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STUDY OF THE FEATURES OF ANGULAR STABILIZATION OF SPACECRAFT WITH FLEXIBLE STRUCTURAL ELEMENTS WITH THE USE OF MOBILE CONTROL METHODS

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Розвиток космічної енергетики є одним із відомих напрямків в ракетно-космічній науці та інноваційних технологіях, що привертає увагу багатьох науковців і дослідників. Досить глибоке науково-теоретичне опрацювання в цьому напрямі мають інженерні рішення щодо розробки конструкцій сонячних електростанцій та засобів безконтактної передачі електроенергії на Землю та із супутника на супутник, а також методи керування енергетичними космічними апаратами. Однак, незважаючи на глибоке науково-теоретичне опрацювання, є потреба удосконалення існуючих методів і підходів щодо створення оптимальної методології проєктування космічних енергетичних апаратів. Одним із напрямів удосконалення підходів щодо створення космічних сонячних електростанцій та енергетичних супутників може бути застосування методів рухомого керування при розробці системи орієнтації, стабілізації та корекції орбіти. Використання таких методів дозволяє зменшити енергію, необхідну для забезпечення операцій керування.

Метою роботи є дослідження особливостей рухомого керування та формування методології розробки системи орієнтації, стабілізації та корекції орбіти космічних енергетичних супутників з використанням алгоритмів рухомого керування. Розглянуто особливості синтезу алгоритмів рухомого керування для забезпечення орієнтації і стабілізації енергетичних космічних апаратів (сонячних електростанцій та енергетичних супутників). Проведено класифікацію задач керування енергетичними космічними апаратами та обґрунтовано доцільність застосування методів рухомого керування. Проведено аналіз проблеми стійкості, що виникає при керуванні енергетичними космічними апаратами з гнучкими елементами. Сформовано методичні рекомендації щодо визначення проєктних параметрів системи керування кутовим рухом сонячних космічних електростанцій та енергетичних космічних апаратів типу безконтактної передачі електроенергії із супутника на супутник. Цю методологію можна використовувати при створенні космічних електроенергетичних супутників.

Ключові слова: *енергетичний космічний апарат, система орієнтації, стабілізації та корекції орбіти, методи рухомого керування, методологія, безконтактна передача енергії.*

The development of space power engineering is one of the well-known lines in rocket and space science and innovative technologies which attracts the attention of many scientists and researchers. Engineering solutions in space-based solar power plant design and wireless space-to-Earth and satellite-to-satellite power transmission and power spacecraft control methods have been substantiated theoretically to sufficient depth. However, despite this, there is a need to improve methods for and approaches to the development of an optimal design methodology for power spacecraft. A way to improve existing approaches to the development of space-based solar power plants and power satellites may be the use of mobile control methods in the development of an attitude and orbit control system. Such methods allow one to reduce power consumption for control operations.

The goal of this paper is to study the features of mobile control and construct a methodology for the development of solar power satellites' attitude and orbit control system (AOCS) using mobile control algorithms. The paper considers the features of mobile control algorithm synthesis for the attitude control and stabilization of solar power spacecraft (solar power plants and power satellites). Power spacecraft control tasks are classified, and the expediency of using mobile control methods is justified. An analysis is made for the stability problem that arises in controlling power spacecraft with flexible elements. The paper presents methodological recommendations on determining the AOCS design parameters for space-based solar power plants and power spacecraft for wireless satellite-to-satellite power transmission. This methodology may be used in power satellite development.

Key words: *power spacecraft, attitude and orbit control system, mobile control algorithms, methodology, wireless power transmission.*

Introduction. The interest to the space solar energy has been observed over the last decades. The fundamental scientific works in this area are usually dedicated to the peculiarities of super large space solar power plants development [1–4]. These super large power plants are proposed for the contactless electricity transmitting from the near-Earth space to the power receivers on the Earth surface. Considering the volume of needed power [1, 4] it can be justified the so large size of these space systems. So, according to the overview of such systems [3, 4] the average size of their cross-section area more than 1 km^2. Considering it, so large constructions have to be launched only by parts with further in-orbit assembly. In turn, given the current state of aerospace technology development, the super large space solar power plants are very complicated for practical implementations and launch. According to analytical analysis [3] the first experimental launches of the large solar power satellites are expected only in the second half of the 21-th century under an optimistic scenario.

Taking into account the current problems of super large space power satellites launch, the creation of mini electrical-power satellites for testing in-space is observed last years [5, 6]. Such satellites are usually proposed in the range of masses and sizes from nano- (several kg) to small (not more than 500 kg). So, it has been offered to test the possibility of contactless power transmitting from satellite to satellite (sat-to-sat) in space using micro satellite-transmitter (with mass – 50 kg) and cubesat – receiver (with mass – 1.33 kg) [5]. According to the estimations the distance of contactless power transmitting is variated from 8 m to 300 m depending on the transmitter and receiver apertures parameters and microwave frequency. Considering the maximum distance of contactless power transmitting the formation flying type of motion is possible for such satellites. In this case, the satellite-transmitter has to keep the relative distance with satellite-receiver within the permissible range (for example less than 300 m).

The next approach [7] is based on the usage of barraging electrical-power satellites without maintaining a constant distance between the receiver and transmitter. In this case, contactless power transmitting is only possible in certain sessions when the relative distance between the receiver and transmitter is less than or equal to the maximum possible contactless power transmitting distance for these satellites.

Thus, having these different types of relative motion between the spacecraft-receiver and the spacecraft-transmitter, it is arisen the problem of determining the most optimal AOCS configuration for these spacecrafts in all cases of relative flight. One of the quasi-optimal controls is based on the use of mobile control methods [8]. Such approaches allow to reduce the energy that is needed for providing control by switching the control loops with determined number of actuators. Thus, the paper proposes to assess the feasibility of such approaches for controlling space solar power plants and power satellites.

Literature review and problem statement. The one of the approaches of super large space solar-power plants control is presented in the paper [9]. The authors proposed the mathematical model that considers the oscillations of flexible elements of the construction and changes in the inertia tensor during deployment in orbit. The obtained results demonstrated the expediency of using this mathematical model of the dynamics of the system with programmatically variable geometry in the analysis of the real small spacecraft behavior in the process of a large radius ring antenna deployment. However, the methodology of control for these transformable space constructions hasn't been presented.

Taking into account the oscillations and vibrations of flexible elements of space solar-power plant construction is also suggested in paper [10]. In this case, instead of the Euler-Lagrange equations (as in paper [9]), it has been proposed the usage of special elasticity coefficients with the help of which the influence of elastic oscillations on the translational and attitude motion of the solar power plant is evaluated. In turn, the oscillations of flexible elements are described by second order differential equations [10]. The use of this approach allowed to calculate the permissible thresholds of mass and inertia tensor deviation at which the control system fulfills the necessary requirements for the accuracy of orientation and stabilization. Also, the use of this methodology also made it possible to formulate requirements for the control system of a solar space power plant, namely for dedicated orientation and stabilization subsystems. However, in [10] the essence concerning the peculiarities of synthesis of control laws for such spacecraft was not revealed. Thus, in this paper [10], the problem of providing the controllability of such systems is undisclosed.

The deeper analysis of solar-power plants dynamics simulation is presented in [11]. The authors of the paper carry out a comparative analysis of 4 main methods that are used for modeling the dynamics of large space structures taking into account the vibrations of flexible elements. These methods are:

– the finite element method;
– the absolute nodal coordinate method;
– the floating frame formulation method;
– the structure-preserving method.

According to [11], the most popular and widely used method is the finite element method. In turn, to use this method more correctly, special software (Solid Works, Catia, etc.) is required. Another problem of finite element methods implementation is connected with integration special software with new developed soft modules such as libraries of the spacecraft 6-dof motion. Also, in [11] it is stated that the other methods also require significant computing power. Considering it, for preliminary analysis of such space constructions motion in order to obtain general estimates it is advisable to use simplified algorithms for flexible elements fluctuations [12, 13]. These algorithms are very convenient for integration with models of spacecraft orbital and attitude motion that allows to carry out complex numerical or analytical analysis.

In turn, existing well-known approaches to control such spatial structures are based on classical controller design methods [14 – 16]. These methods are grounded on traditional control synthesis using proportional-integral differential (PID) controllers, proportional-differential (PD) [14, 16] controllers, linear-quadratic regulators (LQR), *H-inf* [16] controllers. In general, without the use of sampling (discretization), such controllers provide continuous control, that can require significant energy for actuators operation. However, in the case of control discretization implementation for PD controller and pulse-with modulator usage [17] the consumption of onboard energy can be reduced. It was achieved by minimizing the control operations at each time interval. But, applying only discrete control for a distributed actuator system will not be optimal, and can also instigate the resonance when coincide the self-oscillation frequency of the super large space system.

The one of solutions for damping the self-oscillations was proposed in [18]. The new method of using giant magnetostrictive actuator to control the structural vibration of a large space solar power plant in orbit was proposed in the paper [18]. The obtained results have shown the efficiency of this approach, but the additional control of vibrations using electromagnetic actuators will require additional power consumption. Considering it, such approach can't satisfy some control modes requirements (for example the mode of power accumulation with minimal consumption). In turn, the one of the approaches to synthesis controllers that allow to minimize the energy consumption for control operations of the spacecrafts was proposed by professor A. P. Alpatov [8, 19]. These methods were based on synchronous [8, 19] or asynchronous [20, 21] switching of control channels, but in contrast to relay switch the approach of control synthesis was grounded on piecewise continuous functions usage. Control channels in this case mean control subsystems with certain sets of actuators (1 or more). The use of this approach allows reducing the number of active actuators at each control cycle, minimizing the influence of intrinsic disturbances for magnetic control systems, and, as a consequence, reducing the consumption of onboard energy required for control.

Thus, the paper proposes to analyze the peculiarities and feasibility of using mobile control approaches to provide attitude stabilization for the type of space solar power satellite constructions with flexible elements.

The aim and objectives of the study. The aim of the research is the analytical and numerical analysis of mobile-control algorithms implementation for the solar power satellites attitude stabilization that have flexible elements of construction. It can make it possible to understand the possibility of mobile control approaches implementation to the spacecrafts that have elastic oscillations of flexible elements. To achieve this purpose, it has been set the following tasks:

- formalization of mathematical model of angular motion for power satellite with flexible elements;
- analysis of the power satellite perturbative angular motion in stabilize mode in the case of small angular deviations;
- determination of the spacecraft attitude motion stability condition taking into account fluctuations of the flexible elements.

The methods and materials of the study. The object of research is the control of satellite angular motion.

The subject of the research is the theoretical analysis of the satellite angular motion control peculiarities taking into account oscillations of flexible panels and using mobile control methods.

To provide this study the following methods have been used:

1) mathematical modeling and computer simulation (the mathematical model of satellite angular motion has been formalized and the special C++ software modules have been prepared);

2) infinitesimal calculus and multiplicative integrals for condition of stability formulation;

3) differential equations, quaternion algebra and vector analysis for determination of the satellite motion dynamic and kinematic parameters.

Formalization of mathematical model of angular motion for power satellite with flexible elements. The mathematical model of satellite angular motion taking into account oscillations of flexible elements and changing in tensor of inertia is presented in the paper [9]. This model is based on Euler–Lagrange equation, that can be written as follows:

$$J\frac{d\omega}{dt}+\omega\frac{dJ}{dt}+\omega\times\left(J\cdot\omega\right)+\omega\times\mathbf{K_0}=\mathbf{M}^{\mathbf{cont.}}+\mathbf{M}^{\mathbf{pert.}}+\mathbf{M}^{\mathbf{flct.}}, \tag{1}$$

where J is the tensor of inertia of the power satellite with flexible elements; $\boldsymbol{\omega}$ is the vector of power satellite angular velocity in Body Reference Frame (BRF), relative to J2000 inertial reference frame [22]; $\mathbf{K_0}$ is the resulting angular momentum generated by the oscillations of the satellite flexible parts in BRF; $\mathbf{M}^{\mathbf{cont.}}$ is the vector of control torque in BRF; $\mathbf{M}^{\mathbf{pert.}}$ is the sum of perturbative torques vectors in BRF; $\mathbf{M}^{\mathbf{flct.}}$ is the perturbative torque that generates by the fluctuations of satellite construction flexible elements.

In addition to BRF and J2000 inertial reference frame, WGS-84, Orbital reference frame (ORF), Local vertical local horizontal (LVLH) reference frame and STW reference frame are also used in the study. These reference frames are described in the publications [22, 23].

The quaternion equation is proposed to use for obtaining full group of kinematic attitude parameters (angular velocities and quaternion). This equation can be written in the matrix form as follows:

$$\begin{bmatrix} \frac{dQ_0}{dt} \\ \frac{dQ_1}{dt} \\ \frac{dQ_2}{dt} \\ \frac{dQ_3}{dt} \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}, \quad (2)$$

where Q_0, Q_1, Q_2, Q_3 are components of the quaternion $Q_{J2000->BRF}$ of transition from J2000 inertial reference frame to BRF in BRF coordinates; ω_x, ω_y, ω_z are the components of the angular velocity vector $\boldsymbol{\omega}$.

In turn, to use this quaternion for analysis of attitude orientation is not so convenient. In this regard it is proposed to estimate the quaternion $L_{LVLH->BRF}$ of transition from LVLH reference frame to BRF in BRF coordinates. It can be done using vectors of power satellite position $\boldsymbol{R}_{J2000}$ and velocity $\boldsymbol{V}_{J2000}$ in J2000 inertial reference frame and then using algorithm [24] for determination of the transiting matrix from J2000 inertial reference frame to LVLH. Then, using approach [25] the quaternion of transition from LVLH to J2000 $L_{LVLH->J2000}$ is calculated. Finally, the quaternion $L_{LVLH->BRF}$ is calculated using next equality:

$$L_{LVLH->BRF} = L_{LVLH->J2000} \circ Q_{J2000->BRF} . \quad (3)$$

The models of flexible oscillations are proposed as the model of free oscillations without damping.

The use of free oscillation model in this paper is explained by the fact that this study proposes to investigate the effect of classical harmonic oscillations of structural elements on the power satellite attitude stabilization using mobile control algorithms without taking into account the characteristics of the source that generates these oscillations. Thus, in a general approximation, it is proposed to study the effect of a single oscillating element on the attitude motion of a power satellite.

So, mathematical model of free oscillations without damping can be described by differential equations in the following form [26]:

$$\begin{aligned} m\ddot{\chi}+c_1\chi&=0,\\ m\ddot{\gamma}+c_2\gamma&=0,\\ m\ddot{\tau}+c_3\tau&=0, \end{aligned} \tag{4}$$

where m is the mass of oscillation element; χ is the displacement of center of mass coordinate of the oscillating element along the OX axis of BRF; γ is the displacement of center of mass coordinate of the oscillating element along the OY axis of BRF; τ is the displacement of center of mass coordinate of the oscillating element along the OZ axis of BRF; c_1, c_2, c_3 are the elasticity coefficients of body construction materials at corresponding displacements.

Setting initial time of oscillations $t=0$, the solutions of the equations (4) can be written in the next forms [26]:

$$\begin{aligned} \chi&=\mathrm{A}_\chi\cdot\sin\left(p_1t+\varphi_\chi\right),\\ \gamma&=\mathrm{A}_\gamma\cdot\sin\left(p_2t+\varphi_\gamma\right),\\ \tau&=\mathrm{A}_\tau\cdot\sin\left(p_3t+\varphi_\tau\right), \end{aligned} \tag{5}$$

where A_χ, A_γ, A_τ are the amplitudes of flexible element oscillations along OX, OY and OZ axes of BRF; φ_χ, φ_γ, φ_τ are the corresponding initial phases of these oscillations; $p_1=\sqrt{\frac{c_1}{m}}$; $p_2=\sqrt{\frac{c_2}{m}}$; $p_3=\sqrt{\frac{c_3}{m}}$.

Using these equalities (5) it can be found the velocity and acceleration projections of oscillation element as follows:

$$\begin{aligned} V_\chi&=\mathrm{A}_\chi\cdot p_1\cdot\cos\left(p_1t+\varphi_\chi\right),\\ V_\gamma&=\mathrm{A}_\gamma\cdot p_2\cdot\cos\left(p_2t+\varphi_\gamma\right),\\ V_\tau&=\mathrm{A}_\tau\cdot p_3\cdot\cos\left(p_3t+\varphi_\tau\right),\\ W_\chi&=-\mathrm{A}_\chi\cdot p_1^2\cdot\sin\left(p_1t+\varphi_\chi\right),\\ W_\gamma&=-\mathrm{A}_\gamma\cdot p_2^2\cdot\sin\left(p_2t+\varphi_\gamma\right),\\ W_\tau&=-\mathrm{A}_\tau\cdot p_3^2\cdot\sin\left(p_3t+\varphi_\tau\right), \end{aligned} \tag{6}$$

where V_χ, V_γ, V_τ are the velocity projections of oscillation element relative to the spacecraft center of mass; W_χ, W_γ, W_τ are the acceleration projections of oscillation element relative to the spacecraft center of mass.

The resulting angular momentum generated by the oscillations of the satellite flexible parts in BRF $\mathbf{K_0}$ and perturbative torque that generates by the fluctuations of satellite construction flexible elements $\mathbf{M^{flct.}}$ are calculated using next formulas:

$$\mathbf{F}^{\mathbf{flct}} = \left[-mW_\chi \quad -mW_\gamma \quad -mW_\tau \right]^T,$$
$$\mathbf{r}^{\mathbf{flct}} = [\chi \quad \gamma \quad \tau]^T,$$
$$\mathbf{R}^{\mathbf{sum}} = \mathbf{r}^{\mathbf{flct}} + \mathbf{r}^{\mathbf{cm}}, \tag{7}$$
$$\mathbf{M}^{\mathbf{flct}} = \mathbf{R}^{\mathbf{sum}} \times \mathbf{F}^{\mathbf{flct}},$$
$$\mathbf{K_0} = \begin{bmatrix} m\left(\gamma V_\tau - \tau V_\gamma\right) \\ m\left(\tau V_\chi - \chi V_\tau\right) \\ m\left(\chi V_\gamma - \gamma V_\chi\right) \end{bmatrix},$$

where $\mathbf{F}^{\mathbf{flct}}$ is the perturbative force of inertia that generates during oscillations; $\mathbf{r}^{\mathbf{flct}}$ is the radius-vector of oscillation element center of mass relative to its nominal initial position; $\mathbf{r}^{\mathbf{cm}}$ is the vector that links spacecraft center of mass with the oscillating element center of mass.

Considering the fact that it is proposed to analyze the peculiarities of power satellite with flexible element stabilization at low deflection angles and low angular velocities the following assumptions can be done:

1) $|\boldsymbol{\omega}| \to \min,$ minimum is in the vicinity of zero (the angular velocity of oscillations relative to the spacecraft center of mass is higher than $\boldsymbol{\omega}$);

2) it is proposed to neglect by the values $\boldsymbol{\omega}\dfrac{dJ}{dt} \to 0$ and $\boldsymbol{\omega} \times \left(J \cdot \boldsymbol{\omega}\right) \to 0$ in the equation (1);

3) considering first and second items, the equation (1) can be simplified, as follows:

$$J\,\frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times \mathbf{K_0} = \mathbf{M}^{\mathbf{cont.}} + \mathbf{M}^{\mathbf{pert.}} + \mathbf{M}^{\mathbf{flct.}}. \tag{8}$$

It is proposed to take into account the next components of $\mathbf{M}^{\mathbf{pert.}}$: gravitational perturbative torque, aerodynamic perturbative torque and solar pressure perturbative torque. These torques are proposed to calculate using approaches [22, 27]. Evaluating this equation, it can be obtained:

$$\frac{d\boldsymbol{\omega}}{dt} = A \cdot \left(\mathbf{M}^{\mathbf{cont.}} + \mathbf{M}^{\mathbf{pert.}} + \mathbf{M}^{\mathbf{flct.}}\right) - A \cdot \left(\boldsymbol{\omega} \times \mathbf{K_0}\right), \tag{9}$$

where $A = J^{-1}$, $A = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$, A_{xx}, $A_{xy} \ldots A_{zz}$ are corresponding elements of matrix A.

Decomposing the vector product $\left(\boldsymbol{\omega} \times \mathbf{K_0}\right)$ and then multiply matrix A to this decomposition it can be obtained the following form of equation (2):

$$\frac{d\boldsymbol{\omega}}{dt} = B \cdot \boldsymbol{\omega} + A \cdot \left(\mathbf{M}^{\mathbf{cont.}} + \mathbf{M}^{\mathbf{pert.}} + \mathbf{M}^{\mathbf{flct.}}\right), \tag{10}$$

where $B = \begin{bmatrix} A_{xy}K_{oz} - A_{xz}K_{oy} & A_{xz}K_{ox} - A_{xx}K_{oz} & A_{xx}K_{oy} - A_{xy}K_{ox} \\ A_{yy}K_{oz} - A_{yz}K_{oy} & A_{yz}K_{ox} - A_{yx}K_{oz} & A_{yx}K_{oy} - A_{yy}K_{ox} \\ A_{zy}K_{oz} - A_{zz}K_{oy} & A_{zz}K_{ox} - A_{zx}K_{oz} & A_{zx}K_{oy} - A_{zy}K_{ox} \end{bmatrix}$;

K_{ox}, K_{oy}, K_{oz} are components of the vector $\mathbf{K_0}$.

Assuming that changes of parameters in small limits of angular velocity and quaternion of spacecraft orientation can be considered linear and insignificance of the state vector increment, it is proposed to present control in deviation form:

$$\frac{d\Delta\mathbf{X}}{dt} = \tilde{A}\Delta\mathbf{X} + \tilde{B}\mathbf{U} + \tilde{C}\xi, \qquad (11)$$

where $\Delta\mathbf{X} = \begin{bmatrix} \Delta\omega_x & \Delta\omega_y & \Delta\omega_z & \Delta Q_0 & \Delta Q_1 & \Delta Q_2 & \Delta Q_3 \end{bmatrix}^T$ is the vector of state vector parameters deviations from the program kinematic parameters; $U = \begin{bmatrix} M_x^{cont} & M_y^{cont} & M_z^{cont} & 0 & 0 & 0 & 0 \end{bmatrix}^T$ is the control vector; $\xi = \begin{bmatrix} M_x^{pert} & M_y^{pert} & M_z^{pert} & 0 & 0 & 0 & 0 \end{bmatrix}^T$ is the vector of perturbations;

$\tilde{A} = \begin{bmatrix} B & O_{3\times4} \\ O_{3\times3} & \frac{1}{2}\Omega \end{bmatrix}$ is the state matrix; $\frac{1}{2}\Omega = \frac{1}{2}\cdot\begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}$;

$\tilde{B} = \tilde{C} = \begin{bmatrix} A & O_{3\times4} \\ O_{3\times3} & O_{3\times4} \end{bmatrix}$ are the control and perturbative matrices; M_x^{cont}, M_y^{cont}, M_z^{cont} are the projections of the vector of control torque on BRF axes; M_x^{pert}, M_y^{pert}, M_z^{pert} are the projections of the vector of perturbative torque on BRF axes; $\Delta\omega_x = \omega_x^{pr\cdot} - \omega_x$, $\Delta\omega_y = \omega_y^{pr\cdot} - \omega_y$, $\Delta\omega_z = \omega_z^{pr\cdot} - \omega_z$; $\Delta Q = \bar{Q} \circ Q_{pr}$; $\omega_x^{пр\cdot}$, $\omega_y^{пр\cdot}$, $\omega_z^{пр\cdot}$ are the projections of program values of angular velocity on the BRF axes; Q_{pr} is the quaternion of program orientation.

The control $U = \begin{bmatrix} M_x^{cont} & M_y^{cont} & M_z^{cont} & 0 & 0 & 0 & 0 \end{bmatrix}^T$ is proposed to be synthesized using mobile control methods and proportional-integral-derivative (PID) controller [7]. Considering it, the control algorithm is offered in the next form:

$$\left.\begin{aligned} M_x^{cont} &= -(J_{xx} + J_{xy} + J_{xz})\left(K_1\Delta\omega_x + K_2\Delta Q_1 + K_3\int \Delta Q_1 dt\right) \\ M_y^{cont} &= 0 \\ M_z^{cont} &= 0 \end{aligned}\right\} lp-1,$$

$$\left.\begin{aligned} M_x^{cont} &= 0 \\ M_y^{cont} &= -(J_{yx} + J_{yy} + J_{yz})\left(K_1\Delta\omega_y + K_2\Delta Q_2 + K_3\int \Delta Q_2 dt\right) \\ M_z^{cont} &= 0 \end{aligned}\right\} lp-2,$$

$$\left.\begin{aligned} M_x^{cont} &= 0 \\ M_y^{cont} &= 0 \\ M_z^{cont} &= -(J_{zx} + J_{zy} + J_{zz})\left(K_1\Delta\omega_z + K_2\Delta Q_3 + K_3\int \Delta Q_3 dt\right) \end{aligned}\right\} lp-3,$$

$$\left.\begin{aligned} M_x^{cont} &= 0 \\ M_y^{cont} &= 0 \\ M_z^{cont} &= 0 \end{aligned}\right\} lp-4, \tag{12}$$

where J_{xx}, J_{xy}, J_{xz}, J_{yx}, J_{yy}, J_{yz}, J_{zx}, J_{zy}, J_{zz} are the components of power satellite tensor of inertia; M_x^{cont}, M_y^{cont}, M_z^{cont} are the projections of control torque on the BRF axes; K_1, K_2, K_3 are the gains of PID-controller; $\Delta\omega_x$, $\Delta\omega_y$, $\Delta\omega_z$ are the mismatches by angular velocities; ΔQ_1, ΔQ_2, ΔQ_3 are the mismatches by vector part of quaternion; $lp-1$, $lp-2$, $lp-3$, $lp-4$ are the control loops numbers.

The function that switches control loops can be determined using approach [20] as follows:

$$\text{switch} = \begin{cases} lp-1 & \text{if} \quad 2k-10 \le T < 2k, \\ lp-2 & \text{if} \quad 2k \le T < 2k+10, \\ lp-3 & \text{if} \quad 2k+10 \le T < 2k+20, \\ lp-4 & \text{if} \quad 2k+20 \le T < 2k+30, \end{cases}$$
$$k = 5, 25, 45, 65 \ldots 20n+5, \tag{13}$$
$$n = 0,1,2,3,4 \ldots n_{end},$$

where n is the number of control cycle; n_{end} is the number of last control cycle; T is the time of controller in samples (1 sample is proposed to be equal 0.1 s).

However, to analyze stability of the system (11) using traditional approaches is complicated task considering nonlinearity due to the impact of oscillations. Taking it into account, it is proposed to decompose the analysis of stability into two stages. At first stage it is proposed to analyze the stability of angular velocities, that are estimated using solutions of simplified model (10). After angular velocities stability analysis, it is proposed to analyze spacecraft orientation quaternion deviations from its program values.

In spite of similarity of the model (10) in general form with the system of linear differential equations (LDE), it cannot be classified as LDE mathematical model. It can be explained by the nonlinearity due to the presence of oscillatory components ($\mathbf{M}^{\mathbf{flct.}}$ and $\mathbf{K_0}$) in equation and perturbative torques in (10). Taking it into account the components of matrix B are dependent on time $B = B(t)$. In turn, it is proposed to assume that at small time intervals the changes in the matrix B components will be insignificant (i.e., they can be neglected and the matrix components can be considered constant at these intervals). The same assumption is proposed to the changes of perturbative torques at these small intervals. The width of each time interval is determined according to these assumptions where the system (10) is considered to be linear on this interval, and the changes in the components $\mathbf{M}^{\mathbf{cont.}}, \mathbf{M}^{\mathbf{pert.}}, \mathbf{M}^{\mathbf{flct.}}$ and components of the matrix B are insignificant and their values can be considered constant on this small-time interval. Based on this, it is proposed to use Routh–Hurwitz stability criterion [28] for analysis the zero solution of general solution of equation (10) in each sample. The width of each sample is determined according to the assumption when system (10) is considered to be linear at this interval (the width of sample). In turn, in the case of dividing the whole trajectory by parts and analyzing stability properties of general solution in each part, the criteria of stability similar to Bellman's optimum principle can be formed:

Condition of stability 1. *The general solution of the system of differential equations (10) is stable in whole interval of time in the case if it is stable in each sample of time during motion.*

However, in practice, the system may not be stable for all time samples of the entire time interval while the control assurance requirements can be met. Partial loss of stability of a system with oscillating elements when using mobile control methods may occur in the following cases:

1) at the moments of switching of control loops (using mobile control);

2) in the event of a momentary control shutdown.

Taking it into account the *condition of stability 1* is so strict for stability analysis of the system (10). Thus, it is advisable to change this criterion for more flexible by adding the threshold of maximum percentage of stabile cases in all time samples of whole interval. So, the *condition 1* can be updated using following formulation:

Condition of stability 2. *The general solution of the system of differential equations (10) correspond to the requirements of stability when the difference between whole number of samples* $\mathbf{k}$ *and number of samples with stable solution of system (10)* $\mathbf{l}$ *in the time interval less then predetermined value of threshold* δ: $\mathbf{k - l < \delta}$. In turn to determine value δ for all cases is very complicated task. Considering this, it is reasonable to use threshold in percentage:

$$\frac{\mathbf{k - l}}{\mathbf{k}} \cdot \mathbf{100\% < \varepsilon}, \qquad (14)$$

where ε is the maximum permissible percentage of unstable cases in whole time interval.

The ε is determined depends on control accuracy requirements and properties of power satellite construction flexible oscillations. In turn, it is proposed to use mentioned above Routh–Hurwitz stability criterion for determining l number. To determine the general form of the characteristic polynomial and its coefficients at each time sample, it is proposed to use matrix B (10). Considering that matrix B has size 3×3 the characteristic polynomial that can be determined using inequality $\det\left[B - E\lambda\right] = 0$ (E is identity matrix) will have 3-rd order and can be written in the next form:

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0\,, \qquad (15)$$

where $a_0 \ldots a_3$ are coefficients of characteristic polynomial.

These coefficients are determined from equality $\det\left[B - E\lambda\right] = 0$ (taking into account requirement $a_0 > 0$) as follows:

$a_0 = 1$; $a_1 = -\left(B_{xx} + B_{yy} + B_{zz}\right)$;

$a_2 = B_{xx}B_{yy} + B_{zz}B_{yy} + B_{zz}B_{xx} - B_{xz}B_{zx} - B_{xy}B_{yx} - B_{zy}B_{yz}$;

$a_3 = B_{zy}B_{yz}B_{xx} + B_{xy}B_{yx}B_{zz} + B_{xz}B_{zx}B_{yy} - B_{xx}B_{yy}B_{zz} -$

$-B_{xy}B_{zx}B_{yz} - B_{yx}B_{xz}B_{zy}$;

B_{xx} , B_{xy} , B_{xz} , B_{yx} , B_{yy} , B_{yz} , B_{zx} , B_{zy} , B_{zz} are the components of matrix B .

Taking into account the peculiarities of characteristic polynomial the Routh-Hurwitz stability criterion for this case is following:

1) The main diagonal minors are $\Delta_1 = a_1$, $\Delta_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{vmatrix}$.

2) The general solution is stable in determined time sample if current values of $\Delta_1 > 0$, $\Delta_2 > 0$ and $\Delta_3 > 0$. Otherwise, the general solution is instable.

Thus, the mathematical model of angular motion for power satellite with flexible elements has been formalized (1) – (15). The condition of the simplified system (10) general solution stability has been determined. However, for the full analysis of power satellite attitude stabilization using mobile control methods and synthesis of complex criterion of stability the computer simulation is required.

Computer simulation of the power satellite perturbative angular motion in stabilize mode in the case of small angular deviations. The computer simulation is proposed for the modelling of power satellite with flexible elements 6-dof perturbative motion for two cases:

1) using simplified model of attitude motion (10);

2) using full model of attitude motion (1).

Both models include perturbations of oscillations (4) – (5), gravitational perturbative torque, atmospheric perturbative torque and solar pressure perturbative torque. The purpose of using two models is to verify the adequacy of simplification (10) compared to using model (1) for the case of small attitude deviations. The model of orbital motion that has been used in program simulator is described in [20, 22].

Initial parameters for the simulation:
Orbit:
Focal parameter: 7045254.273 m;
Eccentricity: 0.00141871;
Inclination: 98 degrees;
RAAN: 135.249 degrees;
Argument of perigee: 121.177 degrees;
Argument of latitude: 306.234 degrees.
Mass, size and inertia parameters of the power satellite:
Mass of the spacecraft: 500 kg;
Average cross-section area of the spacecraft: 5 m^2;
Tensor of inertia (at initial time):
$J_{xx} = 40$ $\text{kg}\cdot\text{m}^2$, $J_{xy} = 0.25$ $\text{kg}\cdot\text{m}^2$, $J_{xz} = -0.15$ $\text{kg}\cdot\text{m}^2$, $J_{yx} = 0.25$ $\text{kg}\cdot\text{m}^2$, $J_{yy} = 30$ $\text{kg}\cdot\text{m}^2$, $J_{yz} = -0.3$ $\text{kg}\cdot\text{m}^2$, $J_{zx} = -0.15$ $\text{kg}\cdot\text{m}^2$, $J_{zy} = -0.3$ $\text{kg}\cdot\text{m}^2$, $J_{zz} = 50$ $\text{kg}\cdot\text{m}^2$.
Mismatch between center of mass and center of pressure in BRF:
$\Delta X = 0.3$ m; $\Delta Y = 0.5$ m; $\Delta Z = 0.1$ m.
Parameters of oscillations:
$A_\chi = 0.02$ m; $A_\gamma = 0.02$ m; $A_\tau = 0.05$ m.
$m = 20$ kg; $c_1 = 0.2$ N/m; $c_2 = 0.2$ N/m; $c_3 = 1.0$ N/m.
Initial position of the oscillation element in BRF: $\mathbf{r}^{\mathbf{flct}} = \begin{bmatrix}0.3 & 0.6 & -0.5\end{bmatrix}^T$.
$\varphi_\chi = 20$ deg, $\varphi_\gamma = 20$ deg, $\varphi_\tau = -20$ deg.
Initial deviation in $L_{LVLH->BRF}$ *quaternion:*
$L_{LVLH->BRF.0} = 0.97069171535712184$;
$L_{LVLH->BRF.X} = -0.14305901906629273$;
$L_{LVLH->BRF.Y} = -0.062517963671981605$;
$L_{LVLH->BRF.Z} = 0.18271074138962684$.
Quaternion of program orientation in LVLH reference frame:
$L^{pr}_{LVLH->BRF.0} = 1$; $L^{pr}_{LVLH->BRF.X} = 0$; $L^{pr}_{LVLH->BRF.Y} = 0$;
$L^{pr}_{LVLH->BRF.Z} = 0$.
Initial values of angular velocities:
$\omega_x = 0.3$ deg/s, $\omega_y = -0.2$ deg/s, $\omega_z = 0.2$ deg/s.
Program angular velocities:
$\omega_x = 0$ deg/s, $\omega_y = 0$ deg/s, $\omega_z = 0$ deg/s.
Controller parameters: $K_1 = 9$, $K_2 = 0.9$, $K_3 = 10^{-6}$.
Simulation time: 86400 s; *Initial simulation data and time*: December 12, 2023.

Simulation results:

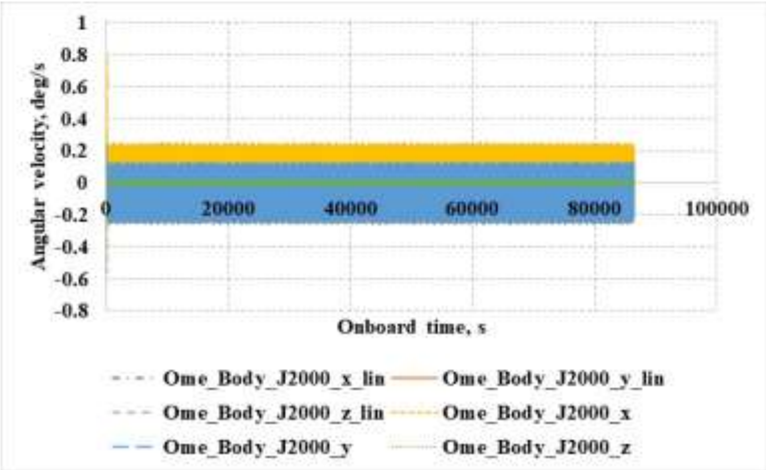

Fig. 1 – Changing in angular velocities ω_x , ω_y , ω_z (Ome_Body_J2000_x, Ome_Body_J2000_y, Ome_Body_J2000_z are the ω_x , ω_y , ω_z that obtained using model (1); Ome_Body_J2000_x_lin, Ome_Body_J2000_y_lin, Ome_Body_J2000_z_lin are the ω_x , ω_y , ω_z that obtained using simplified model (10))

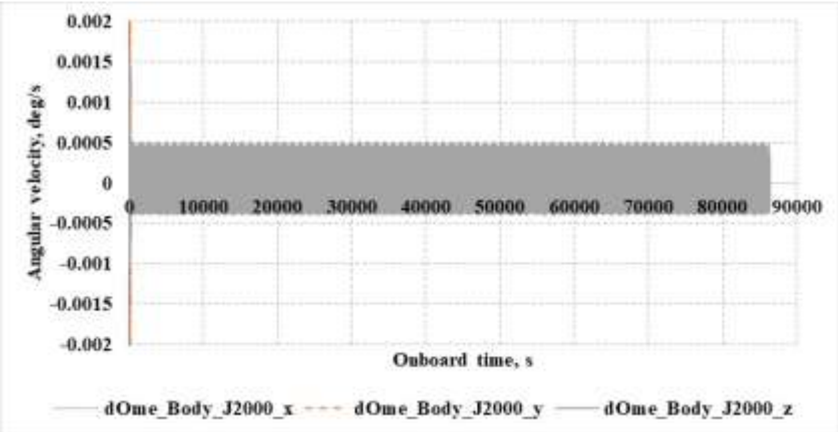

Fig. 2 – Mismatches in angular velocities when using model (1) and model (10)

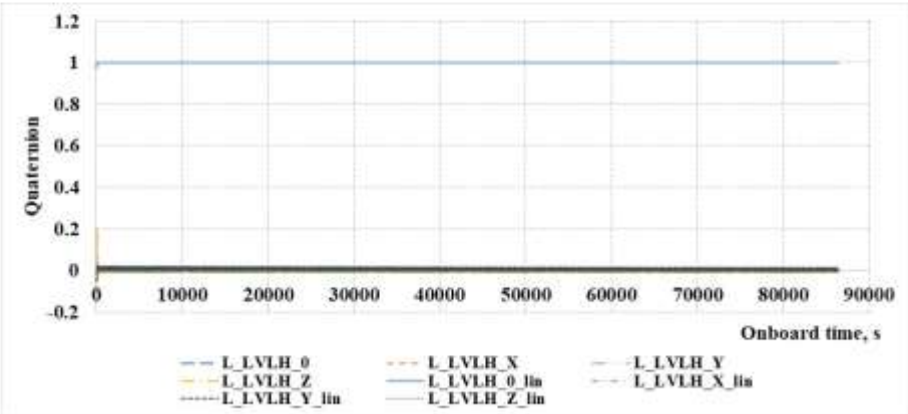

Fig. 3 – Changing in LVLH quaternion during simulation (where L_LVLH_0, L_LVLH_X, L_LVLH_Y, L_LVLH_Z the quaternion $L^{pr}_{LVLH\,->BRF\,Z}$ that obtained using model (1); L_LVLH_0_lin, L_LVLH_X_lin, L_LVLH_Y_lin, L_LVLH_Z_lin the quaternion $L^{pr}_{LVLH\,->BRF\,Z}$ that obtained using simplified model (10))

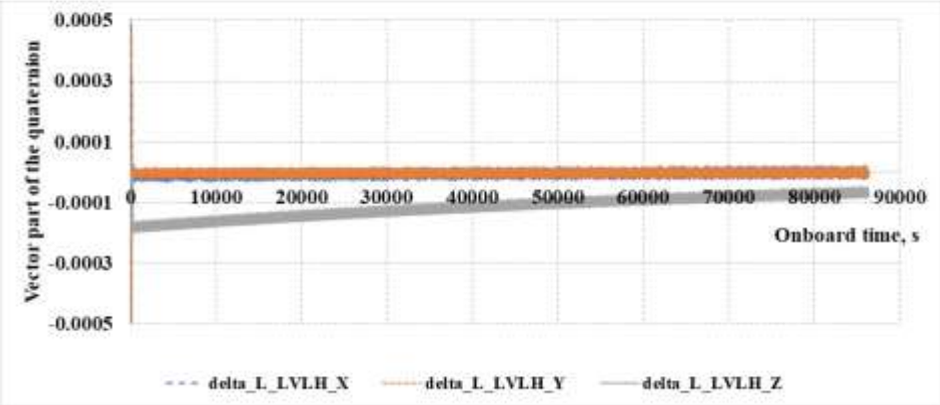

Fig. 4 – Mismatches in vector parts of LVLH quaternion when using model (1) and model (10)

It can be seen from the obtained results (fig. 1–fig. 3) that the simplified model (10) has shown almost the same result as model (1). The largest differences are at the beginning of the trajectory, at the moment of small reorientation (fig. 2, fig. 4). The differences of the models in the angular velocities in stabilize mode are observed at 3 and 4 digits after the dot (fig. 2) and in vector part of the quaternion at 4 and 5 digits after the dot (fig. 4) with further convergence at the end of the trajectory (fig. 4). Thus, it can be concluded that the assumption (10) under the simplification of model (1) is valid at small attitude deviation and can be used for estimation calculations of angular stabilization of satellites with flexible elements.

Testing the ***condition of stability 2*** applicability for analysis the general solution of the equation (10), it has been proposed to estimate the stability of this solution at each time sample using the above-mentioned Routh–Hurwitz criterion. For this purpose, it is proposed to take a certain section of the trajectory in the stabilization mode after reorientation and analyze the stability of the general solution of equation (10). So, it has been selected the part of the trajectory in time period from 10000 s to 11000 s and obtained the result in fig. 5.

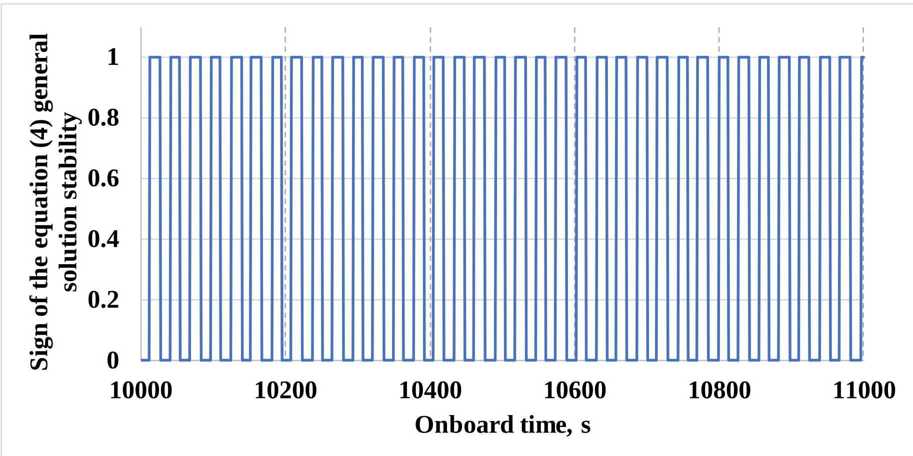

Fig. 5 – The sign of the equation (10) general solution stability at each time sample (1 time sample = 0.1 s) using Routh–Hurwitz criterion, where: ***1 – stable, 0 – instable***.

It can be seen that sign of the general solution stability has periodical changing in time (fig. 5) that corresponds to the requirements of using ***condition of stability 2***. The obtained value of the stable solutions number at this interval **l**=5010 with whole number of samples **k**=10000. So, if to use the formula (14), setting ε = 50%, the requirement of stability is satisfied. Considering it, the ***condition of stability 2*** can be used for analysis of general solution peculiarities.

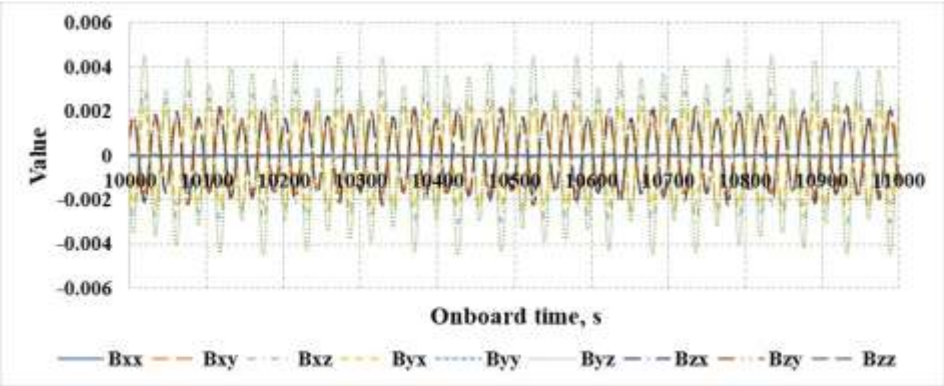

Fig. 6 – The changing of matrix B elements B_{xx}, B_{xy}, B_{xz}, B_{yx}, B_{yy}, B_{yz}, B_{zx}, B_{zy}, B_{zz} on time

Considering, that elements of matrix B of the differential equation system (10) are also have time-periodic character of change (fig. 6), the infinitesimal calculus approaches can be used for their determination [8]. On this basis, the relationship between the elements of the matrix B at successive stable time periods can be expressed using the multiplicative integral in this way [8]:

$$\int_{t_{i,j-1}}^{\frown t_{ij}} \left(E + B_{ij}dt\right) \approx \prod_{k}^{N}\left(E + B_{ijk}\Delta t_{ijk}\right), \quad k = 1,2\ldots N. \tag{16}$$

where B_{ij} is the value of matrix B at i-th control sample of j-th stable time sample according ***condition of stability 2;*** Δt_{ijk} is the k-th time step between B_{ij} and $B_{i,j-1}$ stable general solutions; B_{ijk} is the value of B_{ij} matrix at k-th time step.

However, taking into account the periodicity of changes in the stability of the general solution, these conditions can be used only as necessary conditions of stability, but not as a sufficient one. In the cases of another controller gains the ***condition of stability 2*** could show incorrect result regarding system stability. So, if the controller parameters set equal $K_1 = 1.2$, $K_2 = 0.12$, $K_3 = 10^{-6}$, the modeling results will be the following (fig. 7 and fig. 8). However, the sign of the equation (10) general solution stability at each time sample is similar to the results presented on fig. 5.

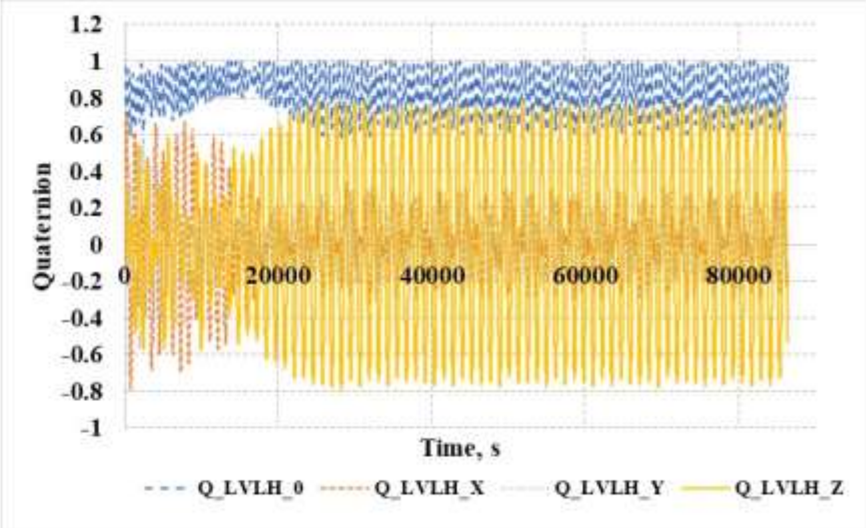

Fig. 7 – Changing in LVLH quaternion in the case of the controller gains $K_1 = 1.2$, $K_2 = 0.12$, $K_3 = 10^{-6}$ (Q_LVLH_0, Q_LVLH_X, L_QVLH_Y, Q_LVLH_Z the quaternion $L^{pr}_{LVLH->BRF\_Z}$ that obtained using model (1))

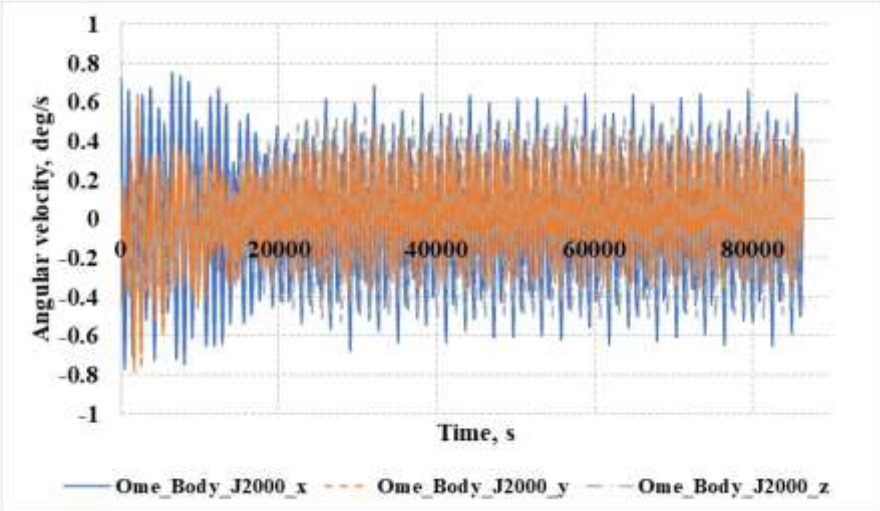

Fig. 8 – Changing in angular velocities ω_x, ω_y, ω_z in the case of the controller gains $K_1 = 1.2$, $K_2 = 0.12$, $K_3 = 10^{-6}$ (Ome_Body_J2000_x, Ome_Body_J2000_y, Ome_Body_J2000_z are the ω_x, ω_y, ω_z that obtained using model (1))

Thus, to determine the full condition of stability it is required to add the quaternion deviation condition.

Determination of the complex stability condition for a power satellite with flexible elements motion when using mobile control. To develop the full criterion of the stability it will be reasonable to use the analysis by the quaternion vector part 2-norm deviation and by linear deviation of quaternion scalar part. It is proposed to use $L_{LVLH->BRF}$ quaternion for analysis of stability. Considering it, the sufficient condition of stability can be written in the next form:

$$\Delta Scalar\left(L_{LVLH->BRF}\right)=\left|L_{LVLH->BRF.0}-L_{LVLH->BRF.0}^{pr}\right|,$$
$$\left\|\Delta Vect\left(L_{LVLH->BRF}\right)\right\|_2=\sqrt{\begin{bmatrix}\left(L_{LVLH->BRF.x}-L_{LVLH->BRF.x}^{pr}\right)^2+\\+\left(L_{LVLH->BRF.y}-L_{LVLH->BRF.y}^{pr}\right)^2+\\+\left(L_{LVLH->BRF.z}-L_{LVLH->BRF.z}^{pr}\right)^2\end{bmatrix}}, \quad (17)$$
$$\Delta Scalar\left(L_{LVLH->BRF}\right)\le\xi,$$
$$\left\|\Delta Vect\left(L_{LVLH->BRF}\right)\right\|_2\le\Upsilon,$$

where $\Delta Scalar\left(L_{LVLH->BRF}\right)$ is the absolute value of linear deviation of $L_{LVLH->BRF}$ quaternion scalar part; $\left\|\Delta Vect\left(L_{LVLH->BRF}\right)\right\|_2$ is the $L_{LVLH->BRF}$ quaternion vector part 2-norm deviation; ξ is the threshold by linear deviation of $L_{LVLH->BRF}$ quaternion scalar part; Υ is the threshold by $L_{LVLH->BRF}$ quaternion vector part 2-norm deviation.

Thus, the complex condition of stability is proposed to formulate as follows:

Complex stability condition: *For control stability of a spacecraft with flexible oscillating elements when using mobile control methods, it is necessary and sufficient to have a certain number of time samples with a stable general solution* $\mathbf{k}$ *of the differential equation system (10) considering (14) where the condition (17) is satisfied.*

Setting $\Upsilon=0.01429$ and $\xi=0.0002$ it has been obtained the following results of the full stability analysis in time period from 10000 s to 11000 s (fig. 5).

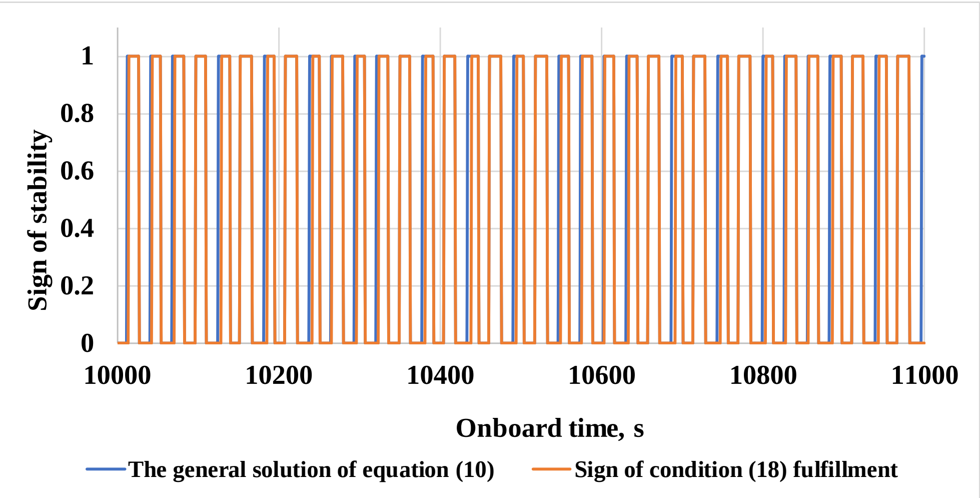

Fig. 9 – The sign of the control stability at each time sample (1 time sample = 0.1 s) using ***Complex stability condition***, where: 1 – stable, 0 – instable.

Analyzing obtained results, it can be seen, that the attitude control system based on mobile control algorithm (12) – (13) of the power satellite with flexible oscillating element provides time-periodical stable solutions at time periods when all conditions of ***Complex stability condition*** are satisfied**.** From one hand, it can be concluded that despite of the time-periodic switching of control loops and the influence of elastic oscillations the control system has time-periodic stability and can maintain it for long time intervals. From the other hand it corresponds to the definition of "Conditionally Stable Systems" when stability conditions are determined according to the proposed ***Complex stability condition.***

Conclusions. The initial studies of the peculiarities of mobile control implementation algorithms for spacecraft with flexible structural elements are carried out. It has been formalized the generalized mathematical model of power spacecraft angular motion considering impact of the flexible elements fluctuations. The influence of elastic oscillations on the control accuracy when using mobile control algorithms is shown. Using mathematical and computer modeling of the spacecraft with elastic oscillating structural element motion it has been formed the general conditions of control stability. These stability conditions are a special case when modeling oscillations as free harmonic for one oscillating element. In the presence of several oscillating elements, these conditions need to be specified. In turn, the proposed approaches for stability evaluations on small time intervals can be adapted to various models of spacecraft dynamics that have elastic oscillating structural elements, which gives directions for further research.

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