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USING THE MONTE CARLO METHOD FOR CALCULATING THE ERROR OF THE MEASUREMENT SYSTEM

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The article considers the Monte Carlo method as one of the possible techniques for calculating the error of a measuring system, which consists of several elements, each of which measures some quantity with its own independent error. Due to its features, the method can be extended to modeling any process affected by random variables. The simplicity of application and the calculation algorithm makes it possible to easily calculate the total error of the system and the probability of its occurrence, while avoiding inflated and unlikely values. The article substantiates the application of the Monte Carlo method for calculating the error of the measuring system, reveals the nature of the distribution of errors, and calculates the value of the error depending on the probability of its occurrence. It is shown that with probability of 0.95 the total error of the system can be taken to be 3 times smaller than the maximum possible error.
References 8, figures 2, table 1.
Keywords*:* Method Monte Carlo method, calculation of system's error.

Introduction. Monte Carlo methods are used to solve many problems by modeling random processes. The theoretical base of such methods was developed long ago, and mathematicians J. von Neumann and S. Ulam [1] are considered the founders of the method. Since the basis of the method is the generation of random values with a given distribution (which is a rather difficult task), the method became widespread only after the appearance of electronic computers.

Features of the method: 1) simple structure of the calculation algorithm; 2) the calculation error is proportional to $\sqrt{D/N}$, where D is the some constant that has the value of the root mean square deviation; N is the number of tests.

That is, it is problematic to obtain a high accuracy of calculation by this method, but where there are random values, and the accuracy of the result of (2-5)% is satisfactory, and therefore the method can be successfully applied [2, 3].

The general structure of the calculation is as follows:

• a computer code is compiled to calculate one random event, the distribution of which depends on the type of specific problem, namely, uniform distribution, distribution subject to the normal law, etc. Usually it is desirable to use standard codes that are already built into most software products, for example MAPLE. Note that an open access software product MAPLE is currently one of the most powerful intelligent computer algebra systems. The computational core of this system is used in another well-known system such as MATLAB. Both systems have been repeatedly tested and validated over the years, the reliability of their components, including the random number generator, is confirmed by long-term experience of use;

• then the test is repeated N times, and each individual event is independent of other events (the final number of events N is set based on the error calculated later);

• test results are combined and averaged.

That is why the Monte Carlo method is sometimes called a statistical testing method.

The range of problems that can be solved by Monte Carlo methods is quite wide because this method allows modeling any process that is affected by random variables and not only that.

These are: 1) tasks of mass service, when a stream of information, distributed with a given probability density, enters the channels of its processing; 2) tasks for calculating the quality and reliability of products; 3) calculations of the most likely measurement error; 4) nuclear physics; 5) calculation of complex integrals, etc. [4-7]; 6) modelling of the joint distribution of several correlated quantities with an arbitrary distribution law [8].

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Problem definition. When measuring any quantities, there is always a measurement error that depends on the error of measuring devices and random values. Therefore, it is always necessary to know what kind of error it is and what is the probability that the error will take one or another value.

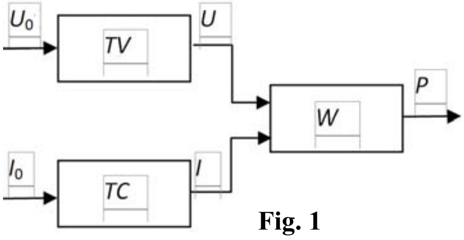

Fig. 1

As an example, consider the diagram of power measurement in the electrical network, which consists of a current transformer *TC*, a voltage transformer *TV* and a power meter *W* (Fig. 1). Moreover, the process can be complicated if current and voltage fluctuations in the network are taken into account.

Suppose that each of the devices measures a value with a relative error that does not depend on the errors of other devices, and then we can write:

$$U = K_u \cdot U_0 \cdot (1 \pm e_1);\ I = K_i \cdot I_0 \cdot (1 \pm e_2)\,, \quad (1)$$

where U, I are the output voltage and current, respectively; U_0, I_0 are the input values of voltage and current, respectively; K_u, K_i are the voltage and current transformation coefficients, respectively; e_1, e_2 are the relative errors in voltage and current, respectively.

Then for power:

$$P = U \cdot I \cdot (1 \pm e_3) = P_0 \cdot (1 \pm e_1) \cdot (1 \pm e_2) \cdot (1 \pm e_3)\,, \quad (2)$$

where P is the output power; $P_0 = K_u \cdot U_0 \cdot K_i \cdot I_0$ is the basic power; e_3 is the relative error of the counter (power meter).

It follows from (2) that the value of the total error has a complex nature, so the question arises: how to estimate the range of power change?

It is possible to estimate the range of power change by choosing the "worst" or "best" parameter values. But, firstly, it is not always known which combination of parameters will be the "worst" and which will be the "best"; secondly, such an estimate may be significantly overestimated, because it is unlikely that all parameters will be the "worst" or "best" at the same time [2-4].

In this connection, the questions arise: 1) what will be the maximum possible value of the error? 2) what is the most likely error and what is the probability of getting it? 3) what will be the value of the error if the probability of its occurrence is given? 4) how much will the error change from the value of the probability of its occurrence?

The answers to all these questions can be obtained using the Monte Carlo method, which allows to model any process that is affected by random variables.

The goal of the article and task of the research. The goal of the article is to substantiate the technique of calculating the error of the measuring system (see Fig. 1) by the Monte Carlo method. The task of the research is to calculate the error of the measuring system using the Monte Carlo method, to identify the nature of the distribution of errors and to calculate the value of the error depending on the probability of its occurrence.

The scientific novelty of the work consists in obtaining new useful research results that allow more accurate calculation of the error at the output of several measuring devices, if the error is distributed according to the normal law.

Calculations carried out, results obtained and their analysis. Let us assume that all the studied parameters (e_1, e_2, e_3) are random variables. It is impossible to analytically calculate the distribution of these values. It is also difficult to do this in practice: for this we need to examine a large batch of finished products.

Therefore, for the application of the Monte Carlo method, we assume that the measured power is a random variable, and the errors are distributed according to the normal law with a mean value (mathematical expectation) of 0 and a variance equal to

$$\sigma_i = e_i / 3\,, \quad (3)$$

where σ_i is the variance; i=1,..,3.

Formula (3) is based on the central limit theorem [2-5], which states that under fairly general conditions, the distribution of a random variable with a sufficiently large number of tests approaches the law of normal distribution.

In practice, this means the following: with a normal distribution of a random variable in one test, it is practically impossible to get an error value greater than $3 \cdot \sigma_i = e_i$.

Without losing the generality of the calculation, we assume that the base power $K_u \cdot U_0 \cdot K_i \cdot I_0$ is equal to one. That is

$$P_0 = K_u \cdot U_0 \cdot K_i \cdot I_0 = 1 \,. \tag{4}$$

Then the algorithm for calculating the possible error will be as follows:

1) we determine the number of test N;

2) we calculate with the help of a random number generator three independent random variables subject to the normal distribution law γ_1, γ_2, γ_2;

3) we calculate the errors $\frac{e_1}{3} \cdot \gamma_1, \frac{e_2}{3} \cdot \gamma_2, \frac{e_3}{3} \cdot \gamma_3$;

4) we calculate the power (2);

5) if the process is not finished, go to item 2; if it is finished, the end.

The generator calculates random numbers subject to the normal distribution law based on the equation for which the mathematical expectation is zero and the variance is one:

$$(2\pi)^{-1/2} \int_{-\infty}^{\xi} e^{-\frac{t^2}{2}} \cdot dt = \gamma \,. \tag{5}$$

The results of calculations for N=50000, e_i=0.1, where i=1, 2, 3.

From calculations were made using MAPLE code, it follows that the values fluctuate with respect to the mathematical expectation of power, which is equal to one, and the maximum estimated deviations are 0.78 and 1.26 (relative values minus 0.22 and 0.26). The maximum theoretical deviation can reach the value ±0.331. That is, even for a very large number of tests (50,000), the error does not reach its maximum value.

It also follows that even with a large number of tests, the error of the measuring system (if it is distributed according to the normal law) is less than the predicted error (maximum error is 0.331, root mean square error is 0.173). From the analysis of data obtained (1 is the base value) it also follows that the smallest error of 0.173 is improbable.

$$\begin{aligned} \varepsilon_{\max} &= (1+\varepsilon_{1\max}) \cdot (1+\varepsilon_{2\max}) \cdot (1+\varepsilon_{3\max}) - 1 = 0.331; \\ \varepsilon_{\sigma} &= \sqrt{\varepsilon_{1\max}{}^2 + \varepsilon_{2\max}{}^2 + \varepsilon_{3\max}{}^2} = 0.173, \end{aligned} \tag{6}$$

where where $\varepsilon_{i\max}$ is the maximum possible relative error of each of the three measuring devices (in the article, according to the conditions of the formulated problem, it is equal to 0.1); $\varepsilon_{\max}$ is the maximum possible relative error; ε_{σ} is the relative root mean square error.

In this connection, the question arises: "What will be the value of the error, if the probability of its occurrence is given?"

To solve this issue, a computer code was developed, the algorithm of which is given below.

Calculation algorithm:

1) we set the initial error value and the required probability value;

2) we set the initial power interval (initial error);

P / probability	0.9	0.92	0.94	0.96	0.98	0.99	0.995
ε / error	0.08	0.09	0.1	0.11	0.13	0.14	0.16

3) we count the number of random hits in the specified power interval ratio to the total number of tests;

4) if the calculated hit probability is greater than the predetermined one, then the calculation is finished;

5) if not, then we set a new power interval, increase the initial value of the error and go to item 2.

The calculation results are shown in Table and in Fig. 2 in the form of a graph of the dependence of the value of the measurement error on the given value of the probability of its occurrence.

From Fig. 2, we can draw a conclusion: with given probability p=0.95, the error value can be assumed to be three times smaller (0.1) than the maximum possible error (0.331), which significantly increases the accuracy of the obtained results.

Finally, it should be noted that if we assume that the errors are uniformly distributed over the interval, then in this

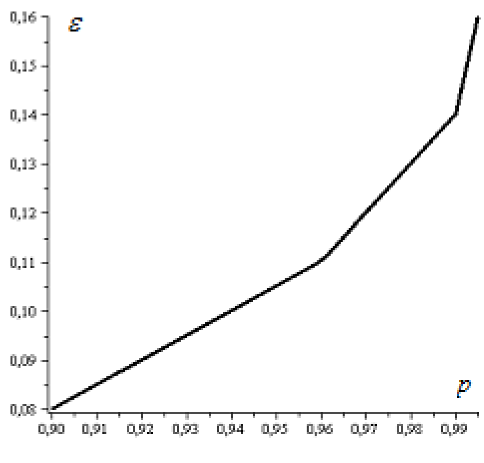

Fig. 2

case, with probability equal to p=0.95, the error value will be 0.17, which is 1.7 times greater than for the law of normal distribution.

Conclusions.

1. A technique for calculating the error of the measuring system using the Monte Carlo method, which, using a fairly simple algorithm, allows to estimate the errors of complex measuring systems with high accuracy is substantiated.

2. Using a random number generator, it is possible to get the error distribution depending on the number of measurements and estimate the real error ranges.

3. The value of the measurement error depends on the value of the given probability of its occurrence and at probability of its occurrence p=0.95 it is three times smaller than the maximum possible one (with the normal law of its distribution).

4. The value of the error at given probability of its occurrence depends on the law of its distribution.

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ВИКОРИСТАННЯ МЕТОДУ МОНТЕ КАРЛО ДЛЯ ОБЧИСЛЕННЯ ПОХИБКИ ВИМІРЮВАЛЬНОЇ СИСТЕМИ

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У статті розглядається метод Монте Карло як один з можливих методів розрахунку похибки вимірювальної системи, яка складається з декількох елементів, кожен з яких вимірює деяку величину зі своєю незалежною похибкою. Завдяки своїм особливостям метод може бути поширено на моделювання будь якого процесу, на який впливають випадкові величини. Простота застосування та алгоритму розрахунку дає можливість легко розрахувати сумарну похибку системи та ймовірність її появи, уникаючи при цьому завищених та мало ймовірних значень. Обґрунтовано застосування методу Монте Карло для розрахунку похибки вимірювальної системи, виявлено характер розподілу похибок та розраховано значення похибки в залежності від ймовірності її появи. Показано, що з ймовірністю 0,95 сумарну похибку системи можна прийняти в 3 рази меншою, ніж максимально можлива похибка. Бібл. 8, рис. 2, табл. 1.

Ключові слова*:* метод Монте-Карло, обчислення похибки системи.

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