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SIMULATION OF NONLINEAR SKIN EFFECT UNDER SINUSOIDAL VOLTAGE SUPPLY BY USING HARMONIC BALANCE FINITE ELEMENT METHOD AND EFFECTIVE MAGNETIC CURVES

I.S. Petukhov*
Institute of Electrodynamics National Academy of Sciences of Ukraine,
Pr. Peremohy, 56, Kyiv, 03057, Ukraine.
E-mail: igor_petu@ukr.net

The nonlinear model of the time-periodic magnetic field in a conducting ferromagnetic medium is presented. The model is based on the combined use of the harmonic balance method and the finite element method. The case of magnetic field excitation by alternating sinusoidal voltage is considered. The values of eddy current losses are compared. They are determined using the method as well as monoharmonic approach with normal magnetization curve and the effective magnetic curves. It is determined that monoharmonic approach with normal magnetization curve gives the eddy current losses greater by about 9% than the losses obtained by harmonic balance method. The monoharmonic approach using effective magnetization curves gives much smaller specific eddy current losses. References 6, figures 3.

Key words: harmonic balance method, finite element method, ferromagnetic conducting medium, eddy current losses.

Introduction. Most electric devices operating in periodic regimes use sinusoidal voltage supply. In such a case nonlinear properties of a device lead to current harmonic generation in supply circuit. If the device contains the magnetic core including solid iron elements it is necessary to take into account skin effect phenomenon. In a wider sense, one can say that in the case when the depth of magnetic field penetration is less than the corresponding dimensions of the magnetic core the skin effect significantly influences the parameters of the device. The complicated geometry of the magnetic core requires field computation. The most effective means for this purpose are the finite element method (FEM) which is implemented in many program packages [1, 2].

In order to simulate time-periodic regime, the complex amplitude monoharmonic method (AM) in combination with FEM is often used. In this method, all variables in mathematical model are supposed to be sinusoidal. The nonlinear properties of a ferromagnetic core are to be taken into consideration. To do this, it is assumed that the normal magnetization curve relates amplitudes of magnetic flux density to the magnetic field strength. In fact, if the magnetic flux density is sinusoidal the magnetic field strength is not sinusoidal due to nonlinear magnetization curve. In consequence of this feature the accuracy of AM simulation decreases. To be specific, the effective field strength and eddy current losses will be determined inaccurately. To avoid the above-mentioned errors of simulation it was proposed to use "effective magnetic curves" instead of normal magnetic curves. It is important to mention that the harmonic model is used to compute the approximate response of the material to a harmonic excitation to prevent expensive numerical integration of transient process.

But any approach based on AM can not accurately take into account a distortion of the magnetic field strength waveform and, as a result, harmonic spectra of the supply current too. For this purpose, the harmonic balance method in combination with the finite element method (HBFEM) was proposed by some authors [3, 4]. HBFEM uses the representation of variables in form of truncated Fourier series and thus can obtain any accuracy according to Fejer's theorem. But the more terms of the Fourier series the higher is the order of equation system and the complexity of the algorithm [3]. This raises the question whether there are significant differences in accuracy of considered approaches and whether complexity increasing of HBFEM algorithm gives more precise simulation result. Therefore, the difference in the results obtained with AM using normal magnetization curve and "effective magnetization curves" as well as results of the HBFEM is the subject of interest of this paper.

Effective magnetic curves. Implementation of the "effective magnetic curves" in FEM simulation of periodic magnetic field proposed by developers of COMSOL software is based on modified normal magnetization curve. These modifications use the "Simple energy method" and the "Average energy method", which are calculated according to the expressions [2]

$$B_{\mathrm{SE}} = \frac{2}{H_m}\int_0^{H_m} B(H)\,dH\,, \tag{1}$$

$$B_{\mathrm{AE}} = \frac{16}{TH_m}\int_0^{T/4}\left(\int_0^{H_m\sin(\omega t)} B(H)\,dH\right)dt\,, \tag{2}$$

* ORCID: https://orcid.org/0000-0003-1416-1174

where H is the magnetic field strength, ω is the angular frequency, T is the time period, B_{SE} determined with the "Simple energy method" and B_{AE} determined with the "Average energy method". To build dependencies (1), (2) in COMSOL 5.2 normal magnetization curve of soft iron is used. The curve is shown in Fig. 1 by black circle markers. The approximated normal curve and "effective magnetic curves" $B_{SE}(H)$ and $B_{AE}(H)$ are shown in Fig. 1 by white markers. And finally, the curves used for calculation of the dependencies of magnetic permeability on magnetic flux density $\mu(B)$ were transferred into the COMSOL 3.4 in the form of data tables. It is necessary to note that the approximation procedure must provide the monotonically increasing function to obtain convergence of an iterative solution process.

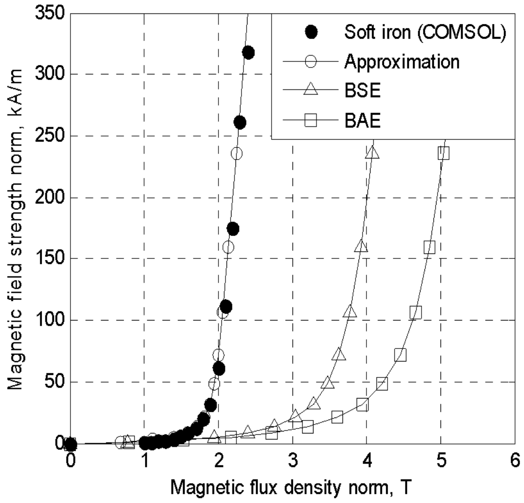

Fig. 1

Mathematical model. Two-dimensional low frequency magnetic field in a conducting medium in the absence of any voltage sources in medium (∇ φ=0) is described in Cartesian coordinate system (x, y, z) by the following equation with respect to the only component of the magnetic vector potential $A = A_z$

$$\frac{\partial}{\partial x}H_y\left(\frac{\partial A}{\partial x},\frac{\partial A}{\partial y}\right)-\frac{\partial}{\partial y}H_x\left(\frac{\partial A}{\partial x},\frac{\partial A}{\partial y}\right)+\gamma\frac{\partial A}{\partial t}=0\,, \qquad (3)$$

where $H_x(\cdot)$ and $H_y(\cdot)$ are the spatial components of the magnetic field strength depending on the derivatives of vector potential, γ is the electrical conductivity, t is the time. Components of the magnetic field strength in an isotropic medium are determined by using the components of the magnetic flux density B_x, B_y including the nonlinear vector function as

$$(H_x,H_y)=\frac{H(B)}{\|B\|}(B_x,B_y)=\frac{H(B)}{\|B\|}\left(\frac{\partial A}{\partial y},-\frac{\partial A}{\partial x}\right), \qquad (4)$$

where function $H(B)$ is magnetization curve. When AM simulation is employed the magnetization curve relates norm of complex amplitude of magnetic flux density to correspondent one of magnetic field strength according to expression

$$\|\dot{\boldsymbol{H}}\| = f\left(\|\dot{\boldsymbol{B}}\|\right), \qquad (5)$$

where $\|\dot{\boldsymbol{H}}\|=\sqrt{|\dot H_{mx}|^2+|\dot H_{my}|^2}$ and $\|\dot{\boldsymbol{B}}\|=\sqrt{|\dot B_{mx}|^2+|\dot B_{my}|^2}$; f is defined as the magnetization curve.

To study the influence of nonlinear properties on the losses and the amplitude values of the field, it is enough to consider the penetration of an alternating magnetic field into the depth of the conducting medium normally to the surface. This study can be done by using the two-dimensional rectangular domain Ω shown in Fig. 2, where ω is the angular frequency, $\tilde I_S(t)$ is the unknown surface excitation current directed along z-coordinate, B_n is the only magnetic flux density component. In the Fig. 2 the boundary conditions for magnetic vector potential are shown too. An example of such a domain is half of a thick plate with infinite thin winding at the surface. Considering that the excitation winding is fed from voltage source it is necessary to express the boundary condition in terms of a voltage.

In accordance to Stoke's theorem, magnetic flux can be expressed as contour integral

$$\Phi=\int_S B_n\,d\boldsymbol{S}=\int_S rot_n\,\boldsymbol{A}\,d\boldsymbol{S}=\oint_l \boldsymbol{A}\,d\boldsymbol{l}\,, \qquad (6)$$

where l is the contour which encloses the area S. Taking into account horizontal symmetric of the domain and the fact of sinusoidal voltage the amplitude of magnetic vector potential A_m (Fig. 2) can be expressed with an amplitude of voltage as

$$A_m=\Delta U_{m0}/2\omega\,, \qquad (7)$$

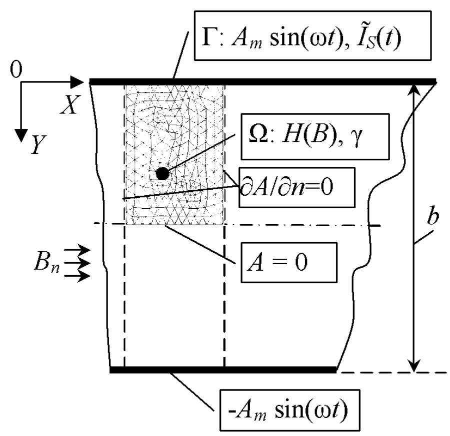

Fig. 2

where ΔU_{m0} is the voltage per meter (along coordinate z) and per coil.

An approximate solution of the magnetic vector potential $\tilde A(x, y, t)$ and other variables was represented by a trigonometric polynomial like the following expression [3, 4, 5]

$$\tilde A(x,y,t)=\sum_{i=1}^{n}\sum_{g=1}^{n_g}\left[A_{cvi}\cos(\nu\omega t)+A_{svi}\sin(\nu\omega t)\right]N_i(x,y)\,, \qquad (8)$$

where n is the number of mesh nodes; n_g is the number of the harmonic basis functions (cosine or sine components), $\nu = [(g + 1)/2]$ is the order of harmonic component, $N_i(x, y)$ is the space basis function of a finite element. Consequently, approximate expressions for the space components of magnetic flux density can be written in compact form as

$$\widetilde{B}_x(\alpha) = \sum_{i=1}^{n}\sum_{g=1}^{n_g} A_{ig}\,\xi_g(\alpha)\frac{\partial N_i}{\partial y};\quad \widetilde{B}_y(\alpha) = -\sum_{i=1}^{n}\sum_{g=1}^{n_g} A_{ig}\xi_g(\alpha)\frac{\partial N_i}{\partial x}\ , \tag{9}$$

where $\alpha = \nu\omega t$, $\xi_g(\alpha)$ is time base function ($\cos(\nu\omega t)$ or $\sin(\nu\omega t)$ in (6) accordantly to g-index).

The assembly of the nonlinear equation system was carried out using the Galerkin method in a weak formulation. Applying this method to equation (1) with boundary condition (Fig. 2) on the period $T = 2\pi/\omega$ and using approximating expressions (5) and (6) one can obtain

$$-\frac{2}{\pi}\int_0^{\pi}\int_{\Omega}\left[\tilde{H}_y\frac{\partial N_j}{\partial x} - \tilde{H}_x\frac{\partial N_j}{\partial y}\right]\xi_h(\alpha)\,d\Omega\,d\alpha + \frac{2}{\pi}\int_0^{\pi}\int_{\Gamma}\tilde{H}_\tau N_j\,\xi_h(\alpha)\,d\Gamma\,d\alpha + \frac{2}{\pi}\int_0^{\pi}\int_{\Omega}\gamma\omega\frac{\partial\tilde{A}}{\partial\alpha}N_j\,\xi_h(\alpha)\,d\Omega\,d\alpha = 0, \tag{10}$$

where $\tilde{H}_x$, $\tilde{H}_y$ are approximate components of the field strength obtained from components of the flux density according to nonlinear vector function (2), $j = 1\ldots n$ is the index of a mesh node, $h = 1\ldots n_g$ is the index of harmonic component (indexes i, g are kept for the expression of approximate solution (5)), Γ is the part of the boundary where field source is situated (Fig. 2), Ω is the domain (ibid), $\tilde{H}_\tau$ is tangential field strength on the surface. In view of field sources antisymmetry (ibid), the magnetic field strength H outside of plate cross section is zero. By these means tangential field strength $\tilde{H}_\tau$ on the external surface inside of the domain is equal to surface current density $\tilde{I}_S$. And jh-contribution of a finite element e in residual vector R can be expressed as

$$R^e_{jh} = \frac{1}{L}\int_e\left\{-\sum_{l=1}^{L}\left[\tilde{H}_y\frac{\partial N_j}{\partial x} - \tilde{H}_x\frac{\partial N_j}{\partial y}\right]\xi_h(\alpha_l)\ + \right. \tag{11}$$

$$\left. + \gamma\omega\sum_{i=i_1,i_2,i_3}\left[(\mp\nu)A_{i,\,g\pm1}\,\xi_h(\alpha_l)\,N_j\right]\xi_h\ (\alpha_l)\ N_i\ \right\}dS_e + \frac{1}{L}\int_{\Gamma^e}\sum_{l=1}^{L} I_{Sh}\ \xi_h(\alpha_l)\,d\Gamma^e,$$

where i_1, i_2, i_3 are the nodes of element e, S_e is the square of the element, $\alpha_l = 2\pi\,(l-1)/L$; ($l = 1\ \ldots\ L$) are the nodes in time period, Γ^e is the part of the boundary belonging to the element e (if such situation occurs).

The specific eddy current losses are determined through the time-derivative of the magnetic vector potential. Consequently, full losses P in meshed region can be determined as sum across all elements and over all harmonics

$$P = \gamma\nu\omega\sum_{e=1}^{n_E}\left(\int_e\left\{\sum_{i_1,i_2,i_3}\sum_{g=1}^{n_g}[-A_{c\nu i}\sin(\nu\,\omega\,t) + A_{s\nu i}\cos(\nu\,\omega\,t)]\right\}N_i(x,y)\,dS_e\right)\ , \tag{12}$$

where n_E is the number of mesh elements, e is the index of a mesh element, i_1, i_2, i_3 are the node indexes of the current element.

Results and discussion. Both HBFEM and AM were applied for computation of eddy current losses. HBFEM implementation from program package GE2D [5] was used, developed at the Institute Electrodynamics NAS of Ukraine. AM using the three types of magnetization curves mentioned above was implemented with package COMSOL 3.4. In Fig. 3 the dependences of the specific eddy current losses are shown. These dependences determined using AM and HBFEM with normal magnetization curve and AM applying two effective magnetization curves mentioned above. The amplitude of the sinusoidal voltage was varied to obtain the magnetic flux density in the surface of iron in the range from 0,7 T to 2,0 T and higher. The frequency of the voltage supply was assumed to be 50 Hz and conductivity was preset to $11.2\cdot10^6$ S/m. As can be seen, the results of HBFEM are positioned between the results obtained by means of AM with normal magnetization curve and the results obtained using magnetization curves calculated by using energy methods. If AM with normal magnetization curve is used the loss value is about 9% higher than in case of using the harmonic balance method (H(B) – normal, Fig. 3). Similar values of eddy current losses for two grades of magnetic steel are obtained in [6]. It should be noted that for small excitation field intensities (<5 kA/m), the results obtained by using the monoharmonic model (COMSOL package) with normal magnetization curve and by means of HBFEM (GE2D package) are identical [5]. At the same time, the use of magnetization curves determined by the "Simple energy method" (BSE) and the "Average energy method" (BAE) gives much lower value of losses compared with HBFEM. To summarize, it can be safely assumed that under the sinusoidal voltage condition, AM, using a normal magnetic curve, gives a loss value closer to the value obtained with HBFEM than the value given by AM using effective magnetic curves.

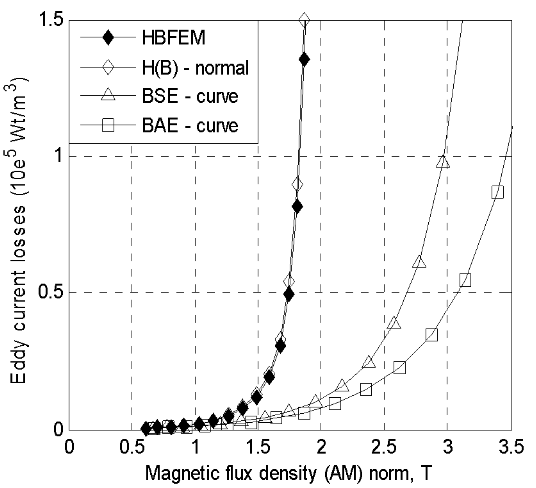

Fig. 3

Conclusions. When the magnetic field is excited by the sinusoidal voltage, the computation of the eddy current losses by using the harmonic balance method combined with the finite element method gives lower specific losses value than the monoharmonic method with amplitude correspondence of the main harmonics of magnetic flux density and magnetic field strength according to the normal magnetization curve.

The use of effective magnetization curves, calculated by means of the "Simplified energy method" and the "Averaged energy method", gives even lower specific loss values than the harmonic balance finite element method. The precise computation of the eddy current loss distribution and the amplitudes of both the magnetic flux density and magnetic field strength simultaneously can be performed only by using the harmonic balance method.

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МОДЕЛЮВАННЯ НЕЛІНІЙНОГО ПОВЕРХНЕВОГО ЕФЕКТУ МЕТОДОМ ГАРМОНІЧНОГО БАЛАНСУ СУМІСНО З МЕТОДОМ СКІНЧЕНИХ ЕЛЕМЕНТІВ ЗА УМОВ ЖИВЛЕННЯ СИНУСОЇДАЛЬНОЮ НАПРУГОЮ ТА ВИКОРИСТАННЯМ ЕФЕКТИВНИХ МАГНІТНИХ ХАРАКТЕРИСТИК

І. С. Пєтухов, докт.техн.наук
Інститут електродинаміки НАН України,
пр. Перемоги, 56, Київ, 03057, Україна. E-**mail:** igor_petu@ukr.net

Представлено математичну модель періодичного у часі магнітного поля, що базується на об'єднанні методів гармонічного балансу та скінченних елементів. Розглянуто випадок збудження магнітного поля синусоїдальною напругою. Виконано порівняння величини втрат на вихрові струми, визначені представленим методом, а також за допомогою моногармонічного підходу, який використовує основну криву намагнічування та криві намагнічування, визначені за спрощеним енергетичним методом та осередненим енергетичним методом. Визначено, що моногармонічний підхід, який використовує основну криву намагнічування, представляє значення втрат на вихрові струми на 9% більше, ніж метод гармонічного балансу. Бібл. 6, рис. 3.

Ключові слова: метод гармонічного балансу, метод скінченних елементів, феромагнітне електропровідне середовище, втрати на вихрові струми

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МОДЕЛИРОВАНИЕ НЕЛИНЕЙНОГО ПОВЕРХНОСТНОГО ЭФФЕКТА МЕТОДОМ ГАРМОНИЧЕСКОГО БАЛАНСА СОВМЕСТНО С МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ ПРИ УСЛОВИИ ПИТАНИЯ СИНУСОИДАЛЬНЫМ НАПРЯЖЕНИЕМ И ИСПОЛЬЗОВАНИИ ЭФФЕКТИВНЫХ ХАРАКТЕРИСТИК НАМАГНИЧИВАНИЯ

И.С. Петухов, докт.техн.наук
Институт электродинамики НАН Украины,
Пр. Победы, 56, Киев, 03057, Украина. E-**mail:** igor_petu@ukr.net

Представлена математическая модель периодического во времени магнитного поля, которая основана на объединении методов гармонического баланса и конечных элементов. Рассмотрен случай возбуждения магнитного поля синусоидальным напряжением. Выполнено сравнение величины потерь на вихревые токи, определенные представленным методом, а также при помощи моногармонического подхода, который использует основную кривую намагничивания и кривые намагничивания определенные упрощенным энергетическим методом и усредненным энергетическим методом. Определено, что моногармонический подход, использующий основную кривую намагничивания, дает значения потерь на вихревые токи на 9% больше, чем метод гармонического баланса. Моногармонический подход, использующий эффективные кривые намагничивания, дает намного большее расхождение в значении удельных потерь по сравнению с методом гармонического баланса. Поэтому для моногармонического моделирования поверхностного эффекта при питании от источника напряжения целесообразно использовать основную кривую намагничивания. Библ. 6, рис. 3.

Ключевые слова: метод гармонического баланса, метод конечных элементов, ферромагнитная проводящая среда, потери на вихревые токи.

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