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THEORY OF SPATIALLY INDIRECT EXCITONS IN NANOSYSTEMS CONTAINING DOUBLE SEMICONDUCTORS QUANTUM DOTS

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In mini-review, deals with the theory of exciton quasimolecules in a nanosystem consisting of double quantum dots of germanium synthesized in a silicon matrix. An exciton quasimolecule was formed as a result of the interaction of two spatially indirect excitons. It is shown that, depending on the distance D between the surfaces of the quantum dots, spatially indirect excitons and of exciton quasimolecules was formedin the nanosystem.The binding energy of the singlet ground state of the exciton quasimolecule has been gigantic exceeding the binding energy of the biexciton in a silicon single crystal by almost two orders of magnitude. The emergence of a band of localized electron states in the band gap of the silicon matrix was found. This band of localized electron states appeared as a result of the splitting of electron levels in the chain of germanium quantum dots.

The nature of formation in the Ge/Si heterostructures was analyzed depending on the distance D between the surfaces of QDs SIEs and of exciton quasimolecules.It was shown that the binding energy of the ground singlet state of an exciton quasimolecule was gigantic, exceeding the binding energy of a biexciton in a silicon single crystal by almost two orders of magnitude.The possibility of using quasimolecules of excitons to create elements of silicon infrared nanooptoelectronics, including new infrared sensors, was established.

The emergence of a band of localized electron states in the band gap of the silicon matrix was found.In this case, the band of localized electron states appeared as a result of the splitting of electron levels in the chain of germanium QDs.It was shown that the movement of an electron along the zone of localized electron states in the linear chain of germanium QDs caused an increase in photoconductivity.The effect of increasing photoconductivity can make a significant contribution in the process of converting the energy of the optical range in photosynthesizing nanosystems.

Keywords*: spatially separated electrons and holes, exciton quasimolecules, binding energy, Coulomb and exchange interaction, electron tunneling, splitting of electron states, charge-transfer exciton, quantum dots.*

Introduction

In the Ge/Si heterostructure of the second type, the spatial separation of electrons and holes was experimentally found. In this case, spatially indirect excitons (SIE) appeared in the heterostructure [1, 2].In the SIE, the electron was localized above the surface of the QD, and the hole was in the QD [1–11].

A giant increase in the binding energy of SIE (by almost two orders of magnitude) for nanosystems containing semiconductor (cadmium sulfide, zinc selenide, germanium) and dielectric (aluminium oxide) QDs has been shown, compared with the exciton binding energy in the corresponding single crystals [12-16]. This circumstance, apparently, will make it possible in the future to create new nanolasers.Such nanolasers will operate on SIE transitions at room temperatures.

Linear germanium QD chains,with an average QD radii not exceeding 30 nm, were obtained on silicon substrates in experimental works [3, 7].In [3,7], a photoluminescence signal was discovered at room temperature in the infrared spectral regionregion

$$(0.20 \text{ to} 1.14) \text{ eV} \tag{1}$$

It has been shown that, in the Ge/Si heterostructuredepending on the distance D between the surfaces of the QDs, SIEs [15, 16] and of exciton quasimolecules [17] was formed. The average distance ($\sim N^{-1/3}$) between the surfaces of germanium QDs at low concentrations N QDs in the Ge/Si heterostructure significantly exceeded the Bohr electron radius ($a_e = 0.63$ nm) in the silicon matrix. In this case, the condition was fulfilled:

Abstract

$$a_e N^{1/3} \ll 1 \tag{2}$$

When (2) is fulfilled, the interaction between electrons and holes localized above the surfaces of germanium QDs can be neglected.In this case, in the Ge/Si heterostructure, SIEs can appearnear the surface of single germanium QDs [15,16].

The appearance of SIE in the Ge/Si heterostructure was due to the Coulomb interaction of an electron with a hole, which depended on the dielectric constant of the QD and the matrix [15, 16]. It was found that the binding energy of an SIE in the Ge/Si heterostructure has been gigantic exceeding the binding energy of the exciton in a silicon single crystal by almost two orders of magnitude [15, 16].

The average distance between QD surfaces in linear chains of germanium QDs significantly decreased with increasing concentration N of QDs. In the case when the distance between the surfaces of the QD has decreased to a_e in the Ge/Si heterostructure, it is necessary take into account the interaction between electrons and holes localized above the surfaces of the QD [17].In this case, the condition should be satisfied

$$a_e N^{1/3} \sim 1 \tag{3}$$

When condition (3) is satisfied, the distance between SIEs localized above the QD surfaces decreases to a size comparable to the Bohr electron radius a_e. This caused a significant increase in the overlapping integral $S(a,D)$ of the electron wave functions. A significant increase in the energy of the exchange interaction of electrons in this case caused the appearance of a coupled state of two SIEs in the nanosystem; i.e., an exciton quasimolecule appeared in the nanosystem [17]. In this case, at a distance between the surfaces of the QDs $D \geq D_c^{(2)}$the exciton quasimolecule disintegrated into two SIEs.It has been shown that the binding energy of the singlet ground state of the exciton quasimolecule in the Ge/Si heterostructure significantly exceeded the binding energy of biexciton in a silicon single crystal by almost two orders of magnitude [17].

In the Ge/Si heterostructure, at distances D between the QD surfaces exceeding $D_c^{(2)}$, the splitting of electron states localized over spherical interfaces (QD is a silicon matrix) occurred.This splitting of electron states was caused by the tunneling of electrons through a

potential barrier separating the double QDs. In this case, a band of localized electronic states appeared in the band gap of the silicon matrix[18, 19–33].
The mini-review considers the appearance of in the Ge/Si heterostructure SIEs and exciton quasimolecules depending on the distance *D* between the surfaces of germanium QDs.It was shown that the binding energy of the singlet ground state of an exciton quasimolecule took on a gigantic value, exceeding the binding energy of a biexciton in a silicon single crystal by almost two orders of magnitude.It was found that, as a result of the splitting of electron levels in the chain of germanium QDs, a band of localized electron states appeared.

Exciton Quasimolecule Containing Double Quantum Dots

The Binding Energy of the Singlet Ground State of Exciton Quasimolecule

In [17], a nanosystem model was studied in which double germanium QD(A) and QD(B) with radius *a* were placed in a silicon matrix. The dielectric constant of the silicon matrix ε_1 = 11.7. The germanium QDs had a dielectric constant ε_2 = 16.3. Holes *h*(A) and *h*(B) with effective mass ((m_h/m_0) = 0.39) were located at the centers of QD(A) and QD(B).Electrons $e(1)$and$e(2)$ with effective mass (($m_e{}^{(1)}/m_0$) = 0.98) were localized above the surfaces QD(A) and QD(B) in the silicon matrix, respectively (m_0 is the electron mass in free space).It was assumed in [17] that electrons did not penetrate into QDs.

Within the framework of the adiabatic approximation and the effective mass approximation, the Hamiltonian of the exciton quasimolecule in [17] was presented in the form:

$$\hat{H} = \hat{H}_{A(1)} + \hat{H}_{B(2)} + \hat{H}_{int} \tag{4}$$

where $\hat{H}_{A(1)}$ and $\hat{H}_{B(2)}$ are the Hamiltonians of the SIEs were localized above the surfaces QD(A) and QD(B), respectively. In the Hamiltonians of the excitons $\hat{H}_{A(1)}$ and $\hat{H}_{B(2)}$the contribution of the energy of polarization interaction with the surfaces QD (A) and QD (B) was not taken into account [15, 16]. In [15, 16], the Hamiltonian SIE localized above the surface QD(A) was written as:

$$\hat{H}_{(A(1)} = -\frac{h^2}{2\mu}\Delta_{(1)} + V_{e(1)h(A)}(r_{A(1)}, r_{h(A)}) + E_g, \tag{5}$$

where the first term is the exciton kinetic energy operator and E_g was the bandgap energy of the germanium ($E_g = 0.661$ eV).

In the Hamiltonian (5) of the SIE $V_{e(1)h(A)}$was the energy of Coulomb interaction between electron *e*(1)and hole *h*(*A*):

$$V_{e(1)h(A)} = -\frac{e^2}{\xi r_{A(1)}} \tag{6}$$

where $\xi = 2\varepsilon_1\varepsilon_2/(\varepsilon_1 + \varepsilon_2)$ is the dielectric constant of the nanosystem,$r_{A(1)}$ was the distance of the electron *e*(1) from the QD(A)center.

The Hamiltonian $\hat{H}_{int}$ did not take into account the interaction energies of electrons *e*(1) and *e*(2) and holes *h*(A) and *h*(B) with polarization fields induced by these charge carriers on the surfaces QD(A) and QD(B) [15 - 17]. Therefore, the Hamiltonian $\hat{H}_{int}$contained only the energies of Coulomb interaction of electron *e*(1) with hole *h(*B*),* and electron *e*(2) with hole *h(*A*)*, as well as that between electrons *e*(1) and *e*(2), and holes *h(*A*)* and *h(*B*)* [17].

We represent the normalized wave function of the ground singlet state of the exciton quasimolecule in the form of a symmetric linear combination of wave functions$\Psi_1(r_{A(1)}, r_{B(2)})$and $\Psi_2(r_{A(2)}, r_{B(1)})$[17]:

$$\Psi_s\left(r_{A(1)},r_{A(2)},r_{B(1)},r_{B(2)}\right)=\left[2\left(1+S^2(D,a)\right)\right]^{-1/2}\left[\Psi_1\left(r_{A(1)},r_{B(2)}\right)+\Psi_2(r_{A(2)},r_{B(1)})\right] \quad (7)$$

where $S(D, a)$ is the overlap integral of single-electron wave functions (D is the distance betweenthesurfaces of the QD). The wave function of the ground singlet state of the exciton quasimolecule (7) was written under the assumption that the spins of the electrons $e(1)$ and $e(2)$ were antiparallel. In [17], the wave functions $\Psi_1(r_{A(1)},r_{B(2)})$ and$\Psi_2(r_{A(2)},r_{B(1)})$(7) were written as a product of single-electron wave functions $\varphi_{A(1)}(r_{A(1)})$and $\varphi_{B(2)}(r_{B(2)})$, as well as $\varphi_{A(2)}(r_{A(2)})$and $\varphi_{B(1)}(r_{B(1)})$,respectively [17].Let us also represent the single-electron wave functions as variational functions of Coulomb type [17]:

$$\begin{aligned}
\varphi_{A(1)}\left(r_{A(1)}\right)&=\tilde{A}\exp\left(-\bar{\mu}(\tilde{a})\left(r_{A(1)}/a_{ex}^{2D}\right)\right),\\
\varphi_{B(2)}(r_{B(2)})&=\tilde{A}exp(-\mu(\tilde{a})(r_{B(2)}/(a_{ex}{}^{2D}))),\\
\varphi_{A(2)}\left(r_{A(2)}\right)&=\tilde{A}\exp\left(-\bar{\mu}\left(\tilde{a}\right)\left(r_{A(2)}/a_{ex}^{2D}\right)\right)\\
\varphi_{B(1)}\left(r_{B(1)}\right)&=\tilde{A}\exp\left(-\bar{\mu}\left(\tilde{a}\right)\left(r_{B(1)}/a_{ex}^{2D}\right)\right).
\end{aligned} \quad (8)$$

where $\bar{\mu}(\tilde{a})$is a variational parameter, $\tilde{a}=(a/a_{ex}^{2D})$, where $a_{ex}^{2D}=\varepsilon(m_0/\mu_{ex}^{2D})(\hbar^2/m_0e^2)$, is the Bohr radius of two – dimensional (2D) exciton localized over the flat interface between the germanium and the matrix of silicon,$\mu_{ex}^{2D}=m_e^{(1)}m_h/\left(m_e^{(1)}+m_h\right)$is the reduced mass of the 2D exciton (of spatially separated electrons and holes), $r_{B(2)}$ is the distance of the electron $e(2)$ from theQD(B)center; $r_{A(2)}$is the distance of the electron $e(2)$ from the QD(A) center;$r_{B(1)}$is the distance of the electron $e(1)$ from the QD(B) center[15, 16].

In [17], the energy of the ground singlet state of an exciton quasimolecule was obtained by the variational method.This energy was determined by the average value of the Hamiltonian $\hat{H}(4)$ over the states described by the wave functions of the zeroth approximation Ψ_s (5) [17]:

$$E\left(D,\bar{\mu}\left(a,D\right),a\right)=\Psi_s(r_{A(1)},r_{A(2)},r_{B(1)},r_{B(2)}\left|\hat{H}\right|\Psi_s(r_{A(1)},r_{A(2)},r_{B(1)},r_{B(2)}) \quad (9)$$

In [17], taking into account the explicit form of wave functions (7), (8), the energy functional of the ground singlet state of an exciton quasimolecule was obtained:

$$E_0\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right),\tilde{a}\right)=2E_{ex}\left(\tilde{a},\bar{\mu}\left(\tilde{a}\right)\right)+\frac{J\left(D,\bar{\mu}\left(\tilde{a},D\right),\tilde{a}\right)+K\left(D,\bar{\mu}\left(\tilde{a},D\right),\tilde{a}\right)}{1+S^2\left(D,\bar{\mu}\left(\tilde{a},D\right),\tilde{a}\right)} \quad (10)$$

here ($\tilde{D}=(D/a_{ex}^{2D})$. In expression (10) $E_{ex}(\tilde{a},\bar{\mu}(\tilde{a}))$ istheenergyfunctional of the SIE ground state:

$$E_{ex}\left(\tilde{a},\bar{\mu}\left(\tilde{a}\right)\right)=\varphi_{A(1)}(r_{A(1)})\left|\hat{H}_{A(1)}\right|\varphi_{A(1)}(r_{A(1)}) \quad (11)$$

The second term in (10) is a functional

$$E_{\hat{a}}(D,a)=\frac{J\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right),\tilde{a}\right)+K\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right),\tilde{a}\right)}{1+S^{2}\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right),\tilde{a}\right)} \quad (12)$$

which described the binding energy of the ground singlet state of an exciton quasimolecule .The functionals $J(\tilde{D},\bar{\mu}(\tilde{a},\tilde{D}),\tilde{a})$ and $K(\tilde{D},\bar{\mu}(\tilde{a},\tilde{D}),\tilde{a})$ in (12) were presented in [17] as follows:

$$J\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right)\tilde{a}\right)=\left\langle\varphi_{A(1)}(r_{A(1)})\ \varphi_{B(2)}(r_{B(2)})\right|H_{int}\left|\varphi_{A(1)}(r_{A(1)})\varphi_{B(2)}(r_{B(2)})\right\rangle \quad (13)$$

$$K\left(\tilde{D},\bar{\mu}\left(\tilde{a},\tilde{D}\right)\tilde{a}\right)=\left\langle\varphi_{B(1)}(r_{B(1)})\ \varphi_{A(2)}(r_{A(2)})\right|H_{int}\left|\varphi_{A(1)}(r_{A(1)})\varphi_{B(2)}(r_{B(2)})\right\rangle \quad (14)$$

Moreover, in [17] the functional $J(\tilde{D},\bar{\mu}(\tilde{a},\tilde{D}),\tilde{a})$ (13) was represented as an algebraic sum of the functionals of the average energies of Coulomb interaction. The functional $K(\tilde{D},\bar{\mu}(\tilde{a},\tilde{D}),\tilde{a})$ (14) was determined by the algebraic sum of the functionals of the averageenergiesof the exchange interaction [17].

Within the framework of the variational method at the first approximation the total energy$E_0(\tilde{D},\tilde{a})$ (9) of ground singlet state of exciton quasimolecule was determined by average value of the Hamiltonian $\hat{H}$(4) for states, which was described by wave functions of the zero approximation$\Psi_s(r_{A(1)},r_{A(2)},r_{B(1)},r_{B(2)})$ (7) [17]:

$$E_0\left(\tilde{D},\tilde{a}\right)=2E_{ex}\left(\tilde{a}\right)+E_{\hat{a}}\left(\tilde{D},\tilde{a}\right) \quad (15)$$

where $E_B(\tilde{D},\tilde{a})$ is the binding energy of the ground singlet state of the exciton quasimolecule and $E_{ex}(\tilde{a})$ is the binding energy of the ground state of the SIE localized over the surface of QD [15, 16].

Numerical Results and Discussions

The binding energy $E_B(\tilde{D},\tilde{a})$ of the of the ground singlet state of the exciton quasimolecule was obtained in [17] in the Ge/Si heterostructure consisting of double germanium QDs with mean radius $\bar{a}_1 = 12.8$ nm(the Bohr radius of the 2D exciton $a_{ex}^{2D} = 2.6$ nm) under the condition:

$$\left(E_{\hat{a}}\left(\tilde{D},\tilde{a}\right)/E_{ex}\left(\tilde{a}\right)\right)\ll 1 \quad (16)$$

SIE binding energy was $E_{ex}(\bar{a}_1)\cong -64$ meV [15, 16]. The ground state energy of the excitonic quasimolecule (15) the in this casewas $E_0(\tilde{D}_1,\bar{a}_1)\approx -134.1$ meV. In this case, condition (16) was fulfilled ($\left(E_B^{(1)}(D_1,\bar{a}_1)/E_{ex}(\bar{a}_1)\right)\cong 0.09$). An exciton quasimolecule with an increase in the distance D between the surfaces of the QD, so that $D\geq D_c^{(2)}\cong 4.4$ nm, decayed into two SIE [17]. In this Ge/Si heterostructure, the binding energy $E_B^{(1)}(D_1,\bar{a}_1)\approx -6.1$ meVof the ground singlet state of the exciton quasimolecule significantly exceeded the binding energy of biexciton in a silicon single crystal by almost two orders of magnitude [17]. The energy of the exchange Such Ge/Si heterostructures was investigated in experimental works [17].

The exciton quasimolecule appeared in the nanosystem at distances $D \geq D_c^{(1)} \cong 2.1$ nm between the surfaces of QD [17].The binding energy $E_B(\tilde{D}, \tilde{\alpha})$ of the exciton quasimolecule ground state in a nanosystem with QD germanium of the mean radius $\bar{\alpha}_1 = 12.8$ nm had a minimum $E_B^{(1)}(D_1, \bar{\alpha}_1) \approx -6.1$ meV (at the distance $D_1 \cong 3.1$ nm) [17]. In this Ge/Si heterostructure, interaction of electrons and holes made the major contribution to the binding energy of an exciton quasimolecule in this Ge/Si heterostructure.This energy significantly exceeded the value of the Coulomb interaction between electrons and holes (i.e. the ratio $\leq$ 0.08) [17]. In experimental works [5, 11] in Ge/Si heterostructure, a blurring of the absorption edge was observed in the infrared wavelength range (1) up to room temperatures. Such blurring of the absorption edge was caused by interband electron transition in SIE [15, 16].

The Splitting of Electron States in Germanium/SiliconNanosystem with Germanium Quantum Dots

Electron Tunneling in the Germanium/Silicon Heterostructure with Germanium Quantum Dots

In the Ge/Si heterostructure model (ref. [17]), electrons *e*(1) and *e*(2) with effective masses $m_e^{(1)}$were localized over the spherical surfaces of QD(A) and QD(B) in potential wells caused by Coulomb attraction$V_{eh}(x)$ (6) electrons and holes (where *x* is the electron distance from the surface of the QD).An exciton quasimolecule with an increase in the distance *D* between the surfaces of the QD (condition (3) is satisfied), so that$D \geq D_c^{(2)} \cong 4.4$ nm, decayed into two SIEs [17]. Such SIEs appeared when the photon with the energy smaller than the width of the bandgap($E_{g(Si)}$= 1.17 eV) of the silicon matrix was absorbed by the nanosystem [15- 19].With an increase in the QD radius *a* (so that $a \geq 22.2$ nm), SIE withenergy $E_0(a)$ turned into 2D SIE with energy $E_0(a) = -E_{ex}^{2D} = -2\hbar^2/\mu_{ex}^{2D}(a_{ex}^{2D})^2$ [15,16]. In this case, $E_{ex}^{2D} = 82$ meV is the binding energy of the 2D SIE and a_{ex}^{2D}= 2.6 nm.In refs. [15,16] and [18,19], the energy of the SIE state $E_0(a)$was measured from the bottom of the conduction band of the silicon matrix ($E_{c(Si)} = E_{g(Si)}$).

The potential barrier

$$U(x) = -\frac{e^2}{\tilde{\varepsilon}((D/2) - x))}/, \qquad 0 \leq x \leq (D/2) \tag{17}$$

$$U(x) = -\frac{e^2}{\tilde{\varepsilon}((D/2) + x))}/, \qquad (-D/2) \leq x \leq 0 \tag{18}$$

separating double QDs was caused by the Coulomb attraction $V_{eh}(x)$ (6) of electrons *e*(1) and *e*(2) to their holes. Electrons *e*(1) and *e*(2) localized above the spherical surfaces of double QDs can tunnel through the potential barrier *U*(*x*) (17), (18).

This electron tunneling resulted in the splitting of the SIE energy level $E_{ex}(a)$ at two close SIE levels $E_{ex}^{(1)}(a)$ and $E_{ex}^{(2)}(a)$. In [18, 19], using the semiclassical approximation, an expression was obtained that describes the splitting $\Delta E_{ex}(a, D) = (E_{ex}^{(1)}(a) - E_{ex}^{(2)}(a))$ of the SIE level ($E_{ex}(a) = -E_0$):

$$\Delta E_e x(a, D) = 2^{-3/2}\{[1 - B(\tilde{E}_0, D)(\tilde{E}_0\tilde{D}^{1/2}](2\tilde{E}_0)^{-3/2} + ln[(\tilde{E}_0\tilde{D})^{1/2} + B(\tilde{E}_0\tilde{D}]\}^{-1} \cdot [(\tilde{E}_0\tilde{D})^{1/2} + B(\tilde{E}_0\tilde{D})]^{(-2\sqrt{2})} exp[-2\tilde{D}^{1/2}B(\tilde{E}_0, \tilde{D})]E_{ex}^{2D} \tag{19}$$

where $\widetilde{E_0} = (E_0/E_{ex}^{2D})$, $\widetilde{D} = (D/a_{ex}^{2D})$ and $B(\widetilde{E_0}, \widetilde{D}) = \left(\widetilde{E_0}\,\widetilde{D} - 1\right)^{1/2}$.Expression $\Delta E_{ex}(a, D)$ (23) was obtained in [18, 19] under the condition of weak splitting (23) of the SIE level $E_{ex}(a)$, that is, with

$$(\Delta E_{ex}(a, D)/E_0) \ll 1 \tag{20}$$

.Numerical Results and Discussions

Tables 1 and 2 (see [18, 19]) show the splitting values $\Delta E_{ex}(a,D)$(19) of SIE levels $(E_{ex}(\bar{a}_1) = -E_0(\bar{a}_1) = -64\ \text{meV})$and $(E_{ex}(\bar{a}_2) = -E_0(\bar{a}_2) = -72\ \text{meV})$ in the Ge/Si heterostructures. Such Ge/Si heterostructures were placed chains of germanium QDs with average radii $\bar{a}_1 = 12.8\ \text{nm}$ and $\bar{a}_2 = 15\ \text{nm}$. These Ge/Si heterostructures were investigated in experimental works [3, 7].From Table 1 it follows that in the case of continuously changing distances *D* between the surfaces of the QDs (in the range from D_1 = 7.8 nm to D_2 = 8.4 nm) splitting (19) also monotonically changes in the range $\Delta E_{ex}(\bar{a}_1, D_1)$ = 8 meV to $\Delta E_{ex}(\bar{a}_1, D_2)$ = 0.16 meV [18, 19]. The values of the splitting (19), with continuously changing distances *D* between the surfaces of the QDs (in the range from D_1 = 7.8 nm to D_2 = 8.4 nm), also monotonically changed in the range from $\Delta E_{ex}(\bar{a}_2, D_1)$ = 8.8 meV to$\Delta E_{ex}(\bar{a}_2, D_2)$ = 0.2 meV(see Table 2) [18,19].Requirements (20) for the smallness of splitings $\Delta E_{ex}(\bar{a}_1, D)$ and $\Delta E_{ex}(\bar{a}_2, D)$ in comparison with the value of the energy of the exciton levels $E_0(\bar{a}_1)$ and $E_0(\bar{a}_2)$ were satisfied [18, 19].It should be noted that the values of splitting (19) were described by a strong exponential dependence on the distance *D* (see Table 1 and Table 2) [18, 19].

In a Ge/Si nanosystem with germanium QDs with radii $\bar{a}_1 = 12.8\ \text{nm}$ and $\bar{a}_2 = 15\ \text{nm}$ SIE levels appeared $(E_{ex}(\bar{a}_1) = -64\ \text{meV})$and $(E_{ex}(\bar{a}_2) = -72\ \text{meV})$. This required light quanta with energies $\hbar\omega_{ex}(\bar{a}_1) = 266$ meV and $\hbar\omega_{ex}(\bar{a}_2) = 258$ meV [18, 19].Such energies $\hbar\omega_{ex}(\bar{a}_1) = 266$ meV and $\hbar\omega_{ex}(\bar{a}_2) = 258$ meV were contained in the infrared spectral region (1), which was observed under experimental conditions up to room temperature [3,7].

The emergence of a band of localized electron states in the band gap of the silicon matrix was found in [18, 19]. This band of localized electron states appeared as a result of the splitting of SIE levels in the chain of germanium QDs.In this case, the width of the band of localized electron states was determined by the order of splitting $\Delta E_{ex}(a, D)$(19) of the SIE levels $E_{ex}(a)$. The position of the zone of localized electron states in the nanosystem was determined by the position of the SIE level $E_{ex}(a)$.In this case, in the linear chain of germanium QDs [3, 7], an charge-transfer exciton moved along the band of localized electronstates [21].In this charge-transfer exciton, the hole was in the valence band of the germanium QDs, and the electron, tunneling between the QDs, moved along the band of localized electronic states [18, 19].

Conclusion

The nature of formation in the Ge/Si heterostructures was analyzed depending on the distance *D* between the surfaces of QDs SIEs and of exciton quasimolecules.It was shown that the binding energy of the ground singlet state of an exciton quasimolecule was gigantic, exceeding the binding energy of a biexciton in a silicon single crystal by almost two orders of magnitude.The possibility of using quasimolecules of excitons to create elements of silicon infrared nanooptoelectronics, including new infrared sensors, was established [34 -38].

The emergence of a band of localized electron states in the band gap of the silicon matrix was found.In this case, the band of localized electron states appeared as a result of the splitting of electron levels in the chain of germanium QDs.It was shown that the movement of an electron along the zone of localized electron states in the linear chain of germanium QDs caused an increase in photoconductivity.The effect of increasing photoconductivity can make a significant contribution in the process of converting the energy of the optical range in photosynthesizing nanosystems [34,38].

Table 1. Dependence of the splitting $\Delta E_{ex}(\bar{a}_1, D)$(19) of the exciton level $(E_0(\bar{a}_1) = -64\ \text{meV})$in a nanosystem that consists of two germanium QDs with average radii $\bar{a}_1 = 12.8\ \text{nm}$, on the distance D between the surfaces of the QD [18,19].

$\bar{a}_1$ nm	$E_0(\bar{a}_1)$ meV	D nm	$\Delta E_{ex}(\bar{a}_1,D)$ meV
12.8	64	7.8	8
12.8	64	8	2.4
12.8	64	8.2	0.64
12.8	64	8.4	0.16

Table 2. Dependence of the splitting $\Delta E_{ex}(\bar{a}_2,D)$(19) of the exciton level $(E_0(\bar{a}_2) = -72\ \text{meV})$in a nanosystem that consists of two germanium QDs with average radii $\bar{a}_2 = 15\ \text{nm}$, on the distance D between the surfaces of the QD [18, 19].

$\bar{a}_2$ nm	$E_0(\bar{a}_2)$ meV	D nm	$\Delta E_{ex}(\bar{a}_2,D)$ meV
15	72	7.8	8.8
15	72	8	2.8
15	72	8.2	0.78
15	72	8.4	0.2

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ТЕОРІЯ ПРОСТОРОВО НЕПРЯМИХ ЕКСИТОНІВ У НАНОСИСТЕМАХ, ЩО МІСТЯТЬ ПОДВІЙНІ НАПІВПРОВІДНИКОВІ КВАНТОВІ ТОЧКИ

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У міні-огляді розглядається теорія екситонних квазімолекул в наносистемі, що складається з подвійних квантових точок германію, синтезованих у кремнієвій матриці. У результаті взаємодії двох просторово непрямих екситонів виникала квазімолекула екситону. Показано, що в залежності від відстані D між поверхнями квантових точок у наносистемі утворюються просторово непрямі екситони та квазімолекули екситонів. Енергія зв'язку основного синглетного стану екситонної квазімолекули виявилася гігантською, майже на два порядки перевищувала енергію зв'язку біекситона в

монокристалі кремнію. Виявлено виникнення смуги локалізованих електронних станів у забороненій зоні кремнієвої матриці. Ця смуга локалізованих електронних станів виникла в результаті розщеплення електронних рівнів у ланцюжку квантових точок германію.

Проаналізовано природу утворення в гетероструктурах Ge/Si в залежності від відстані D між поверхнями просторово непрямих екситонів та екситонних квазімолекул. Встановлено можливість використання квазімолекул екситонів для створення елементів кремнієвої інфрачервоної нанооптоелектроніки, у тому числі нових інфрачервоних сенсорів.

Виявлено виникнення смуги локалізованих електронних станів у забороненій зоні кремнієвої матриці. У цьому випадку смуга локалізованих електронних станів виникла внаслідок розщеплення електронних рівнів у ланцюжку квантових точок германію. Показано, що рух електрона вздовж зони локалізованих електронних станів у лінійному ланцюжку квантових точок германію викликає збільшення фотопровідності. Ефект підвищення фотопровідності може внести істотний внесок у процес перетворення енергії оптичного діапазону у фотосинтезуючих наносистемах.

Ключові слова: *просторово розділений електрони і дірки, екситонні квазімолекули, енергія зв'язку, кулонівська та обмінна взаємодія, тунелювання електронів, розщеплення електронних станів, екситони із перенесенням заряду, квантові точки.*