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LARGE DEVIATION PRINCIPLE FOR ONE-DIMENSIONAL SDES WITH DISCONTINUOUS COEFFICIENTS

We discuss the large deviation principle for one-dimensional SDEs with discontinuous coefficients. It is shown that the discontinuity of coefficients leads, in general, to the LDP asymptotics with a rate function which differs from the rate function in the standard Freidlin–Wentzell theorem.

1. Introduction

In this paper, we discuss the *large deviation principle* (LDP) for a family X^n of solutions to one-dimensional stochastic differential equations (SDEs) of the form

$$dX_t^n = a(X_t^n)dt + \frac{1}{\sqrt{n}}\sigma(X_t^n)dW_t \tag{1}$$

with initial conditions $X_0^n = x_0$ under weak assumptions on the coefficients a and σ. The LDP for a diffusion process with continuous coefficients is a well-known result (see [1], Theorem 4.1). The case of discontinuous coefficients was studied in works of Chiang and Sheu (see [2], [3]), where multidimensional SDEs with coefficients possessing a discontinuity of the jump type along a fixed hyperplane were considered, and Krykun (see [4]), where one-dimensional SDEs were analyzed.
Here, we prove the LDP for a family of solutions to (1) under the conditions on coefficients, which allow rather general forms of discontinuity: to obtain our main result, we made assumptions that the set Δ_σ of discontinuity points of σ has zero Lebesgue measure, and $\frac{a}{\sigma^2}$ has bounded derivative. These conditions are weaker than those in [2] and [3], which mean for (1) just that σ and a have discontinuities of the jump type at a single point. On the other hand, we consider a one-dimensional SDE which makes our model more restrictive than those studied in [2], [3]. The condition for $\frac{a}{\sigma^2}$ to have bounded derivative is a restriction in comparison with [4], where a and σ^2 are not supposed to be adjusted in such a way.
The main idea of the proof of our result is to use the fact that the large deviation principle holds for the family $\{Y^n\}$ of solutions to SDEs with zero coefficient a (see [5]), using a combination of the Bryc formula and the Varadhan lemma (see, e.g., [6], Proposition 3.8). With the use of the Bryc formula $\Lambda(f) = \lim\limits_{n\to\infty} \frac{1}{n}\log Ee^{nf(X^n)}$ for the sequence $\{X^n\}$ and the Girsanov theorem on a change of the measure, the rate transform can be written in terms of the family $\{Y^n\}$. By combinating the Bryc formula and the Varadhan lemma, we will obtain then the rate function for $\{X^n\}$.
The difference from the Freidlin–Wentzell theorem in the case of continuous coefficients consists in that the functional $Q(x) = \frac{1}{2}\int_0^T \frac{(a(x_s)-\dot{x}_s)^2}{\sigma^2(x_s)}ds$, which is the rate function in the continuous case, is not lower semicontinuous, and the resulting rate function is a lower semicontinuous hull of Q.

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Even in the one-dimensional case, this difference shows that, in a more general situation, the rate function may be different from the standard Freidlin–Wentzell one.

2. Main theorem

Let X^n, $n = 1, 2, ...$, be a sequence of $\mathbb{X}$-valued random variables on a complete separable metric space $(\mathbb{X}, \rho)$. We recall some standard definitions (see, e.g., [6], Chapter 3.1).

Definition 2.1. The family $\{X^n\}$ satisfies the *large deviation principle* with the rate function $I : \mathbb{X} \to [0, \infty]$ if, for every open set A,

$$\liminf_{n\to\infty} \frac{1}{n} \log P\{X^n \in A\} \geq -\inf_{x\in A} I(x), \tag{2}$$

and, for every closed set B,

$$\limsup_{n\to\infty} \frac{1}{n} \log P\{X^n \in B\} \leq -\inf_{x\in B} I(x). \tag{3}$$

If (2) holds, and if (3) holds for every compact set only, then $\{X^n\}$ is said to satisfy the *weak large deviation principle.*
The rate function I is called *"good"* if, for every $b \in [0, \infty)$, the set $\{x : I(x) \leq b\}$ is compact.

The main result of this paper is given by the following theorem. Let $\mathbb{X} = \mathcal{C}([0, T])$ with the uniform metric $\rho(x, y) = \sup_{t\in[0,T]} |x(t) - y(t)|$, and let $\{X^n\}$ be a family of random variables in $\mathcal{C}([0, T])$.

Theorem 2.1. *Let $\sigma(x)$ and $a(x)$ be measurable and such that $\frac{a}{\sigma^2}$ has bounded derivative. Assume also that σ is bounded, separated from zero, and such that the set Δ_σ of discontinuity points of σ has zero Lebesgue measure.*
Then the family $\{X^n\}$ satisfies LDP with a good rate function J, which equals

$$J(g) = \liminf_{y\to g} Q(y),$$

where

$$Q(y) = \frac{1}{2} \int_0^T \frac{(a(y_s) - \dot{y}_s)^2}{\sigma^2(y_s)} ds$$

if $y \in \mathcal{C}([0, T])$ is an absolutely continuous function with $y(0) = x_0$, $\dot{y} \in L_2([0, T])$, and $Q(y) = \infty$ otherwise.

Note that the rate function is different here from that in the case of continuous coefficients studied by Freidlin and Wentzell ([1], Chapter 7, §4). The reason for this is the fact that $Q(y) = \frac{1}{2} \int_0^T \frac{(a(y_s)-\dot{y}_s)^2}{\sigma^2(y_s)} ds$ is not lower semicontinuous, in general. This is illustrated by the following example.

Example 2.1. Let $T = 1$, $x_0 = 0$, and

$$\sigma(y) = \begin{cases} c_1, & y < 0 \\ c_2, & y \geq 0 \end{cases}, \quad 0 < c_1 < c_2, \quad a(y) = \sigma^2(y).$$

Then the conditions of Theorem 2.1 are satisfied. Consider a sequence

$$y_n(t) = \begin{cases} -\frac{t}{n}, & t \in [0, \frac{1}{2}] \\ -\frac{1}{2n}, & t \in [\frac{1}{2}, 1] \end{cases}.$$

We have $y_n \to y_0 \equiv 0$ as $n \to \infty$. For this sequence, the function Q is equal to

$$Q(y_n) = \frac{1}{2}\int_0^T \frac{(a(y_n(s)) - \dot{y}_n(s))^2}{\sigma^2(y_n(s))}ds = \frac{1}{2}\int_0^{1/2} \frac{(\sigma^2(-\frac{s}{n}) + \frac{1}{n})^2}{\sigma^2(-\frac{s}{n})}ds + \frac{1}{2}\int_{1/2}^1 \sigma^2(-\frac{1}{2n})ds =$$

$$= \frac{1}{2}\left(\int_0^{1/2} \sigma^2(-\frac{s}{n})ds + \int_0^{1/2} \frac{2}{n}ds + \int_0^{1/2} \frac{1}{n\sigma^2(-\frac{s}{n})}ds + \int_{1/2}^1 \sigma^2(-\frac{1}{2n})ds\right) =$$

$$= \frac{1}{2}\left(c_1^2 + \frac{1}{n} + \frac{1}{2n}\frac{1}{c_1^2}\right) \to \frac{c_1^2}{2} \text{ as } n \to \infty,\ Q(y_0) = \frac{1}{2}\int_0^1 \sigma^2(0)ds = \frac{c_2^2}{2}.$$

So, we obtain $\liminf_{n\to\infty} Q(y_n) < Q(\lim_{n\to\infty} y_n)$, which means that the function Q is not lower semicontinuous.

Theorem 2.1 will be proved in Section 4. We now recall some statements, which will be used below.

3. Prerequisites

3.1. **LDP in the case $a \equiv 0$.** In this section, we formulate LDP for the stochastic differential equations (1) with $a \equiv 0$. Consider the sequence of SDEs of the form

$$dY_t^n = \frac{1}{\sqrt{n}}\sigma(Y_t^n)dW_t, \quad t \in [0,T] \tag{4}$$

with initial conditions $Y_0^n = y_0$. The following theorem gives a result for a family of solutions to (4) under weak assumptions on the diffusion coefficient σ.

Proposition 3.1. *Let σ be measurable, bounded, and separated from zero. Assume also that the set Δ_σ of discontinuity points of σ has zero Lebesgue measure.*
Then the family $\{Y^n\}$ satisfies LDP with a good rate function I, which equals

$$I(g) = \frac{1}{2}\int_0^T \frac{(\dot{g}(t))^2}{\sigma^2(g(t))}dt, \tag{5}$$

if $g \in \mathcal{C}([0,T])$ is an absolutely continuous function with $g(0) = y_0$, $\dot{g} \in L_2([0,T])$, and $I(g) = \infty$ otherwise.

In [5], a similar statement was proved for a sequence of random variables taking values in $\mathbb{X} = \mathcal{C}([0,\infty))$ and defined by the solutions to SDEs similar to (5) considered on the whole semiaxis $[0,\infty)$. In view of this statement, Proposition 3.1 follows directly from the contraction principle (see, e.g., [6], Lemma 3.11) applied to the natural projection from the space $\mathcal{C}([0,\infty))$ onto $\mathcal{C}([0,T])$.

3.2. **The Varadhan lemma and the Bryc formula.** In this section, we recall two statements (see, e.g., [6], Proposition 3.8), which will be used below.
Let $\{X^n\}$ be a sequence of $\mathbb{X}$-valued random variables.

Proposition 3.2. *(Varadhan Lemma) Suppose that $\{X^n\}$ satisfies the large deviation principle with good rate function I. Then, for each $f \in C_b(\mathbb{X})$,*

$$\lim_{n\to\infty}\frac{1}{n}\log Ee^{nf(X^n)} = \sup_{x\in\mathbb{X}}\{f(x) - I(x)\}.$$

Proposition 3.3. *(Bryc formula) Suppose that the sequence $\{X^n\}$ is exponentially tight, and the rate transform*

$$\Lambda(f) = \lim_{n\to\infty}\frac{1}{n}\log Ee^{nf(X^n)} \tag{6}$$

exists for each $f \in C_b(\mathbb{X})$. *Then* $\{X^n\}$ *satisfies the large deviation principle with good rate function*

$$I(x) = \sup_{f \in C_b(\mathbb{X})} \{f(x) - \Lambda(f)\}.$$

4. Proof of Theorem 2.1

In this section, we prove Theorem 2.1. Auxiliary statements, which will be used in the proof, are proved in Appendix A.

The main idea of the proof of the theorem is to use the fact that the large deviation principle holds for the family $\{Y^n\}$ of solutions to (4). We will need the Girsanov theorem on a change of the measure and the statements from section 3.

Let us write the rate transform from the Bryc formula $\Lambda(f) = \lim_{n\to\infty} \frac{1}{n} \log E e^{nf(X^n)}$ for the sequence $\{X^n\}$. Recall that, by the Girsanov theorem (see, e.g., [7], Chapter 4, §4.1), if we take a solution Y^n to (4) for a fixed n with some Wiener process $\widetilde{W}$ and put

$$P(X^n \in B) = E\left(1_{Y^n \in B} \exp\left\{\int_0^T \alpha_n(Y_s^n) d\widetilde{W}_s - \frac{1}{2}\int_0^T \alpha_n^2(Y_s^n)\, ds\right\}\right), B \in \mathcal{B}(\mathcal{C}[0,T]),$$

with $\alpha_n(y) = \sqrt{n}(a(y)/\sigma(y))$, then X^n is a weak solution to (1). Hence, the rate transform $\Lambda(f) = \lim_{n\to\infty} \frac{1}{n} \log E e^{nf(X^n)}$ for the family $\{X^n\}$ (see (6)) can be written as

$$\Lambda(f) = \lim_{n\to\infty} \frac{1}{n} \log E \exp\left\{n\left[f(Y^n) - \frac{1}{2}\int_0^T (\frac{a}{\sigma})^2(Y_s^n) ds\right] + \sqrt{n}\int_0^T \frac{a}{\sigma}(Y_s^n) d\widetilde{W}_s\right\}.$$

Denote $A(x) = \int_0^x \frac{a}{\sigma^2}(y)dy$. Using Itô's formula for A, we obtain the following form of the rate transform:

$$\Lambda(f) = \lim_{n\to\infty} \frac{1}{n} \log E \exp\{n\left[f(Y^n) - \frac{1}{2}\int_0^T (\frac{a}{\sigma})^2(Y_s^n)ds + A(Y_T^n) - A(Y_0^n)\right] -$$

$$-\frac{1}{2}\int_0^T \sigma^2(Y_s^n)(\frac{a}{\sigma^2})'(Y_s^n)ds\}. \tag{7}$$

Under the assumption that $(\frac{a}{\sigma^2})'$ is bounded, we can omit the last term in (7) (see Lemma 1 in Appendix). We know that the sequence $\{Y^n\}$ satisfies the large deviation principle with good rate function I defined by (5). Moreover, for every $f \in C_b(\mathcal{C}[0,T])$, the function

$$g(y) = f(y) + A(y_T) - A(y_0) - \frac{1}{2}\int_0^T (\frac{a}{\sigma})^2(y_s)ds, \quad y \in \mathcal{C}[0,T]$$

belongs to $C_b(\mathcal{C}[0,T])$ as well. Then, applying the Varadhan lemma, we get that the rate transform for the family $\{X^n\}$ has the form

$$\Lambda(f) = \sup_{x\in\mathcal{C}[0,T]} \{g(x) - I(x)\} = \sup_{x\in\mathcal{C}[0,T]} \left\{g(x) - \frac{1}{2}\int_0^T (\frac{\dot{x}_s}{\sigma(x_s)})^2 ds\right\} =$$

$$= \sup_{x\in\mathcal{C}[0,T]} \left\{f(x) + A(x_T) - A(x_0) - \frac{1}{2}\int_0^T (\frac{a}{\sigma})^2(x_s)ds - \frac{1}{2}\int_0^T (\frac{\dot{x}_s}{\sigma(x_s)})^2 ds\right\}.$$

This expression can be transformed as

$$A(x_T) - A(x_0) = \int_{x_0}^{x_T} (\frac{a}{\sigma^2})(y)dy.$$

After the change of variables $y = x_s$, we obtain $A(x_T) - A(x_0) = \int_0^T \frac{a(x_s)\dot{x}_s}{\sigma^2(x_s)} ds$. Therefore, the rate transform takes the form

$$\Lambda(f) = \sup_{x \in \mathcal{C}[0,T]} \left\{ f(x) - \frac{1}{2} \int_0^T \frac{(a(x_s) - \dot{x}_s)^2}{\sigma^2(x_s)} ds \right\} = \sup_{x \in \mathcal{C}[0,T]} \{ f(x) - Q(x) \}.$$

The Bryc formula states that if the rate transform for $\{X^n\}$ exists for each $f \in \mathcal{C}_b(\mathbb{X})$, then the sequence $\{X^n\}$ satisfies the large deviation principle with the good rate function $J(y) = \sup_{f \in \mathcal{C}_b(\mathbb{X})} \{f(y) - \Lambda(f)\}$. Recall that the functional Q is not necessarily lower semicontinuous. Because of that, we introduce the function $\widetilde{Q}(x) = \liminf_{y \to x} Q(y)$, i.e., the lower semicontinuous hull of the functional Q. To finalize the proof, we show that

$$\sup_{x \in \mathcal{C}[0,T]} \{ f(x) - Q(x) \} = \sup_{x \in \mathcal{C}[0,T]} \{ f(x) - \widetilde{Q}(x) \}$$

(see Lemma 2 in Appendix), and that functionals $\widetilde{Q}(y)$ and $J(y)$ are equal (see Lemma 3 in Appendix). Together with Lemma 4 in Appendix, which shows that that the rate function J is good, this completes the proof of Theorem 2.1.

Appendix A. Auxiliary statements

Here, we prove some statements, which were used in the proof of Theorem 2.1. The following Lemma gives the opportunity to omit the last term in (7).

Lemma A.1. *If $\frac{a}{\sigma^2}$ has bounded derivative, then*

$$\lim_{n \to \infty} \frac{1}{n} \log E \exp \left\{ n \left[f(Y^n) - \frac{1}{2} \int_0^T (\frac{a}{\sigma})^2 (Y_s^n) ds + A(Y_T^n) - A(Y_0^n) \right] - \right.$$
$$\left. -\frac{1}{2} \int_0^T \sigma^2(Y_s^n) (\frac{a}{\sigma^2})'(Y_s^n) ds \right\} =$$
$$= \lim_{n \to \infty} \frac{1}{n} \log E \exp \left\{ n \left[f(Y^n) - \frac{1}{2} \int_0^T (\frac{a}{\sigma})^2 (Y_s^n) ds + A(Y_T^n) - A(Y_0^n) \right] \right\}$$

Proof. Suppose that $|\sigma^2(\frac{a}{\sigma^2})'| \le C$. Then $-\frac{C}{2}T \le -\frac{1}{2}\int_0^T \sigma^2(Y_s^n)(\frac{a}{\sigma^2})'(Y_s^n) ds \le \frac{C}{2}T$.

$$E \exp \left\{ n \left[f(Y^n) - \frac{1}{2} \int_0^T (\frac{a}{\sigma})^2 (Y_s^n) ds + A(Y_T^n) - A(Y_0^n) \right] - \right.$$
$$\left. -\frac{1}{2} \int_0^T \sigma^2(Y_s^n) (\frac{a}{\sigma^2})'(Y_s^n) ds \right\} \le$$
$$\le e^{\frac{C}{2}T} E \exp \left\{ n \left[f(Y^n) - \frac{1}{2} \int_0^T (\frac{a}{\sigma})^2 (Y_s^n) ds + A(Y_T^n) - A(Y_0^n) \right] \right\},$$

and the inverse inequality holds true with $e^{-\frac{C}{2}T}$. If we take log of both sides, divide the inequalities by n, and tend $n \to \infty$, then the terms, which correspond to $e^{\pm \frac{C}{2}T}$, vanish. □

Lemma A.2. *The following equality takes place:*

$$\sup_{x \in \mathcal{C}[0,T]} \{ f(x) - Q(x) \} = \sup_{x \in \mathcal{C}[0,T]} \{ f(x) - \widetilde{Q}(x) \}.$$

Proof. Since $\widetilde{Q}(x) \le Q(x)$, we obtain the inequality

$$\sup_{x\in\mathcal{C}[0,T]}\{f(x)-Q(x)\} \le \sup_{x\in\mathcal{C}[0,T]}\{f(x)-\widetilde{Q}(x)\}.$$

To prove the inverse inequality, we denote $s = \sup_{x\in\mathcal{C}[0,T]}\{f(x)-\widetilde{Q}(x)\}$.
For every fixed $\varepsilon > 0$, there exists x_ε, such that

$$f(x_\varepsilon) - \widetilde{Q}(x_\varepsilon) \ge s - \varepsilon.$$

Since $\widetilde{Q}(x_\varepsilon) = \liminf_{y\to x_\varepsilon} Q(y)$, there exists a sequence $y_n \to x_\varepsilon$ such that

$$Q(y_n) \to \widetilde{Q}(x_\varepsilon), \text{ as } n \to \infty.$$

Then $f(y_n) - Q(y_n) \to f(x_\varepsilon) - \widetilde{Q}(x_\varepsilon) \ge s - \varepsilon$, as $n \to \infty$.
Taking the supremum on both sides of the inequality, we obtain $\sup_y\{f(y)-Q(y)\} \ge s-\varepsilon$.
Tending $\varepsilon \to 0$, we obtain

$$\sup_{x\in\mathcal{C}[0,T]}\{f(x)-Q(x)\} \ge s = \sup_{x\in\mathcal{C}[0,T]}\{f(x)-\widetilde{Q}(x)\},$$

and the proof is completed. □

Lemma A.3. *The functionals $\widetilde{Q}(y)$ and $J(y)$ are equal.*

Proof. For each $y \in \mathcal{C}[0,T]$ and $f \in C_b(\mathcal{C}[0,T])$, we have $\Lambda(f) \ge f(y) - \widetilde{Q}(y)$, and, therefore, $f(y) - \Lambda(f) \le \widetilde{Q}(y)$. Taking the supremum w.r.t. f on both sides of the inequality, we obtain

$$J(y) \le \widetilde{Q}(y), \quad y \in \mathcal{C}[0,T].$$

We now show that $J(y) \ge \widetilde{Q}(y)$ for a fixed y. To do that, it is enough to construct, for every fixed $\varepsilon > 0$, a function $f = f_\varepsilon \in C_b(\mathcal{C}[0,T])$ such that

$$f(y) - \Lambda(f) > \widetilde{Q}(y) - \varepsilon. \tag{8}$$

Consider the sequence of functions $h_N \in C_b(\mathcal{C}[0,T]), N \ge 1$, defined by

$$h_N(x) = \begin{cases} -N^3\rho(x,y)^2, & \rho(x,y) < \frac{1}{N} \\ -N, & \rho(x,y) \ge \frac{1}{N} \end{cases}, \quad x \in \mathcal{C}[0,T].$$

Let us show that, for every $\varepsilon > 0$, there exists N such that (8) holds true for $f = h_N$. If this fails, then there exists $\varepsilon_1 > 0$ such that

$$h_N(y) - \Lambda(h_N) \le \tilde{Q}(y) - \varepsilon_1, \quad N \ge 1. \tag{9}$$

Recall that $\Lambda(f) = \sup_{x\in\mathcal{C}[0,T]}(f(x) - Q(x))$. Hence, for every $N \ge 1$, there would exist $x_N \in \mathcal{C}[0,T]$ such that

$$h_N(y) - (h_N(x_N) - Q(x_N)) \le \tilde{Q}(y) - \varepsilon_2, \quad N \ge 1,$$

where $\varepsilon_2 = \varepsilon_1/2 > 0$. We have $h_N(y) = 0$ by the construction of h_N. Hence, we can rewrite the above inequalities as

$$Q(x_N) - h_N(x_N) \le \tilde{Q}(y) - \varepsilon_2, \quad N \ge 1. \tag{10}$$

Next, $h_N(x) \le -N$ if $\rho(x,y) \ge 1/N$. Because $Q(x) \ge 0, x \in \mathcal{C}[0,T]$, this means that

$$\rho(x_N, y) < \frac{1}{N}, \quad N > \tilde{Q}(y) - \varepsilon_2.$$

Therefore, $x_N \to y$ in $\mathcal{C}[0,T]$, and, by the definition of $\tilde{Q}$,

$$\liminf_{N\to\infty} Q(x_N) \ge \liminf_{x\to y} Q(x) = \tilde{Q}(y).$$

The latter relation contradicts (10): because $h_N(x) \le 0, x \in \mathcal{C}[0,T]$, we have, by (10),

$$\liminf_{N\to\infty} Q(x_N) \le \liminf_{N\to\infty} \Big(Q(x_N) - h_N(x_N) \Big) \le \tilde{Q}(y) - \varepsilon_2.$$

This contradiction shows that assumption (9) fails. This proves (8) and completes the proof. □

Lemma A.4. *The rate function J is good.*

Proof. The function $J(g) = \liminf_{x\to g} \frac{1}{2}\int_0^T \frac{(a(x_s)-\dot{x}_s)^2}{\sigma^2(x_s)} ds$ is lower semicontinuous, which means that, for every level set $B = \{x : J(x) \le b\}$, it is closed. Using the Cauchy–Schwarz inequality, one can write an increment of the function x as follows:

$$|x(t) - x(s)| = |\int_s^t \dot{x}_p dp| \le \sqrt{(t-s)\int_s^t (\dot{x}_p)^2 dp}.$$

Since a and σ are bounded, $\int_0^T (\dot{x}_s)^2 ds$ is also bounded for each $x \in B$, and the family B is equicontinuous. Since, for each $x \in B$, $x(0) = 0$, the functions in the family B are bounded at one point. According to the Arzelá-Ascoli theorem, the level set is compact, and the rate function J is good. □

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