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SECOND ORDER PARALLEL TENSORS ON $\mathcal{S}$-MANIFOLDS AND SEMI-PARALLEL HYPERSURFACES OF $\mathcal{S}$-SPACE FORMS

ПАРАЛЕЛЬНІ ТЕНЗОРИ ДРУГОГО ПОРЯДКУ НА $\mathcal{S}$-МНОГОВИДАХ ТА НАПІВПАРАЛЕЛЬНІ ГІПЕРПОВЕРХНІ $\mathcal{S}$-ПРОСТОРОВИХ ФОРМ

We study a second order parallel symmetric tensor in an $\mathcal{S}$-manifold and we deduce that there is no semi-parallel hypersurface in $\mathcal{S}$-space forms $\widetilde{M}^{2n+s}(c)$ with $c \neq s$.

Вивчається паралельний симетричний тензор другого порядку на $\mathcal{S}$-многовиді. Встановлено, що не існує напівпаралельної гіперповерхні в $\mathcal{S}$-просторових формах $\widetilde{M}^{2n+s}(c)$ з $c \neq s$.

1. Introduction. In [19], Yano introduced the notion of φ-structure on a $(2n+s)$-dimensional manifold as a tensor field φ of type $(1,1)$ and rank $2n$ satisfying $\varphi^3 + \varphi = 0$. Almost complex $(s = 0)$ and almost contact $(s = 1)$ structures are well-known examples of f-structure. In the context, Blair [3] defined K-manifolds as the analogue of Kaehlerian manifolds in the almost complex geometry and of quasi-Sasakian manifolds in the almost contact geometry and he showed that the curvature of $\mathcal{S}$-manifolds is completely determined by their φ-sectional curvatures.

In 1923, Eisenhart [11] proved that if a positive definite Riemannian manifold (M, g) admits a second order parallel symmetric covariant tensor other than a constant multiple of the metric tensor, then it is reducible. In [17], Levy proved that a second order parallel symmetric non singular tensor in real space forms is proportional to the metric tensor. Since then, many authors investigated the Eisenhart problem of finding symmetric and skew symmetric parallel tensors on various spaces and obtained fruitful results.

We know from [13] that a second order parallel symmetric tensor in generalized Sasakian space forms is proportional to the metric tensor.

In this paper, we generalize this result for an $\mathcal{S}$-manifold $\widetilde{M}^{2n+s}$ with $s \geq 1$. Further, we investigate the existence of parallel and semi-parallel hypersurface in $\mathcal{S}$-space forms $\widetilde{M}^{2n+s}(c)$ with $c \neq s$.

2. Preliminaries. ***2.1. Semi-parallelism.*** Let $\widetilde{M}^n$ be an n-dimensional Riemannian manifold and M^m an m-dimensional submanifold of $\widetilde{M}^n$. Let g be the metric tensor field on $\widetilde{M}^n$ as well as the metric induced on M^m. We denote by $\widetilde{\nabla}$ the covariant differentiation in $\widetilde{M}^n$ and by ∇ the covariant differentiation in M^m. Let $T(\widetilde{M})$ $\big($resp. $T(M)\big)$ be the Lie algebra of vector field on $\widetilde{M}^n$ (resp. on M^m) and $T(M)^{\perp}$ the set of all vector fields normal to M^m. The Gauss – Weingarten formulas are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \quad \widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \quad X, Y \in T(M),\ V \in T(M)^{\perp},$$

where $\nabla^{\perp}$ is the connection in the normal bundle, σ is the second fundamental form of M^m and A_V is the Weingarten endomorphism associated with V. A_V and σ are related by $g(A_V X, Y) = g(\sigma(X,Y), V)$.

The submanifold M^m is said to be *totally geodesic* in $\widetilde{M}^n$ if its second fundamental form is identically zero and it is said to be *minimal* if $H \equiv 0$, where H is the mean curvature vector defined by $H = \dfrac{1}{m}\,\mathrm{trace}\,(\sigma)$ [6]. We denote by $\widetilde{R}$ and R the curvature tensors associated with $\widetilde{\nabla}$ and ∇, respectively.

The basic equations of Codazzi and Gauss are

$$(\widetilde{R}(X,Y)Z)^{\perp} = \widetilde{\nabla}_X \sigma(Y,Z) - \widetilde{\nabla}_Y \sigma(X,Z)$$

and

$$\widetilde{R}(X,Y)Z = R(X,Y)Z - g(A_V Y, Z)A_V X + g(A_V X, Z)A_V Y,$$

respectively, $X, Y, Z \in T(M)$.

Now, the submanifold is said to be *parallel* if

$$\widetilde{\nabla}_X \sigma(Y,Z) = \nabla^{\perp}_X(\sigma(Y,Z)) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z) = 0 \tag{1}$$

for all $X,\ Y,\ Z \in T(M)$, and *semi-parallel* if

$$\widetilde{R}\,.\,\sigma = (\widetilde{\nabla}_X \widetilde{\nabla}_Y - \widetilde{\nabla}_Y \widetilde{\nabla}_X - \widetilde{\nabla}_{[X,Y]})\sigma = 0.$$

Semi-parallel immersions are defined as extrinsic analogue for semi-symmetric space ($\widetilde{R}\,.\,\widetilde{R} = 0$) and as a direct generalization of parallel immersions [7].

In [12, 13], the authors showed that there is no parallel (and no semi-parallel) hypersurfaces in Sasakian space forms $\widetilde{M}^{2n+1}(c)$ with $c \neq 1$.

2.2. $\mathcal{S}$-manifold. Let $\widetilde{M}^{2n+s}$ be a $(2n+s)$-dimensional Riemannian manifold endowed with an φ-structure [19] (that is a tensor field of type $(1,1)$ and rank $2n$ satisfying $\varphi^3 + \varphi = 0$). If moreover there exist on $\widetilde{M}^{2n+s}$ global vector fields $\xi_1, \ldots, \xi_s$ (called structure vector fields), and their duals 1-forms $\eta_1, \ldots, \eta_s$ such that [14], for all $X, Y \in T(\widetilde{M})$ and $\alpha, \beta \in \{1, \ldots, s\}$,

$$\eta_\alpha(\xi_\beta) = \delta_{\alpha\beta},\ \ \varphi\xi_\alpha = 0,\ \ \eta_\alpha(\varphi X) = 0,\ \ \varphi^2 X = -X + \sum_{\alpha=1}^{s} \eta_\alpha(X)\xi_\alpha, \tag{2}$$

then there exists on $\widetilde{M}$ a Riemannian metric g satisfying

$$g(X,Y) = g(\varphi X, \varphi Y) + \sum_{\alpha=1}^{s} \eta_\alpha(X)\eta_\alpha(Y)$$

and

$$\eta_\alpha(X) = g(X, \xi_\alpha),\ \ \alpha \in \{1, \ldots, s\}.$$

$\widetilde{M}$ is then said to be a metric φ-manifold. The φ-structure is normal if

$$N_\varphi + 2\sum_{\alpha=1}^{s} \xi_\alpha \otimes d\eta_\alpha = 0,$$

where N_φ is the Nijenhuis torsion of φ.

Let ϕ be the fundamental 2-form on M defined for all vector fields X, Y on $\widetilde{M}$ by

$$\phi(X,Y) = g(X,\varphi Y).$$

A normal metric φ-structure with closed fundamental 2-form will be called K-structure and $\widetilde{M}^{2n+s}$ called K-manifold. Finally, if $d\eta_1 = \ldots = d\eta_s = \phi$, then the K-structure is called $\mathcal{S}$-structure and $\widetilde{M}$ is called $\mathcal{S}$-manifold. In the case $s=1$ the $\mathcal{S}$-manifold is a Sasakian manifold.

The Riemannian connection $\widetilde{\nabla}$ of an $\mathcal{S}$-manifold satisfies [3]

$$\widetilde{\nabla}_X \xi_\alpha = -\varphi X, \tag{3}$$

$$(\widetilde{\nabla}_X \varphi)Y = \sum_{\alpha=1}^{s} \left(g(\varphi X, \varphi Y)\xi_\alpha + \eta_\alpha(Y)\varphi^2 X \right), \quad X, Y \in T(\widetilde{M}),$$

and

$$\widetilde{R}(X,Y)\xi_\alpha = \left(\sum_{\beta=1}^{s} \eta_\beta(X)\right)\varphi^2 Y - \left(\sum_{\beta=1}^{s} \eta_\beta(Y)\right)\varphi^2 X \tag{4}$$

for all $\alpha \in \{1,\ldots,s\}$.

A plane section π is called an φ-section if it is determined by a unit vector X, normal to the structure vector fields and φX. The sectional curvature of π is called an *φ-sectional curvature*. An $\mathcal{S}$-manifold is said to be an *$\mathcal{S}$-space form* if it has constant φ-sectional curvature c and it is denoted by $\widetilde{M}^{2n+s}(c)$ $(n>1)$ and its curvature tensor has the form [16]

$$\widetilde{R}(X,Y)Z = \frac{c+3s}{4}\left\{ g(\varphi X,\varphi Z)\varphi^2 Y - g(\varphi Y,\varphi Z)\varphi^2 X \right\} +$$

$$+\frac{c-s}{4}\{g(\varphi Y,Z)\varphi X - g(\varphi X,Z)\varphi Y + 2g(X,\varphi Y)\varphi Z\}+$$

$$+\left(\sum_{\alpha=1}^{s}\eta_\alpha(X)\right)\left(\sum_{\beta=1}^{s}\eta_\beta(Z)\right)\varphi^2 Y - \left(\sum_{\alpha=1}^{s}\eta_\alpha(Y)\right)\left(\sum_{\beta=1}^{s}\eta_\beta(Z)\right)\varphi^2 X +$$

$$+g(\varphi Y,\varphi Z)\left(\sum_{\alpha=1}^{s}\eta_\alpha(X)\right)\left(\sum_{\beta=1}^{s}\xi_\beta\right) - g(\varphi X,\varphi Z)\left(\sum_{\alpha=1}^{s}\eta_\alpha(Y)\right)\left(\sum_{\beta=1}^{s}\xi_\beta\right) \tag{5}$$

for all X, Y, $Z \in T(\widetilde{M})$.

For $s=1$ the $\mathcal{S}$-space form is reduced to Sasakian-space form.

***Example* 2.1** [14]. Let $\mathbb{R}^{2n+s}(-3s)$ be a Euclidean space with Cartesian coordinates $(x^1, \ldots$ $\ldots, x^n, y^1, \ldots, y^n, z^1, \ldots, z^s)$, then an $\mathcal{S}$-structure on $\mathbb{R}^{2n+s}(-3s)$ is defined by

$$\xi_\alpha = 2\frac{\partial}{\partial z^\alpha}, \qquad \alpha = 1, \ldots, s,$$

$$\eta_\alpha := \frac{1}{2}\left(dz^\alpha - \sum_{i=1}^{n} y^i dx^i\right), \qquad \alpha = 1, \ldots, s,$$

$$\varphi X := -\sum_{i=1}^{n} X'^i \frac{\partial}{\partial x^i} + \sum_{i=1}^{n} X^i \frac{\partial}{\partial y^i} - \left(\sum_{i=1}^{n} X'^i y^i\right)\left(\sum_{\alpha=1}^{s} \frac{\partial}{\partial z^\alpha}\right),$$

$$g := \sum_{\alpha=1}^{s} \eta_\alpha \otimes \eta_\alpha + \frac{1}{4}\sum_{i=1}^{n}(dx^i \otimes dx^i + dy^i \otimes dy^i),$$

where

$$X = \sum_{i=1}^{n} X^i \frac{\partial}{\partial x^i} + \sum_{i=1}^{n} X'^i \frac{\partial}{\partial y^i} + \sum_{\alpha=1}^{s} X''^\alpha \frac{\partial}{\partial z^\alpha}.$$

With this structure $\mathbb{R}^{2n+s}$ is an $\mathcal{S}$-manifold of constant φ-sectional curvature $c = -3s$.

Let M^m be an m-dimensional submanifold immersed in $\widetilde{M}^{2n+s}$. For any vector field X tangent to M, we put

$$\varphi X = TX + NX, \tag{6}$$

where TX is the tangential part and NX the normal part of φX. Then T is an endomorphism on the tangent bundle TM and N is a normal bundle valued 1-form on the tangent bundle.

Lemma 2.1 [18]. *Let M be a submanifold of an $\mathcal{S}$-manifold. Then, for X, Y, $\xi_\alpha \in TM$, we have*

$$TX = -\nabla_X \xi_\alpha, \qquad NX = -\sigma(X, \xi_\alpha).$$

M^m is said to be invariant submanifold of $\widetilde{M}$ if all of ξ_α, $\alpha = 1, \ldots, s$, are always tangent to M^m and $\varphi X \in T(M)$ for any $X \in T(M)$. It is easy to show that an invariant submanifold of an $\mathcal{S}$-manifold is an $\mathcal{S}$-manifold too. If M is invariant, then N in (6) vanishes identically. On the other hand, M^m is said to be an antiinvariant submanifold if $\varphi X \in T(M)^\perp$ for any $X \in T(M)$. If M is antiinvariant, then T in (6) vanishes identically.

3. Second order parallel tensor. Let B be a $(0,2)$-symmetric tensor field on $\mathcal{S}$-manifold $\widetilde{M}^{2n+s}$ such that $\widetilde{\nabla} B = 0$, then it follows that

$$B(\widetilde{R}(X,Y)Z, W) + B(Z, \widetilde{R}(X,Y)W) = 0 \tag{7}$$

for arbitrary vector fields $X, Y, Z, W \in T(\widetilde{M})$.

Theorem 3.1. *On an $\mathcal{S}$-manifold $\widetilde{M}^{2n+s}$ a second order parallel symmetric tensor is proportional to the metric tensor if $s = 1$, and it is a linear combination (with constant coefficients) of the underlying metric tensor and 1-forms of structure vector fields if $s \geq 2$.*

Proof. Substituting $X = Z = W = \xi_\gamma$, for all $\gamma \in \{1, \ldots, s\}$ and $s \geq 1$, in (7) gives

$$B(\widetilde{R}(\xi_\gamma, Y)\xi_\gamma, \xi_\gamma) + B(\xi_\gamma, \widetilde{R}(\xi_\gamma, Y)\xi_\gamma) = 0.$$

Then it follows from the symmetry of B that

$$B(\widetilde{R}(\xi_\gamma, Y)\xi_\gamma, \xi_\gamma) = 0 \tag{8}$$

and from (4) we obtain

$$\widetilde{R}(\xi_\gamma, Y)\xi_\gamma = \varphi^2 Y. \tag{9}$$

From (8) and (9) we have

$$B(\varphi^2 Y, \xi_\gamma) = 0.$$

By using (2), we get

$$B(Y, \xi_\gamma) - \sum_{\beta=1}^{s} \eta_\beta(Y) B(\xi_\beta, \xi_\gamma) = 0 \tag{10}$$

for any $\gamma \in \{1, \ldots, s\}$.

By differentiating covariantly along X, we obtain

$$B(\widetilde{\nabla}_X Y, \xi_\gamma) + B(Y, \widetilde{\nabla}_X \xi_\gamma) - \sum_{\beta=1}^{s} \{g(\widetilde{\nabla}_X Y, \xi_\beta) + g(Y, \widetilde{\nabla}_X \xi_\beta)\} B(\xi_\beta, \xi_\gamma) -$$

$$- \sum_{\beta=1}^{s} g(Y, \xi_\beta) \{B(\widetilde{\nabla}_X \xi_\beta, \xi_\gamma) + B(\xi_\beta, \widetilde{\nabla}_X \xi_\gamma)\} = 0 \tag{11}$$

for any $\gamma \in \{1, \ldots, s\}$. Put $Y = \widetilde{\nabla}_X Y$ in (10), we get

$$B(\widetilde{\nabla}_X Y, \xi_\gamma) - \sum_{\beta=1}^{s} g(\widetilde{\nabla}_X Y, \xi_\beta) B(\xi_\beta, \xi_\gamma) = 0. \tag{12}$$

From (11), (12) and (3) we have

$$-B(Y, \varphi X) + g(Y, \varphi X) \sum_{\beta=1}^{s} B(\xi_\beta, \xi_\gamma) + \sum_{\beta=1}^{s} g(Y, \xi_\beta) \{B(\varphi X, \xi_\gamma) + B(\varphi X, \xi_\beta)\} = 0 \tag{13}$$

for any $\gamma \in \{1, \ldots, s\}$. Replacing Y by φX in (10), we obtain

$$B(\varphi X, \xi_\gamma) = 0 \tag{14}$$

for any $\gamma \in \{1, \ldots, s\}$. From (13) and (14) we get

$$-B(Y, \varphi X) + g(Y, \varphi X) \sum_{\beta=1}^{s} B(\xi_\beta, \xi_\gamma) = 0 \tag{15}$$

for any $\gamma \in \{1, \ldots, s\}$. Replace X by φX in (15) and, by using (2) and (10), we have

$$B(X,Y)-g(X,Y)\sum_{\beta=1}^{s}B(\xi_\beta,\xi_\gamma)-\sum_{\alpha,\beta=1}^{s}\eta_\alpha(X)\eta_\beta(Y)B(\xi_\beta,\xi_\alpha)+\sum_{\alpha,\beta=1}^{s}\eta_\alpha(X)\eta_\alpha(Y)B(\xi_\beta,\xi_\gamma)=0 \tag{16}$$

for any $\gamma\in\{1,\ldots,s\}$.

For $s=1$ (that means $\widetilde{M}$ is Sasakian manifold), it suffices to put $\xi_1=\xi_2=\ldots=\xi_s=\xi$ and $\eta_1=\eta_2=\ldots=\eta_s=\eta$, then from (16) we obtain

$$B(X,Y)=B(\xi,\xi)g(X,Y).$$

For $s\geq 2$, since $\widetilde{\nabla}B=0$ and from (14) we can easily show that $B(\xi_\alpha,\xi_\beta)$ is constant, for any $\alpha,\beta\in\{1,\ldots,s\}$, get the Theorem 3.1.

4. Semi-parallel hypersurfaces in an $\mathcal{S}$-space form. In [12] and [13], the authors proved the following theorems.

Theorem 4.1 [12]. *There are not a parallel connected hypersurface in a Sasakian space form $\widetilde{M}^{2n+1}(c)$ with $n\geq 2$ and $c\neq 1$.*

Theorem 4.2 [13]. *There are no semi-parallel hypersurfaces in a Sasakian space form $\widetilde{M}^{2n+1}(c)$ with $c\neq 1$ and $n\geq 2$.*

For a parallel and semi-parallel hypersurfaces in an $\mathcal{S}$-space form, we have the following results.

Theorem 4.3. *Let M be an hypersurface of an $\mathcal{S}$-space form $\widetilde{M}^{2n+s}(c)$, tangent to the structure vector fields with $c\neq s$, then M is not parallel.*

Proof. We suppose that M is a parallel hypersurface of an $\mathcal{S}$-space form $\widetilde{M}^{2n+s}(c)$ and σ is the second fundamental form of M.

Denote by C the unit normal of M in $\widetilde{M}$ and let $U=-\varphi C$. Then, since $\eta_\alpha(C)=0$, for all α,

$$g(U,U)=g(\varphi C,\varphi C)=1$$

and

$$g(U,C)=-g(\varphi C,C)=0.$$

Moreover, if $X\in T(M)$, we have

$$\varphi X=TX+u(X)C, \tag{17}$$

where u and T are tensor fields on M of type (0,1) and (1,1), respectively, also TX represents the tangent part of φX.

In the sequel we set $u\neq 0$, clearly from (17), $u(X)=g(U,X)$. Moreover, it is easy to verify that $\varphi U=C$.

By Codazzi equation, (17) and (5) we obtain

$$0=\widetilde{\nabla}_X\sigma(Y,Z)-\widetilde{\nabla}_Y\sigma(X,Z)=(\widetilde{R}(X,Y)Z)^\perp=$$

$$=\frac{c-s}{4}\{g(\varphi Y,Z)\varphi X-g(\varphi X,Z)\varphi Y+2g(X,\varphi Y)\varphi Z\}^\perp=$$

$$=\frac{c-s}{4}\{g(\varphi Y,Z)u(X)-g(\varphi X,Z)u(Y)+2g(X,\varphi Y)u(Z)\}C=0.$$

If we put $Z=U$, we deduce that

$$\frac{c-s}{4}g(X,TY)=0.$$

Since $c\neq s$, then $TY=0$, so $\dim\varphi T_x(M)=1$ for all $x\in M$. Moreover, we have $T_x(\widetilde{M})=$ $=T_x(M)\oplus T_x(M)^{\perp}$ and $\operatorname{rank}\varphi=2n$, then we obtain

$$2n-1\leq\dim\varphi T_x(M)^{\perp}\leq 2n,$$

which is impossible because $n>1$ and $\dim T_x(M)^{\perp}=1$.

Theorem 4.3 is proved.

Theorem 4.4. *There are no semi-parallel hypersurfaces tangent to the structure vector fields in an $\mathcal{S}$-space form $\widetilde{M}^{2n+s}(c)$ with $c\neq s$.*

Proof. If M is a semi-parallel hypersurface and σ is the second fundamental form of M, we have

$$(\widetilde{R}\,.\,\sigma)(Z,W;X,Y)=-\sigma(\widetilde{R}(X,Y)Z,W)-\sigma(Z,\widetilde{R}(X,Y)W)=0.$$

By using the same argument as in Theorem 3.1, we deduce that

$$\sigma=Kg\quad\text{or}\quad\sigma=K'g+\sum_{\alpha,\beta=1}^{s}K^{\alpha\beta}\eta_\alpha\otimes\eta_\beta-\sum_{\alpha,\beta=1}^{s}K^{\beta\gamma}\eta_\alpha\otimes\eta_\alpha,$$

where K, K', $K^{\alpha\beta}$, $K^{\beta\gamma}$ are constants, so clearly

$$\widetilde{\widetilde{\nabla}}\sigma=0$$

which contradicts Theorem 4.3.

Theorem 4.4 is proved.

***Corollary* 4.1.** *There is no semi-parallel hypersurfaces in $\mathbb{R}^{2n+s}(-3s)$.*

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