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MONADS AND TENSOR PRODUCTS
МОНАДИ ТА ТЕНЗОРНІ ДОБУТКИ

M. Zarichnyi defined an operation of tensor product for each functor that can be complemented to a monad. We investigate the existence of tensor product for functors which cannot be complemented to monads.

М. Зарічний означив операцію тензорного добутку для кожного функтора, що доповнюється до монади. У цій статті досліджено існування тензорного добутку для функторів, які не можна доповнити до монади.

0. Introduction. The general theory of functors acting on the category Comp of compact Hausdorff spaces (compacta) and continuous mappings was founded by Shchepin [9]. He described some elementary properties of such functors and defined the notion of the normal functor which has become very fruitful. The classes of all normal functors include many classical constructions: the hyperspace exp, the space of probability measures P, the space of idempotent measures I, and many other functors (see [4, 10, 11]).

The algebraic aspect of the theory of functors in categories of topological spaces and continuous maps is based, mainly, on the existence of monad (or triple) structure in the sense of S. Eilenberg and J. Moore [2]. This notion turned out, in particular, to be a fruitful tool for investigation of functors in the category Comp (see [8, 10]).

We recall the definition of monad only for the category Comp. A monad $\mathbb{T} = (T, \eta, \mu)$ in the category Comp consists of an endofunctor $T\colon \mathrm{Comp} \to \mathrm{Comp}$ and natural transformations $\eta\colon \mathrm{Id}_{\mathrm{Comp}} \to T$ (unity), $\mu\colon T^2 \to T$ (multiplication) satisfying the relations $\mu \circ T\eta = \mu \circ \eta T = \mathbf{1}_T$ and $\mu \circ \mu T = \mu \circ T\mu$. (By $\mathrm{Id}_{\mathrm{Comp}}$ we denote the identity functor on the category Comp and T^2 is the superposition $T \circ T$ of T.)

The tensor product operation of probability measures is well known and very useful for investigation of the functor P (see, for example, [3], Chapter 8). Zarichnyi has generalized the tensor product of probability measures for any functor which can be completed to a monad [10] (Chapter 3.4). The definition of a functor which admits a tensor product is given in [1].

Definition 0.1 [1]. *We say that a functor $F\colon \mathrm{Comp} \to \mathrm{Comp}$ admits a tensor product if for each family of compacta $\{X_\alpha\}$ there exists a continuous map $\otimes_{\{X_\alpha\}}\colon \prod_\alpha FX_\alpha \to F\left(\prod_\alpha X_\alpha\right)$ which is natural with respect to each argument and for each α we have $F(p_\alpha) \circ \otimes = \mathrm{pr}_\alpha$, where $p_\alpha\colon \prod_\beta X_\beta \to X_\alpha$ and $\mathrm{pr}_\alpha\colon \prod_\beta FX_\beta \to FX_\alpha$ are natural projections. $\Big($By naturality with respect to each argument we mean the following property: for a family of maps $\{f_\alpha\colon X_\alpha \to Y_\alpha\}$ we have $\otimes_{\{Y_\alpha\}} \circ \prod_\alpha F(f_\alpha) = F\left(\prod_\alpha f_\alpha\right) \circ \otimes_{\{X_\alpha\}}.\Big)$*

The question naturally arises whether there exists a functor which can not be completed to a monad but admits a tensor product. Firstly, we investigate the Hartman–Mycielski functor introduced in [7]. Recently it was proved in [6] that this functor can not be completed to a monad. Unfortunately the Hartman–Mycielski functor neither admits a tensor product, what we will prove in Section 1 of this paper. In Section 2 we investigate iterated functors. We will prove that the second iteration of a functor, which admits a tensor product, admits tensor product too. We also will show that the second iteration of hyperspace functor (which is functorial part of hyperspace monad, hence admits a tensor product) can not be completed to a monad.

1. Hartman – Mycielski functor does not admit a tensor product. All functors we consider are endofunctors in Comp. A functor F is called continuous if it preserves the limits of inverse systems. A functor is called monomorphic if it preserves topological embeddings. For monomorphic functor F and an embedding $i\colon A \to X$ we shall identify the space $F(A)$ and the subspace $F(i)(F(A)) \subset \subset F(X)$. For a functor F which preserves monomorphisms the *intersection-preserving* property is defined as follows: $F(\cap\{X_\alpha \,|\, \alpha \in \mathcal{A}\}) = \cap\{F(X_\alpha) \,|\, \alpha \in \mathcal{A}\}$ for every family $\{X_\alpha \,|\, \alpha \in \mathcal{A}\}$ of closed subsets of X. A functor is called seminormal iff it is continuous, monomorphic, preserves empty space, one-point spaces and intersection. In what follows, all functors are assumed to be seminormal. For such a functor there exists a unique natural transformation $\eta\colon \mathrm{Id}_{\mathrm{Comp}} \to F$. Each component $\eta X : X \to FX$ is an embedding [3]. Let us remark that the map ηX has the following property: for each $x \in X$ we have $\eta X(x) = Fi(F\{x\})$, where $i\colon \{x\} \to X$ is the natural embedding.

Let X be a space and d is an admissible metric on X bounded by 1. By $HM(X)$ we shall denote the space of all maps from $[0,1)$ to the space X such that $f\,|\,[t_i, t_{i+1}) \equiv$ const, for some $0 = t_0 \leq \ldots \leq t_n = 1$, with respect to the following metric:

$$d_{HM}(f,g) = \int_0^1 d(f(t), g(t))dt, \quad f, g \in HM(X).$$

The construction of $HM(X)$ is known as the Hartman – Mycielski construction [5]. This construction was considered for any compactum Z in [10] (2.5.2). Let $\mathcal{U}$ be the unique uniformity of Z. For every $U \in \mathcal{U}$ and $\varepsilon > 0$, let

$$\langle \alpha, U, \varepsilon \rangle = \{\beta \in HM(Z) \,|\, m\{t \in [0,1) \,|\, (\alpha(t), \beta(t)) \notin U\} < \varepsilon\}$$

(here m is the Lebesgue measure on $[0,1]$) The sets $\langle \alpha, U, \varepsilon \rangle$ form a base of a topology in HMZ. The construction HM acts also on maps. Given a map $f : X \to Y$ in Comp, define a map $HMX \to HMY$ by the formula $HMF(\alpha) = f \circ \alpha$. In general, HMX is not compact.

Let us fix some $n \in \mathbb{N}$. For every compactum Z consider

$$HM_n(Z) = \Big\{ f \in HM(Z) \,|\, \text{ there exist } 0 = t_1 < \ldots < t_{n+1} = 1 \\ \text{with } f\,|\,[t_i, t_{i+1}) \equiv z_i \in Z,\ i = 1, \ldots, n \Big\}.$$

The constructions HM_n define normal functors in Comp [10] (2.5.2).

Zarichnyi has asked if there exists a normal functor in Comp which contains all functors HM_n as subfunctors (see [10]). Such a functor H was constructed in [7]. The main aim of this section is to show that the functor H does not admit a tensor product. Let us remind the construction of H from [7]. Let X be a compactum. By $C(X)$ we denote the Banach space of all continuous functions $\varphi :$ $X \to \mathbb{R}$ with the usual sup-norm: $\|\varphi\| = \sup\{|\varphi(x)| \mid x \in X\}$. We denote the segment $[0,1]$ by I.

For a compactum X let us define the uniformity of HMX. For each $\varphi \in C(X)$ and $a, b \in [0,1]$ with $a < b$ we define a function $\varphi_{(a,b)} : HMX \to \mathbb{R}$ by the following formula:

$$\varphi_{(a,b)} = \frac{1}{b-a}\int\limits_a^b \varphi\circ\alpha(t)dt \quad \text{for some} \quad \alpha\in HMX.$$

Define

$$S_{HM}(X) = \{\varphi_{(a,b)} \mid \varphi\in C(X) \text{ and } (a,b)\subset(0,1)\}.$$

For $\varphi_1,\ldots,\varphi_n\in S_{HM}(X)$ define a pseudometric $\rho_{\varphi_1,\ldots,\varphi_n}$ on HMX by the formula

$$\rho_{\varphi_1,\ldots,\varphi_n}(f,g) = \max\{|\varphi_i(f)-\varphi_i(g)| \mid i\in\{1,\ldots,n\}\},$$

where $f,g\in HMX$. The family of pseudometrics

$$\mathcal{P} = \{\rho_{\varphi_1,\ldots,\varphi_n} \mid n\in\mathbb{N},\ \text{where } \varphi_1,\ldots,\varphi_n\in S_{HM}(X)\},$$

defines a totally bounded uniformity $\mathcal{U_{HMX}}$ of HMX (see [7]).

For each compactum X we consider the uniform space $(HX,\mathcal{U}_{HX})$ which is the completion of $(HMX,\mathcal{U_{HMX}})$ and the topological space HX with the topology induced by the uniformity $\mathcal{U}_{HX}$. Since $\mathcal{U}_{HMX}$ is totally bounded, the space HX is compact.

Let $f\colon X\to Y$ be a continuous map. Define a map $HMf\colon HMX\to HMY$ by the formula $HMf(\alpha)=f\circ\alpha$ for all $\alpha\in HMX$. It was shown in [7] that the map $HMf\colon(HMX,\mathcal{U}_{HMX})\to$ $\to(HMY,\mathcal{U}_{HMY})$ is uniformly continuous. Hence there exists a continuous map $Hf\colon HX\to HY$ such that $Hf\mid HMX=HMf$. It is easy to see that $H\colon\mathrm{Comp}\to\mathrm{Comp}$ is a covariant functor and HM_n is a subfunctor of H for each $n\in\mathbb{N}$.

Let us remark that the family of functions $S_{HM}(X)$ embeds HMX in the product of closed intervals $\prod_{\varphi_{(a,b)}\in S_{HM}(X)} I_{\varphi_{(a,b)}}$, where $I_{\varphi_{(a,b)}}=[\min_{x\in X}|\varphi(x)|,\max_{x\in X}|\varphi(x)|]$. Thus, the space HX is the closure of the image of HMX. We denote by $p_{\varphi_{(a,b)}}\colon HX\to I_{\varphi_{(a,b)}}$ the restriction of the natural projection. Let us remark that the function Hf could be defined by the condition $p_{\varphi_{(a,b)}}\circ Hf=p_{(\varphi\circ f)_{(a,b)}}$ for each $\varphi_{(a,b)}\in S_{HM}(Y)$.

We will use certain properties of the functor H proved in [7]. Since the functor H preserves embeddings, we can identify the space FA with $Fi(FA)\subset FX$ for each closed subset $A\subset X$, where $i\colon A\to X$ is the natural embedding. We can define for each $\alpha\in HX$ the closed set $\operatorname{supp}\alpha=\cap\{A \text{ is a closed subset of } X \text{ such that } \alpha\in HA\}$. Since H preserves intersection, we have $\alpha\in H(\operatorname{supp}\alpha)$.

By D we denote two-point space $\{0,1\}$ with discrete topology. For $n\in\mathbb{N}$ define α_0^n and $\alpha_1^n\in HMD\subset HD$ as follows: $\alpha_0^n(s)=0$, $\alpha_1^n(s)=1$, if $s\in\left[\dfrac{2i}{2n},\dfrac{2i+1}{2n}\right)$ and $\alpha_0^n(s)=1$; $\alpha_1^n(s)=0$, if $s\in\left[\dfrac{2i+1}{2n},\dfrac{i+1}{n}\right)$, where $i\in\{0,\ldots,n-1\}$, $s\in[0,1)$. For $k,j\in\{0,1\}$ and $n\in\mathbb{N}$ define $\alpha_{k,j}^n\in HM(D\times D)\subset H(D\times D)$ by the formula $\alpha_{k,j}^n(s)=(\alpha_k^n(s),\alpha_j^n(s))$. Denote $\varrho_l=H(\mathrm{pr}_l)\colon H(D\times D)\to HD$, where $l\in\{1,2\}$ and $\mathrm{pr}_l: D\times D\to D$ are natural projections. Obviously, we have $\varrho_1(\alpha_{k,j}^n)=\alpha_k^n$ and $\varrho_2(\alpha_{k,j}^n)=\alpha_j^n$.

Lemma 1.1. *We have* $(\varrho_1)^{-1}(\alpha_k^n)\cap(\varrho_2)^{-1}(\alpha_j^n)=\{\alpha_{k,j}^n\}$.

Proof. We will prove the lemma for $k = j = 0$. The proof is the same for other cases. Consider any $\gamma \in (\varrho_1)^{-1}(\alpha_0^n) \cap (\varrho_2)^{-1}(\alpha_0^n)$. Firstly, let us show that $\operatorname{supp}\gamma \subset \operatorname{supp}(\alpha_{0,0}^n) = \{(0;0),(1;1)\}$. Suppose the contrary. We can assume that $(0;1) \in \operatorname{supp}\gamma$. Consider a function $\psi\colon D \times D \to \mathbb{R}$ such that $\psi(0;1) = 1$ and $\psi(k;l) = 0$ for each $(k;l) \neq (0;1)$. By Lemma 2.5 from [6] there exists $a > 0$ such that $p_{\psi_{(0,1)}}(\gamma) \geq a$. For $r \in \{0,\ldots,n-1\}$ define functions $\varphi_{2r},\ \varphi_{2r+1}\colon D \to \mathbb{R}$ as follows: $\varphi_{2r}(0) = 1,\ \varphi_{2r}(1) = 0$ and $\varphi_{2r+1}(0) = 0,\ \varphi_{2r+1}(1) = 1$. For $k \in \{1,2\}$ and $l \in \{0,\ldots,2n-1\}$ we consider the functions $\varphi_l^k = \varphi_l \circ \mathrm{pr}_k\colon D \times D \to \mathbb{R}$. Choose a neighborhood V of γ defined as follows: $V = \{\gamma' \in H(D \times D) \mid |p_{\psi_{(0,1)}}(\gamma) - p_{\psi_{(0,1)}}(\gamma')| < \dfrac{a}{2}$ and

$$\left| p_{\varphi^k_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\gamma) - p_{\varphi^k_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\gamma') \right| < \frac{a}{4n} \text{ for each } k \in \{1,2\} \text{ and } r \in \{0,\ldots,2n-1\}.$$

Consider any $\gamma_1 \in HM(D \times D) \cap V$. Since $|p_{\psi_{(0,1)}}(\gamma) - p_{\psi_{(0,1)}}(\gamma_1)| < \dfrac{a}{2}$, we have $m\{t \in [0,1) \mid \gamma_1(t) = (0;1)\} > \dfrac{a}{2}$. Hence there exists $r \in \{0,\ldots,2n-1\}$ such that

$$m\left\{ t \in \left(\frac{r}{2n},\frac{r+1}{2n}\right) \,\middle|\, \gamma_1(t) = (0;1) \right\} > \frac{a}{4n}.$$

If $r = 2l$ for some $l \in \{0,\ldots,n-1\}$ we have $p_{\varphi^2_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\gamma_1) = n\displaystyle\int_{\frac{r}{2n}}^{\frac{r+1}{2n}} \varphi_r^2 \circ \gamma_1(t)dt < 1 - \dfrac{a}{2}$. But $p_{\varphi^2_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\gamma) = p_{\varphi_r \circ \mathrm{pr}_{2\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\gamma) = p_{\varphi_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}} \circ \varrho_2(\gamma) = p_{\varphi_{r\left(\frac{r}{2n},\frac{r+1}{2n}\right)}}(\alpha_0^n) = 1$ and we obtain a contradiction with the definition of V.

If $r = 2l+1$ for some $l \in \{0,\ldots,n-1\}$ we obtain a contradiction using the function φ_r^1. Hence we have the inclusion $\operatorname{supp}\gamma \subset \{(0;0),(1;1)\}$.

Consider any $\varphi \in C(D \times D)$ and $(a,b) \subset (0,1)$. Define $\psi \in C(D)$ as follows: $\psi(i) = \varphi(i;i)$ for $i \in D$ and put $\xi = \psi \circ \mathrm{pr}_1$. Since $\operatorname{supp}\gamma \subset \{(i;i) | i \in D\}$, we have $p_{\varphi_{(a,b)}}(\gamma) = p_{\xi_{(a,b)}}(\gamma)$ by Lemma 2.3 from [6]. Then $p_{\varphi_{(a,b)}}(\gamma) = p_{\xi_{(a,b)}}(\gamma) = p_{\psi(a,b)}(\alpha_0^n) = p_{\xi_{(a,b)}}(\alpha_{0,0}^n) = p_{\varphi_{(a,b)}}(\alpha_{0,0}^n)$. Hence $\gamma = \alpha_{0,0}^n$.

Theorem 1.1. *There is no continuous map $t\colon HD \times HD \to H(D \times D)$ such that $\varrho_i \circ t = s_i$ for each $i \in \{1,2\}$, where $s_i\colon HD \times HD \to HD$ is the natural projection.*

Proof. Suppose that there exists such map. It is easy to check that both sequences (α_0^n) and (α_1^n) converge to $\alpha \in HD$ defined as follows $p_{\varphi_{(a,b)}}(\alpha) = \dfrac{1}{2}(\varphi(0) + \varphi(1))$ for each $\varphi \in C(D)$ and $(a,b) \subset (0,1)$. Since t is continuous, both sequences $t(\alpha_0^n,\alpha_0^n)$ and $t(\alpha_0^n,\alpha_1^n)$ converge to $\beta = t(\alpha,\alpha)$. We have $t(\alpha_0^n,\alpha_0^n) = \alpha_{0,0}^n$ and $t(\alpha_0^n,\alpha_1^n) = \alpha_{0,1}^n$ by Lemma 1.1. Consider a function $\varphi\colon D \times D \to \mathbb{R}$ defined as follows: $\varphi(0;0) = \varphi(1;1) = 0$ and $\varphi(1;0) = \varphi(0;1) = 1$. Then we have $p_{\varphi_{(0,1)}}(\alpha_{0,0}^n) = 0 \neq 1 = p_{\varphi_{(0,1)}}(\alpha_{0,1}^n)$ for each $n \in \mathbb{N}$. We obtain a contradiction and the theorem is proved.

2. Iterated functors. Let F be a functor. By F^2 we denote the second iteration $F \circ F$ of the functor F. If we have a family $\{X_\alpha\}$, by $p_\alpha\colon \prod_\beta X_\beta \to X_\alpha$, $\mathrm{pr}_\alpha : \prod_\beta FX_\beta \to FX_\alpha$ and $\mathrm{pr}_\alpha^2 : \prod_\beta F^2X_\beta \to F^2X_\alpha$ we denote the corresponding natural projections.

Theorem 2.1. *Let F be a functor which admits a tensor product. Then F^2 admits a tensor product.*

Proof. Let $\otimes_{\{X_\alpha\}}\colon \prod_\alpha FX_\alpha \to F\left(\prod_\alpha X_\alpha\right)$ be a tensor product for a family $\{X_\alpha\}$ and a functor F. Define a map $\otimes^2_{\{X_\alpha\}}\colon \prod_\alpha F^2X_\alpha \to F^2\left(\prod_\alpha X_\alpha\right)$ by formula $\otimes^2_{\{X_\alpha\}} = F(\otimes_{\{X_\alpha\}}) \circ$ $\circ \otimes_{\{FX_\alpha\}}$. The naturality of $\otimes^2$ is obvious.

For each α we have $F^2p_\alpha \circ \otimes^2_{\{X_\alpha\}} = F^2p_\alpha \circ F(\otimes_{\{X_\alpha\}}) \circ \otimes_{\{FX_\alpha\}} = F(Fp_\alpha \circ \otimes_{\{X_\alpha\}}) \circ \otimes_{\{FX_\alpha\}} =$ $= F(\mathrm{pr}_\alpha) \circ \otimes_{\{FX_\alpha\}} = \mathrm{pr}^2_\alpha$. Hence the operation $\otimes^2$ defines a tensor product for the functor F^2.

Now we consider the hyperspace functor $\exp$. For a compactum X by $\exp X$ we denote the set of nonempty compact subsets of X provided with the Vietoris topology. A base of this topology consists of the sets of the form $\langle U_1,\ldots,U_n\rangle = \left\{A \in \exp\ X \mid A \subset \bigcup_{i=1}^n U_i,\ A\cap U_i \neq \varnothing \text{ for each } i \in \{1,\ldots,n\}\right\}$, where $U_1,\ldots,U_n$ are open in X. The space $\exp\ X$ is called the hyperspace of X.

For a continuous mapping $f\colon X \to Y$ the mapping $\exp\ f\colon \exp\ X \to \exp\ Y$ is defined by the formula $\exp f(A) = fA \in \exp\ Y$, $A \in \exp\ X$. It is easy to see that this defines a functor $\exp:$ $\mathrm{Comp} \to \mathrm{Comp}$ (the hyperspace functor). It is well known that the functor $\exp$ define the natural transformations $s\colon \mathrm{Id}_{\mathrm{Comp}} \to \exp$ and $u\colon \exp^2 \to \exp$ as follows: $sX(x) = \{x\}$ for each $x \in X$, $uX(\mathcal{A}) = \cup\mathcal{A}$, $\mathcal{A} \in \exp^2\ X$. Then the triple $\mathbb{H} = (\exp, s, u)$ is a monad (see [10] for more information). Hence the hyperspace functor admits a tensor product and, by Theorem 2.1, the iterated functor $\exp^2$ admits tensor product too.

Let us remark that there exists a unique natural transformation $\eta\colon \mathrm{Id}_{\mathrm{Comp}} \to \exp^2$ defined as follows $\eta X = s\exp X \circ sX = \exp sX \circ sX$ for each compactum X. We have $\eta X(x) = \{\{x\}\}$ for $x \in X$.

Theorem 2.2. *There is no natural transformation* $\mu\colon \exp^4 \to \exp^2$ *such that* $\mu \circ \exp^2\eta =$ $= \mu \circ \eta\exp^2 = \mathbf{1}_{\exp^2}$.

Proof. Suppose the contrary. Consider $X = \{a,b,c,d\}$ and $\alpha = \{\{\{\{a\},\{b\}\},\{\{d\},\{c\}\}\}\} \in$ $\in \exp^4 X$. Define maps $f_1,\ f_2\colon X \to \{0,1\}$ as follows: $f_1(a) = f_1(b) = 1,\ f_1(c) = f_1(d) =$ $= 0$ and $f_2(a) = f_2(c) = 0,\ f_2(b) = f_1(d) = 1$. Then $\exp^4 f_1(\alpha) = \{\{\{\{0\}\},\{\{1\}\}\}\} =$ $= \exp^2\eta\{0,1\}(\{\{0,1\}\})$ and we have $\mu\{0,1\} \circ \exp^4 f_1(\alpha) = \{\{0,1\}\}$. Since μ is a natural transformation, we obtain that $\exp^2 f_1 \circ \mu X(\alpha) = \{\{0,1\}\}$. On the other hand $\exp^4 f_2(\alpha) =$ $= \{\{\{\{0\},\{1\}\}\}\} = \eta\exp^2\{0,1\}(\{\{0\},\{1\}\})$ and we have that $\exp^2 f_2 \circ \mu X(\alpha) = \{\{0\},\{1\}\}$. It is easy to check that $\mu X(\alpha) = \{\{a,c\},\{b,d\}\}$.

But if we consider maps $g_1, g_2\colon X \to \{0,1\}$ defined by equalities $g_1 = f_1$ and $g_2(a) = g_2(d) =$ $= 0,\ g_2(b) = g_2(c) = 1$, we obtain that $\mu X(\alpha) = \{\{a,d\},\{b,c\}\}$ using the same arguments as before. Hence we have a contradiction.

***Corollary* 2.1.** *The functor* $\exp^2$ *can not be completed to a monad.*

Let us remark that the functors $\exp$ and $\exp^2$ are normal (see, for example, [10]).

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