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EVALUATION OF THE HIGH-FREQUENCY OSCILLATION PARAMETERS OF LIQUID ROCKET ENGINE WITH THE ANNULAR COMBUSTION CHAMBER

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Розвиток високочастотної нестійкості рідинного ракетного двигуна (РРД) при його вогневих випробуваннях часто супроводжується значним зростанням динамічних навантажень на конструкцію камери згоряння, що нерідко призводить до її руйнування. Це динамічне явище може бути також вкрай небезпечним для віброміцності рідинного ракетного двигуна з кільцевою камерою згоряння. Розрахунок параметрів акустичних коливань продуктів згоряння є важливим при проєктуванні і відпрацюванні таких РРД. Мета статті – розробка чисельного підходу до визначення параметрів акустичних коливань продуктів згоряння в кільцевих камерах згоряння рідинних ракетних двигунів з урахуванням особливостей конфігурації вогневого простору і зміни фізичних властивостей газового середовища в залежності від осьової довжини камери. Запропонований чисельний підхід засновано на математичному моделюванні власних коливань замкнутої динамічної системи «оболонкова конструкція кільцевої конфігурації – газ», яка описує високочастотні процеси в такій камері згоряння, за допомогою методу скінченних елементів.

Показано, що на основі розробленої скінченно-елементної моделі спільних просторових коливань конструкції кільцевої камери згоряння і продуктів згоряння, що містяться в ній, можуть бути визначені параметри коливань зазначеної динамічної системи (частоти, форми та ефективні маси) для її домінуючих акустичних мод, амплітуди коливань і віброприскорень стінок камери згоряння, а також виявлені режимні параметри роботи рідинного ракетного двигуна, потенційно небезпечні для розвитку термоакустичної нестійкості робочого процесу в кільцевій камері згоряння двигуна. Для чисельного визначення коефіцієнтів підсилення досліджуваної камери згоряння (по тиску) в скінченно-елементну модель динамічної системи «оболонкова конструкція кільцевої конфігурації – газ» (в елементах початку вогневого простору) вводиться джерело гармонійного збурення тиску.

Тестування розробленого чисельного підходу і подальший аналіз отриманих результатів проведено для двигуна з кільцевою камерою згоряння (при співвідношенні її зовнішнього і внутрішнього діаметрів, рівному 1,5), де в якості паливної пари використовується пара рідкий кисень–метан.

Визначено частоти і форми поздовжніх, тангенціальних і радіальних мод акустичних коливань. Показано, що в разі відносно малої жорсткості стінок конструкції камери згоряння частота першої моди акустичних коливань менше частоти, розрахованої для конструкції з жорсткими стінками, на 40 відсотків.

Ключові слова: *рідинний ракетний двигун, кільцева камера згоряння, паливна пара метан–кисень, високочастотна нестійкість, частоти коливань.*

The high-frequency instability (HF instability) of a liquid-propellant rocket engine (LPRE) during static firing tests is often accompanied by a significant increase in dynamic loads on the combustion chamber structure, often leading to the chamber destruction. This dynamic phenomenon can also be extremely dangerous for the dynamic strength of a liquid-propellant rocket engine with an annular combustion chamber. Computation of the parameters of acoustic combustion product oscillations is important in the design and static firing tests of such rocket engines. The main aim of this paper is to develop a numerical approach to determining the parameters of acoustic oscillations of combustion products in annular combustion chambers of liquid-propellant rocket engines taking into account the features of the configuration of the combustion space and the variability of the physical properties of the gaseous medium depending on the axial length of the chamber. A numerical approach is proposed. The approach is based on mathematical modeling of natural oscillations of a “shell structure of an annular chamber – gas” coupled dynamic system by using the finite element method.

Based on the developed finite-element model of coupled spatial vibrations of the structure of the annular combustion chamber and the combustion product oscillations, the oscillation parameters of the system under consideration (frequencies, modes, and effective masses) for its dominant acoustic modes, the vibration amplitudes of the combustion chamber casing, and the amplitudes of its vibration accelerations can be determined. The operating parameters of the liquid-propellant rocket engine potentially dangerous for the development of thermoacoustic instability of the working process in the annular combustion chamber can be identified. For the numerical computation of the dynamic gains (in pressure) of the combustion chamber, a source of harmonic pressure excitation is introduced to the finite element model of the dynamic system “shell structure of an annular configuration – gas” (to the elements at the start of the chamber fire space).

The developed approach testing and further analysis of the results were carried out for an engine with an annular combustion chamber (with a ratio of the outer and inner diameters of 1.5) using liquid oxygen – methane as a propellant pair.

The system shapes and frequencies of longitudinal, tangential and radial modes are determined. It is shown that the frequency of the first acoustic mode in the case of a relatively low stiffness of the combustion chamber casing walls can be reduced by 40 percent in comparison with the frequency determined for a casing with rigid walls.

***Key words:** liquid propellant rocket engine, annular combustion chamber, methane–oxygen propellant, high-frequency engine instability, oscillation frequencies.*

Introduction. Recently, there has been an increase in interest in the world in researching work processes in liquid-propellant rocket engines with aerospike nozzles due to their inherent promising (in comparison with traditional Laval nozzles) opportunities for adapting the pressure at the nozzle exit to atmospheric pressure that changes during launch vehicle lifting, as well as some other useful characteristics. As follows from modern publications (for example, [1] – [3]), studies of working processes in rocket engines with nozzles of original designs are being carried out for designs with annular combustion chambers.

It is known [for example, 4, 5, 6] that the development of high-frequency instability (HF instability) of liquid-propellant rocket engines (LPRE) with a traditional combustion chamber during firing tests is often accompanied by a significant increase in dynamic loads on the combustion chamber structures, leading in a number of cases to chamber destruction. High-frequency instability in the LPRE combustion chamber develops as a dynamic process caused by the interaction of gas pressure oscillations (at the frequencies of chamber acoustic oscillations) and thermal processes during their combustion. The above phenomena are resonant in nature. Acoustic excitation effect on the burning rate. Oscillations increase the mixing rate of the fuel components and reduce the combustion zone. As a rule [4, 5, 7], dynamic processes in the case of high-frequency instability of the combustion chamber occur mainly at the lowest tangential acoustic modes with oscillation frequencies exceeding 1000 Hz.

Operational modes in combustion chambers of the annular type (as well as in classical cylindrical combustion chambers) can also occur in modes in which high-frequency oscillations are dangerous for the chamber structure strength are realized. Determination of the parameters of high-frequency oscillations (natural frequencies, reduced masses and stability margins) is one of the problems solved in the LPRE design period as part of the development of measures to ensure the engine HF stability.

Calculation methods of the natural acoustic frequencies of chamber combustion products oscillations are based on classical solutions (in particular, [6], [8], obtained by using Bessel functions) for dynamics of combustion products equations written in cylindrical coordinates. Applied to the annular chamber, this solution was obtained for a gas cavity "участоксоaxial cylinder" with external and internal rigid walls.

However, for promising annular structures of combustion chambers, the parameters of natural acoustic oscillations can change significantly due to the influence of the elasticity of the external and internal walls, which have relatively small thicknesses, differences in the spatial configuration of the combustion chamber space from the end of cylindrical part to the critical section of the nozzle. In this case, the tuning accuracy (to chamber dangerous frequencies) of absorbers designed to suppress oscillations in the combustion chamber can be significantly reduced (for example, [6]). It makes the implementation of this constructive measure ineffective. In these circumstances, the problem of developing a numerical approach to determining the parameters of acoustic oscillations of combustion products in annular combustion chambers, taking into account the features of the configuration of the combustion space and changes in the physical properties of the gaseous medium along the chamber axial length, is urgent.

The aim of this paper is to develop an approach to the numerical determination of the parameters of high-frequency acoustic oscillations of combustion products in annular chambers at various operating modes of the liquid-propellant engine.

1. Evaluation of the parameters of high-frequency acoustic oscillations of the LPRE combustion chamber. The proposed approach to calculating the parameters of high-frequency gas acoustic oscillations in the combustion chamber can be attributed to the group of so-called hybrid methods (see, for example, [9]) to analyzing the HF stability of processes in the combustion chamber, for which the parameters of acoustic oscillations of chamber combustion products are calculated separately from the calculation of flame dynamics.

The calculation of the parameters of acoustic oscillations of combustion products is carried out on model of the “shell structure (with annular configuration) – gas” coupled dynamic system. This makes it possible to take into account the effect of the elasticity of the structure on the acoustic properties of the combustion products in the combustion chamber of the configuration under study in the modal analysis of this system. The boundary conditions in solving the problem of free oscillations of the dynamic system are determined by the geometry and method of fastening the combustion chamber structure to the base of the test bench, the conditions associated with the gas flow in the section of the fire bottom and with the supercritical outflow of combustion products from the nozzle.

The parameters of longitudinal and transverse (tangential and radial) acoustic oscillations gas in the engine chamber were determined by using the developed finite element model with the chamber structure features are taken into account.

For developing a design scheme for the finite element analysis of the “shell structure – gas” dynamic system the geometric model of the gas-filled chamber structure under study is formed. This geometric model contains information about the lines that form the surfaces, key points, regions, and volumes of the system. For case constructing it, the available complex volumes in the “shell structure – gas” system are divided into simple ones like pentahedrons or hexagons. Complex areas are divided into flat or spatial triangles or quadrangles. The developed geometric models make it possible to carry out “directional” schematization of the gas-filled annular shell structure with finite element by the CAE – system (Computer Aided Engineering System).

This not only makes it possible to avoid imperfections in the calculation scheme caused by the automatic discretization of complex volumes (or areas), but also to ensure correct consideration of the boundary conditions for the “shell structure – gas” system at the interfaces between the gaseous medium and the structure. Note that in the geometric model of the “shell structure – gas” system, the conjugate surfaces of the obtained simple volumes and the corresponding regions must coincide with each other.

For most structural dynamics problems of a mechanical system, the spatial discretization for the principle of virtual work using the finite element method gives the finite element semi-discrete equation of motion as follows [10]:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\}, \tag{1}$$

where $[M]$, $[C]$, $[K]$ – respectively structural mass matrix , structural damping matrix and structure stiffness matrix, ($n_1 \text{ x } n_1$); n_1 – number of degree-of-freedom (DOF) of (shell structure); $\{X\}$ – nodal structure displacement vector,

$\{X\}=\{X_1,...,X_{n_1}\}$; $\{\dot{X}\}=\{dX_1/dt,...,dX_{n_1}/dt\}, \{\ddot{X}\}=\{d^2X_1/dt^2,...,d^2X_{n_1}/dt^2\}$; t – time; $\{F\}$ – applied load vector: $\{F\}=\{F_1,...,F_{n_1}\}$.

For modeling acoustic fluid-structural interaction (FSI) problems [10], the structural dynamics equation must be considered along with the Navier-Stokes equations of fluid momentum and the flow continuity equation. The fluid momentum (Navier-Stokes) equations and continuity equations are simplified to get the acoustic wave equation using the following assumptions: the gas is compressible (density changes due to pressure variations); there is no mean flow of the gas and combustion process [11, 12].

$$\nabla\left(\frac{1}{\rho}\nabla p\right)-\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2}+\nabla\left(\frac{4\mu}{3\rho}\nabla\left(\frac{1}{\rho c^2}\frac{\partial p}{\partial t}\right)\right)=-\frac{\partial}{\partial t}\left(\frac{Q}{\rho}\right)+\nabla\left(\frac{4\mu}{3\rho}\nabla\left(\frac{Q}{\rho}\right)\right), \quad (2)$$

where c is speed of sound in gas medium; ρ is mean gas density; μ is dynamic viscosity; p is acoustic pressure; Q – mass source in the continuity equation, (mass flow per unit volume of gas [13]).

According to [10, 14, 15] Equation (2) can be written in matrix notation to create the following discretized wave equation:

$$[M_F]\{\ddot{p}_e\}+[C_F]\{\dot{p}_e\}+[K_F]\{p_e\}+\bar{\rho}_0[R]^T\{\ddot{u}_{F,e}\}=\{f_F\}, \quad (3)$$

where $[M_F]=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0 c^2}\{N\}\{N\}^T dv$ – is acoustic fluid mass matrix;

$[C_F]=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{4\mu}{3\rho_0^2 c^2}[\nabla N]^T[\nabla N]dv$ is acoustic fluid damping matrix;

$[K_F]=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0}[\nabla N]^T[\nabla N]dv$ is acoustic fluid stiffness matrix;

$[R]^T=\oiint\limits_{\Gamma_F}\{N\}\{n\}^T\{N'\}^T dS$ is acoustic fluid boundary matrix;

$\{f_F\}=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0}\{N\}\{N\}^T dv\{\dot{q}\}+\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{4\mu}{3\rho_0^2}\cdot[\nabla N]^T[\nabla N]dv\{q\}$ is acoustic fluid load vector; $\bar{\rho}_0$ is acoustic fluid mass density constant; ρ_0 is mean fluid density. $u_{F,e}$ is shell displacement vector at the fluid boundary; $\{P_e\}$ is nodal pressure vector; $\{N\}$ is element shape function for pressure; $\{N'\}$ is element shape function for displacements; $\{n\}$ is outward normal vector at the fluid boundary; $\{q\}$ is nodal mass source vector; $\{\dot{q}\}$ is the first time derivative of nodal mass source vector; dv is volume differential of acoustic domain Ω_F; dS is surface differential of acoustic domain boundary Γ_F.

Equations (1), (3) describe the complete finite element discretized equations for the FSI problem. These equations are written in assembled form [10] as:

$$\begin{bmatrix}[M_S] & 0 \\ \overline{\rho}_0[R]^T & [M_F]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}_e\} \\ \{\ddot{p}_e\}\end{Bmatrix}+\begin{bmatrix}[C_S] & 0 \\ 0 & [C_F]\end{bmatrix}\begin{Bmatrix}\{\dot{u}_e\} \\ \{\dot{p}_e\}\end{Bmatrix}+\begin{bmatrix}[K_S] & -[R] \\ 0 & [K_F]\end{bmatrix}\begin{Bmatrix}\{u_e\} \\ \{p_e\}\end{Bmatrix}=\begin{Bmatrix}\{f_S\} \\ \{f_F\}\end{Bmatrix} \quad (4)$$

The fluid pressure load vector $\{f_e^{pr}\}=[R]\{p_e\}$ at the interface *S* (acoustic gas domain boundary Γ_F) is using in shell structural dynamics and in Equation (4).

For mathematical modeling of free oscillations of the "shell structure – gas" dynamic system, as a rule, energy dissipation is not taken into account [11]. It greatly simplifies the determination of the parameters of the natural oscillations of the system. This approach to modeling is based on the fact that in weakly damped oscillatory systems, energy dissipation practically does not affect the frequencies and modes of oscillations.

For calculating parameters of free oscillations system (4) has the simplest form:

$$\begin{bmatrix}[M_S] & 0 \\ \overline{\rho}_0[R]^T & [M_F]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}_e\} \\ \{\ddot{p}_e\}\end{Bmatrix}+\begin{bmatrix}[K_S] & -[R] \\ 0 & [K_F]\end{bmatrix}\begin{Bmatrix}\{u_e\} \\ \{p_e\}\end{Bmatrix}=\begin{Bmatrix}\{f_S\} \\ \{f_F\}\end{Bmatrix}. \quad (5)$$

Matrix equation (5) has the form:

$$[M_{obF}]\{\ddot{X}_{obF}\}+[K_{obF}]\{X_{obF}\}=0\,. \quad (6)$$

For harmonic solution $\{X\}=\{X_{\max}\}\cos\omega_i t$ equation (6) has the form

$$-\omega^2[M_{obF}]\{X_{\max}\}+[K_{obF}]\{X_{\max}\}=0\,. \quad (7)$$

For calculating parameters of natural oscillations dynamic system "shell structure – gas" without dissipation equation (7) can be written in form:

$$\det\left[[M_{obF}]^{-1}[K_{obF}]-\lambda[E]\right]=0\,,\ \lambda_j=\omega_j^2. \quad (8)$$

This is an eigenvalue problem which may be solved for up to *n* values of ω^2 (*i*-th natural circular frequency (radians per unit time)) and *n* eigenvectors $\{V_i\}$ where *n* is the number of DOFs.

For a linear system (6) free oscillations will be harmonic of the form:

$$\{X_{obFi}\}=\{V_i\}\cos\omega_i t\ ,$$

where $\{X_{obFi}\}$ – nodal displacement vector for *i*-th natural frequency; $\{V_i\}$ – eigenvector representing the mode shape of the *i*-th natural frequency; ω_i – *i*-th natural circular frequency (radians per unit time).

Natural oscillations parameters of dynamic system "shell structure – gas" are calculated using CAE-system [10] Also eigenvalues and eigenvectors of the matrix $[A_{obF}]=[M_{obF}]^{-1}[K_{obF}]$ (см. (8)) are calculated.

Eigenvalue of this matrix $\lambda_j=\omega_j^2$ is representing *j*-th natural frequency (cycles per unit time) f_j of system, and appropriative eigenvector $\{V_i\}$ is representing the mode shape of the *j*-th natural frequency.

$$f_j = \omega_j / 2\pi.$$

Dominant natural frequencies and generalized masses are representing parameters with effective mass in longitudinal and transverse direction of the nozzle. Dominant are modes with effective mass bigger than 0,1 % of dynamic system "shell structure – gas" mass.

M_{aj} is the effective mass for the i[th] mode (which is a function of excitation direction) in direction a; a may be either *X*, *Y*, *Z* or rotations about one of these axes [10]:

$$M_{aj} = \frac{\gamma_{aj}^2}{\{V_j\}^T [M_{obF}] \{V_j\}},$$

where $\gamma_{aj} = \{V_j\}^T [M_{obF}] \{D_a\}$ – the participation factors for the given excitation direction a (for the ith mode), $[M_{obF}]$ – dynamic system structural mass matrix; $\{V_i\}$ – eigenvector representing the mode shape of the ith natural frequency; $\{D_a\} = \{d_1^a, \ldots, d_n^a\}^T$ – vector describing the excitation direction of dynamic system in direction a; n – number of degree-of-freedom (DOF) of dynamic system shell "structure – gas"; $\{d_k^a\}$ – unit vector ($k = 1, \ldots, n$), describing the excitation direction of dynamic system in direction *a*.

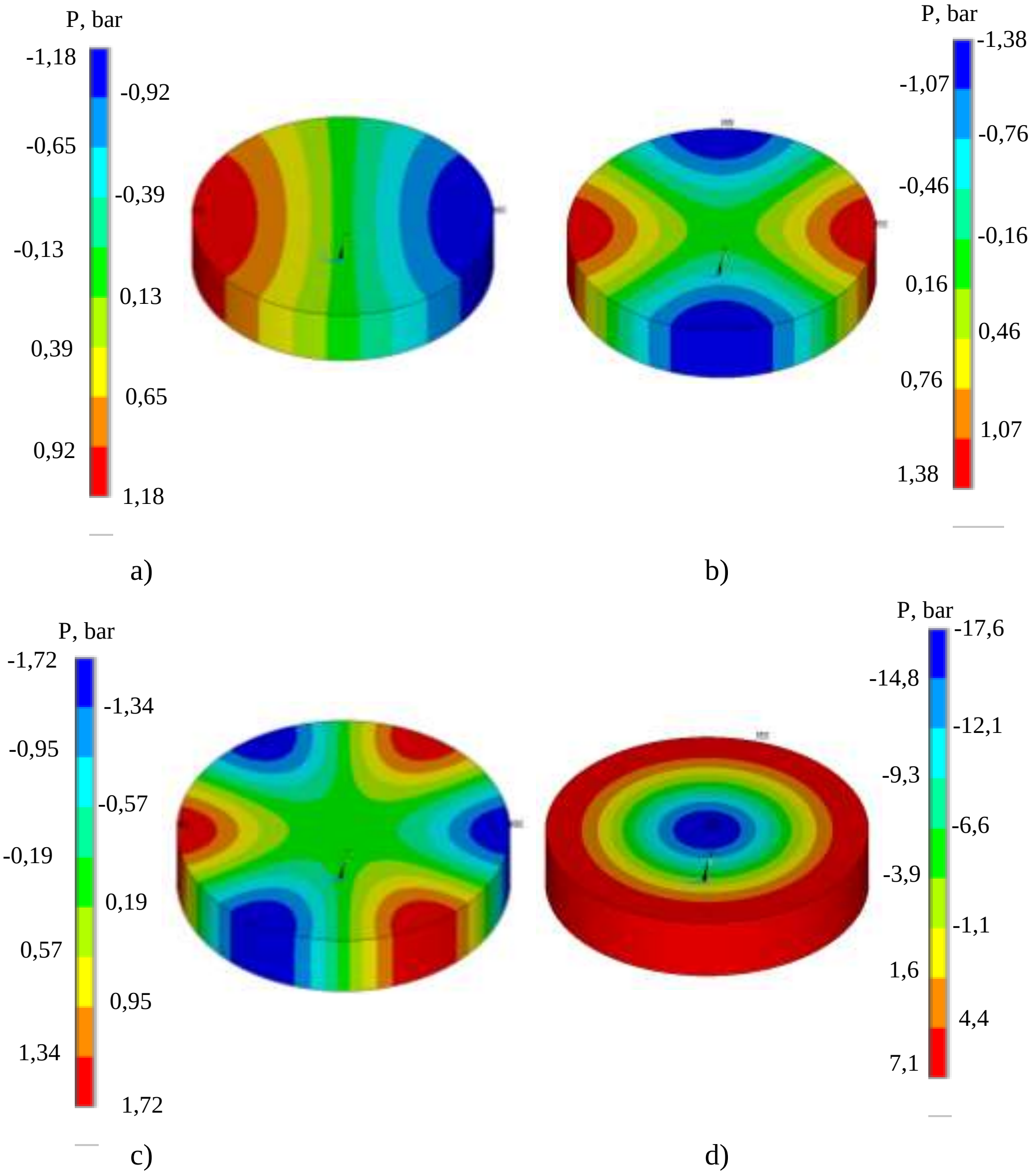
In the proposed approach for the numerical determination of the gaseous medium oscillation amplitudes (as well as for the determination of the pressure dynamic gain) of the combustion chamber under research, at given values of the external harmonic forces (a source of harmonic pressure excitation) is introduced in the finite element model of dynamic system. In this case, the harmonic analysis of the system is carried out taking into account the damping of oscillations of the chamber structure and the gas medium oscillations.

Based on the equation (4), using the means of CAE-systems determines the parameters of forced oscillations under harmonic excitation by pressure in the section of the beginning of the combustion space of the annular chamber, taking into account damping. To determine the amplitudes of chamber longitudinal and radial acoustic oscillations, a symmetric harmonic excitation of the gas medium is introduced. An asymmetric excitation of the pressure of the gas medium is used to determine the amplitudes of tangential acoustic oscillations in various elements of the chamber gas space and the amplitudes of oscillations of the LPRE structures of the outer and inner shells of the combustion chamber.

2. An example of the proposed approach implementation. Results of calculating the parameters of high-frequency acoustic oscillations for the engine annular combustion chamber. The approach was preliminary tested on the basis of a comparison of the results of the numerical and experimental determination of the frequencies and modes of coupled oscillations of a traditional cylindrical combustion chamber (with a diameter of 0,2 m, an axial length of 0,04 m) and combustion products of liquid ethanol and gaseous oxygen as propellant. The experimental frequencies of the dominant oscillation modes were obtained during firing tests of this chamber and are given in [16]. In this case, the experimental value of the speed of sound of the combustion products at the engine operating mode was 657 m/s. The frequencies of the lowest oscillation modes obtained as a result of calculations are in satisfactory agreement with the experimental frequencies $f_{\exp}$. So, the value of the calculated frequency of the first mode of tangential oscillations (1T) was 1929 Hz, and the experimental one is 1930 Hz; the value of the calculated frequency of the first mode of radial oscillations (1R) was 4018 Hz, and the experimental one was 4020 Hz. The calculated frequency of the second mode of tangential oscillations (2T) f_{2T} = 3200 Hz coincided with the experimental one. The calculated and experimental frequencies of the chamber third mode of tangential oscillations (3T) do not differ practically in values (4407 Hz and, correspondingly, 4400 Hz).

Fig. 1 shows the diagrams of the distribution of the pressure oscillations amplitudes in the chamber under tangential acoustic oscillations for the conditions of the LPRE fire tests, obtained from the results of modeling the dynamic system “shell structure – gas” by harmonic pressure excitation of combustion products in chamber (with pressure amplitude of 1 bar).

To test the developed numerical approach to an annular liquid-propellant rocket engine, we performed mathematical modeling of the dynamic system “shell structure of an annular combustion chamber – gas” operating on “liquid oxygen – methane” propellant (the ratio of the outer and inner chamber diameters is 1,5). Note that this rocket propellant is currently considered by EKA and NASA as the most promising in the rocket engine design.



a) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 1T frequency of tangential oscillations of 1929 Hz ;

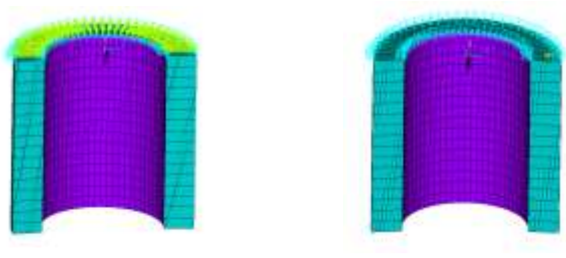
b) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 2T frequency of tangential oscillations of 3200 Hz ;

c) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 3T frequency of tangential oscillations of 4407 Hz

d) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 1R frequency of radial oscillations of 4018 Hz

Fig. 1

To simplify the comparative analysis of the results the finite element model development of the annular combustion chamber, only its cylindrical section was used in the calculations (i.e. the section of the "transition" of the chamber profile to the critical section was not considered) with rigid fixation of the chamber mixture head to the test bench (see picture of finite element the model of the annular combustion chamber of the engine in Fig. 2). The parameters of combustion products (speed of sound, temperature, pressure were calculated by the "ASTRA" software [17] for the case of thermodynamic equilibrium of the combustion process of a "methane – oxygen" pair. The value of the calculated equilibrium speed of sound in the section of the chamber, close to the critical one, was 1273 m/sec.



a) b)

a) is symmetric pressure excitation;
b) is asymmetric pressure excitation

Fig. 2

The frequencies of acoustic longitudinal, tangential and radial oscillations, calculated for the frequency range up to 10 kNz, are given in Tables 1, 2. The following designations are adopted in the Tables: the first and second modes of longitudinal oscillations are designated by symbols 1L and 2L; the first, second and third modes of tangential oscillations – 1T, 2T, 3T; the first mode of radial oscillations is 1R. The designations of the mixed oscillation modes are entered in the Tables through the corresponding combinations of the indicated symbols. For the acoustic modes of the "shell structure of the annular combustion chamber - gas" system, given in Tables 1 and 2, the shapes of longitudinal, tangential and radial oscillations were determined. When performing the calculations, it was assumed that the system harmonic excitation was applied to the section corresponding to the beginning of the fire space of the combustion chamber. The distribution diagrams of the gas dynamic pressure in the annular chamber of the engine at a pressure excitation equal to 1 bar are shown in fig. 3 and 4. In particular, the distribution of the gas dynamic pressure in the annular chamber of the engine under harmonic excitation at the longitudinal (1L) oscillation frequency of 3611 Hz is

Table 1 – Parameters of the dominant longitudinal modes of the engine annular chamber (direction X)

Mode number j	Frequency $f_{X_0 j}$, Hz	Effective mass $M_{X_0 j}$, kg	Mode designation
1	3611	0,34E-20	**1L**
2	5711	5,55	**2L**
3	7699	0,74E-01	**1R**
4	8325	0,62	**3L**

Table 2 – Parameters of the dominant transverse and longitudinal-transverse modes of the engine annular chamber (direction Y)

Mode number j	Frequency $f_{Y_0 j}$, Hz	Effective mass $M_{Y_0 j}$, kg	Mode designation
1	1661	0,39E-01	**1T**
2	2008	5,91	**1T - 1L**
3	3554	0,97E-01	**2T - 1L**
4	5548	1,14	**2T-2L**
5	6562	0,6E-10	**2T**
6	9406	0,77E-08	**3T**

shown in Fig. 3, a).

Note that the values of the frequency of the tangential mode 1T, obtained as a result of the system modeling, turned out to be noticeably lower than the values calculated by the formula

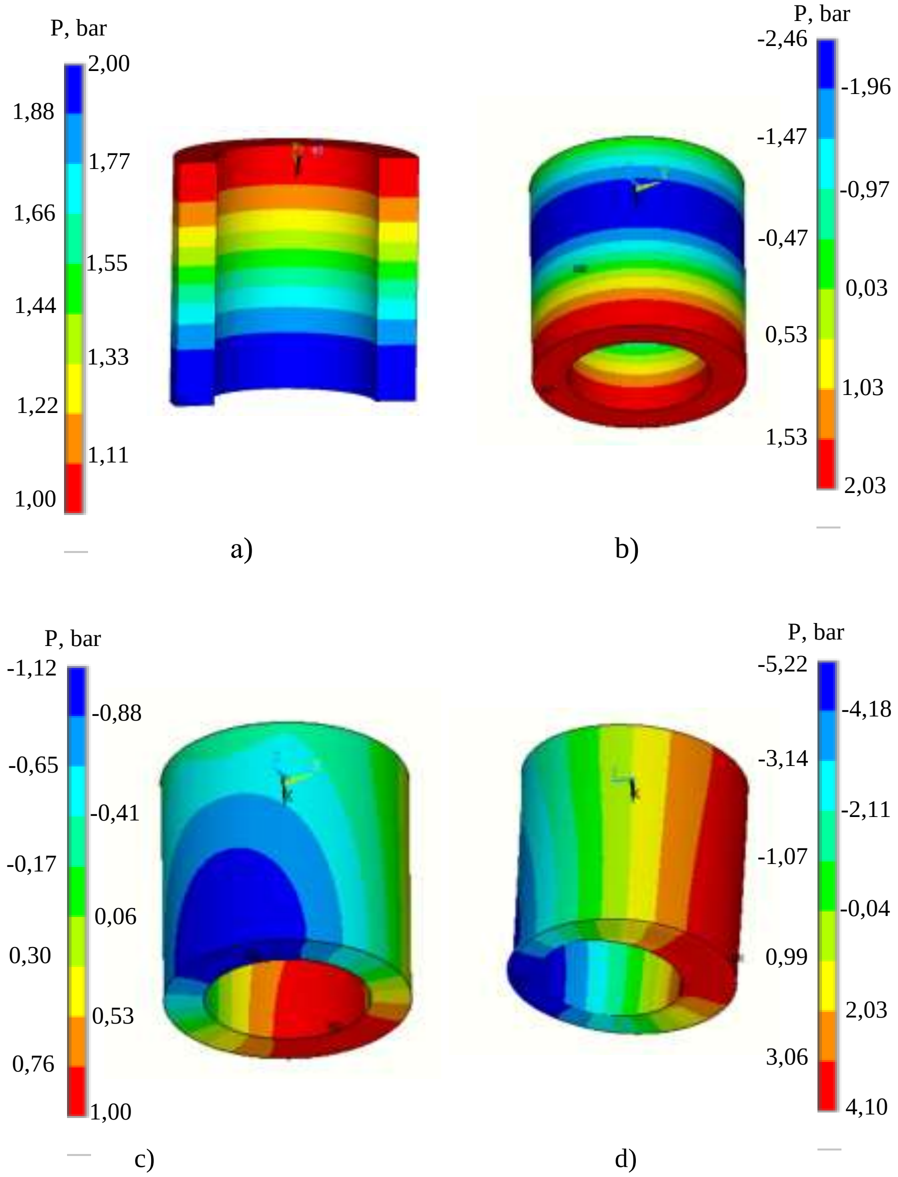
$$f_{\mathrm{tang}} = \frac{2 * C}{\pi(D_{in} + D_{out})}, \tag{9}$$

given in the NASA manual [8] and used the development of high-frequency oscillations in a liquid-propellant rocket engine with an annular combustion chamber. Formula (9) follows from the theoretical solution of the problem of gas oscillations in an annular gas cavity for conditions of rigid walls.

In particular, at the above-mentioned speed of sound of combustion products, the value of the frequency of the 1T tangential mode of the tested annular chamber, calculated by formula (9), is 2968 Hz. This is more than one and a half times more than the value of the corresponding frequency in Table 2. This result is a consequence of the influence on the oscillation frequency of the compliance of the walls of the combustion chamber casing.

Comparing the frequency and mode of oscillations of the first tangential mode 1T, calculated for two options - when the modulus of elasticity of the material of the chamber walls is twice that of stainless steel (Fig. 3d)), and when it is half that of stainless steel (Fig. 3, c)), it can be concluded that with an increase in the stiffness of the walls, the system 1T oscillation frequency approaches its value $f_{1\mathrm{tan}g}$ for rigid walls. So, for a relatively low stiffness of the walls of the combustion chamber structure, the frequency 1T tangential mode of acoustic oscillations (Fig. 3, c) and d)) can be 40 % less than for a structure with rigid walls.

As follows from Fig. 4, a), an increase in the level of pressure amplitudes is possible for oscillations of combustion products with mixed shapes in comparison with system oscillations at individual oscillation modes. For example, for the 2T-1L oscillations at frequency of 3554 Hz the dynamic pressure (amplitudes) reaches 5,73 bar. Thus, the presented approach makes it possible to evaluate the acoustic properties of the combustion chamber during acoustic oscillations at similar frequencies and different shapes (for example, with longitudinal-transverse oscilations). Also As follows from Fig. 4, d), the presented approach makes it possible to evaluate the stress intensity distribution diagram of the amplitudes of oscillations in the combustion chamber during acoustic oscillations.



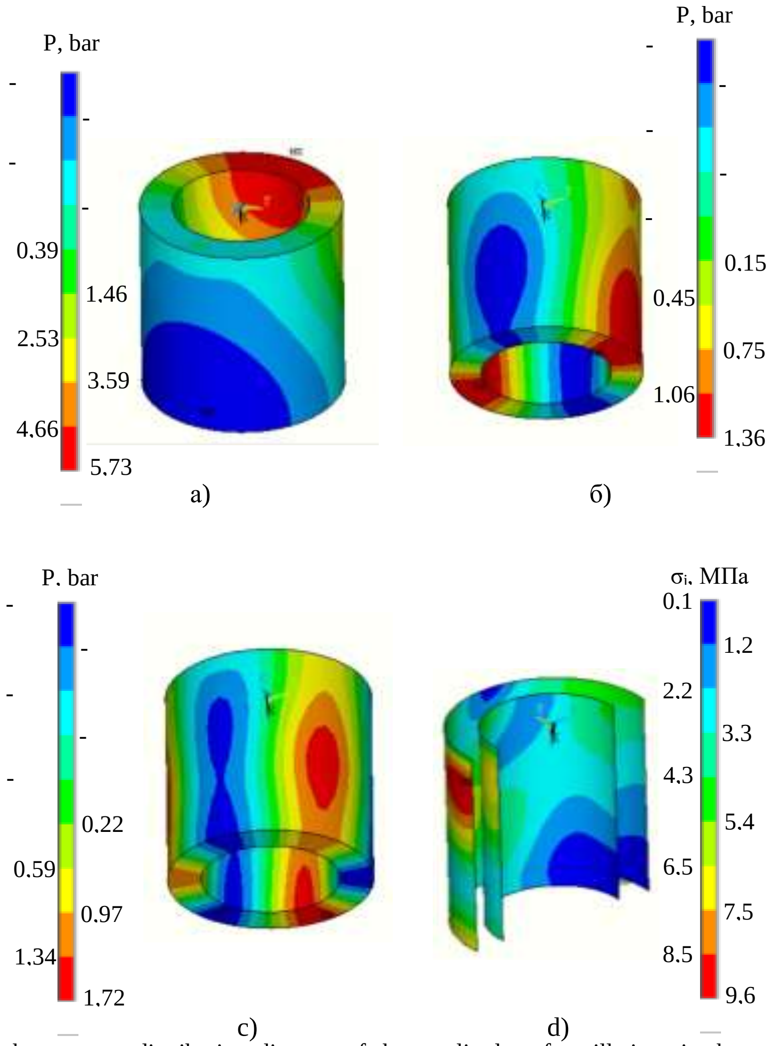
a) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 1L frequency of longitudinal oscillations of 3611 Hz;

b) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 2L frequency of longitudinal oscillations of 5712 Hz;

c) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 1T frequency of tangential oscillations of 1661 Hz (the modulus of elasticity of the material of the chamber walls is half that of stainless steel);

d) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 1T frequency of tangential oscillations of 2682 Hz (the modulus of elasticity of the material of the chamber walls is twice that of stainless steel);

Fig. 3



a) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the (2T-1L) frequency of mixed oscillations of 3554 Hz;
b) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 2T frequency of tangential oscillations of 6562 Hz;
c) is the pressure distribution diagram of the amplitudes of oscillations in the combustion chamber at the 3T frequency of tangential oscillations of 9406 Hz;
d) is the dynamic stress intensity distribution diagram (amplitudes of oscillations) in the combustion chamber at the (2T-1L) frequency of mixed oscillations of 3554 Hz;

Fig. 4

Conclusions. A numerical approach to evaluating the acoustic oscillation parameters of combustion products in LRPE annular combustion chambers is proposed, taking into account the features of the configuration of the combustion space and the changes in the physical properties of the gaseous medium depending on the chamber axial length. In accordance with the proposed approach, the calculation of the acoustic oscillation parameters of combustion products to ensure the rocket engine operational stability is carried out on the basis of a developed finite element model of "shell structure of an annular configuration – gas" coupled dynamic system.

It is shown that on the basis of the developed finite element model of coupled spatial oscillations of the annular combustion chamber structure and the oscillation combustion products, the parameters of the natural oscillations of the system (its frequencies, shapes and effective masses) for the dominant acoustic modes, the parameters of oscillations (displacement) and oscillation acceleration of the combustion chamber walls, and also revealed the operating parameters of the liquid-propellant rocket engine, potentially dangerous from the point of view of the development of engine thermoacoustic instability.

Testing of the numerical approach and the subsequent analysis of the results were carried out for an engine with an annular combustion chamber (with a ratio of the outer and inner diameters of 1,5), which uses a liquid oxygen - methane propellant pair. The shapes and frequencies of the dominant longitudinal, tangential and radial oscillation modes have been determined. It is shown that the frequency of the first mode of acoustic tangential oscillations for a relatively low stiffness of the chamber casing walls can be significantly (up to 40 percent) lower than the corresponding frequency in the case of a chamber with rigid walls.

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