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O. Basdouri (Dep. Math., Faculté Sci. Gafsa, Tunisia),
A. Braghtha (Univ. Bourgogne, Inst. Math. Bourgogne, France),
S. Hammami (Univ. Sfax, Tunisia)

THE SECOND COHOMOLOGY SPACES $\mathcal{K}(2)$ WITH COEFFICIENTS IN THE SUPERSPACE OF WEIGHTED DENSITIES

ПРОСТОРИ ДРУГОЇ КОГОМОЛОГІЇ $\mathcal{K}(2)$ З КОЕФІЦІЄНТАМИ, ЩО НАЛЕЖАТЬ ДО СУПЕРПРОСТОРУ ЗВАЖЕНИХ ЩІЛЬНОСТЕЙ

Over the $(1,2)$-dimensional supercircle, we investigate the second cohomology space associated the lie superalgebra $\mathcal{K}(2)$ of vector fields on the supercircle $S^{1|2}$ with coefficients in the space of weighted densities. We explicitly give 2-cocycle spanning these cohomology spaces.

Над $(1,2)$-вимірним суперколом вивчаються простори другої когомології, які пов'язані з супералгеброю Лі $\mathcal{K}(2)$ векторних полів на суперколі $S^{1|2}$ з коефіцієнтами у просторі зважених щільностей. Ми явно отримали 2-коцикл, що охоплює ці простори когомології.

1. Introduction. Let $\mathfrak{g}$ be a Lie algebra and M a $\mathfrak{g}$-module. We shall associate a cochain complex known as the **Chevalley – Eilenberg differential**. The nth space of this complex will be denoted by $C^n(\mathfrak{g},\ M)$.

It is trivial if $n < 0$, and if $n > 0$, it is the space of n-linear antisymmetric mappings of $\mathfrak{g}$ into M: they will be called n-cochains of $\mathfrak{g}$ with coefficients in M. The space of 0-cochains $C^0(\mathfrak{g}, M)$ reduces to M. The differential δ^n will be defined by the following formula: for $c \in C^n(\mathfrak{g},\ )$, the $(n+1)$-cochain $\delta^n(c)$ evaluated on $g_1, g_2, \ldots, g_{n+1} \in \mathfrak{g}$ gives

$$\delta^n c(g_1, \ldots, g_{n+1}) = \sum_{1\leq s<t\leq n+1} (-1)^{s+t-1} c\left([g_s, g_t], g_1, \ldots, \hat{g}_s, \ldots, \hat{g}_t, \ldots, g_{q+1}\right) +$$

$$+ \sum_{1\leq s\leq n+1} (-1)^s g_s c\left(g_1, \ldots, \hat{g}_s, \ldots, g_{n+1}\right),$$

the notation $\hat{g}_i$ indicates that the ith term is omitted.

We check that $\delta^{n+1} \circ \delta^n = 0$, so we have a complex

$$0 \to C^0(\mathfrak{g}, M) \to \ldots \to C^{n-1}(\mathfrak{g}, M) \overset{d^{n-1}}{\to} C^n(\mathfrak{g}, M) \to \ldots .$$

We note by $H^n(\mathfrak{g}, M) = \ker d^n / \mathrm{Im} d^{n-1}$ the quotient space. This space is called the space of n-cohomology from $\mathfrak{g}$ with coefficients in M. We denoted by:

$Z^n(\mathfrak{g}, M) = \ker \delta_n$: the space of n-cocycles,

$B^n(\mathfrak{g}, M) = \Im \delta_{n-1}$: the space of n-coboundaries.

For $M = \mathbb{R}$ (or $\mathbb{C}$) considered as a trivial module, we denote the cohomology in this case, $H^n(\mathfrak{g})$.

We shall now recall classical interpretations of cohomology spaces of low degrees:

The space $H^0(\mathfrak{g}, M) \simeq \mathrm{Inv}_{\mathfrak{g}}(M) := \{m \in M;\ X.m = 0\ \ \forall X \in \mathfrak{g}, \}$.

The space $H^1(\mathfrak{g}, M)$ classifies derivations of $\mathfrak{g}$ with values in M modulo inner ones. This result is particularly useful when $M = \mathfrak{g}$ with the adjoint representation. In this case, a derivation is a map $\varrho\colon \mathfrak{g} \longrightarrow \mathfrak{g}$ such that

$$\varrho([X,Y]) - [\varrho(X), Y] - [X, \varrho(Y)] = 0,$$

while an inner derivation is given by the adjoint action of some element $Z \in \mathfrak{g}$.

The space $H^2(\mathfrak{g}, M)$ classifies extensions of Lie algebra $\mathfrak{g}$ by M, i.e., short exact sequences of Lie algebras

$$0 \to M \to \hat{\mathfrak{g}} \to \mathfrak{g} \to 0,$$

in which M is considered as an Abelian Lie algebra. We shall mainly consider two particular cases of this situation which will be extensively studied in the sequel:

If M is a trivial $\mathfrak{g}$-module (typically $M = \mathbb{R}$ or $\mathbb{C}$), $H^2(\mathfrak{g}, M)$ classifies central extensions modulo trivial ones. Recall that a central extension of $\mathfrak{g}$ by $\mathbb{R}$ produces a new Lie bracket on $\hat{\mathfrak{g}} = \mathfrak{g} \oplus M$ by setting that

$$[(X,\lambda),(Y,\mu)] = ([X,Y], c(X,Y)).$$

It is trivial if the cocycle $c = dl$ is a coboundary of a 1-cochain l, in which case the map $(X,\lambda) \to (X, \lambda - l(X))$ yields a Lie isomorphism between $\hat{\mathfrak{g}}$ and $\mathfrak{g} \oplus M$ considered as a direct sum of Lie algebras.

If $M = \mathfrak{g}$ with the adjoint representation, then $H^2(\mathfrak{g}, \mathfrak{g})$ classifies infinitesimal deformations modulo trivial ones. By definition, a (formal) series

$$(X,Y) \to \Phi_\lambda(X,Y) := [X,Y] + \lambda f_1(X,Y) + \lambda^2 f_2(X,Y) + \ldots$$

is a deformation of Lie bracket $[,]$ if Φ_λ is a Lie bracket for every λ, i.e., is an antisymmetric bilinear form in X, Y and satisfies the Jacobi identity. If one sets simply that

$$[X,Y]_\lambda = [X,Y] + \lambda c(X,Y),$$

c being a 2-cochain with values in $\mathfrak{g}$ and λ being a scalar, then this bracket satisfies Jacobi identity modulo terms of order $O\left(\lambda^2\right)$ if and only if c is a 2-cocycle. Thus, one gets what is called an infinitesimal deformation of the bracket of $\mathfrak{g}$, which is trivial if c is a coboundary, by which we mean (as in the case of central extensions) that an adequate linear isomorphism from $\mathfrak{g}$ to $\mathfrak{g}$ transforms the initial bracket $[,]$ into the deformed bracket $[,]_\lambda$. The infinitesimal deformation associated to a cocycle c does not always give rise to an actual deformation coinciding with the infinitesimal deformation to order 1, i.e., such that $f_1 = c$, as one may check by looking inductively for functions $f_2, f_3, \ldots$ which satisfy Jacobi's identity to order $2,\ 3, \ldots$ Cohomological obstructons to prolongations of deformations are contained in $H^3(\mathfrak{g}, \mathfrak{g})$.

The natural generalization of the Virasoro algebra is given by extensions of the Lie algebra $\mathfrak{vect}(S^1)$ of the vector fields on the circle by modules $\mathcal{F}_\lambda$ of λ-densities on the circle. The problem of classifying such extensions is equivalent to that of the calculation of the cohomology $H^2\left(\mathrm{Vect}\left(S^1\right);\mathcal{F}_\lambda\right)$. In [4, 5], V. Ovsienko, C. Roger and P. Marcel, calculated the space $\mathrm{H}^2\left(\mathrm{Vect}\left(S^1\right);\mathcal{F}_\lambda\right)$ and where $\mathrm{Vect}\left(S^1\right)$ is the algebra of smooth vector field on the circle S^1 and $\mathcal{F}_\lambda$ is the space of λ densities. Following V. Ovsienko and C. Roger, B. Agrebaoui, I. Basdouri and M. Boujelben [1] computed $H^2_{\mathrm{diff}}\left(\mathcal{K}(1);\mathfrak{F}^1_\lambda\right)$, where $\mathcal{K}(1)$ is the lie superalgebra of contact vector fields on the supercircle $S^{1|1}$ with coefficients in the space of weighted densities.

In this paper, we explicitly compute $H^2_{\mathrm{diff}}\left(\mathcal{K}(2);\mathfrak{F}^2_\lambda\right)$, where $\mathcal{K}(2)$ is the lie superalgebra of contact vector fields in $S^{1|2}$ with coefficients in the spaces of weighted densities $\mathfrak{F}^2_\lambda$.

The present paper is organized as follows. After some preliminary definitions and explanation of notation in Section 2. In Section 3, we compute the 2-cohomology space $H^2_{\mathrm{diff}}\left(\mathcal{K}(2);\mathfrak{F}^2_\lambda\right)$, we classify the extensions of a Lie superalgebra $\mathcal{K}(2)$ by $\mathfrak{F}^2_\lambda$.

2. Preliminaries. In this section, we recall some tools pertaining to the problem of cohomology such as weighted densities, superfunctions, contact projective vector fields on $S^{1|n}$.

2.1. Standard contact structure on $S^{1|n}$. Let $S^{1|n}$ be the supercircle with coordinates $(x,\theta_1,\ldots$ $\ldots,\theta_n)$, where x is an even indeterminate and $\theta_1,\ldots,\theta_n$ are odd indeterminate: $\theta_i\theta_j=-\theta_j\theta_i$. This superspace is equipped with the standard contact structure given by the distribution $D=\langle\overline{\eta}_1,\ldots,\overline{\eta}_n\rangle$ generated by the vector fields $\overline{\eta}_i=\partial_{\theta_i}-\theta_i\partial_x$. That is, the distribution D is the kernel of the following 1-form:

$$\alpha_n=dx+\sum_{i=1}^{n}\theta_i d\theta_i.$$

2.2. Superfunctions on $S^{1|n}$. We define the geometry of the superspace $S^{1|n}$, where $n\in\mathbb{N}$, by describing its associative supercommutative superalgebra of superfunctions on $S^{1|n}$ which we denote by $C^\infty(S^{1|n})$ which is the space of functions F of the form

$$F=\sum_{1\le i_1<\ldots<i_k\le n} f_{i_1,\ldots,i_k}(x)\theta_{i_1}\ldots\theta_{i_k},\quad \text{where}\quad f_{i_1,\ldots,i_k}\in C^\infty\left(S^1\right). \tag{2.1}$$

Of course, even (respectively, odd) elements in $C^\infty\left(S^{1|n}\right)$ are the functions (2.1) for which the summation is only over even (respectively, odd) integer k. Note $p(F)$ the parity of a homogeneous function F. On $C^\infty\left(S^{1|n}\right)$, we consider the contact bracket

$$\{F,G\}=FG'-F'G-\frac{1}{2}(-1)^{p(F)}\sum_{i=1}^{n}\overline{\eta}_i(F)\overline{\eta}_i(G),$$

where the superscript $'$ stands for $\dfrac{\partial}{\partial x}$.

2.3. Vector fields on $S^{1|n}$. A vector field on $S^{1|n}$ is a superderivation of the associative supercommutative superalgebra $C^\infty\left(S^{1|n}\right)$. In coordinates, it can be expressed as

$$X=f\partial_x+\sum_{i=1}^{n}g_i\partial_{\theta_i},$$

where f and g_i are the elements of $C^\infty\left(S^{1|n}\right)$.

The superspace of all vector fields on $C^\infty\left(S^{1|n}\right)$ is a Lie superalgebra which we shall denote by $\mathrm{Vect}\left(C^\infty\left(S^{1|n}\right)\right)$.

2.4. Lie superalgebra of contact vector fields on $S^{1|n}$. Consider the superspace $\mathcal{K}(n)$ of contact vector fields on $S^{1|n}$. That is, $\mathcal{K}(n)$ is the superspace of vector fields on $S^{1|n}$ with respect to the 1-form α_n. The Lie superalgebra of contact vector fields is by definition

$$\mathcal{K}(n)=\left\{X\in \mathrm{Vect}\left(S^{1|n}\right)\Big| \quad \text{there exists} \quad F_X\in C^\infty\left(S^{1|n}\right) \quad \text{such that} \quad \mathfrak{L}_{X_F}(\alpha_n)=F\alpha_n\right\}.$$

Let us define the vector fields η_i and $\overline{\eta_i}$ by $\eta_i=\partial_{\theta_i}+\theta_i\partial_x$ $\overline{\eta_i}=\partial_{\theta_i}-\theta_i\partial_x$. Then any contact vector field on $S^{1|n}$ can be written in the following explicit form:

$$X_F=F\partial_x-\frac{1}{2}(-1)^{p(F)}\sum_{i=1}^{n}\overline{\eta}_i(F)\overline{\eta}_i, \quad \text{where} \quad F\in C^\infty\left(S^{1|n}\right).$$

The $\mathcal{K}(n)$ acts on $S^{1|n}$ through

$$\mathfrak{L}_{X_F}(X_G)=F\partial_x X_G+(-1)^{p(F)+1}\frac{1}{2}\sum_{i=1}^{n}\overline{\eta}_i(F)\overline{\eta_i}(G).$$

The vector field X_F has the same parity as F. The bracket in $\mathcal{K}(n)$ can be written as

$$[X_F,X_G]=X_{\{F,G\}}.$$

The Lie superalgebra $\mathfrak{osp}(2|n)$ is called the Lie superalgebra of the contact projective vector fields. Thus $\mathfrak{osp}(2|n)$ is a $(n+2|2n)$-dimensional Lie superalgebra spanned by the following contact projective vector fields:

$$\left\{X_x,X_{x^2},X_1,2X_{\theta_i\theta_j},X_{\theta_i},X_{x\theta_i},i,j=1,\ldots,n\right\}.$$

2.5. Modules of weighted densities. Now, consider the 1-parameter action of $\mathcal{K}(n)$ on $C^\infty\left(S^{1|n}\right)$ given by the rule

$$\mathfrak{L}^{\lambda}_{X_F}=X_F+\lambda F'.$$

We denote this $\mathcal{K}(n)$-module by $\mathfrak{F}^n_\lambda$, the space of all weighted densities on $S^{1|n}$ of weight λ:

$$\mathfrak{F}^n_\lambda=\left\{F\alpha_n^\lambda \mid F\in C^\infty(S^{1|n})\right\}.$$

The superspace $\mathfrak{F}^n_\lambda$ has $\mathcal{K}(n)$-module structure defined by the Lie derivative:

$$\mathfrak{L}^{\lambda}_{X_G}\left(F\alpha_n^\lambda\right)=\left(X_G+\lambda G'\right)(F)\alpha^n_\lambda,$$

where $G':=\dfrac{\partial G}{\partial x}$. Obviously, $\mathcal{K}(n)$ is isomorphic to $\mathfrak{F}^n_{-1}$ as $\mathcal{K}(n)$-module and

$$\mathfrak{F}^n_\lambda\simeq\mathfrak{F}^{n-1}_\lambda\oplus\Pi\left(\mathfrak{F}^{n-1}_{\lambda+\frac{1}{2}}\right),$$

where Π is the change of parity function.

3. Space $\mathrm{H}^2\left(\mathcal{K}(2);\mathfrak{F}_\lambda^2\right)$. In this paper, we study the differential cohomology spaces $\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_\lambda^2\right)$. That is, we consider only cochains $(X_F, X_G) \to \Omega(F,G)\alpha_\lambda^2$ where Ω is a differential operator.

3.1. Main theorem. The main result of this paper is the following theorem.

Theorem 3.1.

$$\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_\lambda^2\right) \simeq \begin{cases} \mathbb{K}, & \textit{if} \quad \lambda = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, 3, \\ 0 & \textit{otherwise}. \end{cases}$$

The nontrivial spaces $\mathrm{H}^2\left(\mathcal{K}(2);\mathfrak{F}_\lambda^2\right)$ *are spanned by the following* 2*-cocycles:*

$$\Omega_0(X_F, X_G) = \left(\bar\eta_1(F)\bar\eta_2(G) - \bar\eta_2(F)\bar\eta_1(G)\right)\theta_1\theta_2,$$

$$\Omega_{\frac{1}{2}}(X_F, X_G) = \frac{1}{2}\left(\bar\eta_1\bar\eta_2(F)\bar\eta_1(G) - \bar\eta_1(F)\bar\eta_1\bar\eta_1(G)\right)\theta_1\theta_2,$$

$$\Omega_1(X_F, X_G) = \left(F\bar\eta_1\bar\eta_2(G) - \bar\eta_1\bar\eta_2(F)G + \bar\eta_1(F)\bar\eta_2(G) + \bar\eta_2(F)\bar\eta_1(G)\right)\theta_1\theta_2,$$

$$\Omega_{\frac{3}{2}}(X_F, X_G) = \left(\bar\eta_1\bar\eta_2(F)\bar\eta_1(G) + \bar\eta_1\bar\eta_2(F)\bar\eta_2(G) - \bar\eta_1(F)\bar\eta_1\bar\eta_2(G) - \bar\eta_2(F)\bar\eta_1\bar\eta_2(G)\right)\theta_1\theta_2,$$

$$\Omega_2(X_F, X_G) = \bar\eta_1\bar\eta_2\left(F'\right)\bar\eta_1\bar\eta_2\left(G'\right),$$

$$\Omega_3(X_F, X_G) = \Big((-1)^{|F|}\left(\bar\eta_1\left(F''\right)\bar\eta_1\left(G''\right) + \bar\eta_2\left(F''\right)\bar\eta_2(G'')\right) +$$

$$+2\left(\bar\eta_1\bar\eta_2\left(F'\right)\bar\eta_1\bar\eta_2\left(G''\right) - \bar\eta_1\bar\eta_2\left(F''\right)\bar\eta_1\bar\eta_2\left(G'\right)\right)\Big).$$

Corollary **3.1.**

$$\mathrm{H}^2_{\text{diff}}(\mathcal{K}(2),\, \mathcal{K}(2)) \simeq 0. \tag{3.1}$$

3.2. Relationship between $\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2),\mathfrak{F}_\lambda^2\right)$ and $\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(1),\mathfrak{F}_\lambda^1\right)$. Before proving the Theorem 3.1 we present some results illustrating the relation between the cohomology space in supercircle $S^{1|1}$ and $S^{1|2}$.

Proposition 3.1 [1]**.**

$$H^2_{\text{diff}}\left(\mathcal{K}(1);\mathfrak{F}_\lambda^1\right) \simeq \begin{cases} \mathbb{K}, & \textit{if} \quad \lambda = 0, 3, 5, \\ \mathbb{K}^2, & \textit{if} \quad \lambda = \frac{1}{2}, \frac{3}{2}, \\ 0 & \textit{otherwise}. \end{cases}$$

The nontrivial space $H^2\left(\mathcal{K}(1);\mathfrak{F}_\lambda^1\right)$ *are spanned by the* 2*-cocycles:*

$$\omega_0(X_F, X_G) = FG' - F'G - \left(\frac{1}{4} + \frac{3}{4}(-1)^{p(F)p(G)}\right)\bar\eta_1(F)\eta_1(G),$$

$$\omega_{\frac{1}{2}}(X_F, X_G) = (-1)^{p(F)+p(G)}\left(F'\eta_1(G') - \eta_1(F')G'\right)\alpha_1^{\frac{1}{2}},$$

$$\widetilde\omega_{\frac{1}{2}}(X_F, X_G) = \left(\frac{1}{2} + \frac{1}{4}\left(1 + (-1)^{p(F)p(G)}\right)\right)(-1)^{p(F)+p(G)}\left(F\eta_1(G') - \eta_1(F')G\right)\alpha_1^{\frac{1}{2}},$$

$$\omega_{\frac{3}{2}}(X_F,X_G)=\Big(\bar{\eta}_1\left(F''\right)G-(-1)^{p(F)}F'\bar{\eta}_1\left(G''\right)\Big)-\frac{1}{2}\theta_1\left(\eta_1(F)\eta_1\left(G''\right)+\eta_1\left(F''\right)\eta_1(G)\right)\alpha_1^{\frac{3}{2}},$$

$$\widetilde{\omega}_{\frac{3}{2}}(X_F,X_G)=\left(F'\bar{\eta}_1\left(G''\right)-\bar{\eta}_1\left(F''\right)G'\right)\alpha_1^{\frac{3}{2}},$$

$$\omega_3(X_F,X_G)=\left(\eta_1\left(F''\right)\bar{\eta}_1\left(G''\right)G'\right)\alpha_1^3,$$

$$\omega_5(X_F,X_G)=\Big(\Big(F^{(3)}G^{(4)}F^{(4)}G^{(3)}\Big)+\frac{3}{2}\Big(\eta_1(F^{(4)})\eta_1\Big(G^{(2)}\Big)-\eta_1\Big(F^{(2)}\Big)\eta_1\Big(G^{(4)}\Big)\Big)-$$
$$-4\eta_1\Big(F^{(3)}\Big)\eta_1\Big(G^{(3)}\Big)\Big)\alpha_1^5.$$

The following lemma gives the general form of each Ω.

Lemma 3.1. *The* 2*-cocycle* Ω *belongs to* $\mathrm{Z}^2\left(\mathcal{K}(2),\mathfrak{F}^2_\lambda\right)$. *Up to a coboundary, the map* Ω *is given by*

$$\Omega(X_F,X_G)=\sum_{i,j,k,l}a_{i,j,k,l}\overline{\eta}_1^i\overline{\eta}_2^j(F)\overline{\eta}_1^k\overline{\eta}_2^l(G)\alpha_2^\lambda,$$

where $a_{i,j,k,l}$ *depends only on* $\theta_1,\ \theta_2$ *and the parity of* F *and* G.

Proof. Every differential operator Ω can be expressed in the form

$$\Omega(X_F,X_G)=\sum a_{i,j,k,l}\overline{\eta}_1^i\overline{\eta}_2^j(F)\overline{\eta}_1^k\overline{\eta}_2^l(G)\alpha_2^\lambda,$$

where the coefficients $a_{i,j,k,l}$ are arbitrary function. By using the 2-cocycle equation, we can show that $\dfrac{\partial}{\partial x}a_{i,j,k,l}=0$. The dependence on the parity of F and G comes from the fact that Ω is skew-symmetric:

$$a_{i,j,k,l}(F,G)=(-1)^{\varepsilon_{ij}(F,G)}a_{i,j,k,l}(F,G),$$

where

$$\varepsilon_{ij}(F,G)=ij(p(F)+1)(p(G)+1)+p(F)p(G)+1.$$

Lemma 3.1 is proved.

Now, to prove Theorem 3.1, we also need to compute the cohomology space vanishing on $\mathcal{K}(1)$. We will be interested in cohomology space vanishing on $\mathcal{K}(1)$, that is, we assume

$$\Omega(X,Y)=0,\quad \text{if}\quad X,Y\in\mathcal{K}(1).$$

Therefore, the relevant cohomology space is

$$\mathrm{H}^2_{\mathrm{diff}}\left(\mathcal{K}(2),\mathcal{K}(1),\mathfrak{F}^2_\lambda\right).$$

Theorem 3.2. *The space*

$$\mathrm{H}^2_{\mathrm{diff}}\left(\mathcal{K}(2),\mathcal{K}(1),\mathfrak{F}^2_\lambda\right)\simeq\begin{cases}\mathbb{K}, & \textit{if}\quad \lambda=2,\\ 0 & \textit{otherwise.}\end{cases}\tag{3.2}$$

Proof. Let Ω a 2-cocycle of $\mathcal{K}(2)$ vanishing on $\mathcal{K}(1)$. The expressions of Ω are given by Lemma 3.1. We check with “MATHEMATICA” that the 2-cocycle condition has the solution

$$\Omega(X_F, X_G) = \begin{cases} 0 & \text{if} \quad \lambda \neq 2, \\ \nu \overline{\eta}_1 \overline{\eta}_2 \left(F'\right) \overline{\eta}_1 \overline{\eta}_2 \left(G'\right) \alpha_2^2, & \text{if} \quad \lambda = 2, \end{cases}$$

where ν is constant. Assume that the map Ω is trivial 2-cocycle vanishing on $\mathcal{K}(1)$. Thus, there exists an even operator $b \colon \mathcal{K}(2) \to \mathfrak{F}_2^2$, given by

$$b(X_F) = \left(\sum_k \kappa_k(x, \theta_1, \theta_2) \eta_1 \eta_2 \left(F^{(k)}\right) + \sum_l \mu_l(x, \theta_1, \theta_2) F^{(l)} \right) \alpha_2^\lambda,$$

where the coefficients $\kappa_k(x, \theta_1, \theta_2)$ and $\mu_l(x, \theta_1, \theta_2)$ are arbitrary such that Ω is equal to $\delta(b)$, that is

$$\Omega(X_F,\, X_G) := (-1)^{p(X_F)p(b)} \mathfrak{L}^2_{X_F}(b(X_G)) -$$

$$-(-1)^{p(X_G)(p(X_F))} \mathfrak{L}^2_{X_G}(b(X_F)) - b([X_F, X_G]). \tag{3.3}$$

The condition (3.3) implies that its coefficients are constant.

We check with “MATHEMATICA” that the condition (3.3) has no solution. We can see that the expression (3.2) never appears on the right-hand side of (3.3). This is a contradiction with our assumption.

Theorem 3.2 is proved.

***Proof of Theorem* 3.1.** Consider a 2-cocycles $\Omega \in Z^2_{\text{diff}}\left(\mathcal{K}(2); \mathfrak{F}_\lambda^2\right)$. If $\Omega_{|\mathcal{K}(1)\otimes\mathcal{K}(1)}$ is trivial then the 2-cocycle Ω is completely described by Theorem 3.2. Thus, assume that $\Omega_{|\mathcal{K}(1)\otimes\mathcal{K}(1)}$ is nontrivial. Of course, by considering Proposition 3.1, we deduce that nontrivial space $\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2); \mathfrak{F}_\lambda^2\right)$ only can appear if $\lambda \in \left\{\dfrac{-1}{2}, 0, \dfrac{1}{2}, 1, \dfrac{3}{2}, \dfrac{5}{2}, \dfrac{9}{2}, 3, 5\right\}$. The $\mathcal{K}(1)$-isomorphism:

$$\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(1); \mathfrak{F}_\lambda^2\right) \simeq \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(1); \mathfrak{F}_\lambda^1\right) \oplus \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(1); \prod \left(\mathfrak{F}^1_{\lambda+\frac{1}{2}}\right)\right).$$

Together with Proposition 3.1 that describes up to a coboundary and up to a scalar factor the restriction of any 2-cocycle Ω to $\mathcal{K}(1)$. In inception, we consider separately the even and odd cases. Even cohomology spaces only can appear if $\lambda \in \{0, 1, 3, 5\}$ and odd cohomology spaces only can appear if $\lambda \in \left\{\dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{3}{2}, \dfrac{5}{2}, \dfrac{9}{2}\right\}$.

In each case, the restriction of Ω to $\mathcal{K}(1)$ is a linear combination of corresponding 2-cocycles given in Proposition 3.1. First, the operators Ω labeled by semi-integer are odd and given by

$$\Omega(X_F, X_G) = \sum_{i,j,k,l} a_{i,j,k,l} \overline{\eta}_1^i \overline{\eta}_2^j (F) \overline{\eta}_1^k \overline{\eta}_2^l (G) \alpha_2^\lambda,$$

where $i + j + k + l \in \{1, 3\}$ and the coefficient a_{ijkl} are arbitrary functions independent on the variable x, but they are depending on θ and parity of F and G. By using “MATHEMATICA”, we will investigate the dimension of the space of operators that satisfy the 2-cocycle condition:

$$\delta(\Omega)(X_F, X_G, X_H) := (-1)^{p(F)} X_F.\Omega(X_G, X_H) - (-1)^{p(G)(1+p(F))} X_G.\Omega(X_F, X_H) +$$

$$+(-1)^{p(H)(1+p(G)+p(F))}X_H.\Omega(X_F,X_G)-\Omega([X_F,X_G],X_H)+$$

$$+(-1)^{p(G)p(H)}\Omega([X_F,X_H],X_G)-(-1)^{p(F)(p(G)+p(H))}\Omega([X_G,X_H],X_F)=0, \qquad (3.4)$$

where $X_F.\Omega(X_F,X_H)=\mathfrak{L}^{\lambda}_{X_F}(\Omega(X_G,X_H))$ and $F,G,H\in\mathcal{C}^{\infty}\left(S^{1|2}\right)$.

The number of variables generating any 2-cocycle is much smaller than the number of equations coming out from the 2-cocycle condition for particular values of a_{ijkl}. We have:

For $\lambda=\dfrac{1}{2}$:

Therefore, by a direct computation, we can see that the 2-cocycle condition is always satisfied with particular values:

$$a_{1000}=0,\quad a_{0100}=0,\quad a_{0010}=0,\quad a_{0001}=0,$$

$$a_{1110}=\frac{1}{2}\,\theta_1\theta_2,\quad a_{1101}=0,\quad a_{0111}=0,\quad a_{1011}=-\frac{1}{2}\,\theta_1\theta_2.$$

We will study all trivial 2-cocycles, namely, operators of the form δb, where b is a linear operator given by

$$b(X_F)=(\kappa\ \eta_1\eta_2(F)+\mu\ F)\alpha_2^{\lambda}.$$

A direct computation proves that

$$\delta b(X_F,X_G)=\frac{1}{2}\Big(\kappa\left(3f_1(x)g_1(x)+3f_2(x)g_2(x)+g_0(x)f_0'(x)-f_0(x)g_0'(x)\right)-$$

$$-\kappa\theta_1\big(3f_{12}(x)g_2(x)-3f_2(x)g_{12}(x)-3g_1(x)f_0'(x)+3g_0(x)f_1'(x)+f_1(x)g_0'(x)-f_0(x)g_1'(x)\big)-$$

$$-\kappa\theta_2\big(3f_{12}(x)g_1(x)-3f_1(x)g_{12}(x)-6g_2(x)f_0'(x)-g_0(x)f_2'(x)+6f_2(x)g_0'(x)-f_0(x)g_2'(x)\big)+$$

$$+\kappa\theta_1\theta_2\big(g_{12}(x)f_0'(x)+2g_2(x)f_1'(x)-2g_1(x)f_2'(x)+$$

$$+g_0(x)f_{12}'(x)-f_{12}(x)g_0'(x)+2f_2(x)g_1'(x)-2f_1(x)g_2'(x)-f_0(x)g_{12}'(x)\big)+$$

$$+\mu\big(3g_{12}f_0'(x)+2g_2(x)f_1'(x)-2g_1(x)f_2'(x)-3f_{12}(x)g_0'+4f_2(x)g_1'(x)-4f_1(x)g_2'(x)\big)+$$

$$+\mu\theta_1\big(-g_{12}(x)f_1'(x)+2g_1(x)f_{12}'(x)+f_2'(x)g_0'(x)-$$

$$-f_{12}(x)g_1'(x)-f_0'(x)g_2'(x)-4f_1(x)g_{12}'(x)-4g_2(x)f_0''(x)+4f_2(x)g_0''(x)\big)+$$

$$+\mu\theta_2\big(-g_{12}(x)f_2'(x)+4g_2(x)f_{12}'(x)-f_1'(x)g_0'(x)+f_0'(x)g_1'(x)+$$

$$+3f_{12}(x)g_2'(x)-4f_1(x)g_{12}'(x)+2g_1(x)f_0''(x)-2f_2(x)g_{12}''(x)\big)+$$

$$+\mu\theta_1\theta_2\big(-g_{12}(x)f_{12}'(x)-2f_1'(x)g_1'(x)-2f_2'(x)g_2'(x)+f_{12}(x)g_{12}'(x)+g_0'(x)f_0''(x)+$$

$$+2g_1(x)f_1''(x)-g_2(x)f_2''(x)-f_0'(x)g_0''(x)-2f_1(x)g_1''(x)-2f_2(x)g_2''(x)\big)\Big).$$

It is now easy to check that the equation $\Omega-\delta b=0$ has no solutions. So the 2-cocycle is nontrivial and $\dim\mathrm{H}^2_{\mathrm{diff}}(\mathcal{K}(2);\mathfrak{F}^2_{\lambda})=\dim\mathrm{Z}^2_{\mathrm{diff}}\left(\mathcal{K}(2);\mathfrak{F}^2_{\lambda}\right)$. Hence, the cohomology space is one-dimensional.

For $\lambda = \dfrac{3}{2}$:

Therefore, by a direct computation, we can see that the 2-cocycle condition is always satisfied with particular values:

$$a_{1000} = 0, \quad a_{0100} = 0, \quad a_{0010} = 0, \quad a_{0001} = 0,$$

$$a_{1110} = \theta_1\theta_2, \quad a_{1101} = \theta_1\theta_2, \quad a_{0111} = -\theta_1\theta_2, \quad a_{1011} = -\theta_1\theta_2.$$

Let us study the triviality of this 2-cocycle. We can see that any coboundary $\delta b(X_F, X_G)$ can be expressed as follows:

$$\delta b(X_F, X_G) = \kappa\left(\frac{3}{2}\,f_1(x)g_1(x) + \frac{3}{2}\,f_2(x)g_2(x) + \frac{3}{2}\,g_0(x)f_0'(x) - \frac{3}{2}\,f_0(x)g_0'(x)\right) +$$

$$+\kappa\theta_1\left(-\frac{1}{2}\,f_{12}(x)g_2(x) + \frac{3}{2}f_2(x)g_{12}(x) + 4g_1(x)f_0I(x) - \frac{3}{2}g_0(x)f_1I(x) +\right.$$

$$\left.+4f_1(x)g_0I(x) + \frac{3}{2}\,f_0(x)g_1I(x)\right)\kappa\theta_2\left(-\frac{1}{2}\,f_{12}(x)g_1(x) + \frac{3}{2}\,f_1(x)g_{12}(x) + 4g_2(x)f_0'(x) +\right.$$

$$\left.+\frac{3}{2}\,g_0(x)f_2'(x) - 4f_2(x)g_0'(x) + \frac{3}{2}\,f_0(x)g_2'(x)\right) + \kappa\theta_1\theta_2\left(\frac{3}{2}\,g_{12}(x)f_0'(x) + \frac{3}{2}\,g_0(x)f_{12}'(x) +\right.$$

$$\left.+\frac{3}{2}\,f_{12}(x)g_0'(x) + f_2(x)g_1'(x) - f_1(x)g_2'(x) - \frac{3}{2}\,f_0(x)g_{12}'(x)\right) +$$

$$+\mu\left(\frac{5}{2}\,g_{12}(x)f_0'(x) + g_2(x)f_1'(x) - g_1(x)f_2'(x) - \frac{5}{2}\,f_{12}(x)g_0'(x) + 2f_2(x)g_1'(x) - 2f_1(x)g_2'(x)\right) +$$

$$+\mu\theta_2\Bigg(-\frac{3}{2}\,g_{12}(x)f_2'(x) + 2g_2(x)f_{12}'(x) - \frac{3}{2}\,f_1'(x)g_0'(x) + \frac{3}{2}\,f_0'(x)g_1'(x) + \frac{5}{2}\,f_{12}(x)g_2'(x) -$$

$$-f_2(x)g_{12}'(x) + g_1(x)f_0''(x) - f_1(x)g_0''(x)\Bigg) + \mu\theta_1\left(-\frac{3}{2}\,g_{12}(x)f_1'(x) + g_1(x)f_{12}'(x) +\right.$$

$$\left.+\frac{3}{2}\,f_2'(x)g_0'(x) + \frac{3}{2}\,f_{12}(x)g_1'(x) - \frac{3}{2}\,f_0'(x)g_2'(x) - 2f_1(x)g_{12}'(x) - 2g_2(x)f_0''(x) + 2f_2(x)g_0''(x)\right) +$$

$$+\mu\theta_1\theta_2\left(\frac{1}{2}\,g_{12}(x)f_{12}'(x) - 3f_1'(x)g_1'(x) - 3f_2'(x)g_2'(x) - \frac{1}{2}\,f_{12}(x)g_{12}'(x) - \frac{1}{2}\,g_0'(x)f_0''(x) -\right.$$

$$\left.-g_1(x)f_1''(x) - g_2(x)f_2''(x) + \frac{1}{2}\,f_0'(x)g_0''(x) - f_1(x)g_1''(x) - f_2(x)g_2''(x)\right).$$

So, in the same way as before, the equation $\Omega - \delta b = 0$ has no solutions. So the 2-cocycle is nontrivial and $\dim \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2); \mathfrak{F}^2_\lambda\right) = \dim \mathrm{Z}^2_{\text{diff}}\left(\mathcal{K}(2); \mathfrak{F}^2_\lambda\right)$. We deduce that the cohomology space is one-dimensional.

For $\lambda \in \left\{ \frac{-1}{2}, \frac{5}{2}, \frac{9}{2} \right\}$, the equation (3.4) has no solutions. Then

$$\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2), \mathfrak{F}^2_\lambda\right) \simeq 0.$$

By applying the 2-cocycles equation to Ω, using "MATHEMATICA", we deduce the expressions of Ω. To be more precise, we get

$$\Omega = \begin{cases} \frac{1}{2}\left(\bar\eta_1\bar\eta_2(F)\bar\eta_1(G) - \bar\eta_1(F)\bar\eta_1\bar\eta_1(G)\right)\theta_1\theta_2, & \text{if} \quad \lambda = \frac{1}{2}, \\ \left(\bar\eta_1\bar\eta_2(F)\bar\eta_1(G) + \bar\eta_1\bar\eta_2(F)\bar\eta_2(G) - \bar\eta_1(F)\bar\eta_1\bar\eta_2(G) - \bar\eta_2(F)\bar\eta_1\bar\eta_2(G)\right)\theta_1\theta_2, & \text{if} \quad \lambda = \frac{3}{2}. \end{cases}$$

Next, the proof here is the same as in odd 2-cocycle. The operators Ω labeled by integer are even and given by

$$\Omega(X_F, X_G) = \sum_{i,j,k,l} a_{i,j,k,l}\bar\eta_1^i\bar\eta_2^j(F)\bar\eta_1^k\bar\eta_2^l(G)\alpha_2^\lambda,$$

where $i+j+k+l \in \{0,2,4\}$ and the coefficient a_{ijkl} are arbitrary functions independent on the variable x, but they are depending on θ and the parity of F and G.

Using "MATHEMATICA", this map satisfies the 2-cocycles equation

$$\delta(\Omega)(X_F, X_G, X_H) := X_F.\Omega(X_G, X_H) - (-1)^{p(G)p(F)}X_G.\Omega(X_F, X_H) +$$

$$+(-1)^{p(H)(p(G)+p(F))}X_H.\Omega(X_F, X_G) - \Omega([X_F, X_G], X_H) +$$

$$+(-1)^{p(G)p(H)}\Omega([X_F, X_H], X_G) - (-1)^{p(F)(p(G)+p(H))}\Omega([X_G, X_H], X_F) = 0, \qquad (3.5)$$

where $F, G, H \in \mathcal{C}^\infty\left(S^{1|2}\right)$.

For $\lambda = 0$:

Therefore, by a direct computation, we can see that the 2-cocycle condition is always satisfied with particular values:

$$a_{0000} = 0, \quad a_{1100} = 0, \quad a_{0011} = 0, \quad a_{1001} = \theta_1\theta_2,$$

$$a_{0110} = -\theta_1\theta_2, \quad a_{1010} = 0, \quad a_{0101} = 0, \quad a_{0111} = 0, \quad a_{1111} = 0.$$

On the other hand, we can see that the coboundary $\delta b(X_F, X_G)$ can be expressed as follows:

$$\delta b(X_F, X_G) = \kappa\left(-\frac{1}{2}f_1(x)g_1(x) - \frac{1}{2}f_2(x)g_2(x)\right) +$$

$$+\kappa\theta_1\left(\frac{1}{2}f_{12}(x)g_2(x) + \frac{1}{2}f_2(x)g_{12}(x) - \frac{1}{2}g_1(x)f_0'(x) + \frac{1}{2}f_1(x)g_0'(x)\right) +$$

$$+\kappa\theta_2\left(\frac{1}{2}f_{12}(x)g_1(x) + \frac{1}{2}f_1(x)g_{12}(x) - \frac{1}{2}g_2(x)f_0'(x) + \frac{1}{2}f_2(x)g_0'(x)\right) +$$

$$+\kappa\theta_1\theta_2\left(-\frac{1}{2}g_2(x)f_1'(x) + \frac{1}{2}g_1(x)f_2'(x) + \frac{1}{2}f_2(x)g_1'(x) - \frac{1}{2}f_1(x)g_2'(x)\right) +$$

$$+\mu\left(g_{12}(x)f_0'(x)-f_{12}(x)g_0'(x)+f_2(x)g_1'(x)-f_1(x)g_2'(x)\right)+$$

$$+\mu\theta_2\left(-g_2(x)f_{12}'(x)+f_{12}(x)g_2'(x)+2f_2(x)g_{12}'(x)+2g_1(x)f_0''(x)-2f_1(x)g_0''(x)\right)+$$

$$+\mu\theta_1\left(-g_1(x)f_{12}'(x)-f_{12}(x)g_1'(x)+f_1(x)g_{12}'(x)-f_1'(x)g_{12}(x)-g_2(x)f_0''(x)+f_2(x)g_0''(x)\right)+$$

$$+\mu\theta_1\theta_2\left(-g_{12}(x)f_{12}'(x)+f_1'(x)g_1'(x)+2f_2'(x)g_2'(x)+f_{12}(x)g_{12}'(x)+g_0'(x)f_0''(x)+\right.$$

$$\left.+2g_1(x)f_1''(x)+2g_2(x)f_2''(x)-f_0'(x)g_0''(x)+2f_1(x)g_1''(x)+2f_2(x)g_2''(x)\right).$$

So, the cohomology space is one-dimensional since the equation $\Omega-\delta b=0$ has no solutions. Hence, the 2-cocycle is nontrivial and $\dim \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_0^2\right)=\dim \mathrm{Z}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_0^2\right)=1$.

For $\lambda=1$:

Therefore, by a direct computation, we can see that the 2-cocycle condition is always satisfied with particular values:

$$a_{0000}=0,\quad a_{1100}=-\theta_1\theta_2,\quad a_{0011}=\theta_1\theta_2,\quad a_{1001}=\theta_1\theta_2,$$

$$a_{0110}=\theta_1\theta_2,\quad a_{1010}=0,\quad a_{0101}=0,\quad a_{1111}=0.$$

By a direct computation, we get

$$\delta b(X_F,X_G)=\kappa\left(-\frac{1}{2}f_1(x)g_1(x)-\frac{1}{2}f_2(x)g_2(x)+g_0(x)f_0'(x)-f_0(x)g_0'(x)\right)+$$

$$+\kappa\theta_1\left(\frac{1}{2}f_{12}(x)g_2(x)+\frac{1}{2}f_2(x)g_{12}(x)+\frac{1}{2}g_1(x)f_0'(x)+\right.$$

$$\left.+g_0(x)f_1'(x)-\frac{1}{2}f_1(x)g_0'(x)-f_0(x)g_1'(x)\right)+$$

$$+\kappa\theta_2\left(\frac{1}{2}f_{12}(x)g_1(x)+\frac{1}{2}f_1(x)g_{12}(x)+\frac{1}{2}g_2(x)f_0'(x)+\right.$$

$$\left.+g_0(x)f_2'(x)-\frac{1}{2}f_2(x)g_0'(x)-f_0(x)g_2'(x)\right)+$$

$$+\kappa\theta_1\theta_2\left(g_{12}(x)f_0'(x)+\frac{1}{2}g_2(x)f_1'(x)-\frac{1}{2}g_1(x)f_2'(x)+g_0(x)f_{12}'(x)-\right.$$

$$\left.-f_{12}(x)g_0'(x)+\frac{3}{2}f_2(x)\,g_1'(x)-\frac{3}{2}f_1(x)g_2'(x)-f_0(x)g_12'(x)\right)+$$

$$+\mu\left(2g_{12}(x)f_0'(x)-2f_{12}(x)g_0'(x)+f_2(x)g_1'(x)-f_1(x)g_2'(x)\right)+$$

$$+\mu\theta_2\left(g_{12}(x)f_2'(x)-g_2(x)f_{12}'(x)-f_1'(x)g_0'(x)+f_0'(x)g_1'(x)+\right.$$

$$\left.+2f_2(x)g_{12}'(x)+2g_1(x)f_0''(x)-2f_1(x)g_0''(x)\right)+\mu\theta_1\left(g_{12}(x)f_1'(x)-g_1(x)f_{12}'(x)+\right.$$

$$+f_2'(x)g_0'(x) - 2f_{12}(x)g_1'(x) - f_0'(x)g_2'(x) + f_1(x)g_{12}'(x) - g_2(x)f_0''(x) + f_2(x)g_0''(x)\big) +$$

$$+\mu\theta_1\theta_2\left(2f_1'(x)g_2'(x) + 2f_2'(x)g_1'(x) + 2g_1(x)f_1''(x) +\right.$$

$$\left. +2g_2(x)f_2''(x) + 2f_1(x)g_1''(x) + 2f_2(x)g_2''(x)\right).$$

Hence, we deduce that the cohomology space is one-dimensional since the equation $\Omega - \delta b = 0$ has no solutions. So, the 2-cocycle is nontrivial and $\dim \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_1^2\right) = \dim \mathrm{Z}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_1^2\right)$.

For $\lambda = 3$, the equation (3.5) has a single solution Ω. It is now easy to check that the equation $\Omega - \delta b = 0$ has no solutions. So the 2-cocycle is nontrivial and $\dim \mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_3^2\right) = \dim \mathrm{Z}^2_{\text{diff}}\left(\mathcal{K}(2);\mathfrak{F}_3^2\right) = 1$.

For $\lambda = 5$, the equation (3.5) has no solutions. Then

$$\mathrm{H}^2_{\text{diff}}\left(\mathcal{K}(2),\mathfrak{F}_5^2\right) \simeq 0.$$

By using “MATHEMATICA”, that the condition of 2-cocycle has solutions, we deduce the expressions of Ω. To be more precise, we get

$$\Omega = \begin{cases} \left(\bar\eta_1(F)\bar\eta_2(G) - \bar\eta_2(F)\bar\eta_1(G)\right)\theta_1\theta_2, & \text{if} \quad \lambda = 0, \\ \left(F\bar\eta_1\bar\eta_2(G) - \bar\eta_1\bar\eta_2(F)G + \bar\eta_1(F)\bar\eta_2(G) + \bar\eta_2(F)\bar\eta_1(G)\right)\theta_1\theta_2, & \text{if} \quad \lambda = 1, \\ \left((-1)^{|F|}(M(F,G)) + 2(N(F,G))\right), & \text{if} \quad \lambda = 3, \end{cases}$$

where

$$M(F,G) = \bar\eta_1\left(F''\right)\bar\eta_1\left(G''\right) + \bar\eta_2\left(F''\right)\bar\eta_2\left(G''\right),$$

$$N(F,G) = \bar\eta_1\bar\eta_2\left(F'\right)\bar\eta_1\bar\eta_2\left(G''\right) - \bar\eta_1\bar\eta_2\left(F''\right)\bar\eta_1\bar\eta_2\left(G'\right).$$

Theorem 3.1 is proved.

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