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APPROXIMATION OF EXACT MASSIVE SOLENOID PROFILE FOR GENERATING PULSED MAGNETIC FIELD

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Approximation of exact massive single-turn solenoid profile for generating given magnetic field distribution on the surface of long coaxial conductive cylinder by polygon is proposed and validated. The accuracy of the method for determining the exact profile based on using the Green function is confirmed by solution of the integral equation for surface current density in ideal surface effect approximation. Errors of the induction distribution due to using approximate profile solenoids and external inductance of solenoid-cylinder system are calculated. Influence on these values of number polygon sides on the profile main part and external surface solenoid radius is investigated. References 7, figures 4.

Key words: pulsed magnetic field, profile of massive solenoid, ideal skin effect, surface current density, inductance of solenoid-cylinder system.

Introduction. The profile of massive solenoid for generating given distribution of strong pulsed magnetic field can be found by solving the inverse problem for magnetic flux from boundary surface (we shall call such profile and its contour exact) [4, 7]. Magnetic field lines are used to find main part of profile which faces boundary surface. Its peripheral parts are approximated by straight line or curvilinear segments [3]. The rest part of contour is completed according to constructive ideas. Such solenoid has rather complicated form, that makes it hard to manufacture. Objective is approximation of exact profile main part and estimation of errors connected with this approximation.

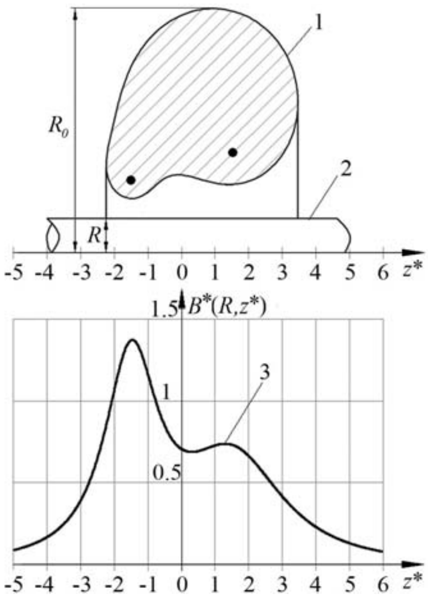

Fig. 1

Validation of the solenoid exact profile. Let the exact profile of massive single-turn solenoid *1* for generating given magnetic induction distribution B (curve *3*) on cylindrical surface *2* with ideal skin effect approximation is found by method [4], based on solution of the problem of the magnetic field continuation from cylindrical surface by using two ring conductors with currents I and Green's function (fig. 1, location of the ring conductors is shown by points). Designations: $B^* = 2\pi RB/(I_s\mu_0)$, $z^* = z/R$, z – axial cylindrical coordinate, R – cylinder radius, μ_0 – magnetic constant, I_s – current in the solenoid. Notice, that curve *3* is close to curve *5* from [4, p. 12, fig. 3], which has been calculated using another baseline magnetic flux compared to accepted in this paper.

To calculate magnetic induction distribution on cylinder surface and current density in the solenoid with ideal skin-effect approximation numerical solution of the integral equation is used [5]

$$\int_l \eta(M)\sqrt{r_M}\,K(P,M)\,dl_M = \begin{cases} \dfrac{\Phi_0}{\mu_0\sqrt{r_P}}, & P\in l_1, \\ 0, & P\in l_2, \end{cases} \tag{1}$$

where η is the surface current density, P, M is the observation point and point with current coordinates, $P, M \in l$, $l=l_1 + l_2$, l is the total contour, l_1 is the solenoid profile contour, l_2 is the generatrix of the cylinder, dl_M is the an element of total contour, r_P, r_M is the radial coordinates of P and M respectively, $K(P,M)=(2/k-k)K(k) - (2/k)E(k)$ [6, p. 304], $K(k)$, $E(k)$ is the first and second kind complete elliptic integrals of module k, Φ_0 is the magnetic flux on the solenoid surface, constant.

In the right-hand side of equation (1) for solenoid with exact profile magnetic flux $\Phi_0=\Phi_e=1{,}22873\mu_0 RI_s$, $I_s=2I$. Magnetic induction on cylinder surface is found from

$$B(M) = -\mu_0 \eta(M), \tag{2}$$

using solution of equation (1). Equation (2) is consequence of boundary condition on interface between ideal conductor and nonconducting nonmagnetic medium [7]. There is minus in right-hand side (2) because if Φ_0>0 for $M \in l_2$ then values η (M)<0 and B(M)>0 (fig. 1, curve *3*).

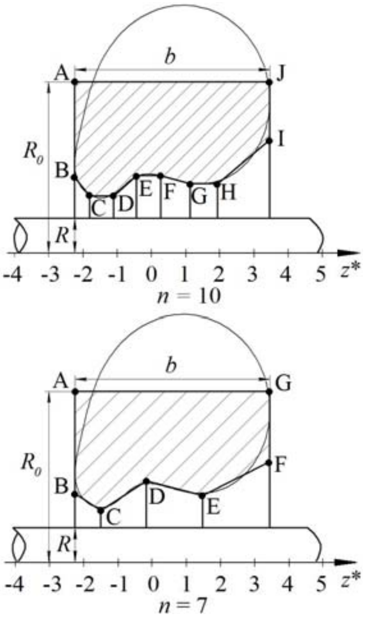

Fig. 2

Calculations executed by (1) and (2) show that magnetic induction distribution B on cylinder surface for exact profile differs from given distribution (fig. 1, curve *3*) by value, which less than one percent. It confirms correctness of the exact profile and the method [4].

Approximation of solenoid exact profile. Exact profile of **s**olenoid is approximated by polygon with n sides and corners (fig. 2, b, R_o – length and radius of solenoid outer cylindrical surface; contour of exact profile is shown by thin line). Main part of exact profile contour corresponds to polygonal chain *BCDEFGHI* of seven (n = 10) or *BCDEF* of four (n = 7) segments of polygon closed boundary. Let the rest part of profile slightly affects boundary distribution of magnetic field and its contour is approximated by three line segments with two right angles in both cases. Evidently profiles depicted in fig. 2 make design and manufacture much easier.

To solve (1) for solenoid with approximated profile value Φ_0 is obtained by following condition of constant current I_s with profile variation. This leads to obvious formula

$$\Phi_0 = \Phi_e L_n / L_e , \tag{3}$$

where L_e, L_n is the inductance of solenoid-cylinder system for solenoids with exact and approximated profiles.

Dimensional and dimensionless (L_e*, L_n*) inductances are

$$L_{e,n} = \Phi_0 / I_s , \qquad L^*_{e,n} = \frac{L_{e,n}}{\mu_0 R} = 2\pi \Phi^*_0 / I^*_s , \tag{4, 5}$$

where $\Phi_0^* = \Phi_0/(\mu_0 R I_s)$, for exact profile $\Phi_0^* = \Phi_e^*$ = 1,22873,

$$I_s = \oint_{l_1} \eta(M) dl_M , \qquad I^*_s = \oint_{l_1} \eta^*(M) dl^*_M . \tag{6, 7}$$

In equation (7) $\eta^*(M) = 2\pi R \eta(M)/I_s$, $dl_M^* = dl_M/R$, $I_s^* = \pi I_s / I$. Next relations are valid for solenoid with exact profile

$$I^*_s = 2\pi, \qquad L^*_e = \Phi^*_e . \tag{8}$$

Profile	h_s/R	R_o/R	L_e*, L_n*	L_n/L_e	I_s*	
					$\Phi_0=\Phi_e$	Φ_0 – (3)
Exact	0,6000	7,2432	1,22873	-	6,28319 / 6,28467	-
Polygon n = 10	0,6532	6	1,22444	0,99651	6,30522	6,28321
		5	1,23206	1,00271	6,26624	6,28322
		4	1,24440	1,01275	6,20409	6,28320
		3,3083	1,26062	1,02595	6,12426	6,28318
Polygon n = 7	0,5032	5	1,20145	0,9778	6,42585	6,28320

Results of calculations L_e*, L_n*, L_n /L_e and I_s* for exact and approximated profiles are presented in table (h_s – minimum gap between the solenoid and the cylinder). Here and in further calculations b = 5,7R, length of cylinder generatrix equals 16R. For each profile after numerical solution of integral equation (1) two values of I_s* are calculated by (7): first – $\Phi_0 = \Phi_e$ (value for exact profile is in denominator, theoretical value which equals 2π according to (8) is in numerator); second – Φ_0 is obtained by (3). Notice, that Φ_0 doesn't affect L_e, L_n because of linearity of equation. They depend only on shape and sizes of system conductors. If n = 7, inductance L_n* is somewhat less and $\Phi_0 = \Phi_e$, current I_s* will be higher than with n = 10 and R_o = 5R. This can be explained by decrease in h_s. For profile n = 10 inductance is some decreased (~ to 2,6%) with decreasing R_o to minimum – radial coordinate of point I on shape (fig. 2). Accordingly, current I_s* is decreased with $\Phi_0 = \Phi_e$ due to current redistribution in solenoid. This data shows that approximation of profile main part by several line segments and almost halving of radius R_o slightly affect external inductance of solenoid-cylinder system.

Distributions of magnetic induction generated by solenoids with approximated profiles on cylinder surface with (3) and their relative differences ξ from given distribution are shown in fig. 3 ($R_o = 5R$; for curves *1*, *3* – $n = 10$, *2*, *4* – $n = 7$). Here and in fig. 4 solenoid butts are shown by dotted lines.

First of all, we note that condition (3) is satisfied for all approximated profiles with absolute error ~10^{-5} compared to the exact profile (last column in the table). We see that obtained distributions are sufficiently smooth despite polygonal shape of the solenoids. With $n = 10$ relative differences don't exceed couple percent on major part of the cylinder. The maximum ξ is located near the maximum of magnetic induction and slightly exceeds four percent. For solenoid with $n = 7$ relative differences are significantly increased to eight percent in the area of magnetic induction maximum and on the right-hand side of distribution. In both cases it can be explained by decrease in approximation accuracy of exact profile's main part. Increase and decrease in maximum induction are related with variation of h_s (table) which leads to amplification (particularly, under point C with $n = 7$, fig. 2) or attenuation of proximity effect [1, 6].

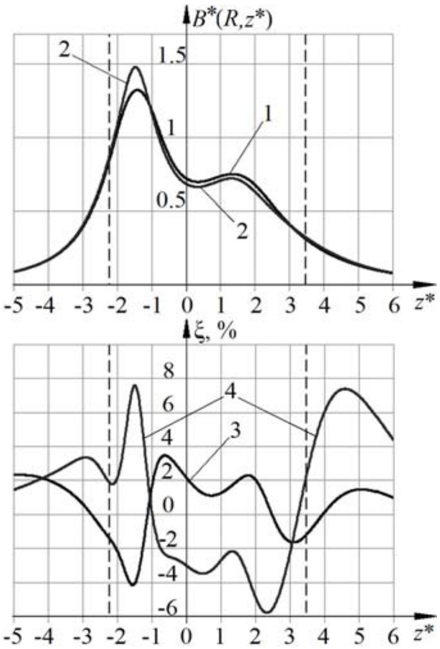

Fig. 3

In fig. 4 relative differences ξ of obtained magnetic induction distributions from given distribution (fig. 1, curve *3*) are shown for approximated profile $n = 10$ and variation of R_o (curve *1* – $R_o/R = 6$; *2* – 4; *3* – 3,3083). On part of cylinder generatrix located under the solenoid for all R_o (see also curve *3*, fig. 3) ξ doesn't exceed few percent, but outside it rises to ten and more percent with decrease in R_o. However magnetic induction $B^*(R,z^*)$ is small in the area of increasing of ξ, which can be explained by current redistribution in the solenoid.

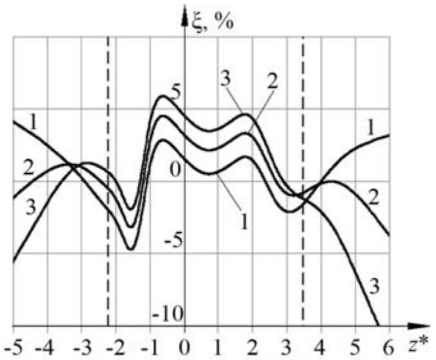

Fig. 4

Conclusions. 1. Relative differences between given distribution and distributions obtained with approximated profile solenoids are decreased with increasing approximation accuracy of profile's main part, which faces the cylinder. The differences don't exceed a few percent with main part approximated by polygonal chain of seven line segments.

2. Shape and sizes of the profile rest part, which complements main part, slightly affect boundary induction distribution and external inductance of solenoid-cylinder system, that can be used for solenoid construction approximation.

3. Taking into account previous conclusion to determine solenoid profile from magnetic field continuation problem, magnetic field line segments, which make up contour of profile main part, is enough. This reduces amount of work needed for the computations and increases the possibility to employ different methods.

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АПРОКСИМАЦІЯ ТОЧНОГО ПРОФІЛЮ МАСИВНОГО СОЛЕНОЇДА ДЛЯ СТВОРЕННЯ ІМПУЛЬСНОГО МАГНІТНОГО ПОЛЯ

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Запропоновано та обґрунтовано апроксимацію точного профілю масивного одновиткового соленоїда, що забезпечує заданий розподіл індукції магнітного поля на поверхні довгого співвісного циліндра, многокутником. Розв'язком інтегрального рівняння для поверхневої густини струму в наближенні ідеального поверхневого ефекту підтверджена правильність методу визначення точного профілю за допомогою функції Гріна. Обчислені похибки розподілу індукції, що створюються соленоїдами апроксимованого профілю, та зовнішня індуктивність системи соленоїд-циліндр у залежності від числа сторін многокутника на головній частині контура і радіуса зовнішньої поверхні соленоїда. Бібл. 7, рис. 4.
Ключові слова: імпульсне магнітне поле, профіль масивного соленоїда, ідеальний поверхневий ефект, поверхнева густина струму, індуктивність системи соленоїд-циліндр.

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АППРОКСИМАЦИЯ ТОЧНОГО ПРОФИЛЯ МАССИВНОГО СОЛЕНОИДА ДЛЯ СОЗДАНИЯ ИМПУЛЬСНОГО МАГНИТНОГО ПОЛЯ

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Предложена и обоснована аппроксимация точного профиля массивного одновиткового соленоида, обеспечивающего заданное распределение индукции импульсного магнитного поля на поверхности соосного длинного проводящего цилиндра, многоугольником. Решением интегрального уравнения для поверхностной плотности тока в приближении идеального поверхностного эффекта подтверждена правильность метода определения точного профиля при помощи функции Грина. Вычислены погрешности распределения индукции, получаемой при помощи соленоидов аппроксимированного профиля, и внешняя индуктивность системы соленоид-цилиндр в зависимости от числа сторон многоугольника на главной части контура профиля и радиуса внешней поверхности соленоида. Библ. 7, рис. 4.
Ключевые слова: импульсное магнитное поле, профиль массивного соленоида, идеальный поверхностный эффект, поверхностная плотность тока, индуктивность системы соленоид-цилиндр.

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