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DETERMINATION OF PARAMETERS OF A DISSOCIATED SUPERSONIC RAREFIED PLASMA FLOW BY CURRENT-VOLTAGE CHARACTERISTICS OF ISOLATED SYSTEM OF CYLINDRICAL PROBES

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Метою роботи є розробка процедури ідентифікації параметрів беззіштовхувальної плазми дисоційованого двохатомного газу за вольтамперною характеристикою (ВАХ) ізольованої зондової системи. Вимірювальна зондова система представляє собою поперечно обтічні циліндричний зонд та опорний електрод, що складається з декількох циліндрів. З використанням відомих теоретичних і експериментальних залежностей іонного та електронного струмів на циліндр побудовано математичну модель збирання струму зондовою системою у струмені газорозрядного джерела лабораторної дисоційованої плазми. Модель включає розрахунок рівноважного потенціалу опорного електрода при зміні напруги зміщення зонду.

Отримані аналітичні співвідношення, що дозволяють визначити ступінь дисоціації іонів у струмені плазми за результатами вимірів зондових струмів в області насичення електронів при зміні площі поверхні опорного електрода. При прийнятих допущеннях вірогідність визначення ступеню дисоціації плазми залежить тільки від точності вимірювань зондового струму. Сформульовано обмеження на розміри зондової системи та на потенціали зміщення зонду щодо застосовності запропонованої методики вимірювання ступеню дисоціації плазми. Концентрація заряджених частинок та електронна температура дисоційованої плазми у струмені газорозрядного джерела визначаються на основі побудованої математичної моделі за раніше розробленою авторами процедурою інтерпретації ВАХ. Процедура передбачає визначення параметрів плазми, за якими теоретична ВАХ найкращим чином описує експериментальну ВАХ.

Проведено числові дослідження впливу похибок вимірювань зондових струмів на відновлення параметрів плазми. В рамках прийнятих припущень отримані оцінки вірогідності відновлення ступеню дисоціації плазми в залежності від похибок вимірювань зондових струмів. Отримані результати можуть бути використані у діагностиці лабораторної плазми.

Ключові слова: *струмінь беззіштовхувальної дисоційованої плазми, ізольована зондова система, циліндричні електроди, математична модель збирання струму, рівноважний потенціал, ступінь дисоціації.*

Целью работы является разработка процедуры идентификации параметров бесстолкновительной плазмы диссоциированного двухатомного газа по вольтамперной характеристике (ВАХ) изолированной зондовой системы. Измерительная зондовая система представляет собой поперечно обтекаемые цилиндрический зонд и опорный электрод, составленный из нескольких цилиндров. С использованием известных теоретических и экспериментальных зависимостей ионного и электронного токов на цилиндр построена математическая модель собирания тока зондовой системой в струе газоразрядного источника лабораторной диссоциированной плазмы. Модель включает расчет равновесного потенциала опорного электрода при изменении напряжения смещения зонда.

Получены аналитические соотношения, позволяющие определить степень диссоциации ионов в струе плазмы по результатам измерений зондовых токов в области насыщения электронов при изменении площади поверхности опорного электрода. При принятых допущениях достоверность определения степени диссоциации плазмы зависит только от точности измерений зондового тока. Сформулированы ограничения на размер зондовой системы и на потенциалы смещения зонда для применимости предложенной методики измерения степени диссоциации плазмы. Концентрация заряженных частиц и электронная температура диссоциированной плазмы в струе газоразрядного источника определяются на основе построенной математической модели по ранее разработанной авторами процедуре интерпретации ВАХ. Процедура основана на определении значений параметров плазмы, при которых теоретическая ВАХ наилучшим образом описывает экспериментальную ВАХ.

Проведены численные исследования влияния погрешностей измерения зондовых токов на восстановление параметров плазмы. В рамках принятых допущений получены оценки достоверности восстановления степени диссоциации плазмы в зависимости от точности измерения зондовых токов. Полученные результаты могут быть использованы в диагностике лабораторной плазмы.

Ключевые слова: *струя бесстолкновительной диссоциированной плазмы, изолированная зондовая система, цилиндрические электроды, математическая модель собирания тока, равновесный потенциал, степень диссоциации.*

The aim of this work is to develop a procedure for identifying the parameters of a collisionless plasma of a dissociated diatomic gas from the current-voltage characteristic of an isolated probe system. The measuring probe

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system is a cylindrical probe and a reference electrode made up of several cylinders, the electrodes being placed perpendicularly to the flow. Using the familiar theoretical and experimental relationships for the ion and the electron current to a cylinder, a mathematical model of current collection by the probe system in a jet of a laboratory gas-discharge dissociated plasma is constructed. The model includes the calculation of the equilibrium potential of the reference electrode as a function of the probe bias voltage.

Analytical relationships are obtained for determining the degree of plasma jet ion dissociation from the measured probe currents in the electron current saturation region with varying the reference electrode surface area. Under the adopted assumptions, the reliability of determination of the plasma dissociation degree depends only on the probe current measurement accuracy. The paper formulates restrictions on the probe system dimensions and the probe bias voltage under which the proposed method for measurement of the plasma dissociation degree is applicable. The density of charged particles and the electron temperature in a dissociated gas-discharge plasma jet are determined using the mathematical model constructed and the authors' procedure of current–voltage characteristic interpretation developed earlier. The procedure is based on determining the plasma parameter values such that the theoretical current–voltage characteristic best fits the experimental one.

The paper presents the results of a numerical study of the effect of the probe current measurement error on the accuracy of plasma parameter determination. Under the adopted assumptions, the reliability of determining the plasma dissociation degree is estimated as a function of the probe current measurement accuracy. The results obtained may be used in laboratory plasma diagnostics.

Keywords*: collisionless dissociated plasma jet, isolated probe system, cylindrical electrodes, mathematical model of current collection, equilibrium potential, degree of dissociation.*

Introduction. Laboratory modeling of ionospheric conditions, testing and calibration of scientific on-board equipment is an important stage in the development (preparation) of space experiments and technological processes in plasma. Using diatomic gases (nitrogen, oxygen, hydrogen) in rarefied plasma sources, dissociated gas flow with parameters close to required conditions [1, 2]. Laboratory modeling of ionospheric measurements, technological plasma processes involve a complete diagnosis of laboratory plasma.

The most developed and commonly used diagnostic method to date remains the method of a cylindrical Langmuir probe [3]. An effective procedure for interpreting the I-V characteristic of a single cylindrical probe immersed in a stream of a three-component (consisting of neutrals, positive ions, and electrons) collisionless plasma is proposed in [4]. This procedure is based on a comparison of the theoretical approximation of the I-V characteristic with the results of probe current measurements. A priori information about plasma properties and experimental conditions is specified as restrictions on the parameters of the theoretical I-V characteristic. In [5], this procedure is extended to a system of isolated cylindrical probes with an arbitrary ratio of current-collecting surface areas of the probe and the reference electrode. Developed procedure it capable to interpret probe measurements on nanosatellites in the approximation of three-component plasma.

In this work, the procedure for interpreting probe measurements [4, 5] was adapted for the diagnosis of laboratory dissociated plasma containing atomic and molecular ions of the working gas of a plasma source.

Formulation of the problem. Let us consider the rarefied plasma flow produced by a gas-discharge plasma source by ionization of a diatomic gas (nitrogen, oxygen, hydrogen) and acceleration of ions in the electric field of a jet flowing into vacuum [1, 2]. Plasma in the jet is considered to be four-component, consisting of neutral particles, atomic ions having the mass $m_i/2$, molecular ions having the mass m_i and electrons.

The core region of the jet (a region with an uniform distribution of plasma parameters such as density n_α, temperature T_α of the charged particles of the kind

α) is placed in the vicinity of the jet's axis and is limited to a cylindrical surface with the base radius of R_{jet}. In the core region, the degree of ions dissociation is characterized by the parameter $\eta = \frac{n_{i,1}}{n_{i,1} + n_{i,2}} \equiv \frac{n_{i,1}}{n_e}$, where $n_{i,1}$, $n_{i,2}$ are the density of atomic and molecular ions, respectively, n_e is the density of electrons (the condition of plasma quasineutrality follows $n_{i,1} + n_{i,2} = n_e$).

Since ions are accelerated in the electric field of the jet, mass velocities of atomic $V_{i,1}$ and molecular $V_{i,2}$ ions satisfy the relation $V_{i,1}/V_{i,2} = \sqrt{m_{i,2}/m_{i,1}}$ in the core region.

A measuring probe system is placed in the core region ofthe jet. The probe system consists of transversely streamlined cylindrical electrodes having areas ratio of $S_s = S_{cp}/S_p$, where S_p is the probe area, S_{cp} is the area of the reference electrode, $S_{cp} >> S_p$ is assumed. The base radii of the probe r_p and the reference electrode r_{cp} are significantly smaller than their lengths, the end surfaces of the reference electrode are isolated from the plasma, the electrostatic and gas-dynamic influence of the electrodes on each other in plasma is negligible, emission currents from the electrode surfaces are absent. The plasma in the core region of the jet is quasineutral, the flow around the electrodes is collisionless, the influence of the magnetic field on the probe current is not significant, and the velocity distribution of particles of the same kind is Maxwellian. The temperatures of atomic and molecular ions are assumed to be the equal, $T_{i,1} = T_{i,2} = T_i$.

We assume that the probe system does not introduce a significant gasdynamic and electrodynamic perturbations into the plasma flow. To ensure this the following restrictions are accepted

$$r_{cp} << R_{jet},$$
$$I_{e,sat} << I_{i,jet}, \tag{1}$$

where r_{cp} is the base radius of the reference electrode, $I_{e,sat}$ is electron saturation current collected by the probe; $I_{i,jet}$ is the ion current through the core region of the jet.

It is required to determine the degree of ion dissociation in the core region of the jet from the results of measurement of the I-V characteristic of the proposed probe system – the dependence of the probe current I_p on the probe potential U_{iz} with respect to the reference electrode potential.

Mathematical model of current collection. The electric and gas-dynamic interaction of the cylinder with the plasma flow is characterized by the ion velocity ratio $S_i = V_{i,2}/u_i$, ratio of the cylinder's characteristic size to the Debye length $\xi = r_c/\lambda_d$, dimensionless electric potential of the cylinder φ (normalized by kT_e/e where e is the elemental charge) relative to the unperturbed plasma potential, ratios of masses $\mu = m_e/m_i$ and temperatures $\beta = T_e/T_i$ of charged particles, degree of ion dissociation η. Here $u_i = \sqrt{2kT_i/m_i}$ is the thermal velocity of the ions, k stands for the Boltzmann constant, r_c is the cylinder base radius, λ_d is the Debye length.

In prior works [4, 5], the approximation of the current collected by the cylinder in a flow of a collisionless three-component plasma is used obtained on the basis of the classical asymptotic Langmuir's relations [6], analytical studies [7] and calculations [4, 8 – 10]. Preliminary qualitative calculations performed accordingly to the method of [4, 11] for a four-component plasma show that the presence of ions of different kinds (atomic and molecular) in the supersonic flow does not lead to a significant change in the self-consistent electric field in the vicinity of the streamlined cylinder. In a plasma stream containing both atomic and molecular ions of a diatomic gas, the total current on the cylinder with applied potential φ relative to the plasma potential, is estimated by the following dimensionless relations (the electronic current on a cylinder is assumed to be positive):

$$\bar{I}_c(\varphi) = \bar{I}_e(\varphi) - (1 + 0{,}414\eta)\sqrt{\mu/\beta} \cdot \bar{I}_i(\varphi),$$

$$\bar{I}_e(\varphi) = \begin{cases} 2/\sqrt{\pi} \cdot \sqrt{\pi/4 + \varphi}, & \varphi > 0; \\ \exp(\varphi), & \varphi \le 0 \end{cases} \tag{2}$$

$$\bar{I}_i(\varphi) = \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^{\,2}\right), & \varphi \ge S_i^{\,2}/\beta; \\ 2/\sqrt{\pi}\sqrt{1/2 + S_i^{\,2} - \beta\varphi}, & \varphi < S_i^{\,2}/\beta \end{cases},$$

where $\bar{I}_c$, $\bar{I}_e$ stand for total and electronic currents, respectively, on a cylinder, normalized to the thermal electronic current; $\bar{I}_i$ – ion current on a cylinder, normalized to the thermal ion current of the corresponding ions kind. The thermal current of particles of a kind α is determined by the expression $I_{\alpha,0} = j_{\alpha,0} \cdot S_c$, where $j_{\alpha,0} = en_\alpha u_\alpha / 2\sqrt{\pi}$ is the thermal current density, $u_\alpha = \sqrt{2kT_\alpha/m_\alpha}$ is the thermal velocity, m_α is the mass of particles, S_c is the area of the cylinder's current collecting surface. Here the index $\alpha = i,1$ corresponds to atomic ions, $\alpha = i,2$ – to molecular ions, $\alpha = e$ – to electrons.

In dimensional form, the dependence of the cylinder current I_c on its potential U relative to the unperturbed plasma potential is determined via the dimensionless current $\bar{I}_c$ as follows:

$$I_c(U) = j_{e,0} \cdot S_c \cdot \bar{I}_c(eU/kT_e).$$

We imply the following restrictions on the radii of probe r_p and reference electrode r_{cp} [5]:

$$\xi_p = r_p/\lambda_d \le 1\ ,\ \ \xi_{cp} = r_{cp}/\lambda_d \le 10\,.$$

Direct problem. Measuring I-V characteristic, we obtain the dependence of the current I_p in the circuit "probe–plasma–reference electrode" on the probe bias potential U_{iz} relative to the reference electrode ($U_{iz} = U_p - U_{cp}$ where U_p, U_{cp}

are potentials of the probe and reference electrode, respectively, relative to the unperturbed plasma potential). Probe potential with respect to plasma potential is $U_p = U_{iz} + U_{cp}$.

The proposed probe system is isolated. For each value of the bias voltage U_{iz} there is a corresponding equilibrium potential of the reference electrode U_{cp}, which provide zero total current of charged particles through all collecting surfaces of the probe system. For the reference electrode, the equation of current balance in a dimensionless form writes

$$S_s \cdot \bar{I}_{cp}(\varphi_{iz}) + \bar{I}_p(\varphi_{iz}) = 0 . \tag{3}$$

Here, the dimensionless currents to the reference electrode $\bar{I}_{cp}(\varphi_{iz}) = \bar{I}_c(\varphi_{cp})$ and to the probe $\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\varphi_{iz} + \varphi_{cp})$ are determined by relations (2). For each value of the bias potential φ_{iz} the solution of the nonlinear equation (3) for the potential of the reference electrode φ_{cp} gives the dependence of the equilibrium potential of the reference electrode on the probe's bias potential – $\varphi_{cp} = \Phi(\varphi_{iz})$. In a dimensional form, the equilibrium potential of the reference electrode is determined as follows

$$U_{cp}(U_{iz}) = \Phi(eU_{iz}/kT_e) \cdot kT_e/e .$$

Thus, the I-V characteristic of the probe in a dimensionless form is given by the formula

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\Phi(\varphi_{iz}) + \varphi_{iz}),$$

and in dimensional form:

$$I_p(U_{iz}) = j_{e0} \cdot S_p \cdot \bar{I}_c(\Phi(eU_{iz}/kT_e) + eU_{iz}/kT_e).$$

Since the dependence of the current on the potential of the electrodes and parameters η, μ, β, S_i is a continuous single-valued function (2), the solution $\Phi(\varphi_{iz})$ of the nonlinear equation (3) exists and it is unique for all considered values of the bias potential φ_{iz}, it can be found using iterative method [5].

Under a sufficiently large positive bias voltage φ_{iz}, when the probe potential relative to the plasma one satisfies $\varphi_p = \varphi_{iz} + \varphi_{cp} >> 1$, the probe current is mostly electronic:

$$\bar{I}_c(\varphi_{iz} + \varphi_{cp}) \approx 2/\sqrt{\pi} \cdot \sqrt{\pi/4 + \varphi_{iz} + \varphi_{cz}} ,$$

and the reference electrode attracts ions:

$$\bar{I}_c(\varphi_{cp}) \approx -(1 + 0{,}414\eta)\sqrt{\mu/\beta}\ 2/\sqrt{\pi} \cdot \sqrt{1/2 + S_i^2 - \beta\varphi_{cp}} .$$

In this case, the current balance equation (3) allows us to obtain an analytical solution

$$\varphi_{cp} = -\frac{(\pi/4+\varphi_{iz}) - S_s{}^2\mu(1+0{,}414\eta)^2 (1/2+S_i{}^2)/\beta}{1+S_s{}^2\mu(1+0{,}414\eta)^2}. \tag{4}$$

The dimensionless I-V characteristic of the probe in the electron saturation regime is determined as follows

$$\bar{I}_p(\varphi_{iz}) \approx \frac{2}{\sqrt{\pi}} \cdot \left(\frac{1}{S_s{}^2\mu(1+0{,}414\eta)^2} + 1 \right)^{-1/2} \cdot \sqrt{(1/2+S_i{}^2)/\beta + \pi/4 + \varphi_{iz}}. \tag{5}$$

It can be seen that, in contrast to a single Langmuir probe, in the proposed system the electron saturation current depends on the ion flow velocity S_i and the degree of plasma nonisothermality β. This is due to the insulation of the probe system, as well as the shape of the reference electrode and plasma flow pattern around it.

Relations (2), (3) that determine the parametric representation of the I-V characteristic of the “probe – plasma – reference electrode” system, include dimensionless parameters η, μ, β, S_i, S_s, φ_{iz} defined through the following parameters of the unperturbed plasma, probe, and reference electrode: n_e, T_e, m_i, η, T_i, $V_{i,2}$, S_p, S_{cp}, U_{iz}.

The dependences on the bias potential φ_{iz} of the probe current $\bar{I}_p$ is shown on Fig. 1 and of the equilibrium potential φ_{cp} is shown on Fig. 2 for the ratio of the electrode areas S_s = 50, 100, 200, 300, 400. Three curves correspond to each S_s: thin solid curve is calculated for the degree of dissociation η = 0, dotted curve – η = 0.2 , dashed – η = 0.5. The thick solid curve is calculated at S_s = 1000, η =0. The dots in the figures show the results of calculations by the formulas (4), (5) for the corresponding S_s and η =0 (dots 1), η =0.2 (dots 2), η =0.5 (dots 3). The calculations are performed for such a parameters $S_i = 4.6$, $\mu = 2.7 \cdot 10^{-5}$, $\beta = 4.2$ that correspond to the laboratory plasma used for modeling the flow conditions in the ionosphere [12].

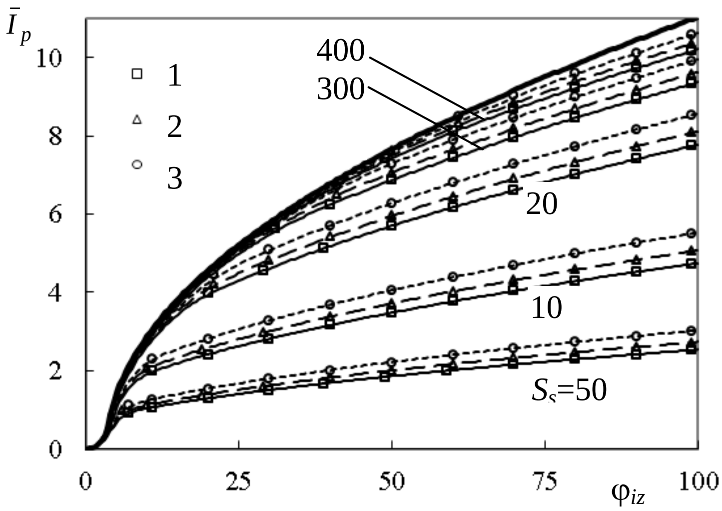

Рис.1

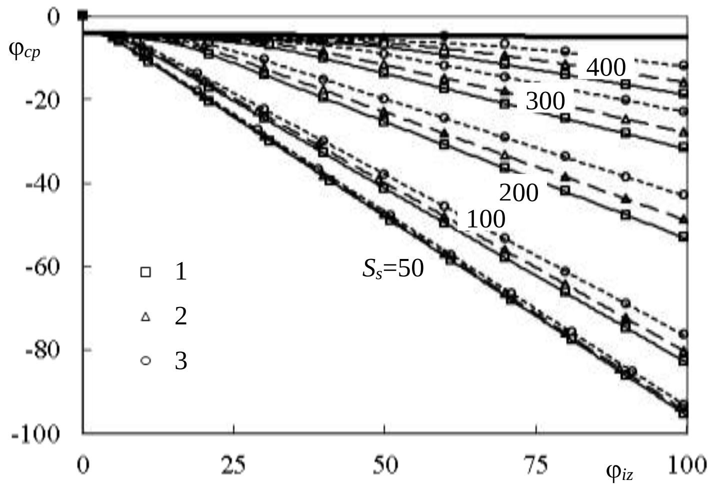

Fig. 2

The approximation (5) for the electron saturation current is applicable if the following condition satisfy:

$$\varphi_z > \varphi_{iz}^{\min} = 6\sqrt{S_i^2/\beta}\left[S_s^2\mu(1+0.414\eta)^2 + 0.14\right] + 6.5\,. \tag{6}$$

From (6) one can see that an increase in the ratio of electrode's areas S_s leads to an increase in the necessary bias voltage φ_{iz} for achieving the electron saturation regime. At the same time, the bias voltage is limited by conditions (1). The ion current in the core region of the supersonic plasma jet can be estimated as

$$I_{i,\,jet} \approx e n_e V_{i,2} \pi R_{jet}^2\,.$$

Then considering (5), the restriction on the probe size (1) writes:

$$r_p l_p << \sqrt{\frac{\mu}{\beta}}(1+0.414\eta)\frac{S_i}{\sqrt{\varphi_{iz}^{\max}}}\frac{\pi R_{jet}^2}{2}\,, \tag{7}$$

where l_p is the probe length, $\varphi_{iz}^{\max}$ is the largest bias potential applied to the probe in measurements.

Inverse problem. Let the reference electrode to consist of a series of parallel cylinders and each cylinder can be connected or disconnected from the electrical measurement circuit. Such a measuring probe system makes it possible to simultaneously measure the I-V characteristic for various values of the area ratio S_s. Let $S_s{}^*$ and $S_s{}^{**}$ be the two different values of area ratio. We assume that the local flow parameters do not change when the area of the reference electrode changes. Then, to each bias potential φ_{iz} corresponds the probe current I_p^* in the measuring system with $S_s = S_s{}^*$ and current I_p^{**} in system with $S_s = S_s{}^{**}$. Substituting the measured currents in a dimensionless form in (5) and considering the two obtained equalities relative to the parameters η, S_i, β, we find

$$\eta = 2.415 \cdot \left(\frac{1}{\sqrt{\mu}} \sqrt{\frac{\left[I_p^{*}/S_s^{*}\right]^2 - \left[I_p^{**}/S_s^{**}\right]^2}{\left[I_p^{**}\right]^2 - \left[I_p^{*}\right]^2}} - 1 \right). \tag{8}$$

Note that (8) defines the degree of dissociation η only through the dimensional values of currents and does not depend on other parameters of plasma flow.

The degree of dissociation η is determined on the basis of (8) using the standard method for processing the results of measurements of probe currents I_p^{*} and I_p^{**} for various values of the bias potential U_{iz} in the electron saturation region

$$\varphi_{iz}^{\min} < U_{iz} \frac{e}{kT_e} < \varphi_{iz}^{\max}.$$

The lower boundary of the acceptable range of the bias potential is found from (6) at the largest value of the parameter S_s, the upper boundary is determined by relation (7) and is limited by the probe dimensions.

Numerical simulation of determination of the dissociation degree η by the measurement of probe currents I_p^{*} and I_p^{**} at parameter $S_s \in [50,400]$ confirmed the reliability of the obtained values of η:

$$\delta_\eta < K(\overline{\eta}, p_s) \cdot \delta_I ,$$

$$K(\overline{\eta}, p_s) \approx \frac{0.214}{\overline{\eta} + 0.01} \left(1 - \frac{(\overline{\eta} + 9)\overline{\eta}^2}{\left[(p_s + 2.065)\overline{\eta} - 0.035\right]^2} \right)^{-1} + 0.2 \frac{\overline{\eta} - 1.39}{\overline{\eta} + 0.09},$$

where δ_η is the relative error in obtained η, $\overline{\eta}$ is the "precise" value of the degree of dissociation, δ_I is the relative error in measurements of probe current, $p_s = S_s^{**}/S_s^{*} > 1$.

Thus, the problems of determination of the dissociation degree η and other plasma flow parameters n_e, T_e, S_i, β are independent. Having the η value determined accordingly to the proposed procedure using (8), the kinetic parameters of the plasma flow might be found using the mathematical model of current collection (2), (3) accordingly to the method [5] or the model (2) of current collection by the single cylindrical Langmuir probe [4].

Conclusions. A procedure for determining the parameters of charged particles in a jet of a gas-discharge source of a collisionless dissociated plasma of a diatomic gas by the I-V characteristics of a probe system with cylindrical electrodes is developed. An isolated probe system allows measurements with a discretely variable surface area of the reference electrode. A mathematical model of current collection by the probe system in a high-speed flow of the dissociated plasma is constructed. Obtained analytical relations allow to determine the degree of ions dissociation in a plasma jet by the results of measurements of probe currents in the electron saturation regime at various surface area of the reference electrode.

It is shown that the task of obtaining the degree of dissociation and the task of determining the density of charged particles and the electron temperature of the dissociated plasma are independent. Restrictions on the probe system size and on the probe bias potential are formulated as the condition of applicability of the proposed procedure for measuring the degree of plasma dissociation. Within the accepted assumptions, the reliability of determining the degree of plasma dissociation is estimated depending on the accuracy of probe current measurements.

The obtained results can be used in planning and interpreting the experiments in laboratory plasma.

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