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RECURRENT EXPRESSIONS FOR RELIABILITY INDICATORS OF COMPOUND ELECTROPOWER SYSTEMS

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Main reliability indicators of unrestorable compound electropower systems are examined in this paper. A method of investigation of reliability indicators for compound electropower systems by means of generating functions is developed taking account of aging of the system's output elements. Recurrent expressions are worked out for the failure probability in the prescribed availability condition, the failure frequency in the prescribed availability condition and the failure rate in the prescribed availability condition provided that the lifetime of ageing output elements is described by the Rayleigh distribution. References 5, figure 1.
Key words: reliability indicators, electropower systems, Rayleigh distributed ageing elements.

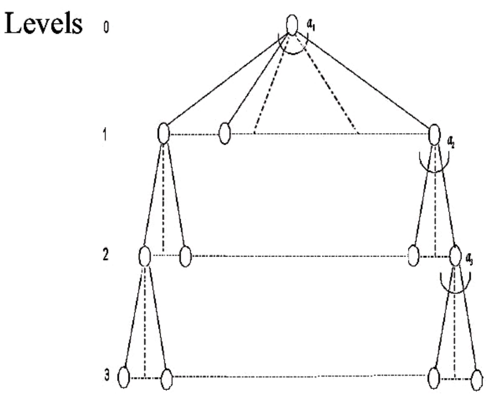
Introduction. Network modelling is an important approach to solve problems for such systems such as supply chains, telecommunication networks, electric-power transmission systems. The complexity of electropower systems and their safety requirements have risen significantly.

None of such formalisms as Petri nets, block diagrams, finite state machines, Markov graphs is suitable to looped systems that arise typically in reliability analysis of electrical networks. Electrical networks can be abstracted as graphs whose nodes are subject to failures.

There exist different methods of investigation for reliability indicators of power systems [1-3]. But existing traditional methods of reliability evaluation are not able to satisfy requirements of investigations of complicated systems such as compound electropower systems.

Models of Reliability Indicators. The problem of reliability investigation of the systems arose up a long ago together with becoming of the engineering approach to industry. Every engineering object must contain the sign of reliability. At a choice among competitive projects, reliability indicators occupy an important place in the list of requirements. But reliability prognostication is difficult because of multivariate and statistical nature of this phenomenon.

Over the past decade with the development of compound systems manufacturing technologies, necessity has emerged for building new and improving traditional methods and models in the process of solving problems of designing these systems [4].



Up to now for systems with complicated structures such as compound electropower systems there were no mathematical models in the form of analytic expressions for main reliability indicators.

Let us consider a symmetric electropower system ramified to level 3 with ageing output elements (Figure), where a_1 elements of level 1 are subordinate to an element of level 0, a_2 elements of level 2 are subordinate to every element of level 1, a_3 elements of level 3 are subordinate to every element of level 2. a_1 is a coefficient of ramification to level 1, a_2 is a coefficient of ramification to level 2 and a_3 is a coefficient of ramification to level 3.

By analogy with [5] we obtain the following generation function of the system shown in Figure:

$$S_3(z)=p_0\sum_{x_1=0}^{a_1} C_{a_1}^{x_1} p_1^{x_1} q_1^{a_1-x_1} \sum_{x_2=0}^{a_2x_1} C_{a_2x_1}^{x_2} p_2^{x_2} q_2^{a_2x_1-x_2} z^{x_2} \sum_{x_3=0}^{a_3x_2} C_{a_3x_2}^{x_3} p_3^{x_3} q_3^{a_3x_2-x_3} z^{x_2} + q_0, \qquad (1)$$

where p_0, q_0, p_1, q_1, p_1, p_2, q_2, p_3, q_3, p_0 are probabilities of trouble-free operation and failure probabilities of elements on levels 0,1,2,3 correspondingly, z is an arbitrary parameter.

Expressions for probability distribution calculation of count of operating output elements of the system are deduced on the basis of the generation function (1). Expressions for reliability indicators of compound electropower systems ramified to level 2 and 3, with Rayleigh distributed ageing output elements, may be extended to such systems ramified to level n, where n is any natural number greater than 1.

We use $a_{l(l=\overline{1,n})}$ to denote coefficients of ramification to level *l*, in other words a_l is count of elements of level *l* that are subordinate to every element of level *(l-1)*. A total number of output elements of the system is defined by:

$$N_n = \prod_{i=1}^{n} a_i \tag{2}$$

Let $P_{nR}(x_n,t)$ be the probability that there are exactly x_n operating output elements provided the probability of failure-free operation of ageing output elements are circumscribed by the Rayleigh distribution.

Under condition $0<x_n \le N_n$ we obtain the following recurrent expressions

$$P_{nR}(x_n,t) = e^{-\lambda_0 t} \sum_{x_1=y_1}^{a_1} C_{a_1}^{x_1} e^{-\lambda_1 x_1 t}\left(1-e^{-\lambda_1 t}\right)^{a_1-x_1} S_{2R}, \qquad y_{n-1} = ceil\left(x_n/a_n\right), \tag{3, 4}$$

$$y_l = ceil\left(\frac{y_{l+1}}{a_{l+1}}\right),\ l=\overline{1,n-2}, \qquad S_{lR} = \sum_{x_l=y_l}^{a_l x_{l-1}} C_{a_l x_{l-1}}^{x_l} e^{-\lambda_l x_l t}\left(1-e^{-\lambda_l t}\right)^{a_l x_{l-1}-x_l} S_{l+1,R},\ l=\overline{2,n-1}, \tag{5, 6}$$

$$S_{nR} = C_{a_n x_{n-1}}^{x_n} e^{-\frac{x_n t^2}{2\sigma^2}}\left(1-e^{-t^2/2\sigma_n^2}\right)^{a_n x_{n-1}-x_n}. \tag{7}$$

Let $Q_{nR}(k,t)$ be the failure probability in the prescribed availability condition *k* provided the probability of failure-free operation of ageing output elements is circumscribed by the Rayleigh distribution. We obtain

$$Q_{nR}(k,t) = 1 - \sum_{x_n=k}^{N_n} P_{nR}(x_n,t), \tag{8}$$

where N_n is calculated according to equation (2) and $P_{nR}(x_n,t)$ is calculated according to the recurrent expressions (3-7).

We use $a_{nR}(k,t)$ to denote the failure frequency in the prescribed availability condition *k* provided the probability of failure-free operation of ageing output elements is circumscribed by the Rayleigh distribution. It is determined as the derivative of the failure probability with respect to time *t*.

Under condition $0<x_n \le N_n$ we obtain the following recurrent expressions

$$a_{nR}(k,t) = e^{-\lambda_0 t} \sum_{x_n=k}^{N_n} \sum_{x_1=y_1}^{a_1} C_{a_1}^{x_1} U_{2R}, \qquad U_{lR} = \sum_{x_l=y_l}^{a_l x_{l-1}} C_{a_l x_{l-1}}^{x_l} U_{l+1,R},\ l=\overline{2,n-1}, \tag{9, 10}$$

$$U_{nR} = C_{a_n x_{n-1}}^{x_n} \sum_{j_1=0}^{a_1-x_1} C_{a_1-x_1}^{j_1}(-1)^{j_1} \prod_{m=2}^{n} \sum_{j_m=0}^{a_m x_{m-1}-x_m} C_{a_m x_{m-1}-x_m}^{j_m}(-1)^{j_m} \times$$
$$\times\left(\frac{x_n+j_n}{\sigma_n^2}t + \lambda_0 + \sum_{r=1}^{n-1}\lambda_r(x_r+j_r)\right) e^{-\left(\sum_{r=1}^{n-1}\lambda_r(x_r+j_r)\right)t} e^{-\frac{x_n+j_n}{2\sigma_n^2}t^2}. \tag{11}$$

During designing compound electropower systems, decisions about structures of such systems should be made on the basis of results of valuing the failure probability, the failure frequency and the failure rate in the prescribed availability condition.

The main thrust of this paper is to reduce the computational time and complexity when valuing reliability indicators of compound electropower systems. It is necessary to work out methods of reliability prognostication with regard for systems' specific features.

Conclusions. Calculation of reliability indicators is very important during the developmental stage and applying of systems of different kinds in industry. It is amply difficult due to a great number of factors and general statistical nature of this phenomenon.

Recurrent expressions for main reliability indicators of unrestorable systems in the case when the lifetime of ageing output elements is circumscribed by the Rayleig*h* distribution make it possible to calculate the availability function of the system under availability condition, to carry out timing simulation of availability function under various availability conditions of the system. Varying coefficient of ramification, it is possible to vary the system's structure and to choose its optimal variant under prescribed operation conditions.

Prediction of reliability at stages of design of compound electropower systems makes possible to evaluate probabilistic and time characteristics of systems, to compare reliabilities of possible variants of systems' structures depending on requirements of production process.

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РЕКУРРЕНТНЫЕ ВЫРАЖЕНИЯ ДЛЯ ПОКАЗАТЕЛЕЙ НАДЕЖНОСТИ СЛОЖНЫХ ЭЛЕКТРОЭНЕРГЕТИЧЕСКИХ СИСТЕМ

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В статье рассматриваются основные показатели надежности невосстанавливаемых сложных электроэнергетических систем. Метод исследования показателей надежности сложных электроэнергетических систем с помощью производящих функций распространен на случай старения выходных элементов системы. Разработаны рекуррентные выражения для вероятности отказов при заданном условии готовности, частоты отказов при заданном условии готовности и интенсивности отказов при заданном условии готовности для случая, когда безотказность стареющих выходных элементов описывается распределением Релея. Библ. 5, рис. 1.

Ключевые слова: показатели надежности, электроэнергетические системы, стареющие по закону Релея выходные элементы.

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РЕКУРЕНТНІ ВИРАЗИ ДЛЯ ПОКАЗНИКІВ НАДІЙНОСТІ СКЛАДНИХ ЕЛЕКТРОЕНЕРГЕТИЧНИХ СИСТЕМ

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Розглянуто основні показники надійності невідновлюваних складних електроенергетичних систем. Метод дослідження показників надійності складних електроенергетичних систем за допомогою твірних функцій поширено на випадок старіння вихідних елементів системи. Розроблено рекурентні вирази для ймовірності відмов за заданої умови готовності, частоти відмов за заданої умови готовності та інтенсивності відмов за заданої умови готовності для випадку, коли безвідмовність старіючих вихідних елементів описується розподілом Релея. Бібл. 5, рис. 1.

Ключові слова: показники надійності, електроенергетичні системи, старіючі за законом Релея вихідні елементи.

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