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ASSESSMENT OF THRUST CHAMBER STABILITY MARGINS TO HIGH-FREQUENCY OSCILLATIONS BASED ON MATHEMATIC MODELING OF COUPLED ‘INJECTOR – ROCKET COMBUSTION CHAMBER’ DYNAMIC SYSTEM

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Високочастотна нестійкість рідинного ракетного двигуна (РРД) при статичних вогневих випробуваннях часто супроводжується значним збільшенням динамічних навантажень на конструкцію камери згяряння, що нерідко призводить до її руйнування. Це динамічне явище також може бути надзвичайно небезпечним для динамічної міцності РРД. Розрахунок параметрів акустичних коливань продуктів згоряння важливий при проєктуванні та статичних вогневих випробуваннях таких ракетних двигунів. Визначення параметрів коливання (власних частот та запасів стійкості по декременту коливань) є однією із задач, що вирішуються в період проєктування РРД в рамках розробки заходів із забезпечення стійкості двигуна.

Основною метою роботи є розробка розрахункового підходу до визначення параметрів акустичних коливань продуктів згоряння в камері згоряння РРД з урахуванням особливостей конфігурації камер згоряння та змінності фізичних властивостей газового середовища в залежності від осьової довжини камер, акустичного імпедансу в критичній горловині та ефектів дисипації (експериментальні значення демпфування) в конструкції оболонки і газових середовищах в камері. Підхід, заснований на математичному моделюванні зв'язаної динамічної системи «оболонкова конструкція камери – газ» з використанням методу скінчених елементів і системи CAE (Computer Aided Engineering).

Проведено тестування розробленого підходу та подальший аналіз результатів для двигуна РД 253 з використанням тетраоксиду азоту та несиметричного диметилгідразину в якості паливної пари. Визначено форми і частоти поздовжніх, тангенціальних і радіальних мод динамічної системи. Результати математичного моделювання динамічної системи свідчать про задовільне узгодження розрахункових декрементів першого тону поздовжніх коливань і третього тону тангенціальних коливань з експериментальними декрементами, отриманими за допомогою вогневих випробувань. З системного гармонічного аналізу камери згоряння випливає, що коефіцієнт динамічного підсилення тиску газу в камері на частоті першого тону в 1,6 рази більше, ніж коефіцієнт динамічного підсилення системи для тангенціальної моди. При цьому декремент коливання для тангенціальної моди системи в 2 рази менше, ніж декремент коливаннь для першої поздовжньої моди. Це означає, що тангенціальні коливання більш небезпечні і можуть призвести до нестабільності горіння РРД.

Теоретично показано вплив динамічних властивостей форсунки на стійкість камери згоряння до термоакустичних коливань та можливість часткового придушення високочастотних коливань шляхом настроювання динаміки форсунки.

Ключові слова: рідинний ракетний двигун, камера згоряння, високочастотна нестійкість, частоти коливань, логарифмічні декременти коливань.

High-frequency instability of a liquid-propellant rocket engine (LRE) during static firing tests is often accompanied by a significant increase in dynamic loads on the combustion chamber structure, often leading to a chamber destruction. This dynamic phenomenon can also be extremely dangerous for the dynamic strength of a liquid-propellant rocket engine. The calculation of acoustic combustion product oscillation parameters is important in the design and static firing tests of such rocket engines. The determination of the oscillation parameters (natural frequencies and stability margins such as oscillation decrement) is one of the problems solved in the LRE design period as part of the development of measures to ensure the engine stability.

The main aim of the paper is to develop a numerical approach to determining the parameters of acoustic oscillations of combustion products in liquid-propellant rocket engines combustion chambers taking into account the features of combustion space configuration and the variability of gaseous medium physical properties depending on the axial length of the chamber, acoustic impedance in critical throat and dissipation effects (damping experimental values) in the shell structure and the gas media in the chamber. The approach is based on mathematical modeling of the coupled ‘chamber shell structure – gas’ dynamic system by using the finite element method and the CAE (Computer Aided Engineering) system.

The developed approach testing and further analysis of the results for the RD 253 engine using nitrogen tetroxide and unsymmetrical dimethylhydrazine as a propellant pair were carried out. The dynamic system shapes and frequencies of longitudinal, tangential and radial modes are determined. The results of mathematical modeling of the dynamic system indicate a satisfactory agreement of the calculated decrements of the first longitudinal oscillation mode and third tangential oscillation mode with the experimental decrements obtained by hot-fire tests data. From system harmonic analysis of the thrust chamber, it follows that the dynamic pressure gain factor of the gas media in the chamber at the first longitudinal mode frequency is 1.6 times greater than the system dynamic gain in the tangential mode. At the same time, the oscillation decrement for the system tangential mode is 2 times smaller than that of the first longitudinal mode. This means that the thrust chamber tangential mode is more dangerous and can lead to rocket engine combustion instability.

The effect of the injector on the high-frequency stability of the combustion chamber and the possibility of partial suppression of combustion chamber thermoacoustic oscillations by adjusting the high-frequency dynamics of the injector are shown theoretically.

Key words: *liquid-propellant rocket engine, combustion chamber, high-frequency instability, oscillation frequencies, logarithmic decrements (oscillation decrements).*

1. Introduction. It is known [for example, 1, 2, 3] that the development of high-frequency instability (HF instability) of liquid-propellant rocket engines (LRE) with traditional cylindrical combustion chamber during fire tests is often accompanied by a significant increase in dynamic loads on combustion chamber structures, leading in a number of cases to chamber destruction. HF instability in the LRE combustion chamber is developed as a dynamic process caused by the interaction of gas pressure oscillations (at the frequencies of chamber acoustic oscillations) and thermal processes during combustion. The above phenomena are resonant in nature. Acoustic excitation effect on the burning rate. Oscillations increase the mixing rate of the propellant components and reduce the burning zone. As a rule, [1, 2, 4], dynamic processes in the case of combustion chamber HF instability occur mainly at the lowest tangential acoustic modes with oscillation frequencies exceeding 1000 Hz.

HF stability evaluation and determination of the parameters of HF oscillations (natural frequencies, reduced masses and stability margins such as decrement [2, 5]) is one of the problems solved in the LRE design period as part of the development of measures to ensure the engine HF stability. In different work [2, 4, 5] the experimental HF stability margins of combustion chamber (such as logarithmic decrements) are calculated by width of 0.5-maximum of pressure spectral density in combustion chamber based on the dependencies recorded during the fire tests of the chamber. Also HF stability margins of combustion chamber are calculated by the values of the engine operating parameters closeness to the stability boundaries plotted in the ‘pressure – propellant components ratio’ plane of the operating parameters [2, 4, 5].

Methods theoretical (numerical) calculation of operation process parameters in combustion chamber based on approaches used Computational fluid dynamics (CFD) or more simple phenomenological models of dynamic process. First differ extremely unwieldy approaches that require significant time for calculating the flow parameters, combustion chemical reactions parameters [6, 7]. Using of 3D numerical models of the unsteady processes of motion and combustion of propellant components, scientists can analyze physical essence of the phenomenon more fully than in case of using analytical chemical reaction laws. Also, it allows to identify the leading feedback mechanisms with the input parameters of the chamber and to evaluate the influence of a large number of operating and design factors on the parameters of oscillation motion and burning.

Calculation methods of the natural acoustic frequencies of chamber combustion product oscillations are based on classical solutions (in particular, [3, 8], obtained by using Bessel functions) for dynamics of combustion products equations written in cylindrical coordinates.

Modern conception about the mechanism of HF oscillations development suggest the influence of dynamic processes in the chamber feed system and in the combustion chamber injectors on the quantitative indicators of stability of operation process in the combustion chamber [2, 9, 10]. For example, in M. Dranovsky paper [2] carried out an experimental evaluation of influence of injector head and the pressure drop across the injectors on the HF stability of the combustion chamber. Also, the Klein's experimental research [4] have shown the possibility of suppressing the HF instability of the chamber by installing special (resonators) dampers on injectors. It should be noted the influence of injector head dynamics and the propellant feed system on the development of the Chug dynamic phenomenon [11]. In this case, the theoretical approach to evaluating the stability of the combustion chamber should take into account the interrelated dynamic processes during the propellant flow in the feed system, chemical combustion reactions and acoustic phenomena in the combustion chamber.

The aim of this paper is to develop a numerical approach to evaluating the HF stability margins of LRE combustion chambers taking into account the features of combustion space configuration and the variability of gaseous medium physical properties depending on the axial length of the chamber, acoustic impedance chamber at critical throat section and dissipation effects (damping experimental values) in chamber structure and gas media in chamber.

2. Numerical approach to evaluation stability of liquid rocket thrust chamber. The proposed numerical approach to evaluation stability of liquid rocket thrust chamber consists of calculating the dynamic gain of the coupled 'injector – combustion chamber' dynamic system, which describes the dynamic processes of the liquid propellant flow in injectors, mixing and combustion of propellant components, flow of combustion products in the subcritical part of combustion chambers during harmonic disturbance of the propellant flow at the inlet of combustion chamber in the frequency range of propagation of acoustic waves in the combustion chamber. (in dimensionless variables of pressure and flow).

In this case, the total dynamic gain of the coupled 'injector – combustion chamber' dynamic system is calculated. Total dynamic gain $W_{sys}(j\omega)$ of 'injector – combustion chamber' coupled dynamic system is defined as complex value of multiplication of dynamic gains of three consistently elements: zone of oxidizer flows in the injector $W_{in}(j\omega)$, burning zone $W_{bn}(j\omega)$ and zone of oscillation motion of combustion products in the combustion chamber $W_{cc}(j\omega)$:

$$W_{sys}(j\omega) = W_{in}(j\omega) \times W_{bn}(j\omega) \times W_{cc}(j\omega). \quad (1)$$

2.1 Liquid rocket engines (LRE) injector dynamic gain. LRE injectors was considered as complex hydraulic tubes as a system with distributed parameters. Mathematical model of LRE injector fluid dynamics was developed and based on the equations of unsteady motion and continuity of fluid [12]:

$$\begin{cases} \dfrac{\partial p}{\partial z} + \dfrac{1}{g \cdot F} \cdot \dfrac{\partial G}{\partial t} + \dfrac{k}{g \cdot F} \cdot G = 0, \\ \dfrac{\partial G}{\partial z} + \dfrac{g \cdot F}{c^2} \cdot \dfrac{\partial p}{\partial t} = 0, \end{cases} \quad (2)$$

where p, G are pressure and weight fluid flow rate; t is time; z is axial coordinate of the injector length; F is area of the injector cross section; k is reduced coefficient of linear friction per injector length unit; g is gravity acceleration; c is the fluid sound speed in a injector with elastic walls.

However, the use of equations (2) in partial derivatives even in linear mathematical models with a further approximate solution is associated with cumbersome computations. Therefore, to calculate the injector dynamic gains, taking into account the axial distribution of injector parameters, the impedance method was used [12]. In this case, the solution of partial differential equations (2) can be represented, for example, by a passive quadrupole of the following form:

$$\begin{cases} \delta\overline{p}_2 = b_{11}\cdot\delta\overline{p}_1 + b_{12}\cdot\delta\overline{G}_1, \\ \delta\overline{G}_2 = b_{21}\cdot\delta\overline{p}_1 + b_{22}\cdot\delta\overline{G}_1, \end{cases} \tag{3}$$

where $\delta\overline{p}_1$, $\delta\overline{G}_1$, $\delta\overline{p}_2$, $\delta\overline{G}_2$ are deviations of fluid pressure and weight flow rate from their values at steady state conditions at the inlet and outlet of the quadrupole; b_{11}, b_{12}, b_{21} and b_{22} are elements of the transfer matrix of the hydraulic pipeline with distributed parameters

$$b_{11} = ch(\gamma\cdot l),\ \ b_{12} = -Z_B\cdot sh(\gamma\cdot l),\ \ b_{21} = -\frac{sh(\gamma\cdot l)}{Z_B},\ \ b_{22} = ch(\gamma\cdot l), \tag{4}$$

where γ is complex 'constant' of wave propagation per unit of pipeline length; Z_B is wave resistance of the hydraulic pipeline; l is the injector length.

Using a passive quadrupole of the form (3), (4), for the injector with a combustion chamber burning element input impedance $Z_{1in}(j\omega)$ (as a result of calculating the frequency characteristics of the combustion process and the gas-dynamic flow of combustion products in the chamber by the finite element method) to calculate the injector output impedance $Z_{2in}(j\omega)$, the injector dynamic gain $W_{in}(j\omega)$ can determine by formulas:

$$W_{in}(j\omega) = \frac{\delta\overline{p}_{2in}(j\omega)}{\delta\overline{p}_{1in}(j\omega)} = b_{11} + b_{12}\frac{1}{Z_{1in}(j\omega)}, \tag{5}$$

$$Z_{1in}(j\omega) = \frac{\delta\overline{p}_{1in}(j\omega)}{\delta\overline{G}_{1in}(j\omega)} = \frac{b_{12} - b_{22}\cdot Z_{2in}(j\omega)}{b_{21}\cdot Z_{2in}(j\omega) - b_{11}}. \tag{6}$$

2.2 Modelling of combustion process with time-varying delay. For combustion chambers with good organization of the operating process, the conversion of liquid propellant into combustion products is mainly completed in a small area adjacent to the injector head. To describe the combustion of liquid propellant, the combustion phenomenological model [1] in a combustion chamber with time-varying delay τ, depending on the pressure p_{cc} in the combustion chamber, is used. On the basis of this model, the dynamics of the burning zone (for simplest case), can be described in the form of a passive quadrupole. The equations of passive quadrupole couple the pressures and flow rates at the inlet and outlet from the burning zone:

$$\begin{cases} \delta\overline{p}_{2bn} = a_{11}\cdot\delta\overline{p}_{1bn} + a_{12}\cdot\delta\overline{G}_{1bn}, \\ \delta\overline{G}_{2bn} = a_{21}\cdot\delta\overline{p}_{1bn} + a_{22}\cdot\delta\overline{G}_{1bn}, \end{cases} \tag{7}$$

where δp_{1bn}, δG_{1bn}, δp_{2bn}, δG_{2bn} are deviations of fluid pressure and weight flow rate from their values at steady state conditions at the inlet and outlet of the quadrupole of combustion process; a_{11}, a_{12}, a_{21} and a_{22} are elements of the transfer matrix of the burning zone $a_{11}=1.0$, $a_{12}=\frac{1}{d_b}(e^{-j\omega\tau}-1.0)$, $a_{21}=0$ and $a_{22}=e^{-j\omega\tau}$, parameter d_b is factor depended on chamber pressure p_{cc} and characterize the rate of the process of converting liquid propellant into combustion products with a change in the delay time τ. Notice that p_{1cc} is pressure in the inlet of acoustic oscillations zone of combustion products in chamber, equal to pressure p_{2bn} in the outlet of burning zone.

Using (7) the burning zone dynamic gain $W_{bn}(j\omega)$ can determine by formulas:

$$W_{bn}(j\omega)=\frac{\delta p_{2bn}(j\omega)}{\delta p_{1bn}(j\omega)}=a_{11}+a_{12}\frac{1}{Z_{1bn}(j\omega)}, \tag{8}$$

where $\frac{1}{Z_{1bn}(j\omega)}$ is the admittance of the combustion chamber at the inlet of the burning zone (equal to the total admittance of the injectors in the accepted schematization of the chamber process).

2.3 Evaluation of the dynamic gains of the LRE combustion chamber at frequencies of acoustic oscillations. The proposed approach to calculating the parameters of HF gas acoustic oscillations in the combustion chamber can be attributed to the group of so-called hybrid methods (see, for example, [9]) to analyzing the HF stability of processes in the combustion chamber, for which the parameters of acoustic oscillations of chamber combustion products are calculated separately from the calculation of flame dynamics.

The significant feature of approach is the calculation of the parameters of acoustic oscillations of combustion products based on model of the coupled ‘chamber structure – gas’ dynamic system. This makes it possible to take into account the effect of the elasticity of the combustion chamber structure on the acoustic properties of the combustion products in the combustion chamber of the configuration under study by the modal analysis of this system. The boundary conditions in solving the problem of natural oscillations of the dynamic system are determined by the geometry and method of fastening the combustion chamber structure to the base of the test bench, the conditions associated with the gas flow in the section of the injector head bottom and with the supercritical outflow of combustion products from the nozzle.

The parameters of axial (longitudinal) and transverse (tangential and radial) gas acoustic oscillations in the engine chamber were determined by using the developed finite element model with the chamber structure features are taken into account.

For developing a design scheme for the finite element analysis of the ‘chamber structure – gas’ dynamic system the geometric model of the gas-filled chamber structure under study is formed. This geometric model contains information about the lines that form the surfaces, key points, regions, and volumes of the system. For case constructing it, the available complex volumes in the ‘chamber structure – gas’ system are divided into simple ones like pentahedrons or hexagons. Complex areas are divided into flat or spatial triangles or quadrangles. The developed geometric models make it possible to carry out ‘directional’ schematization of the gas-filled chamber structure with finite element (Fig. 1) by the CAE – system (Computer Aided Engineering System) [13].

This not only makes it possible to avoid imperfections in the computation scheme caused by the automatic discretization of complex volumes (or areas), but also to ensure correct consideration of the boundary conditions for the ‘chamber structure – gas’ system at the interfaces between the gaseous medium and the structure. Note that in the geometric model of the ‘chamber structure – gas’ system, the conjugate surfaces of the obtained simple volumes and the corresponding regions must coincide with each other.

For most structural dynamic problems, the spatial discretization for the principle of virtual work using the finite element method gives the finite element semi-discrete equation of motion as follows:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\}, \qquad (9)$$

where $[M]$, $[C]$, $[K]$ are respectively structural mass matrix , structural damping matrix and structure stiffness matrix, ($n_1 \text{x} n_1$); n_1 is number of degree-of-freedom (DOF) of chamber structure; $\{X\}$ is nodal structure displacement vector, $\{X\}=\{X_1,...,X_{n_1}\}$; $\{\dot{X}\}=\{dX_1/dt,...,dX_{n_1}/dt\}$; $\{\ddot{X}\}=\{d^2X_1/dt^2,...,d^2X_{n_1}/dt^2\}$; t is time; $\{F\}$ is applied load vector: $\{F\}=\{F_1,...,F_{n_1}\}$.

For modeling acoustic fluid – structural interaction (FSI) problems [13], the structural dynamics equation must be considered along with the Navier–Stokes equations of fluid momentum and the flow continuity equation. The fluid momentum (Navier–Stokes) equations and continuity equations are simplified to get the acoustic wave equation using the following assumptions: the gas is compressible (density changes due to pressure variations); there is no mean flow of the gas and combustion process [14, 15].

$$\nabla\left(\frac{1}{\rho}\nabla p\right)-\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2}+\nabla\left(\frac{4\mu}{3\rho}\nabla\left(\frac{1}{\rho c^2}\frac{\partial p}{\partial t}\right)\right)=-\frac{\partial}{\partial t}\left(\frac{Q}{\rho}\right)+\nabla\left(\frac{4\mu}{3\rho}\nabla\left(\frac{Q}{\rho}\right)\right), \qquad (10)$$

where c is speed of sound in gas medium; ρ is mean gas density; μ is dynamic viscosity; p is acoustic pressure; Q is mass source in the continuity equation, (mass flow per unit gas volume [16]).

According to [13, 17, 18] the equation (10) can be written in matrix notation to create the following discretized wave equation:

$$[M_F]\{\ddot{p}_e\}+[C_F]\{\dot{p}_e\}+[K_F]\{p_e\}+\bar{\rho}_0[R]^T\{\ddot{u}_{F,e}\}=\{f_F\}, \qquad (11)$$

where $[M_F]=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0 c^2}\{N\}\{N\}^T dv$ is acoustic fluid mass matrix; $[C_F]=\bar{\rho}_0\iiint\limits_{\Omega_F}\frac{4\mu}{3\rho_0^2 c^2}[\nabla N]^T[\nabla N]dv$ is acoustic fluid damping matrix;

$[K_F]=\overline{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0}[\nabla N]^T[\nabla N]dv$ is acoustic fluid stiffness matrix;

$[R]^T=\oiint\limits_{\Gamma_F}\{N\}\{n\}^T\{N'\}^T dS$ is acoustic fluid boundary matrix;

$\{f_F\}=\overline{\rho}_0\iiint\limits_{\Omega_F}\frac{1}{\rho_0}\{N\}\{N\}^T dv\{\dot{q}\}+\overline{\rho}_0\iiint\limits_{\Omega_F}\frac{4\mu}{3\rho_0^2}\cdot[\nabla N]^T[\nabla N]dv\{q\}$ is acoustic fluid load vector; $\overline{\rho}_0$ is acoustic fluid mass density constant; ρ_0 is mean fluid density. $u_{F,e}$ is shell displacement vector at the fluid boundary; $\{P_e\}$ is nodal pressure vector; $\{N\}$ is element shape function for pressure; $\{N'\}$ is element shape function for displacements; $\{n\}$ is outward normal vector at the fluid boundary; $\{q\}$ is nodal mass source vector; $\{\dot{q}\}$ is the first time derivative of nodal mass source vector; dv is volume differential of acoustic domain Ω_F; dS is surface differential of acoustic domain boundary Γ_F.

Equations (9), (11) describe the complete finite element discretized equations for the FSI problem. These equations are written in assembled form [13] as:

$$\begin{bmatrix}[M_S] & 0\\ \overline{\rho}_0[R]^T & [M_F]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}_e\}\\ \{\ddot{p}_e\}\end{Bmatrix}+\begin{bmatrix}[C_S] & 0\\ 0 & [C_F]\end{bmatrix}\begin{Bmatrix}\{\dot{u}_e\}\\ \{\dot{p}_e\}\end{Bmatrix}+\begin{bmatrix}[K_S] & -[R]\\ 0 & [K_F]\end{bmatrix}\begin{Bmatrix}\{u_e\}\\ \{p_e\}\end{Bmatrix}=\begin{Bmatrix}\{f_S\}\\ \{f_F\}\end{Bmatrix}. \quad (12)$$

The fluid pressure load vector $\{f_e^{pr}\}=[R]\{p_e\}$ at the interface *S* (acoustic gas domain boundary Γ_F) is using in shell structural dynamics and in equation (12).

In the proposed approach for the numerical determination of the gaseous medium oscillation amplitudes (as well as for the determination of the pressure dynamic gain) of the combustion chamber under research, at given values of the external harmonic forces (a source of harmonic pressure excitation) is introduced in the finite element model of dynamic system. In this case, the harmonic analysis of the system is carried out taking into account the damping of the chamber structure and the gas medium oscillations.

Based on the equation (12), using by the means of CAE–systems determines the parameters of forced oscillations under harmonic excitation by pressure in the section of the beginning of the combustion space of the chamber, taking into account damping. To determine the amplitudes of chamber longitudinal and radial acoustic oscillations, symmetric harmonic excitation of the gas medium is introduced. The asymmetric excitation of the pressure of the gas medium is used to determine the amplitudes of tangential acoustic oscillations in various elements of the chamber gas space and the amplitudes of oscillations of the LRE structures of the outer and inner shells of the combustion chamber.

Proposed model of combustor chamber dynamic is used for calculations of acoustic parameters taking into account impedance $Z_{2cc}(j\omega)=\frac{\delta\overline{p}_{2cc}(j\omega)}{\delta\overline{G}_{2cc}(j\omega)}$ ($Z_{2cc}(j\omega)$ mean complex ratio of pressure and mass flow) in throat. Meanwhile model is taking into account dissipation effect in critical intersection. More than finite element model is taking into account damping ratio in each structural material and combustion gases. For calculations damping ratio is 2 % for elastic structure element and 0.1 % for 3D acoustic fluid (this values are corresponding experimental data).

Dynamic pressure gain $W_{cc}(j\omega)$ is characterized oscillation motion of combustion products in cylindrical part of chamber. Depending on the oscillation mode (tangential, radial or longitudinal) this dynamic pressure gain is considered as the ratio of the complex pressure value $\delta P_{2cc}(j\omega)$ at the inlet of chamber throat to complex value of pressure $\delta P_{cc}(j\omega)$ at the inlet of cylindrical part (immediately after the combustion section):

$$W_{cc}(j\omega) = \frac{\delta P_{2cc}(j\omega)}{\delta P_{1cc}(j\omega)}, \qquad (13)$$

where ω is imposed circular frequency; j is square root of -1; symbol δ is characterized for marking of deviation from the steady state value of the parameter.

In case of combustion chamber tangential oscillations study, the harmonical disturbance by pressure at the inlet of chamber acoustic zone the sectoral perturbation $\delta P_{1cc}(j\omega)$ of its section in longitudinal direction is made [19]. In case of axial oscillation study, the harmonical disturbance by pressure at the chamber inlet in the form of axial pressure perturbation $\delta P_{1cc}(j\omega)$ of the entire transverse section is introduced (Fig. 1).

2.4 Calculating of oscillation decrement by total dynamic gain factor of frequency characteristic of 'injector – combustion chamber' coupled dynamic system $W_{sys}(j\omega)$. During fire tests of the combustion chamber with the recording of pressure pulsations, the value of decrement δ is calculated by using the time dependence of the pressure of the dynamic process under study [2]. At the same time, the experimental evaluation of the stability of the operating process in the combustion chamber to pressure disturbances is based on the method of numerical determining the system decrement from the spectral density of the dynamic process and its comparison with the maximum allowable value typical for this type of chamber. Advantage of using the oscillation decrement as an indicator of the stability margin of 'injector – combustion chamber' dynamic system due to the fact that at the stability system boundary in the engine operating parameters space the system stability margin is $\delta = 0$ by definition.

Frequency characteristics are used to calculate oscillation parameters of complex dynamic systems, based on the analysis of the frequency characteristics of the constituent parts [20]. In analysis of the calculated frequency characteristics, the stability of the 'injector - combustion chamber' dynamic system can be estimated based on the calculation of the oscillation decrement of dynamic system. The frequency full width Δf at 0.5-maximum of the modulus of dynamic gain characteristic $\left|W_{sys}(j \times 2\pi f_i)\right|$ at system resonant frequency f_i corresponds to the damping constant (decrement) δ. This is absolutely for a single-mass oscillatory system [20]

$$\delta = 2\pi \times \frac{\Delta f}{f_i}. \qquad (14)$$

For determining the system oscillation increments (in the case of instability of a coupled system), the oscillation increment is carried out taking into account the phase of the frequency response $W_{sys}(j\omega)$.

3. An example of the proposed approach implementation. Evaluation of the parameters of HF acoustic oscillations of the LRE combustion chamber RD253

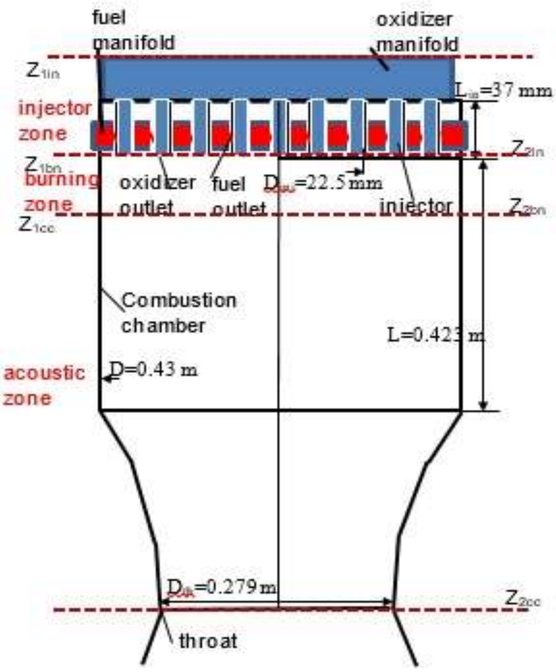
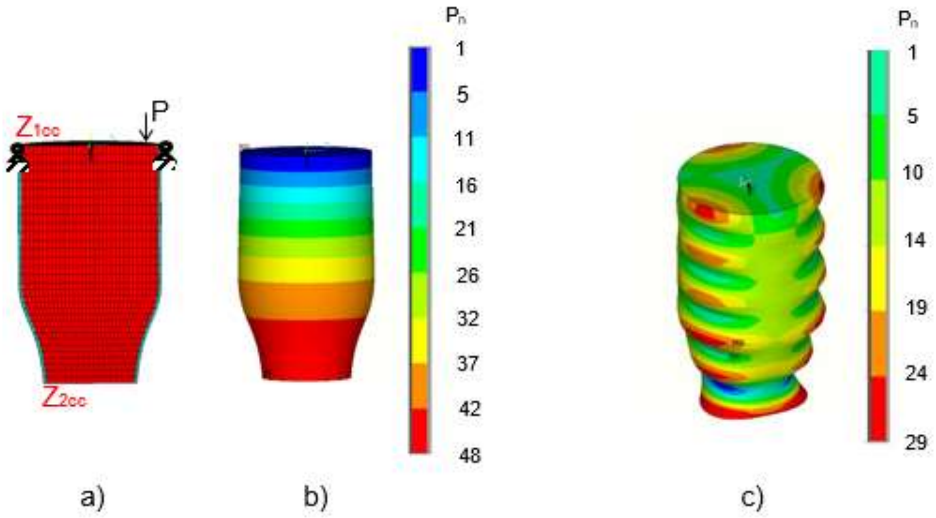


Fig. 1

The proposed approach was tested on the basis of a comparison of the results of the numerical and experimental computation of the frequencies and modes of coupled oscillations of the traditional cylindrical combustion chamber RD253 (with a diameter of 0.43 m, an axial length (cylindrical) of 0.423 m and nitrogen tetroxide and unsymmetrical dimethyl hydrazine as propellant) The RD253 engine schematic is shown in Fig.1).

According to proposed approach parameters of forced harmonically oscillations of RD253 chamber were calculated. Finite element model and boundary condition are shown in Fig. 2 (the impedances at the injector head bottom and at the throat, applied pressure, fixed chamber structure displacement). Calculated normalized dynamic pressure P_n diagrams for acoustic longitudinal and tangential oscillations of the cylindrical combustion chamber RD253 under a single harmonic external excitation are shown in Fig. 2. Total dynamic gains $W_{sys}(j\omega)$ of coupled 'injector – combustion chamber' dynamic system are calculated by use of (1), (5), (6), (8), (13) in the frequency range from 1000 to 1700 Hz for longitudinal oscillation modes, and from 4000 to 4500 Hz for the chamber third tangential mode of acoustic oscillations. These total dynamic gains $W_{sys}(j\omega)$ are shown in Fig. 3, a), b).



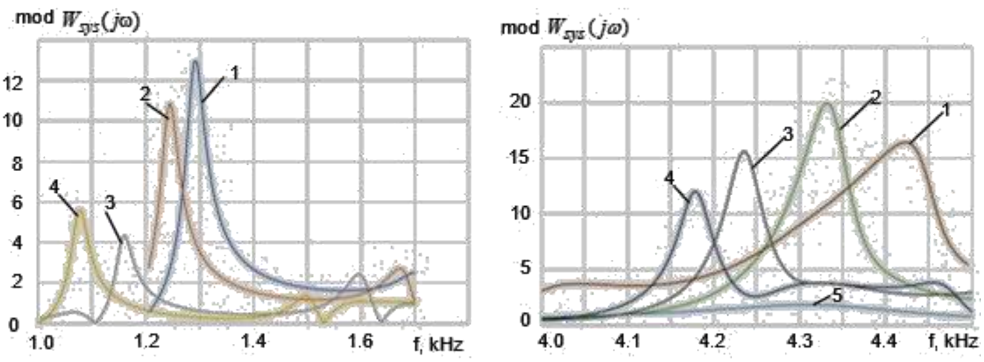
a) is finite element model schematics

b) is normalized dynamic pressure P_n diagram for longitudinal mode of f_{1L}=1291 Hz for static pressure P =99.5 bar in combustion chamber

c) is normalized dynamic pressure diagram for tangential mode of f_{3T}=4175 Hz for static pressure P =134.5 bar

Fig. 2

The curves number from 1 to 4 denote the frequency characteristics in Fig. 3,a) are the modulus of the total dynamic gains $W_{sys}(j\omega)$ of coupled 'injector – combustion chamber' dynamic system for pressure in combustion chamber equal to 99.5 bar, 112 bar, 135 bar, 158 bar. The curves number from 1 to 4 denote the frequency characteristic in Fig. 3,b) are the modulus of the total dynamic gains $W_{sys}(j\omega)$ of coupled 'injector – combustion chamber' dynamic system for oxidizer-to-fuel ratio Km equal to 1.95, 2.6, 3.1, 3.38. The curve number 5 in Fig. 3,b) denotes the frequency characteristic is the modulus of the injector dynamic gain factor $W_{in}(j\omega)$.

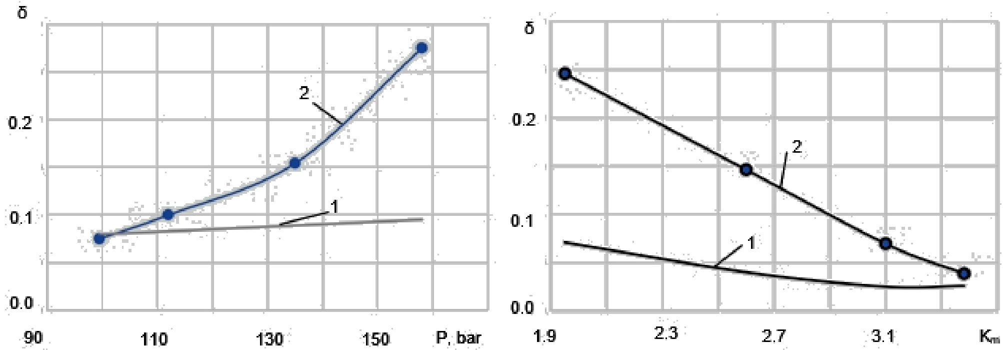


a) b)

a) is total dynamic gains $W_{sys}(j\omega)$ of 'injector – combustion chamber' coupled dynamic system in case of longitudinal oscillations

b) is total dynamic gains $W_{sys}(j\omega)$ of 'injector – combustion chamber' coupled dynamic system in case of tangential oscillations

Fig. 3



a) b)

a) is dependences of oscillation decrements (14) vs. engine RD 253 pressure in combustion chamber *P*

b) is dependences of oscillation decrements (14) vs. engine RD 253 oxidizer-to - fuel ratio K_m

1 – calculated data; 2 – experimental data

Fig. 4

Based dynamic gain calculation results $W_{sys}(j\omega)$, the oscillation decrement values δ of 'injector – combustion chamber' coupled dynamic system under study are computed by using (14).

The calculated dependences (curve 1) of oscillation decrements of the engine RD 253 vs. operating parameters (pressure in combustion chamber and oxidizer-to-fuel ratio K_m) are shown in Fig. 4. The experimental values of the oscillation decrements (curve 2) obtained by calculated spectral densities of operating process in the chamber RD253 pressure for fire tests [2] are shown in Fig. 4. As follows from the analysis of Fig. 4 there is satisfactory agreement of the above dependences in the range of operating parameters change closed to the engine dynamic system stability boundary.

4. Discussion. The parameters of combustion chamber dynamic system natural acoustic oscillations (the frequencies, shapes, effective masses and decrements) for dominant acoustic modes, oscillation parameters (displacements, pressures) and dynamic acceleration of combustion chamber structure walls are calculated on the basis of the developed finite element model of coupled dynamic processes of combustion chamber structure spatial vibrations and oscillation processes of combustion products. As a result of the analysis of complex dynamic gain of the 'injector – burning zone – cylindrical combustion chamber' system, the operating parameters of the LRE chamber, potentially dangerous from the point of view of the development of LRE thermoacoustic instability, were identified.

The analysis results of the dynamic interaction of the RD253 combustion chamber structure and combustion products showed that the effect of wall thickness of combustion chamber cylindrical part is limited by oscillation frequencies up to 1 kHz. In turn, the effect of sound speed in the combustion chamber is more decisive for the system oscillation frequencies above 1 kHz.

From analysis of the results presented above, it follows (see Fig. 3) that the tangential and radial modes under study, as a rule, have the higher oscillation frequencies than the frequencies of longitudinal oscillation modes. But total dynamic gain factor $W_{sys}(j\omega)$ for the tangential mode has large values in comparison with similar values calculated for the longitudinal oscillation mode (see Fig. 3), taking into account the modulus of the injector dynamic gain factor $W_{in}(j\omega)$. Moreover, the minimum values of calculated and experimental oscillation decrements for the longitudinal oscillation mode in two times greater than similar values for the decrements of tangential mode oscillations. This follows from a comparison of the results presented in Fig. 4, a) and Fig. 4, b). The experiment value of decrement is 0.075 and calculated value is 0.0827 for longitudinal frequency f1L in frequency range (from 1.26 kHz to 1.36 kHz), corresponds to static pressure Pcc =99.5 bar in Fig. 4, a). The value of oscillation decrements of engine RD 253 chamber for tangential frequency f3T at frequency range from 3.76 kHz to 4.2 kHz is 0.03846 in experiment data and is 0.0376 in calculation data for static pressure P= 134.5 bar and oxidizer-to-fuel ratio K=3.386.

Thus, the system tangential modes can lead the system to the stability limit in case of smallest values of oscillation decrement. It can be dangerous in case of engine combustion instability.

The injector as element of dynamic system, depended on its resonant adjustable frequency (resonant maximum of its frequency gain), can either increase in the total dynamic gain factor $W_{sys}(j\omega)$ (and, consequently, reduce the stability margin of the 'injector – combustion chamber' dynamic system, i.e., decrement δ), or decrease in module value of $W_{sys}(j\omega)$. It means from the dependencies shown in Fig. 4, b) (curve number 5, the modulus of the injector dynamic gain factor $W_{in}(j\omega)$). In this case, as follows from the Fig. 4, there are frequency ranges characterized by modulus $W_{in}(j\omega)$ don't exceeded 1.0. Moreover, it is necessary to take into account in analysis the phase relationships between $W_{in}(j\omega)$ and $W_{cc}(j\omega)$). That is why, the injector frequency characteristic can have a stabilizing effect on the 'injector – combustion chamber' dynamic system. This property of injector HF dynamics can be used to suppress the thermoacoustic oscillations of the combustion products in the combustion chamber.

It can be noted the overestimated values of some experimental oscillation decrements obtained by working with the results of the RD253 engine fire tests. It is possible that such overestimated values of the decrements were determined by the software imperfection for processing the results of experiments on LRE firing tests in the seventies of the last century. There is the satisfactory agreement of experimental and calculated decrements dependences in the range of operating parameters change closed to the dynamic system stability boundary. These facts allow us to conclude that it is possible to use the approach for theoretical prediction of HF stability boundaries of the rocket engines.

5. Conclusions. The numerical approach to evaluate HF stability of combustion chamber of the LRE is proposed. This approach takes into account features of chamber structure and oscillation motion of combustion products in the chamber. Accordingly, to the proposed approach, the evaluation of rocket engine stability margins (logarithmic decrements) based on the analysis of the 'injector - combustion chamber' coupled dynamic system. This dynamic system describes dynamic processes of liquid propellant flow in injectors, mixing and burning propellants, flow of combustion products in the subcritical part of combustion chambers during harmonic excitation of the propellant flow at injector inlet at the frequency range of acoustic waves propagation in the rocket engine combustion chamber.

The satisfactory agreement of experimental and calculated decrements dependences in the range of operating parameters change closed to the dynamic system stability boundary allow us to conclude that it is possible to use the approach for theoretical prediction of HF stability boundaries of the rocket engines.

The effect of the injector on the stability of the combustion chamber and the possibility to partial suppress of combustion chamber thermoacoustic oscillations by adjusting the injector HF dynamics are shown theoretically.

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