К О Р О Т К І   П О В І Д О М Л Е Н Н Я

UDC 512.5

A. Wang (Northwest Univ., China and Chongqing Univ. Technology, China),
L. Wang (Chongqing Univ. Technology, China)

CONGRUENCES ON REGULAR SEMIGROUPS WITH Q-INVERSE TRANSVERSALS*

КОНГРУЕНЦІЇ НА РЕГУЛЯРНИХ НАПІВГРУПАХ З Q-ОБЕРНЕНИМИ ТРАНСВЕРСАЛЯМИ

We give congruences on a regular semigroup with a Q-inverse transversal S^o by the congruence pair (abstractly), which consists of congruences on the structural component parts R and Λ. We prove that the set of all congruences for this kind of semigroups is a complete lattice.

Наведено конгруенції на регулярних напівгрупах з Q-оберненою трансверсаллю S^o щодо пари конгруенцій (абстрактно), сформованої з конгруенцій на частинах структурних компонент R і Λ. Доведено, що множина всіх конгруенцій для такого типу напівгруп є повною ґраткою.

1. Introduction and preliminaries. The multiplicative inverse transversals of a regular semigroup were first introduced by Blyth and McFadden in 1982 [1]. An inverse subsemigroup S° of a regular semigroup S is an *inverse transversal* if $|V(x)\cap S^\circ| = 1$ for any $x \in S$, where $V(x)$ denoted the set of inverses of x. In this case, the unique element of $V(x)\cap S^\circ$ is denoted by x° and $(x^\circ)^\circ$ is denoted by $x^{\circ\circ}$. Throughout this paper S denotes a regular semigroup with an inverse transversal S° and $E(S^\circ) = E^\circ$ denotes the semilattice of idempotents of S°. An inverse transversal S° is a *multiplicative inverse transversal* if $x^\circ xyy^\circ \in E^\circ$, and S° is a Q*-inverse transversal* if $S^\circ S S^\circ \subseteq S^\circ$.

Let S° be a Q-inverse transversal, and let

$$R = \{x \in S \mid x^\circ x = x^\circ x^{\circ\circ}\},\ L = \{a \in S \mid aa^\circ = a^{\circ\circ}a^\circ\},$$

and

$$I = \{e \in E(S) \mid ee^\circ = e\},\ \Lambda = \{f \in E(S) \mid f^\circ f = f\},$$

where $E(S) = \{x \in S \mid x^2 = x\}$ which is the set of idempotents of S. It was shown in [6] that R and L are orthodox subsemigroups of S with transversal S° which is a right ideal of R and a left ideal of L and that $E(R) = I,\ E(L) = \Lambda$. Moreover, E° is a multiplicative inverse transversal of I and Λ. In [3], McAlister and McFadden show that I and Λ are $\mathcal{R}$-unipotent and $\mathcal{L}$-unipotent subbands respectively of S. Saito gave the structure theory of a regular semigroup with a Q-inverse transversal in [6]. The congruences on regular semigroups with inverse transversals were studied using the congruence triple by Wang and Tang (see [5, 9, 10]). In this paper, we give the congruences on regular semigroups with Q-inverse transversals by the congruence pair and prove that the set of all congruences on this kind of semigroups is a complete lattice.

We list already obtained results in [3 – 6], which will be used in this paper.

* Research was supported by the Project Foundation of Chongqing Municipal Education Committee (KJ1500925).

Lemma 1.1. *Let S° be a Q-inverse transversal. Then, for $e \in E^\circ,\ f \in I$ $[resp.\ g \in \Lambda]$, $e^\circ f = e^\circ f^\circ [resp.\ ge^\circ = g^\circ e^\circ]$.*

Lemma 1.2. *Let S° be a Q-inverse transversal. Then $(xy)^\circ = (x^\circ xy)^\circ x^\circ = y^\circ (xyy^\circ)^\circ$ for every $x, y \in S$.*

Lemma 1.3. *S° is a Q-inverse transversal if and only if for any $s, t \in S^\circ,\ x \in S,\ (sxt)^\circ = t^\circ x^\circ s^\circ$.*

Lemma 1.4. *Let S be a band with an inverse transversal S°. If S° is a left ideal of S, then $e^\circ e = e$ for every $e \in S$. In this case, S is an $\mathcal{L}$-unipotent band.*

For a regular semigroup S, $\mathrm{Con}\,(S)$ denotes the complete lattice of congruences on S. For $\rho \in \mathrm{Con}\,(S)$, let $\rho^\circ = \rho|_{S^\circ},\ \rho_I = \rho|_I,\ \rho_\Lambda = \rho|_\Lambda$.

Lemma 1.5. *Let S° be a Q-inverse transversal. For any $\rho \in \mathrm{Con}\,(S)$ and for $x, y \in S,\ x\rho y$ implies $x^\circ \rho^\circ y^\circ$.*

Lemma 1.6. *Let R be a regular semigroup with a right ideal inverse transversal S°. Suppose that Λ is a band with a left ideal inverse transversal E°. Let $\Lambda \times R \longrightarrow S^\circ$ described by $(\lambda, x) \longrightarrow \lambda * x$ be a mapping, such that for any $x, y \in R$ and for any $\lambda, \mu \in \Lambda$:*

(Q_1) $(\lambda * x)y = \lambda * (xy)$ and $\mu(\lambda * x) = (\mu\lambda) * x$;

(Q_2) if $x \in E^\circ$ or $\lambda \in E^\circ$, then $\lambda * x = \lambda x$.

Define a multiplication on the set

$$\Gamma \equiv R| \times |\Lambda = \{(x, \lambda) \in R \times \Lambda : x^\circ x = \lambda^\circ\}$$

by

$$(x, \lambda)(y, \mu) = (x(\lambda * y), (\lambda * y)^\circ (\lambda * y)\mu).$$

Then Γ is a regular semigroup with a Q-inverse transversal which is isomorphic to S°.

Conversely, every regular semigroup with a Q-inverse transversal can be constructed in this way.

2. Main results. In this section, we first establish a characterization of congruences abstractly by congruence pair. We describe a congruence pair of the form (ρ^R, ρ^Λ) with $\rho^R \in \mathrm{Con}\,(R)$ and $\rho^\Lambda \in \mathrm{Con}\,(\Lambda)$ satisfying some conditions in order that they produce a congruence on S naturally.

Let S° be a Q-inverse transversal. For $\rho \in \mathrm{Con}\,(S)$, let $\rho_R = \rho|_R$. The following lemma shows that ρ_R is determined uniquely by its restrictions to S° and I.

Lemma 2.1. *For $\rho, \sigma \in \mathrm{Con}\,(S),\ \rho_R \subseteq \sigma_R \Leftrightarrow \rho^\circ \subseteq \sigma^\circ,\ \rho_I \subseteq \sigma_I$. Therefore,*

$$\rho_R = \sigma_R \Leftrightarrow \rho^\circ = \sigma^\circ,\ \rho_I = \sigma_I.$$

Proof. Suppose that $\rho^\circ \subseteq \sigma^\circ, \rho_I \subseteq \sigma_I$. Then for any $x, y \in S$, by Lemma 1.5,

$$x\rho_R y \Longrightarrow x^\circ \rho^\circ y^\circ,\ x, y \in R \Longrightarrow$$

$$\Longrightarrow x^{\circ\circ} \rho^\circ y^{\circ\circ},\ x^\circ x^{\circ\circ} \rho^\circ y^\circ y^{\circ\circ},\ xx^\circ \rho_I yy^\circ,\ x, y \in R \Longrightarrow$$

$$\Longrightarrow x^{\circ\circ} \rho^\circ y^{\circ\circ},\ x^\circ x \rho^\circ y^\circ y,\ xx^\circ \rho_I yy^\circ,\ x, y \in R \Longrightarrow$$

$$\Longrightarrow x^{\circ\circ} \sigma^\circ y^{\circ\circ},\ x^\circ x \sigma^\circ y^\circ y,\ xx^\circ \sigma_I yy^\circ,\ x, y \in R \Longrightarrow$$

$$\Longrightarrow x = xx^\circ x^{\circ\circ} x^\circ x \sigma yy^\circ y^{\circ\circ} y^\circ y = y,\ x, y \in R \Longrightarrow x\sigma_R y.$$

So $\rho_R \subseteq \sigma_R$. The reverse implication is obvious.

Lemma 2.1 is proved.

Suppose ρ^R and ρ^Λ are congruences on R and Λ, respectively. Then (ρ^R, ρ^Λ) is called a *congruence pair* for Γ if the following conditions hold:

(C_1) $\rho^R|_{E^\circ} = \rho^\Lambda|_{E^\circ}$;

(C_2) $(\forall\, z \in R)(\forall\, \lambda, \mu \in \Lambda)\ \lambda\rho^\Lambda\mu \Rightarrow (\lambda * z)\rho^R(\mu * z)$;

(C_3) $(\forall\, \nu \in \Lambda)(\forall\, x, y \in R)\ x\rho^R y \Rightarrow (\nu * x)\rho^R(\nu * y)$.

Define $\rho^{(\rho^R,\rho^\Lambda)}$ on Γ by

$$(x,\lambda)\rho^{(\rho^R,\rho^\Lambda)}(y,\mu) \Leftrightarrow x\rho^R y,\ \lambda\rho^\Lambda\mu.$$

Theorem 2.1. *Let Γ be a regular semigroup having a Q-inverse transversal as in Lemma* 1.6, *and (ρ^R, ρ^Λ) be a congruence pair on Γ. Then $\rho^{(\rho^R,\rho^\Lambda)}$ is a congruence on Γ.*

Conversely, every congruence on Γ can be constructed in the above manner.

Proof. Let (ρ^R, ρ^Λ) be a congruence pair on Γ. Obviously, $\rho^{(\rho^R,\rho^\Lambda)}$ is an equivalence on Γ. For $(x,\lambda), (y,\mu) \in \Gamma$, with $(x,\lambda)\rho^{(\rho^R,\rho^\Lambda)}(y,\mu)$, we have $x\rho^R y,\ \lambda\rho^\Lambda\mu$. Let $z \in R$ and $\nu \in \Lambda$ be such that $(z,\nu) \in \Gamma$. By Lemmas 1.2, 1.3 and condition (C_2), we have

$$(x(\lambda * z))^\circ = (x^\circ x(\lambda * z))^\circ x^\circ =$$

$$= (\lambda * z)^\circ (x^\circ x(\lambda * z)(\lambda * z)^\circ)^\circ x^\circ \rho^\circ (\mu * z)^\circ (y^\circ y(\mu * z)(\mu * z)^\circ)^\circ y^\circ =$$

$$= (y(\mu * z))^\circ$$

and

$$(x(\lambda * z))(x(\lambda * z))^\circ \rho^I (y(\mu * z))(y(\mu * z))^\circ.$$

Thus, by Lemma 2.1,

$$x(\lambda * z)\rho^R y(\mu * z).$$

From condition $(C_2,)$ we have $(\lambda * z)\rho(\mu * z)$ and so $(\lambda * z)^\circ\rho^\circ(\mu * z)^\circ$. It follows that

$$(\lambda * z)^\circ(\lambda * z)\rho^\Lambda(\mu * z)^\circ(\mu * z).$$

Hence

$$(\lambda * z)^\circ(\lambda * z)\nu\rho^\Lambda(\mu * z)^\circ(\mu * z)\nu.$$

Thus

$$(x,\lambda)(z,\nu)\rho^{(\rho^R,\rho^\Lambda)}(y,\mu)(z,\nu).$$

Next, by $x\rho^R y$ and condition (C_3), we have

$$(\nu * x)\rho^R(\nu * y).$$

It follows that

$$z(\nu * x)\rho^R z(\nu * y)$$

and

$$(\nu * x)^\circ(\nu * x)\lambda\rho^\Lambda(\nu * y)^\circ(\nu * y)\mu.$$

Hence

$$(z(\nu * x), (\nu * x)^\circ(\nu * x)\lambda)\rho^{(\rho^R,\rho^\Lambda)}(z(\nu * y), (\nu * y)^\circ(\nu * y)\mu).$$

That is,

$$(z,\nu)(x,\lambda)\rho^{(\rho^R,\rho^\Lambda)}(z,\nu)(y,\mu).$$

Therefore $\rho^{(\rho^R,\rho^\Lambda)}$ is a congruence on Γ.

Conversely, assume that ρ is a congruence on Γ. We define the following equivalences on R and Λ, respectively:

$$(\forall\ x, y \in R) \quad x\rho_R y \Leftrightarrow (x, x^\circ x)\rho(y, y^\circ y),$$
$$(\forall\ \lambda, \mu \in \Lambda) \quad \lambda\rho_\Lambda\mu \Leftrightarrow (x^\circ x, \lambda)\rho(y^\circ y, \mu).$$

Since ρ is a congruence on Γ, we have ρ_R and ρ_Λ are equivalences on R and Λ, respectively.

Let $(x, \lambda), (y, \mu), (x_1, \lambda_1), (y_1, \mu_1) \in \Gamma$. If $x\rho_R y$ and $x_1\rho_R y_1$, then

$$(x, x^\circ x)\rho(y, y^\circ y) \text{ and } (x_1, x_1^\circ x_1)\rho(y_1, y_1^\circ y_1).$$

Now we immediately get

$$(x, x^\circ x)(x_1, x_1^\circ x_1)\rho(y, y^\circ y)(y_1, y_1^\circ y_1).$$

And this implies that

$$(x(x^\circ x * x_1), (x^\circ x * x_1)^\circ (x^\circ x * x_1)x_1^\circ x_1)\rho(y(y^\circ y * y_1), (y^\circ y * y_1)^\circ (y^\circ y * y_1)y_1^\circ y_1).$$

Then, by Lemma 1.2,

$$(xx_1, (xx_1)^\circ xx_1)\rho(yy_1, (yy_1)^\circ yy_1).$$

So we have proved that $xx_1\rho_R yy_1$.

Suppose that $\lambda\rho_\Lambda\mu$ and $\lambda_1\rho_\Lambda\mu_1$, then we obtain

$$(x^\circ x, \lambda)\rho(y^\circ y, \mu) \text{ and } (x_1^\circ x_1, \lambda_1)\rho(y_1^\circ y_1, \mu_1).$$

Hence

$$(x^\circ x, \lambda)(x_1^\circ x_1, \lambda_1)\rho(y^\circ y, \mu)(y_1^\circ y_1, \mu_1).$$

That is,

$$(x^\circ x(\lambda * x_1^\circ x_1), (\lambda * x_1^\circ x_1)^\circ (\lambda * x_1^\circ x_1)\lambda_1)\rho(y^\circ y(\mu * y_1^\circ y_1), (\mu * y_1^\circ y_1)^\circ (\mu * y_1^\circ y_1)\mu_1).$$

By Lemma 1.4, we have

$$(\lambda\lambda_1^\circ, \lambda\lambda_1)\rho(\mu\mu_1^\circ, \mu\mu_1).$$

Thus, by Lemma 1.1,

$$(\lambda_1^\circ\lambda^\circ, \lambda\lambda_1)\rho(\mu_1^\circ\mu^\circ, \mu\mu_1).$$

So we have proved $\lambda\lambda_1\rho_\Lambda\mu\mu_1$.

And we have the following cases:

(1) $\rho_R|_{E^\circ} = \rho_\Lambda|_{E^\circ}$ is obvious. So condition (C_1) holds.

(2) Let $x,\ y \in R$ and $x\rho_R y$. Then

$$(x, x^\circ x)\rho(y, y^\circ y).$$

Hence

$$(z^\circ z, \nu)(x, x^\circ x)\rho(z^\circ z, \nu)(y, y^\circ y).$$

That is,

$$(z^\circ z(\nu * x), (\nu * x)^\circ (\nu * x)x^\circ x)\rho(z^\circ z(\nu * y), (\nu * y)^\circ (\nu * y)y^\circ y).$$

It follows that

$$z^\circ z(\nu * x)\rho_R z^\circ z(\nu * y).$$

By condition (Q_1) and $z^\circ z = \nu^\circ$, we get

$$(\nu * x)\rho_R(\nu * y).$$

Now condition (C_3) holds.

(3) Let $\lambda,\ \mu \in \Lambda$ and $\lambda\rho_\Lambda\mu$. Then

$$(x^\circ x, \lambda)\rho(y^\circ y, \mu).$$

Hence

$$(x^\circ x, \lambda)(z, \nu^\circ)\rho(y^\circ y, \mu)(z, \nu^\circ).$$

It follows that

$$(x^\circ x(\lambda * z), (\lambda * z)^\circ(\lambda * z)\nu^\circ)\rho(y^\circ y(\mu * z), (\mu * z)^\circ(\mu * z)\nu^\circ).$$

Thus

$$x^\circ x(\lambda * z)\rho_R y^\circ y(\mu * z).$$

By condition (Q_1) and Lemma 1.4, we have

$$(\lambda * z)\rho_R(\mu * z).$$

Now condition (C_2) holds.

Now from the above proof, (ρ_R, ρ_Λ) is a congruence pair on Γ.

By the direct part, $\rho^{(\rho_R,\rho_\Lambda)}$ is a congruence. If $(x, \lambda)\rho^{(\rho_R,\rho_\Lambda)}(y, \mu)$, then we have

$$x\rho_R y, \lambda\rho_\Lambda\mu.$$

Thus

$$(x, x^\circ x)\rho(y, y^\circ y), (x^\circ x, \lambda)\rho(y^\circ y, \mu).$$

Hence

$$(x(x^\circ x * x^\circ x), (x^\circ x * x^\circ x)^\circ(x^\circ x * x^\circ x)\lambda)\rho(y(y^\circ y * y^\circ y), (y^\circ y * y^\circ y)^\circ(y^\circ y * y^\circ y)\mu).$$

By Lemma 1.4,

$$(x, \lambda)\rho(y, \mu).$$

Thus, $\rho^{(\rho_R,\rho_\Lambda)} \subseteq \rho$. Since $\rho \subseteq \rho^{(\rho_R,\rho_\Lambda)}$ is obvious, $\rho^{(\rho_R,\rho_\Lambda)} = \rho$.

Theorem 2.1 is proved.

***Example* 2.1.** Let $R = \{a, b, c\}$ and $\Lambda = \{e, f, g\}$ be semigroups whose multiplication tables given respectively by

.	a	b	c
a	a	b	a
b	b	b	b
c	c	b	c

.	e	f	g
e	e	f	g
f	f	f	f
g	e	f	g

It is clear that R is a regular semigroup with a right ideal inverse transversal $S^\circ = \{b, c\}$. The equivalence ρ^R on R with class $\{a, c\}, \{b\}$ is a nontrivial congruence. Λ is a band with a left ideal inverse transversal $\Lambda^\circ = \{f, g\}$. The equivalence ρ^Λ on Λ with class $\{e, g\}, \{f\}$ is a nontrivial congruence. Moreover, $S^\circ = E(S^\circ) \cong \Lambda^\circ$.

By Lemma 1.6, we have $\Gamma = \{(a, e), (a, g), (b, f), (c, e), (c, g)\}$ is a regular semigroup with a Q-inverse transversal which is isomorphic to S°. It is easy to see that (ρ^R, ρ^Λ) satisfies the conditions (C_1), (C_2) and (C_3), thus (ρ^R, ρ^Λ) is a congruence pair. Using Theorem 2.1, we have the equivalence ρ on Γ with class $\{(a, e), (a, g), (c, e), (c, g)\}$, $\{(b, f)\}$ is a nontrivial congruence.

We denote the set of all congruences on Γ and the set of all congruence pairs on Γ constructed as in Theorem 2.1 by $C(\Gamma)$ and $CT(\Gamma)$.

Lemma 2.2. *If* $(\rho_1^R, \rho_1^\Lambda), (\rho_2^R, \rho_2^\Lambda) \in CT(\Gamma)$, *then*

$$\rho^{(\rho_1^R, \rho_1^\Lambda)} \subseteq \rho^{(\rho_2^R, \rho_2^\Lambda)} \Leftrightarrow \rho_1^R \subseteq \rho_2^R, \rho_1^\Lambda \subseteq \rho_2^\Lambda.$$

Proof. Suppose $\rho^{(\rho_1^R, \rho_1^\Lambda)} \subseteq \rho^{(\rho_2^R, \rho_2^\Lambda)}$. Let $x \rho_1^R y$. By the proof of Theorem 2.1,

$$((x, x^\circ x), (y, y^\circ y)) \in \rho^{(\rho_1^R, \rho_1^\Lambda)} \subseteq \rho^{(\rho_2^R, \rho_2^\Lambda)}.$$

Hence $x \rho_2^R y$, and immediately we get $\rho_1^R \subseteq \rho_2^R$. Similarly, we have $\rho_1^\Lambda \subseteq \rho_2^\Lambda$.

The reverse implication is obvious.

Lemma 2.2 is proved.

Define $\leq$ on $CT(\Gamma)$ by

$$(\rho_1^R, \rho_1^\Lambda) \leq (\rho_2^R, \rho_2^\Lambda) \Leftrightarrow \rho_1^R \subseteq \rho_2^R, \rho_1^\Lambda \subseteq \rho_2^\Lambda.$$

Then $CT(\Gamma)$ is a partial ordered set with respect to $\leq$. By Theorem 2.1 and Lemma 2.2, we can easily see that $C(\Gamma)$ and $CT(\Gamma)$ are isomorphic as partial ordered set.

Proposition 2.1. *Let* $\Omega \subseteq C(\Gamma)$ *and* $T_\rho = (\rho^R, \rho^\Lambda)$ *where* $\rho \in \Omega$. *Then*

$$T_{(\cap_{\rho \in \Omega} \rho)} = \left(\bigcap_{\rho \in \Omega} \rho^R, \bigcap_{\rho \in \Omega} \rho^\Lambda \right)$$

and

$$T_{(\bigvee_{\rho \in \Omega} \rho)} = \left(\bigvee_{\rho \in \Omega} \rho^R, \bigvee_{\rho \in \Omega} \rho^\Lambda \right).$$

Proof. The first equality is obvious, we only need to prove the second equality. Let $x, y \in R$ be such that $x (\bigvee_{\rho \in \Omega} \rho)^R y$. Then

$$i = (x, x^\circ x) \bigvee_{\rho \in \Omega} \rho (y, y^\circ y) = j.$$

Hence, there exist $\rho_i \in \Omega$ and $a_i = (x_i, {x_i}^\circ x_i) \in \Gamma$ such that

$$i \rho_1 a_1 \rho_2 a_2 \ldots a_{n-1} \rho_n j.$$

This implies that

$$x\rho_1^R x_1\rho_2^R x_2\ldots x_{n-1}\rho_n^R y.$$

We have proved that

$$\left(\bigvee_{\rho\in\Omega}\rho\right)^R\subseteq\bigvee_{\rho\in\Omega}\rho^R.$$

$\bigvee_{\rho\in\Omega}\rho^R\subseteq(\bigvee_{\rho\in\Omega}\rho)^R$ is obvious. The dually equality can be proved similarly.

Proposition 2.1 is proved.

Now, by summing up the above results, we obtain the following theorem.

Theorem 2.2. *Let* Γ *be constructed in Lemma* 1.6. *Then* $CT(\Gamma)$ *forms a complete lattice with respect to* $\leq$ *and* $C(\Gamma)$ *is isomorphic to* $CT(\Gamma)$ *as a complete lattice.*

For a semigroup S, the equality relations on S are denoted by ϵ_S. The following theorem describes the idempotent-separating congruences.

Theorem 2.3. *Let* π *be an idempotent-separating congruence on* R. *Then* $\rho^{(\pi,\epsilon_\Lambda)}$ *is an idempotent-separating congruence on* Γ, *and every such congruence may be obtained in this way.*

Proof. Since π is an idempotent-separating congruence, $\pi|E^\circ=\epsilon_{E^\circ}$. It is easy to see that (π,ϵ_Λ) is a congruence pair. For any $(x,\lambda),(y,\mu)\in E(\Gamma)$ with $(x,\lambda)\rho^{(\pi,\epsilon_\Lambda)}(y,\mu)$, we have $\lambda=\mu$, $xx^\circ=yy^\circ$, $x^\circ x=y^\circ y$. So $x^\circ=x^\circ x(\lambda*x)x^\circ=y^\circ y(\mu*y)y^\circ=y^\circ$. Hence $x=xx^\circ x^{\circ\circ}x^\circ x=$ $=yy^\circ y^{\circ\circ}y^\circ y=y$. Therefore $\rho^{(\pi,\epsilon_\Lambda)}$ is an idempotent-separating congruence. It is easy to show the reverse implication.

References

1. *Blyth T. S., McFadden R. B.* Regular semigroups with a multiplicative inverse transversal // Proc. Roy. Soc. Edinburgh A. – 1982. – **92**. – P. 253 – 270.
2. *Tang X. L.* Regular semigroups with inverse transversals // Semigroup Forum. – 1997. – **55**, № 1. – P. 24 – 32.
3. *McAlister D. B., McFadden R. B.* Regular semigroups with inverse transversals // Quart. J. Math. Oxford. – 1983. – **34**, № 2. – P. 459 – 474.
4. *McAlister D. B., McFadden R. B.* Regular semigroups with inverse transversals as matrix semigroups // Quart. J. Math. Oxford. – 1984. – **35**, № 2. – P. 455 – 474.
5. *Wang L. M.* On congruence lattice of regular semigroups with Q-inverse transversals // Semigroup Forum. – 1995. – **50**. – P. 141 – 160.
6. *Saito T.* Structure of regular semigroups with a quasi-ideal inverse transversals // Semigroup Forum. – 1985. – **31**. – P. 305 – 309.
7. *Petrich M.* The structure of completely semigroups // Trans. Amer. Math. Soc. – 1974. – **189**. – P. 211 – 236.
8. *Petrich M., Reilly N.* Completely regular semigroups. – New York: Wiley, 1999.
9. *Tang X. L., Wang L. M.* Congruences on regular semigroups with inverse transversals // Communs Algebra. – 1995. – **23**. – P. 4157 – 4171.
10. *Wang L. M., Tang X. L.* Congruence lattices of regular semigroups with inverse transversals // Communs Algebra. – 1998. – **26**. – P. 1234 – 1255.
11. *Saito T.* A note on regular semigroups with inverse transversals // Semigroup Forum. – 1986. – **33**. – P. 149 – 152.

Received 22.04.13,
after revision — 06.03.16