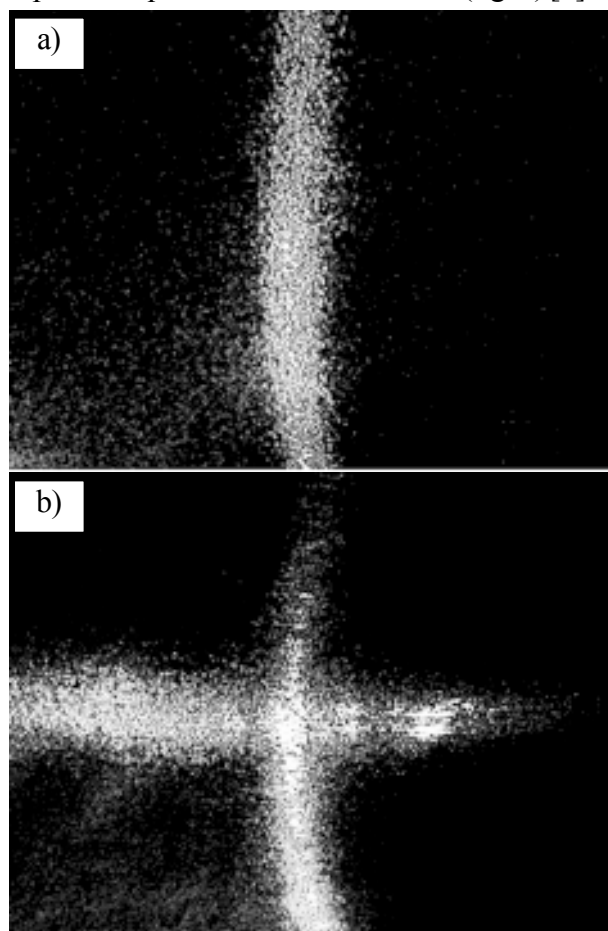

Fig. 3. Diffraction patterns received from surfaces after mechanical machining. a) – one polishing direction, b) – polishing was made in two directions.

Diffraction maxima from each of gratings form a diffraction curve. The quantity of diffraction curves in a diffraction pattern corresponds to the quantity of the diffraction gratings which have taken part in formation of a roughness of this surface. The considered model of roughness gives the grounds to apply the diffraction methods based on approach of Fresnel-Kirchhoff [2, 3]. For the solution of a number of tasks, as well, the geometrical theory of diffraction is used [4].

The result of diffraction depends on a ratio of the spatial sizes of a beam and object. When a diameter of a beam is much smaller then the spatial extent of object, the result of section of a beam and a surface represents the tangent plane to this surface. In this case the object surface unambiguously is defined by a set of the tangent planes in each point of a surface. For this case the primary measuring information in the analysis of a geometrical form are

parameters of a normal to a surface in a point of lighting and coordinate of this point.

In this work the theoretical and experimental research of the diffraction picture, received as a result of interaction of coherent radiation with a surface, was made for the purpose of measurement of parameters of a normal and coordinates of a point of lighting.

Two options of the achievement of objective point are possible. The first: the diffraction picture in the aperture plane of the receiver is analyzed. In this case the contradiction between the sizes of the receiver and the picture demands permission. The second option assumes formation of a diffraction picture on the coordinate screen. In this work the second case, as more interesting from the practical point of view, is considered.

Let's consider diffraction of coherent radiation on one-dimensional reflecting grating for inclined falling of a beam. Thus, the beam is focused arbitrarily in relation to strokes of a grating (fig. 4). The flat wave, which direction of distribution is defined by corners θ and φ, falls on a grating. The angular spectrum of diffracted light is described by the following expression [5]:

$$F(k_x, k_y) = \frac{1}{(2\pi)^2} \int\limits_{-\infty}^{\infty} \int u(x, y, 0)\exp[-i(k_x x + k_y y)]dxdy$$

(1)

There $u(x, y, 0)$ – an intensity of electric field in a point $(x, y, 0)$;

$k_x = k \cdot \cos\theta$, $k_y = k \cdot \cos\varphi$, $k = 2 \cdot \pi/\lambda$;

λ – wave length of the used radiation.

The angular range of a falling flat wave looks like:

$$F_1(k_x, k_y) = \delta(k_x) \cdot \delta(k_x).$$ (2)

The angular spectrum of a field after diffraction on a grating is equal to convolution of an angular spectrum of a falling wave F_1 and function of reflection amplitude coefficient of a grating $\Phi(x)$ [5]:

$$F(k_x, k_y) = \int\limits_{-\infty}^{\infty} \int F_1(\xi, \eta)\Phi(k_x - \xi, k_y - \eta)d\xi d\eta$$

(3)

For infinite uniform periodic structure the range of coefficient of reflection is:

$$\Phi(k_x, k_y) = \sum_{n=-\infty}^{n=\infty} a_n \delta(k_y)\delta\left(k_x - \frac{2\pi \cdot n}{d}\right),$$ (4)

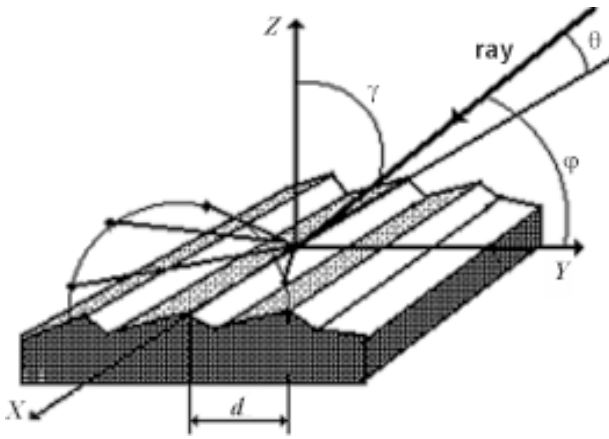where n – a number of a diffraction order; d – a
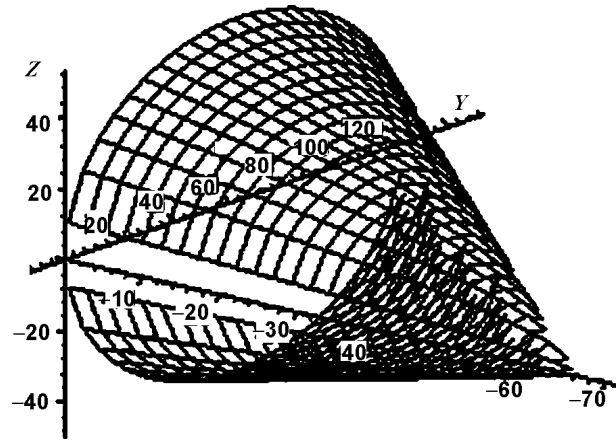
Fig. 4. Diffraction on a grating.

Fig. 5. Surface, formed by diffraction orders.

grating period; a_n – coefficient of Fourier number of amplitude coefficient of reflection of a grating:

$$a_n = \frac{1}{d} \int_{-d/2}^{d/2} R(x) \exp\left[-i\frac{2\pi \cdot n \cdot x}{d}\right] dx \; .$$

$R(x)$ – is defined by concrete model of diffraction structure of a surface and the direction of beam falling on it. Substituting (2) and (4) in (3) and carrying out integration, we will receive an angular spectrum of a field after diffraction on a grating [6]:

$$F\left(k_x, k_y\right) = \sum_{n=-\infty}^{\infty} a_n \delta\left(k_y - k\left(\cos\varphi - \frac{2\pi \cdot n}{d}\right)\right) \cos\left(k_x - k\cos\theta\right).$$

From the last expression we define directing cosines of diffraction orders:

$$\cos\theta_n = \cos\theta, \; \cos\varphi_n = \cos\varphi - n\lambda/d,$$

$$\cos\eta_n = \sqrt{\sin^2\theta - \left(\cos\varphi - \frac{n\lambda}{d}\right)^2} \; . \qquad (5)$$

Apparently from (5), next ratio is true for any orders: there k_n – a single vector of n – order of diffraction in the direction of distribution, $i(1, 0, 0)$ – ort in the direction of the *OX* axis (fig. 4).

Thus, at normal falling of a beam on a surface the diffraction maxima lie on a straight line. In this case formation of diffraction orders is described by classical laws of diffraction of a beam on a grating [7, 8]. At diffraction of randomly focused beam all diffraction orders lie on a surface of the circular angle, which axis coincides with the OX axis, and the corner at top is equal $2 \cdot \beta$ (fig. 5). It was confirmed completely by experimental check [9].

Section of a similar surface by the plane of the screen leads to that diffraction orders on the plane

of the receiver form diffraction maxima as the curve of the second order (fig. 6).
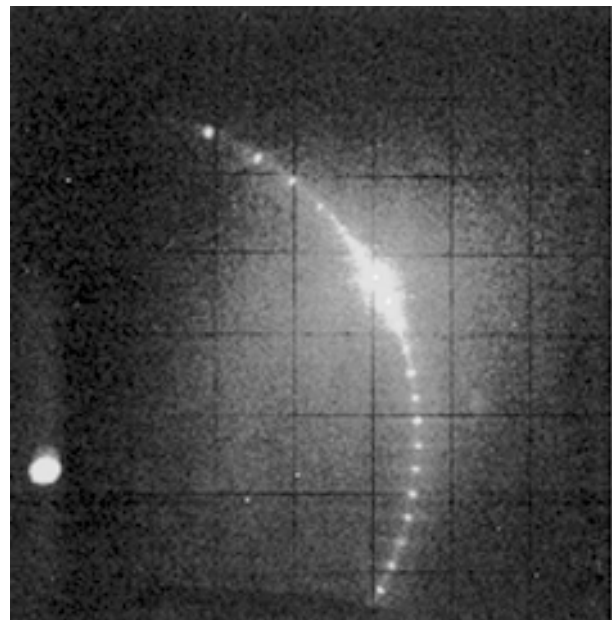

Fig. 6. Diffraction maxima at arbitrary falling of a beam on a grating.

The third formula in (5) defines the quantity of diffraction orders at a beam assumed position concerning grating strokes. Recognizing that a radicand shouldn't be negative, the maximum number of diffraction orders is defined by the following expression:

$$N = \frac{d}{\lambda}\left(\sin\theta + \cos\varphi\right).$$

The system of coordinates, in which it is necessary to define the position of a normal to a surface and coordinates of a point of lighting, is formed by the plane of the screen, on which the diffraction pattern is formed, and a perpendicular to it. The position of a beam in this system of coordinates is described by the straight line equation:

$$\frac{x - x_b}{u_{xb}} = \frac{y - y_b}{u_{yb}} = \frac{z - z_b}{u_{zb}},$$

there x_b, y_b, z_b – coordinates of a point of the falling of a beam on the screen; u_{xb}, u_{yb}, u_{zb} – directing cosines of the straight line, containing a beam.

The perpendicular to the screen forms an axis X, the beginning of coordinate lies on the screen. Therefore $x_b = 0$. Directing cosines of diffraction orders (5) are found in the system of coordinates formed by a diffraction grating. The analysis of a diffraction pattern is kept in system of coordinates formed by the screen. The equations of the plane, that is tangent to a surface, also are in this coordinate system. Therefore for the processing of a diffraction pattern it is necessary to find coordinates of directing vectors of diffraction orders in screen system of coordinates.

The solution of this task assumes the definition of a matrix of transition from one system of co-ordinates to another. Thus, some properties of the diffraction curves should be noted, allowing connect coordinates of diffraction maxima on the screen with coordinates of a point of lighting and parameters of the tangent plane to a surface. As it was already noted, diffraction orders form a circular conic surface in space. As a result of a section of this surface by a flat screen diffraction curves represent second order curves. In this task the spatial position of a beam was set so that diffraction curves have ellipse forms. Cases of a parabola, hyperbole and straight line are almost unrealizable. The parabolic form turns out in case of a screen arrangement strictly parallel to a forming conic surface, hyperbolic – strictly parallel to a cone axis, the straight line turns out only when the projection of an incident beam to a grating is parallel to its strokes.

For transformation of coordinates of directing vectors of diffraction orders the oblique-angled system of coordinates formed by the screen and a beam of the laser was interpolated. At first coordinates were transformed from the system of coordinates connected with a grating in system of coordinates, connected with a beam, and then from this system of coordinates to system of coordinates formed by the screen. Such two-level approaching allowed us considerably reduce time of computing procedures.

The matrix of transition T_{b0} from the oblique-angled system of coordinates, formed by a beam and the screen, in rectangular system, formed by the screen and a perpendicular to it, is:

$$T_{b0} = \begin{bmatrix} \cos\delta_x & \cos\delta_y & \cos\delta_z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

There $\delta_x, \delta_y, \delta_z$ – the angles between a falling beam and axes of rectangular system of coordinates.

The following step is calculation of a matrix of transition T from system of coordinates of a grating in system of coordinates of a beam. For this purpose it is necessary to find the equation of the straight line containing diameter of an ellipse on the screen. The ellipse, as a second order curve, unambiguously is defined by coordinates of five points. The beam is generator of conic surface. Therefore its point of intersection with the screen lies on a considered ellipse. Thus, for a complete definition of an ellipse it is necessary to measure coordinates of four maxima on a diffraction curve. The equation of the ellipse passing through five points with coordinates (y_1, z_1), (y_2, z_2), (y_3, z_3), (y_4, z_4), (y_5, z_5) looks like [10]:

$$\begin{vmatrix} y^2 & yz & z^2 & y & z & 1 \\ y_1^2 & y_1 z_1 & z_1^2 & y_1 & z_1 & 1 \\ y_2^2 & y_2 z_2 & z_2^2 & y_2 & z_2 & 1 \\ y_3^2 & y_3 z_3 & z_3^2 & y_3 & z_3 & 1 \\ y_4^2 & y_4 z_4 & z_4^2 & y_4 & z_4 & 1 \\ y_5^2 & y_5 z_5 & z_5^2 & y_5 & z_5 & 1 \end{vmatrix} = 0$$

Calculation of determinant of this matrix gives the ellipse equation.

There A, B, C, D, E – equation coefficients. Passing simple, but quite bulky mathematical calculations, we will write down a matrix of transition T from rectangular system of coordinates of a grating to the oblique-angled system of coordinates formed by a beam and the plane of the screen, consisting of directing cosines of the axes, that form system of coordinates, connected with a grating, in the system of coordinates, connected with a beam:

$$T_{gb} = \begin{bmatrix} u_{xx} & u_{xy} & u_{xz} \\ u_{yx} & u_{yy} & u_{yz} \\ u_{zx} & u_{zy} & u_{zz} \end{bmatrix}.$$

To find coordinates of directing vectors of diffraction orders in the system of coordinates, formed by the plane of the screen and a perpendicular to it, the following equation must be solved

$$X = T_{b0}^{-1} \left[T_{gb}^t \cdot X_g \right]. \qquad (6)$$

There $X(u_x, u_y, u_z)$ – a required directing vector;

$X_p(u_{xg}, u_{yg}, u_{zg})$ – a directing vector of diffraction orders in system of grating coordinates, it is defined by expressions (5).

Angles θ and φ, which define the position of a normal to a surface in a lighting point, are in the equation (6). Knowing coordinates (y_n, z_n) of n-th diffraction maximum on the screen, it is possible to write down the equation of n-th diffraction order

$$\frac{x}{u_x(\varphi, \theta, n)} = \frac{y - y_n}{u_y(\varphi, \theta, n)} = \frac{z - z_n}{u_y(\varphi, \theta, n)}. \quad (7)$$

The point of illumination of a surface with coordinates (x_b, y_b, z_b) belongs to a straight line (7), therefore the following equalities have to be carried out

$$\frac{x_b}{u_x(\varphi, \theta, n)} = \frac{y_b - y_n}{u_y(\varphi, \theta, n)} = \frac{z_b - z_n}{u_y(\varphi, \theta, n)}. \quad (8)$$

From expression (8) it follows that coordinates of three diffraction maxima completely define the equation of the plane tangent to a surface in a lighting point, and coordinates of this point. I.e. the system decision

$$\begin{cases} \dfrac{x_b}{u_x(\varphi, \theta, n)} = \dfrac{y_b - y_n}{u_y(\varphi, \theta, n)}, \\[2mm] \dfrac{x_b}{u_x(\varphi, \theta, n)} = \dfrac{z_b - z_n}{u_z(\varphi, \theta, n)}, \\[2mm] \dfrac{x_b}{u_x(\varphi, \theta, m)} = \dfrac{y_b - y_m}{u_y(\varphi, \theta, m)}, \\[2mm] \dfrac{y_b - y_m}{u_y(\varphi, \theta, m)} = \dfrac{z_b - z_m}{u_z(\varphi, \theta, m)}, \\[2mm] \dfrac{y_b - y_h}{u_y(\varphi, \theta, h)} = \dfrac{z_b - z_h}{u_z(\varphi, \theta, h)}, \end{cases}$$

there n, m, h – the numbers of diffraction maxima, allows to calculate (x_b, y_b, z_b) and φ, θ – angles.

Substituting the found values of θ and φ angles in the equation (6) we find coordinates (u_{xk}, u_{yk}, u_{zk}) of directing vector of a normal to the tangent plane. The required equation of the tangent plane is.

Thus, the measured coordinates of three diffraction maxima in a diffraction pattern allow calculate the coordinates of laser lighting point on a surface and directing cosines of a vector of a normal to a surface in a lighting point.

The roughness of a surface is approximated by a reflective diffraction grating. As model the reflective diffraction grating with the period of 5 microns was

used. Ne-Ne LG-208 laser forms a beam with a diameter of 1.5 mm and divergence 1.2 mrad. Wave length of the laser radiation is 0.6328 microns. The screen represents diffusely reflecting plane, with the coordinate grid put on it. In the experiment the graph paper pasted on glass was used. The screen size is 350 to 280 mm.

The laser beam incidence was normal to the screen. It allowed create in space three-dimensional system of coordinates in which all measurements have been carried out. The point of a beam incidence on the screen corresponded to the origin of coordinates (fig. 7).
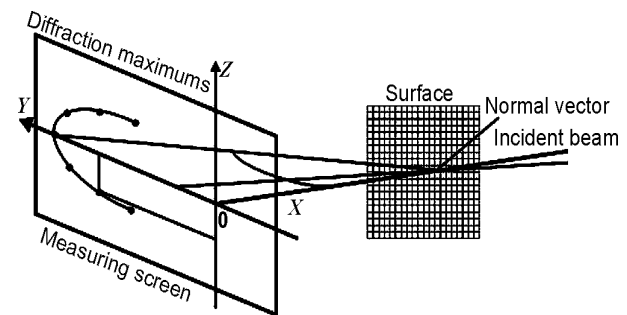

Fig. 7. Scheme of experiment.

The diffraction grating was located on a small rotary table perpendicular to the XOY plane. The angle г was fixed by means of indication of a small rotary table. It allowed install a grating with exact knowledge of its spatial parameters. At this relative positioning of a beam and the screen the attitude of a point of lighting is defined completely by distance l from the origin of coordinates to it. Thus, the point of lighting has coordinates (l, 0, 0).

The true attitude of the grating plane is defined by the equation.

True coordinates of a point of illumination on a grating surface $B(141.5, 0, 0)$.

As a result of diffraction the diffraction curve is formed on the screen. The measured coordinates of diffraction maxima are specified in tab. 1.

Table 1

Y, mm	144.5	203.7	210	142.5
Z, mm	82.5	31.5	0	−82.5

On the basis of these points coordinates and a point of intersection of a beam with the screen the equation of the ellipse approximating a diffraction curve (fig.8) was received

$$-0.444y^2 + 0.018yz - 0.623z^2 + 93.32y - 2.236z = 0.$$

The algorithm of restoration of the equation of the plane is based on theoretical conclusions produced above.
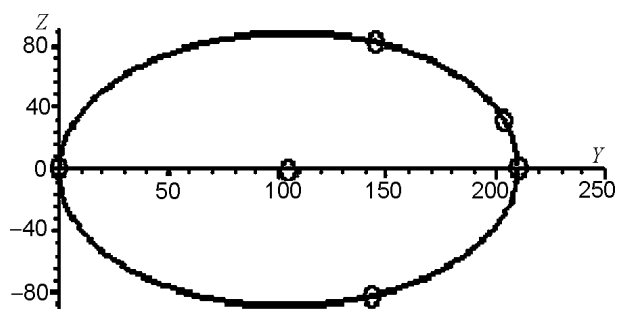
Fig. 8. Diffraction curve.

As a result the following equation of the plane of a grating (fig. 9) has turned out. The values of the measured coordinates of a point of lighting are (139.2, 0, 0). So, the relative error of measurement of coordinates was 1,6%. For the estimation of an error of the restored plane position next corners were calculated:

– between the true position of the plane and measured one – 0.36, 0;

– between the restored plane and the plane of coordinates XOY (the true plane is perpendicular to XOY plane) – 90.29, 0.
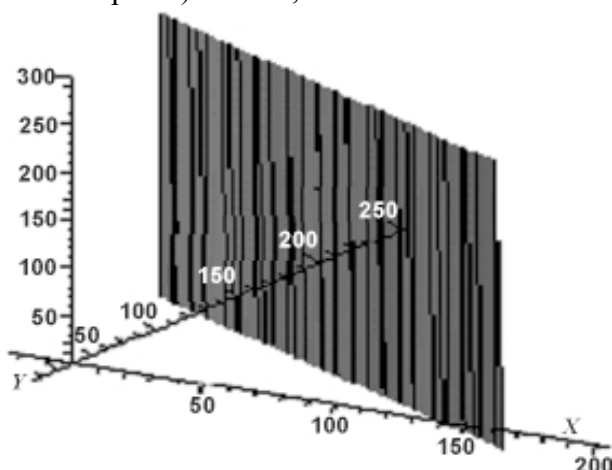

Fig. 9. Restored plane.

The made experiments on restoration of the equation of the plane in space completely confirmed theoretical conclusions. Distinctions in spatial positions of the true and restored planes are caused by the following factors:

– inaccuracy of measurement of coordinates of diffraction maxima on the screen because of their ellipticity;

– divergence of laser radiation wasn't considered in calculations;

– intensity distribution in diffraction maxima wasn't considered in calculations;

– inaccuracy of measurement of true position of a grating.

CONCLUSION

Approximation of roughness of a surface by integration of reflective diffraction gratings allowed to measure normal parameters in a point of lighting by a beam of the laser and coordinate of this point. Measurements were made on the basis of the analysis of the diffraction field, received as a result of interaction of coherent radiation with a surface. These data from all surface of object allow measure its geometrical form unambiguously.

The material presented in article can be of interest for developers of monitoring systems of a geometrical form of products of mechanical engineering, for example shovels of gas-turbine engines.

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