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COMPUTATION OF THE POGO SELF-OSCILLATION PARAMETERS IN THE DYNAMIC "PROPULSION – ROCKET STRUCTURE" SYSTEM BY USING A 3D STRUCTURAL MODEL

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На основі скінченно-елементної дискретизації автоколивальної системи «Рідинна ракетна двигунна установка (РРДУ) – корпус ракети-носія (РН)» із використанням тривимірних і одновимірних кінцевих елементів розроблено математичну модель, що описує нелінійну динамічну взаємодію корпусу двоступеневої РН (як складної оболонкової конструкції, що містить рідину) і її маршової РРДУ на активній частині польоту РН, і розвинено підхід до визначення параметрів автоколивань рідинної РН при її поздовжньої нестійкості (POGO).

У запропонованому підході корпус рідинної ракети розглядається як складна багатозв'язкова дисипативна система «конструкція РН – рідке паливо в баках» і схематизується тривимірними скінченними елементами, що дозволяє досліджувати просторові коливання корпусу РН і рідкого палива в його баках. Моделювання низькочастотної динаміки насосів РРДУ виконується на основі розробленої в Інституті технічної механіки Національної академії наук України і Державного космічного агентства України (ІТМ НАНУ і ДКАУ) теорії кавітаційних автоколивань в насосних системах. При моделюванні низькочастотних процесів в динамічній системі «РРДУ – корпус РН» враховуються нелінійності, які є найбільш суттєвими при чисельному рішенні нелінійної задачі про POGO-коливання рідинної ракети – нелінійна залежність об'єму і постійної часу кавітаційних каверн від режимних параметрів насосів та нелінійна залежність декрементів коливань корпусу РН від амплітуд його коливань.

Проведено чисельне моделювання POGO-автоколивань двоступеневої РН загальною масою 165 т із масою рідини в баках окислювача і пального першого ступеня 130 т. Для розрахункового випадку резонансної взаємодії корпусу РН із РРДУ визначено параметри граничного циклу динамічної системи «РРДУ – корпус РН». Показано, що при автоколиваннях переміщення елементів конструкції корпусу ракети, тисків та витрат пального в елементах рідинного ракетного двигуна відбуваються з частотою 15,9 Гц, близькою до частоти II -го тону власних поздовжніх коливань корпусу.

Розроблене науково-методичне забезпечення може бути використано для теоретичного визначення параметрів автоколивань перспективних рідинних РН (в тому числі ракет, які мають складну просторову конфігурацію корпусу) по відношенню до пружних поздовжніх і поперечних коливань конструкції, а також для оцінки динамічного навантаження конструкцій РН.

На основе конечно-элементной дискретизации автоколебательной системы «Жидкостная ракетная двигательная установка (ЖРДУ) – корпус ракеты-носителя (РН)» с использованием трехмерных и одномерных конечных элементов разработана математическая модель, описывающая нелинейное динамическое взаимодействие корпуса двухступенчатой РН (как сложной оболочечной конструкции с жидкостью) и ее маршевой ЖРДУ на активном участке полета РН, и развит подход к определению параметров автоколебаний жидкостной РН при ее продольной неустойчивости (POGO).

В предлагаемом подходе корпус жидкостной ракеты рассматривается как сложная многосвязная диссипативная система «конструкция РН – жидкое топливо в баках» и схематизируется трехмерными конечными элементами, что позволяет исследовать пространственные колебания корпуса РН и жидкого топлива в его баках. Моделирование низкочастотной динамики насосов ЖРДУ выполняется на основе разработанной в Институте технической механики Национальной академии наук Украины и Государственного космического агентства Украины (ИТМ НАНУ и ГКАУ) теории кавитационных автоколебаний в насосных системах. При моделировании низкочастотных процессов в динамической системе «ЖРДУ – корпус РН» учитываются нелинейности, наиболее существенные при численном решении нелинейной задачи о POGO-колебаниях жидкостной ракеты – нелинейная зависимость объема и постоянной времени кавитационных каверн от режимных параметров насосов и нелинейная зависимость декрементов колебаний корпуса РН от амплитуд его колебаний.

Проведено численное моделирование POGO-автоколебаний двухступенчатой РН общей массой 165 т с массой жидкости в баках окислителя и горючего первой ступени 130 т. Для расчетного случая резонансного взаимодействия корпуса РН и ЖРДУ определены параметры предельного цикла динамической системы «ЖРДУ – корпус РН». Показано, что при автоколебаниях перемещения элементов конструкции корпуса ракеты, давлений и расходов в элементах жидкостного ракетного двигателя происходят с частотой 15,9 Гц., близкой к частоте II -го тона собственных продольных колебаний корпуса.

Разработанное научно-методическое обеспечение может быть использовано для теоретического определения параметров автоколебаний перспективных жидкостных РН (в том числе ракет, имеющих сложную пространственную конфигурацию корпуса) по отношению к упругим продольным и поперечным колебаниям конструкции, а также для оценки динамического нагружения конструкций РН.

A mathematical model describing the nonlinear dynamical interaction of a launch vehicle (LV) and its marching liquid-propellant engine in the active phase of the LV flight is developed on the basis of the finite-element discretization of the "propulsion – LV structure" self-oscillating system using three-dimensional and one-dimensional finite elements. An approach to the computation of the liquid-propellant launch vehicle self-oscillation parameters self-oscillations of the liquid launch vehicle under POGO instability is developed.

In the proposed approach, the rocket structure is considered as a complex multiply connected dissipative system "LV structure – liquid propellant in tanks" and is schematized by three-dimensional finite elements, which allows investigating the spatial vibrations of the LV structure and the liquid propellant in the tanks. Modeling of the low-frequency dynamics of the rocket engine pumps is performed on the basis of the theory of cavitation self-oscillations in pumping systems developed at the Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine (ITM of NASU and SSAU). The most significant nonlinearities in the numerical solution of the non-linear problem of liquid-propellant rocket POGO oscillations, namely, the nonlinearity of the dependence of the cavitation volume and the cavitation time constant on the pump operational parameters and the nonlinearity of the dependence of the LV structure oscillation decrements on the LV structure vibration amplitudes, were taken into account in the model of the system low-frequency dynamics.

Numerical modeling of POGO self-oscillations of a two-staged LV with a total mass of 165 tons and with a mass of 130 tons of the propellant in the first stage tanks is carried out. For the computation case of the resonant interaction of the LV structure and the liquid-propellant rocket engine (LPRE), the limiting cycle parameters of the dynamic "LPRE – LV structure" system are determined. It is shown that in the case of LV POGO self-oscillations the structural elements vibrate and the pressures and the flow rates in the liquid-propellant rocket engine oscillate at a frequency of 15.9 Hz, which is close to the natural frequency of the second mode of the structural longitudinal oscillations.

The scientific software developed may be used in the theoretical determination of the POGO self-oscillation parameters of prospective liquid-propellant rockets (including rockets whose structure has a complex spatial configuration) with respect to elastic longitudinal and transverse oscillations of the LV structure and in assessing dynamic loads on LV structures.

Keywords: *POGO instability, liquid-propellant rocket, self-oscillations, longitudinal structural vibrations, mathematical modeling, pump cavitation, low-frequency propulsion system dynamics, three-dimensional finite elements.*

1. Introduction. Longitudinal oscillations of the launch vehicle structure (POGO-oscillations according to the NASA terminology) with frequencies from 3 Hz to 50 Hz – 100 Hz, arising due the loss of liquid rocket longitudinal stability during the flight, can lead to disruption of the integrity of the launch vehicle (LV), to operation failure of the LV control system and to other emergency situations [1]. Hence, ensuring the longitudinal stability of liquid-propellant rockets is an urgent task, requiring its solution in the design stage of new LV and in modernization stage of old-designed LV [1 – 6].

At present, the problem of POGO oscillations as a problem of theoretical prediction of the amplitude limiting level of longitudinal self-excited vibration of liquid rockets in the leading research aerospace institutes is not solved. For example, the American Aerospace Corporation [7], which for many years carries out the most responsible research on POGO oscillations of liquid rockets, consider the solution of this problem by mathematical modeling methods is impossible because of the poor knowledge of nonlinear dynamic processes in systems and assemblies of liquid rocket propulsion system (mainly, in cavitating pumps).

The problem of determining the amplitudes of the POGO oscillations at the LV longitudinal instability on the base of a nonlinear mathematical model of the "liquid propellant propulsion – rocket structure" dynamic system was first considered by Dr. M. Natanzon. He substantiated the "mechanism" of the longitudinal self-oscillations of the rocket structure and showed the determining role of the nonlinearities in the characteristics of LRE pumps for the development of longitudinal oscillations of liquid rockets [1]. However, his POGO nonlinear analysis of a liquid rocket was based on qualitative methods and estimates.

The nonlinear theory of longitudinal oscillations of liquid LVs was created and further developed in works of the ITM NASU and SSAU under of Academician V. Pilipenko [in particular, 8 – 11]. Mathematical models of low-frequency dynamics of cavitation pumps of liquid rocket engines have been developed, which allow to carry out not only qualitative but also quantitative analysis of the effect of cavitation phenomena in LRE pumps on the dynamic characteristics of a propulsion system [8], and also carry out theoretical forecasts of longitudinal stability and determine the parameters of longitudinal oscillations of liquid rockets in the LV flight. It was shown in [9] that the nonlinearities of the characteristics of LRE cavitating inducer-centrifugal pumps are a factor limiting the growth of the amplitudes of longitudinal oscillations of liquid rockets. An approach to the numerical solution of the problem of determining the parameters of the limiting cycle of longitudinal oscillations of a liquid rocket using the method of harmonic linearization was proposed in [10].

In the papers [11–12], the longitudinal components of the LV load factor oscillations were determined on the basis of nonlinear mathematical models of the low-frequency dynamics of the system "liquid propellant propulsion – rocket structure". However, the LV structure longitudinal oscillations were described in these models on the basis of the generally accepted simplification [1], i.e. by equations in generalized normal coordinates, taking into account several (mostly three) lower vibration modes, and the structural shapes of free longitudinal oscillations of the investigated LV and liquid propellants in LV tanks were calculated without accounting for damping and interaction of propellant with an LV elastic structures. In work [13] the developed mathematical model of low-frequency dynamics of the "liquid propellant propulsion – rocket structure" system is proposed, in which longitudinal oscillations of the LV structure are described as oscillations of a branched chain of oscillators with damping effects. However, the model presented was linear. It took into account only the lowest modes of natural longitudinal oscillations of liquid propellant in the tanks. Thus, perspective directions of the further development of the nonlinear mathematical model of low-frequency dynamics of the "liquid propellant propulsion – rocket structure" system used to the study of LV longitudinal stability and dynamic loads are the allowance for the interaction of various shapes of fluid oscillations in LV tanks with oscillations of LV elastic structure and dissipative losses at these oscillations.

2. Statement of the problem. The aim of this work is the mathematical modeling of LV oscillations at which to describe the spatial oscillatory motion of LV structure holds its three-dimensional finite element schematization, and the one-dimensional finite elements schematization are used to describe the low-frequency dynamics of liquid propellant rocket engines (LRE).

In accordance with the proposed approach, the development of a mathematical model of POGO self-oscillations of a liquid LV is based on the following positions:

– parameters of self–excited oscillations of a liquid LV are determined on the basis of a nonlinear mathematical model of the low-frequency dynamics of the closed "LRE – LV structure" system with "frozen" coefficients;

– modeling of the spatial vibrational motion of the LV structure with liquid propellant in tanks is carried out on the basis of the three-dimensional finite element model of the system of liquid filled shell structures using modern computer means of finite element analysis;

– spatial fluctuations of liquid propellants in the tanks are modeled taking into account the motion of the free surface of the liquid and the interaction of the liquid with the tank structure [15];

– when describing the LRE low-frequency dynamics, aggregates, hydraulic and gas paths are considered as one-dimensional finite elements [2];

– allowance for the energy dissipation of liquid propellant oscillations and elastic vibrations of the LV structure is carried out on the basis of the viscous friction model with the use of experimental values of the vibration decrements of the LV structure and the nonlinear dependences of the decrements of its oscillations on the vibration amplitudes;

– cavitation phenomena in LRE pumps are taken into account on the basis of the hydrodynamic model of cavitation oscillations in pumping systems [8];

– the main nonlinearities are taken into account in model of the low–frequency processes in the «LRE – LV structure» dynamic system: the nonlinear dependences of the volume of the cavitation caverns and the cavitation time constant on the operational parameters of the pumps [8], the dependence of the decrements of the LV structure oscillations on the vibration amplitudes [17].

The proposed approach allows to investigate the POGO oscillations of liquid LVs and determine the parameters of limiting cycle taking into account the complex character of the interaction of the spatial oscillations of liquid propellant in tanks, the elastic vibrations of the LV structure with nonlinear dissipative forces in the spatial deformations of the LV structure and low-frequency processes in the LPE.

3. Mathematical modeling of spatial vibrations of the liquid LV elastic structure by the use of the finite element method. The liquid LV structure is a complex hydromechanical system "the LV structure – liquid propellant in tanks". As a rule, it has a complex configuration, is performed with variable wall thickness of load-carrying structures of propellant tanks and includes various load-bearing elements.

Using the capabilities of modern CAE–systems [14, 15], based on the finite element method, allows in most cases to take into account the design features of the LV structure during mathematical modeling of its dynamic interaction with the LRE. In accordance with the proposed approach, the problem of numerical modeling and analysis of the spatial vibrations of a liquid rocket structure with frequencies up to 100 Hz is solved in the following formulation:

– the LV structure is considered as a complex system of filled liquid shell structures;

– the LV structure spatial vibrations are modeled as vibrations of the "LV structure – liquid propellant in tanks" dissipative system using finite element methods and modern computer finite element analysis [13].

A mathematical model describing the LV structure spatial vibrations taking into account energy dissipation is represented by the matrix equation

$$\mathrm{M}_s \ddot{\mathrm{u}}_s(t) + \mathrm{D}_s \dot{\mathrm{u}}_s(t) + \mathrm{K}_s \mathrm{u}_s(t) = 0, \quad (1)$$

where M_s, K_s, D_s are mass matrix, stiffness matrix and, accordingly, damping factors matrix of the system "LV structure – liquid propellant in tanks", having a size of $6n_s \times 6n_s$; n_s is number of nodes of the finite element model of the "LV structure – liquid propellant in tanks" system; u_s is vector of system node movements: $\mathrm{u}_s = [\mathrm{u}_{si}]_{i=1}^{n_S}$, $\mathrm{u}_{si} = [u_{si}^x,\ u_{si}^y,\ u_{si}^z,\ u_{si}^{(y,z)}, u_{si}^{(x,z)},\ u_{si}^{(x,y)}]$; x, y, z are axis of the Cartesian coordinate system, whose center is located on the axis of symmetry of the LV structure at any point $A(x_A, y_A, z_A)$, convenient for analyzing the disturbed motion (the axis z coincides with the LV longitudinal axis and is the axis of symmetry of the LV structure); $u_{si}^{(x)}, u_{si}^{(y)},\ u_{si}^{(z)}$ are projections of the displacement of the *n-th* node on the axes x, y, z; $u_{si}^{(y,z)}, u_{si}^{(x,z)},\ u_{si}^{(x,y)}$ are angles of rotation of the n-th node around the axes x, y, z; t is time.

Here and in the paper, the first derivative of a function with respect to time is denoted by a variable with a point, and the second derivative by a variable with two points: $\dot{\mathrm{u}}_s(t) = d\mathrm{u}_s(t)/dt$; $\ddot{\mathrm{u}}_s(t) = d^2\mathrm{u}_s(t)/dt^2$.

The values of the damping coefficients for describing the vibrational motions of the LV structure and the liquid propellants in equation (1) were selected in accordance with [16].

In modeling the spatial oscillations of liquid propellant in tanks, the following generally accepted assumptions were made regarding the properties and nature of liquid motion [1, 14]:

– liquid propellant is a homogeneous ideal compressible liquid;

– the motion of the fluid is irrotational;

– the forces of surface tension of liquid propellants are small, and their influence on the parameters of fluid and structure vibrations can be neglected.

The feature of modeling of spatial oscillations of the liquid propellants in LV tanks was account the interaction of the liquid propellants with the tank elastic structure and interaction of longitudinal and transverse oscillations of the liquid in the tanks.

For modeling the spatial vibrations of the LV elastic structure, it was assumed that the condition of joint deformation of the liquid and the tank structure is fulfilled for the LV tanks (i. e. the condition of "sticking" of liquid on the tank wall). This means that for nodes of the finite-element model of the system "LV structure – liquid propellant in tanks" corresponding to the interface between the media "wetted surface of the tank shell – liquid in the tank", displacement of finite elements "3D-liquid" [14] and shell elements of the tank structure are equal to each other in the directions of the three axes of the local coordinate system. Furthermore, it was assumed that the condition of equality of nodal displacements in the direction of the normal to the tank shell (the condition of ideal sliding of the liquid along the wall of the tank) is fulfilled at the nodes of the finite element model system "LV structure – liquid propellant in the tanks", corresponding media interface of "wetted surface of the tank shell – the free surface of the liquid in the tank" for elements «3D-liquid" and shell elements of the tank structure.

The pressure of liquid propellant in the nodes of the finite elements "3D liquid" used in the schematization of liquid filling in the LV tank was determined in the developed model by the equation

$$\mathrm{p}_T = \mathrm{G\,B\,u}_f, \tag{2}$$

where p_T is vector by $6n_f$ length, the components of which are the pressure values in the finite elements "3D liquid"; n_f is total number of finite element nodes

"3D liquid" in tanks; G, B are matrixes with size $6n_f \times 6n_f$: G is bulk elasticity matrix, B is transition matrix, connecting deformations and node movements of "3D-liquid" elements; $\mathrm{u}_f = [\mathrm{u}_{fi}]_{i=1}^{n_F} = [u_{fi}^x, u_{fi}^y, u_{fi}^z, u_{fi}^{(y,z)}, u_{fi}^{(x,z)}, u_{fi}^{(x,v)}]_{i=1}^{n_f}$ is vector of nodal displacements of the "3D fluid" elements, which is part of the vector of nodal displacements of the system (1): $\mathrm{u}_f \subset \mathrm{u}_s$.

4. Development of a mathematical model of the dynamic interaction of the first stage LV rocket engine and the LV structure. At flight conditions with POGO phenomena **t**he elastic LV structure and liquid propulsion forms a self-oscillating "LRE – LV structure" dynamic system with positive feedback on the thrust of the rocket engine [1], [2]. In accordance with [1], the elastic longitudinal vibrations of the LV structure, caused by the longitudinal dynamic part of the engine thrust. This vibrations cause pressure fluctuations on the bottom of the tank, which lead to propellant pressure oscillations at the entrance to the engine. The latter cause oscillations in the LRE combustion chamber pressure and engine thrust oscillations that excite the LV structure vibrations. At unfavorable amplitude and phase relationships between the LV structure oscillations and the thrust of the liquid rocket engine (such relationships are realized in the resonant interaction of the LV structure with the LPRE), these oscillations increase and the "LRE – LV structure" dynamic system loses stability. In case of stability loss of a linear dynamical system, the amplitudes of oscillations of this system increase without limit. The influence of nonlinear factors in the dynamic "LRE – LV structure" system with a stable limit cycle consists mainly of the fact that: as the LV oscillations amplitudes increase to a certain level, the resonant frequency of propellant oscillations in the LRE feed line decreases and moves away from the frequency of the longitudinal oscillations of the LV structure [1]. Accordingly, a "detuning" of the resonance relations occurs in the dynamic system, which leads to a decrease in the intensity of the feedback, and consequently, to a decrease in the oscillating energy influx. When the influx of energy into the "LRE – LV structure" system is completely compensated for by dissipative losses, the system sets to the self-oscillation mode.

The nonlinear mathematical model of the low-frequency dynamics of the sustainer LRE, considered as an autonomous dynamic system "feed lines – LRE", can be represented in general form by the equation

$$\mathrm{F}_e(\ddot{\mathrm{u}}_e, \dot{\mathrm{u}}_e, \mathrm{u}_e, t) = 0, \tag{3}$$

where $\mathrm{u}_e = [u_{e1}, \ldots, u_{en_e}]$ is vector of variables of the system under consideration; $\mathrm{F}_e(\ddot{\mathrm{u}}_e, \dot{\mathrm{u}}_e, \mathrm{u}_e, t) = [F_{ei}(\ddot{u}_{e1}, \ldots, \ddot{u}_{en_e}, \dot{u}_{e1}, \ldots, \dot{u}_{en_e}, u_{e1}, \ldots, u_{en_e}, t)]^{\mathrm{т}}$ is column vector consisting of nonlinear functions F_{ei} ($i = 1, \ldots, n_e$), which describe the low-frequency processes in the LRE (superscript «т» denotes the vector transposition operation).

A feature of the mathematical model of the low-frequency dynamics of the propulsion system used in modeling its dynamic interaction with the LV structure was the description of the cavitation phenomena in the LRE pumps based on the nonlinear hydrodynamic model of cavitation oscillations in systems with high-speed centrifugal pumps with inducer [8]. The mathematical model of the dynamics of cavitating pumps includes the equation of the dynamics of cavitation caverns, the equation of fluid continuity in the flowing part of the pump, and the equation for determining the fluid pressure at the pump outlet:

$$p_1 = p_{CP} + k^*(V_K, G_1) \cdot \frac{\rho W_{1CP}^2}{2} + B_1 T_K \frac{dV_K}{dt}, \qquad (4)$$

$$\rho g \frac{dV_K}{dt} = G_2 - G_1, \qquad (5)$$

$$p_2 = p_1 + p_H \cdot \tilde{p}_H(\tilde{V}_K) - J_H \frac{dG_2}{dt}, \qquad (6)$$

where p_1, G_1 are pressure and flow rate of liquid at the pump inlet; p_{CP} is the pump breakdown pressure; V_K – volume of cavitation cavity; $k^*(V_K, G_1)$ is cavitation number dependence k^* from the volume of cavitation caverns and the flow rate of fluid at the pump inlet; $\frac{\rho W_{1CP}^2}{2}$ is speed head at the pump inlet; B_1, T_K are cavitation elasticity and cavitation time constant; ρ is fluid density; g is free fall acceleration; p_2, G_2 are the pressure and flow rate of liquid at the pump outlet; p_H is pump head; $\tilde{p}_H(\tilde{V}_K)$ is cavitation function of pump; $\tilde{V}_K$ is relative cavitation cavity volume; J_H is coefficient of inertial resistance of fluid in the flowing part of the pump.

The main nonlinear factors in this model are the dependences of the number of cavitation k^* and the cavitation time constant T_K (or variable $E_1 = T_K \cdot B_1$) in the flowing part of the pump on the pressure p_1 and flow rate G_1 of the propellant at the pump inlet.

The force effect of the LV structure on the marching LRE in the nonlinear model of their dynamic interaction was taken into account in the equations of the unsteady isothermal motion of a real compressible fluid on all sections of LRE the hydraulic tract.

In the case, when the modeling is carried out taking into account only the longitudinal components of the oscillations of the parameters of the "LPRE – LV structure" closed dynamic system, the motion of the liquid in the *k-th* section of the hydraulic tract of the LPRE is described by the following equations [2]:

$$p_k - p_{k-1} + I_k \dot{G}_k + a_k G_k^2 = \rho g\, h_k (n_{CT} + \ddot{u}_{sj}^k / g), \qquad (7)$$

$$C_k \dot{p}_k = G_k - G_{k+1}, \qquad (8)$$

where p_k, G_k are pressure and weight flow rate of fluid in the k-th element of the LRE hydraulic path; h_k is the projection length r of the k *-th* element on the longitudinal axis of the LV; C_k, a_k, I_k – the concentrated compliance, the coefficient of hydraulic and inertial resistance for k-th section, respectively; $\ddot{u}_{sj}^k$ is longitudinal acceleration of j-th the LV structural finite element, to which is rigidly fastened the k *-th* element of the LRE hydraulic tract.

The propellant pressure p_B at the tank outlet into the feed lines during LV flight time this structural elastic oscillations was determined as follows:

$$p_B = p_{HB} + p_{Ti} = p_{HB} + P_{TB}^{din} + \rho H_B n_{CT}\ , \tag{9}$$

where p_{HB} the supercharge pressure of the tank; p_{Ti} is pressure in the finite «3D-fluid» element, corresponding to the lower pole of the propellant tank; ρ is propellant density; H_B is liquid propellant height at tank; n_{CT} is the LV quasi-static overload; p_{TB}^{din} is dynamic component of propellant pressure at tank bottom, due to nodal movements u_f of finite «3D-fluid» element at the LV structure elastic vibrations.

It follows from (3), (7) – (9), that low-frequency processes in the marching LRE under the action of perturbations due to elastic longitudinal oscillations of the LV structure are described by a system of equations

$$\mathrm{F}_e(\ddot{\mathrm{u}}_e, \dot{\mathrm{u}}_e, \mathrm{u}_e, t) = \hat{\mathrm{F}}_s(\ddot{\mathrm{u}}_s, t)\,, \tag{10}$$

where $\hat{\mathrm{F}}_s(\ddot{\mathrm{u}}_s, t) = [\hat{F}_{si}(\ddot{u}_{s1}, \ldots, \ddot{u}_{s6n_s}, t)]^{\grave{O}}$ is vector of perturbations acting in the LV flight from the LV structure side on the LRE.

The mathematical model of the LV structure elastic oscillations arising from the action of perturbing forces on it from the side of the operating LRE can be written in the following form:

$$\mathrm{M}_s \ddot{\mathrm{u}}_s(t) + \mathrm{D}_s \dot{\mathrm{u}}_s(t) + \mathrm{K}_s \mathrm{u}_s(t) = \hat{\mathrm{F}}_e(\mathrm{u}_e, t)\,, \tag{11}$$

where $\hat{\mathrm{F}}_e(\mathrm{u}_e, t)$ is the vector of disturbing forces acting to the LV structure from the side of the working liquid rocket engine.

The basic perturbing force in equation (11) is applied to the elements of the LV structure in the place where the propulsion system is attached to rocket structure. This force is due to the deviation of the LRE thrust force R from its quasistatic value $\overline{R}_C$:

$$R - \overline{R}_C = \overline{R}_C \cdot P_C^{din} / \overline{P}_C\,, \tag{12}$$

where $\overline{P}_C$, P_C^{din} are the steady-state value of the pressure and the dynamic pressure in the combustion chamber.

In addition, the perturbing forces act on the LV structure in the elements of the LRE hydraulic tract. The dynamic components F_k of these forces for the k-th element of the hydraulic tract are determined by the expression

$$F_k = p_k A_k - p_{k+1} A_{k+1} - \frac{\dot{G}_k}{g} L_k - m_{fk}\, \ddot{u}_{si}^z - \frac{G_k^2}{\rho g^2}\left(\frac{1}{A_{k+1}} - \frac{1}{A_k}\right), \tag{13}$$

where p_k, G_k are pressure and flow rate in the k-th element of the hydraulic path; A_k, A_{k+1}, L_k are the the area of the inlet and outlet cross-section and, respectively, the length of the k-th element of the hydraulic tract; m_{fk} is mass of propellant in the k-th element of the hydraulic tract; i is number of finite element in the LV structural model, with which is in the force interaction with the k-th element of the hydraulic tract; u_{si}^{z} is the displacement projection of the i-th element of the LV structure on the longitudinal axis of the rocket.

A nonlinear mathematical model of the low-frequency dynamics of a closed self-oscillating system "LRE – LV structure", determined the dynamic interaction of the liquid rocket propulsion and the LV structure in the active part of flight, is described by the system of equations (10) and (11). In the equation of forced elastic vibrations of the rocket structure (8), taking into account the main non-linear factor – the non-linear dependence of the amplitude of the structure oscillations on the decrements – is carried out by means of a corresponding change in the values of the matrix elements of the damping coefficients D_s using the data given in [17].

3. An example of the implementation of the proposed approach to the computation of the LV self-oscillation parameters in case of longitudinal instability the liquid launch vehicle. Numerical modeling of the dynamic processes caused by the interaction of the LV structure and its first stage LRE was carried out with reference to the hypothetic two-staged LV with a spacecraft. The LV first stage is equipped with a liquid propulsion system. The length of the considered LV is 38 m; diameter is 3.9 m; the total mass of the launch vehicle is 165 tons; the mass of the liquid propellant in the oxidant and the fuel tanks of the LV first stage is about 130 tons; The mass of the spacecraft is 3 tons, the total mass of the shell structures of LV is 10 tons.

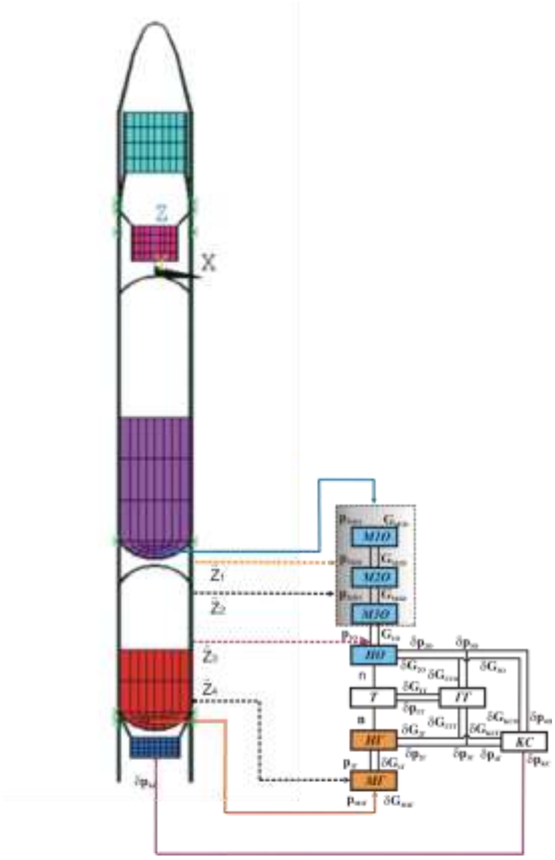

Fig. 1

The finite element model of the LV structure with liquid propellant in the tanks included 1440 "elastic shell" elements, 496 "3D liquid" elements, 576 "3D mass" elements, 96 elements of "concentrated mass" and the calculated grid obtained in the CAE system consisted of Ns = 2795 nodes. This model was supplemented by a model of the LRE low-frequency dynamics, in which the hydraulic and gas paths, LREP pumps were represented by one-dimensional finite elements (Fig. 1). The thrust force of the LRE was relied on the "3D mass" elements imitating the vibrational motion of the propulsion engine.

The Table 1 presents the calculated values of the natural frequencies f_j of the LV structure free elastic vibrations in the frequency range up to 50 Hz and the values of the corresponding effective (generalized) masses m_j^z in the longitudinal direction $j = 1, \ldots, 10$. From these results it follows that the modes of the LV housing with natural frequencies f_1 = 12.4 Hz, f_2 = 16 Hz and f_6 = 39.3 Hz have the largest generalized masses in the longitudinal direction.

The frequencies f_1, f_2, f_6 are in the range of the oscillation frequencies of the liquid propellant in the oxidizer and fuel feed lines, therefore, in this case the dynamic interaction of the LRE and the LV structure can lead to the development of longitudinal (POGO) LV self-induced oscillations with amplitudes that are unacceptable for the integrity of the LV and working capacity its systems

Analysis of the LV POGO stability of the LV performed on the basis of the linearized model of the dynamic "LRE – LV structure" system for the relative operating time of the first stage LRE t/T_{max} = 0.5 showed that the system is unstable at the frequency close to 15.9 Hz. The loss of longitudinal stability of the liquid rocket under investigation resulted from the resonance interaction of the longitudinal oscillations of the LV structure and fluid oscillations in the oxidant feed line (with the natural structure oscillation frequency of 16 Hz, and the frequency of the fluid in the oxidant feed line of 16.1 Hz).

To determine the parameters of the limit cycle of longitudinal self-oscillations of the LV, the mathematical model of the dynamic "LRE – LV structure" system (7) – (8) was used, in which the matrix equation (7) included nonlinear equations of low-frequency dynamics of cavitating pumps. To describe the cavitation phenomena in pumps, the nonlinear hydrodynamic model of V. Pilipenko was used [8]. Parameters of the limiting cycle of POGO oscillations were determined by numerical integration of the nonlinear system of ordinary differential equations (7) – (8) by the Runge–Kutta method.

Table 1

j	f_j, Hz	m_j^z, kg
1	12,4	42117
2	16,0	45633
3	25,2	548
4	33,2	1254
5	37,6	498
6	39,3	32461
7	41,0	1103
8	41,4	209
9	41,8	7408
10	46,3	135

As a result of the system numerical integration, it is established that the considered "LRE – LV structure" dynamic system goes to the limit cycle with a frequency of self-oscillations of 15.8 Hz, close to the natural frequency of the second mode of the structural longitudinal oscillations. The parameters of auto-oscillations of this system are determined: the dynamic components of the pressure in the LV tanks, the movement and vibration acceleration of the elements of the LV structure, as well as the flow rates and pressures in the hydraulic elements of the liquid rocket engine.

Below are the calculated dependencies of a number of parameters of the "LRE – LV structure" dynamic system versus time at the self-excited oscillation operating mode (in the resonance interaction of the liquid propulsion system and the LV structure). Comparing the calculated dependences of the oxidant pressure P_1 at the entrance to the main rocket engine (Fig. 2) and the dynamic component of the pressure P_{kc} in the combustion chamber (Fig. 3) versus time for auto-oscillations of the LV, it is easy to see that the pressure oscillations at the entrance to the propulsion system and the pressure in the combustion chamber occur with the same frequency of 15.8 Hz – the frequency of POGO self-oscillations.

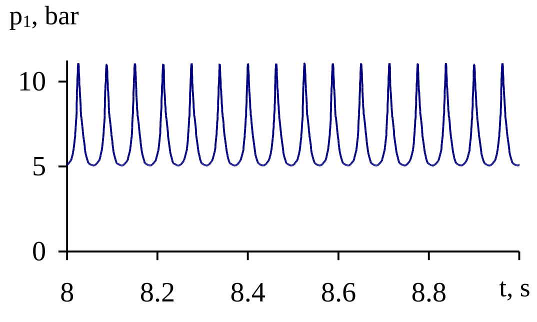

Fig. 2

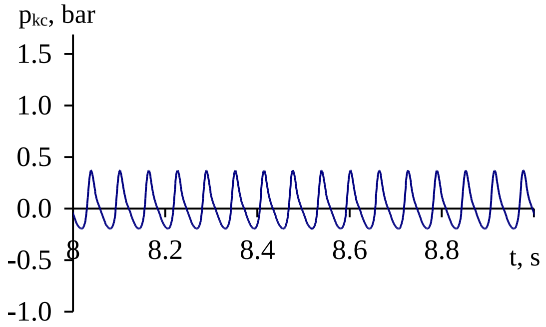

Fig. 3

The oscillations of the above calculated parameters have a non-harmonic (impulse) character, since nonlinear dependences of the volume V_K of cavitation caverns in the flowing part of the cavitating inducer (Figure 4) and the catitation time constant T_K (or the parameter E_1 given in Fig. 5) from the cavitation number k and the flow rate parameter q ($q_1 < q_2 < q_3$) were used in the mathematical model (7) - (8) of the dynamic system. Note that the cavitation number and flow rate are determined by the pressure and flow rate of the liquid at the pump inlet [8, 9].

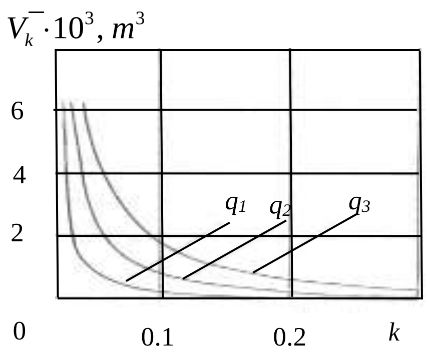

Fig. 4

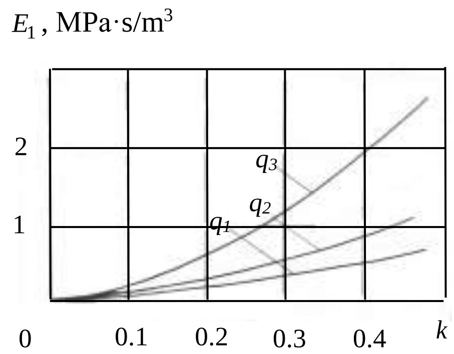

Fig. 5

As can be seen from Fig. 2, 3, the non-harmonic nature of the liquid oscillations is particularly pronounced in the oscillations of the oxidant pressure at the engine inlet. It should be emphasized that this type of pressure oscillations at the inlet to the cavitating pump corresponds to the flight data of liquid rockets in which the POGO phenomenon was recorded at [18].

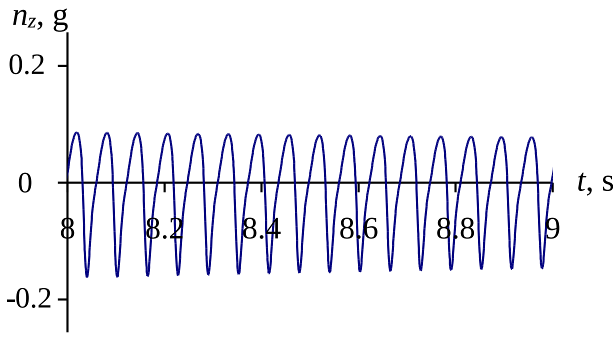

Fig. 6

As a result of the system numerical study, the POGO amplitudes of various LV structure elements, including the spacecraft (in the cross section of its center of mass), are determined at the self-excited oscillation operating mode. It is shown that after the "LRE – LV structure" dynamic system has entered the limit cycle mode, the values of vibration amplitudes of the spacecraft reach 0.13 g (Fig. 6).

Conclusions. An approach to the mathematical modeling of POGO self-oscillations of liquid rockets by use of three-dimensional finite-element discretization for the LV structure with liquid filling of its propellant tanks, the propulsion systems by use of by one-dimensional finite elements discretization and with help of nonlinear hydrodynamic model of cavitation phenomena in LRE pumps is proposed.

Applied to the liquid two-staged rocket, a nonlinear mathematical model of the low-frequency dynamics of the "LRE – LV structure" system is developed, which describes the interaction of spatial oscillations of liquid propellant in tanks, the LV elastic structural vibrations and low-frequency processes in the LRE on the active part of the LV flight. It has been determine that the considered "LRE – LV structure" dynamic system goes to the limit cycle with the self-oscillations frequency of 15.8 Hz, close to the natural frequency of the LV structural second modes of longitudinal oscillations. The parameters of self-oscillations of this system are determined: the dynamic components of the pressure in the LV tanks, the movement and vibration acceleration of the elements of the LV structure, as well as the flow rates and pressures in the hydraulic elements of the liquid rocket engine.

The proposed approach makes it possible to investigate the POGO oscillations of liquid rockets (including rockets having a complex spatial configuration of propellant tanks and an asymmetric arrangement of stages) and to determine the parameters of the limit cycle of the rocket POGO self-oscillations. The use of this approach allows one to take into account the interaction of the spatial oscillations of liquid propellant in tanks, the elastic vibrations of the LV structure and low-frequency processes in the LRE, and also the nonlinear dissipative forces in the elastic deformations of structure.

The developed approach can be used to estimate the LV oscillation amplitudes and the dynamic loads of the rocket structures of liquid launch vehicles under design, which, according to the results of the POGO linear analysis, are unstable with respect to longitudinal oscillations at flight time phase.

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