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APPROXIMATE SOLUTION OF A DOMINANT SINGULAR INTEGRAL EQUATION WITH CONJUGATION

НАБЛИЖЕНИЙ РОЗВ'ЯЗОК ДОМІНАНТНОГО СИНГУЛЯРНОГО ІНТЕГРАЛЬНОГО РІВНЯННЯ ЗІ СПРЯЖЕННЯМ

In the present paper, the method of successive approximations and Faber polynomials are used to derive an approximate solution of a dominant singular integral equation with Hölder continuous coefficients and conjugation on the Lyapunov curve. Moreover, conditions of convergence in the L_2 and $H(\alpha)$ spaces are presented.

У цій роботі за допомогою методу послідовних наближень та поліномів Фабера отримано наближений розв'язок домінантного сингулярного інтегрального рівняння з неперервними за Гельдером коефіцієнтами та спряженням на кривій Ляпунова. Крім того, запропоновано умови збіжності у просторах L_2 та $H(\alpha)$.

1. Introduction. Let us consider a singular equation with Cauchy kernel of the form

$$a_1(t)\varphi(t) + b_1(t)\frac{1}{\pi i}\int\limits_L \frac{\varphi(\tau)}{\tau - t}\,d\tau + a_2(t)\overline{\varphi(t)} + b_2(t)\overline{\frac{1}{\pi i}\int\limits_L \frac{\varphi(\tau)}{\tau - t}\,d\tau} = f(t), \quad t \in L, \tag{1.1}$$

where $a_1(t),\ b_1(t),\ a_2(t),\ b_2(t),\ f(t)$ are given complex-valued Hölder continuous functions on a Lyapunov curve L and $\varphi(t)$ is an unknown function. We seek the solution in the class of Hölder continuous functions.

Equation (1.1) has numerous applications [3, 4, 8, 11, 12, 17]. It is a generalization of the well-known from the literature dominant singular integral equation [7, 11]. An explicit solution of the dominant equation was found by I. N. Vekula [16]. He reduced it to the Riemann problem which had been solved previously by F. D. Gakhov [6]. Equation (1.1) contains a conjugate of unknown function. As the exact solution of such a equation can be found only in some rare particular cases [10, 18] approximate methods are developed. In [14] the computational schemes of collocation and mechanical quadrature methods for solving (1.1) are proposed when L is a smooth simple closed contour containing inside it the point $z = 0$. In [2] approximation methods for singular integral operators with continuous coefficients and conjugation on curves with corners are investigated with respect to their stability.

In the present paper solution of equation (1.1) is based on the solution of some boundary problem [13] being a generalization of Riemann problem [7, 11]. Next the method of successive approximation and Faber polynomials are used to construct an approximate solution of equation (1.1) on a closed Lyapunov curve. The Faber polynomials and their modifications play an important role in modern methods of approximation of complex functions [1, 5, 9, 15, 19, 20].

Let us recall that a simple continuous curve is called Lyapunov curve if it satisfies the following conditions:

at every point of L there exists a well-defined tangent, the angle $\theta(s)$ between OX axis and the tangent to L at the point M whose distance from a fixed point, measured along the curve L, is equal to s, satisfies Hölder condition

$$\left|\theta(s') - \theta(s'')\right| \leq K|s' - s''|^{\mu},$$

where $K > 0$, $0 < \mu \leq 1$ are constants independent of the position of the points s', s''.

It is known [15, p. 90] that in such case every Cauchy-type integral

$$f(z) = \frac{1}{2\pi i}\int\limits_L \frac{\varphi(\zeta)}{\zeta - z}\, d\zeta, \quad z \in G,$$

is expandable in uniformly convergent Faber series.

2. Approximate solution. Let us consider the Cauchy-type integral

$$\phi(z) = \frac{1}{2\pi i}\int\limits_L \frac{\varphi(\tau)}{\tau - z}\, d\tau, \quad z \in D^{\pm},$$

where D^+ and D^- are interior and exterior domains with the boundary L, respectively, and Sokhotski – Plemelj formulae [7]

$$\varphi(t) = \phi^+(t) - \phi^-(t), \quad t \in L, \tag{2.1}$$

$$\frac{1}{\pi i}\int\limits_L \frac{\varphi(\tau)}{\tau - t}\, d\tau = \phi^+(t) + \phi^-(t), \quad t \in L. \tag{2.2}$$

Substituting the left-hand sides of equations (2.1) and (2.2) into equation (1.1) we get

$$\alpha_1(t)\phi^+(t) + \beta_1(t)\overline{\phi^+(t)} = \alpha_2(t)\phi^-(t) + \beta_2(t)\overline{\phi^-(t)} + f(t), \tag{2.3}$$

$$\phi^+(\infty) = 0, \tag{2.4}$$

where

$$\alpha_1(t) = a_1(t) + b_1(t), \quad \beta_1(t) = a_2(t) + b_2(t),$$

$$\alpha_2(t) = a_1(t) - b_1(t), \quad \beta_2(t) = a_2(t) - b_2(t).$$

Thus, equation (1.1) has been reduced to the boundary problem of finding two analytic functions $\phi^+(z)$, $z \in D^+$, and $\phi^-(z)$, $z \in D^-$, which limit values fulfil (2.3) on the curve L and, moreover, function $\phi^+(z)$ vanishes at infinity. Taking into account the system consisting of equation (2.3) and its conjugation, and then eliminating the function $\overline{\phi^-(t)}$, we obtain

$$\phi^+(t) = A(t)\phi^-(t) + B(t)\overline{\phi^+(t)} + C(t), \quad t \in \Gamma, \tag{2.5}$$

where

$$A(t) = \frac{|\alpha_2(t)|^2 - |\beta_2(t)|^2}{\alpha_1(t)\overline{\alpha_2(t)} - \overline{\beta_1(t)}\beta_2(t)}, \quad B(t) = \frac{\overline{\alpha_1(t)}\beta_2(t) - \overline{\alpha_2(t)}\beta_1(t)}{\alpha_1(t)\overline{\alpha_2(t)} - \overline{\beta_1(t)}\beta_2(t)},$$

$$C(t) = \frac{\overline{\alpha_2(t)}f(t) - \beta_2(t)\overline{f(t)}}{\alpha_1(t)\overline{\alpha_2(t)} - \overline{\beta_1(t)}\beta_2(t)}.$$

Let $A(t) \neq 0$ $\forall t \in \Gamma$ and the origin of a coordinate system $\mathbb{C}_z$ belong to the area D^+. Moreover, let $\operatorname{Ind} A(t) = \kappa \geq 0$. Then by [7] the canonical functions $X^+(z),\ z \in D^+$, and $X^-(z),\ z \in D^-$, of the linear conjugation problem

$$X^+(t) = A(t)X^-(t), \quad t \in \Gamma, \tag{2.6}$$

have the forms

$$X^+(z) = \exp\big(\Gamma^+(z)\big), \quad z \in D^+,$$

and

$$X^-(z) = z^{-\kappa}\exp\big(\Gamma^-(z)\big), \quad z \in D^-,$$

respectively. Here

$$\Gamma^\pm(z) = \frac{1}{2\pi i}\int\limits_\Gamma \frac{\ln\left(\tau^{-\kappa}A(\tau)\right)}{\tau - z}\,d\tau, \quad z \in D^\pm.$$

Taking into account (2.6), the boundary problem (2.5) can be rewritten in the form

$$\frac{\phi^+(t)}{X^+(t)} - \frac{\phi^-(t)}{X^-(t)} = G(t)\overline{\frac{\phi^+(t)}{X^+(t)}} + g(t), \quad t \in \Gamma,$$

where

$$G(t) = B(t)\frac{\overline{X^+(t)}}{X^+(t)}, \quad g(t) = \frac{C(t)}{X^+(t)}.$$

Furthermore, by setting

$$F^\pm(z) = \frac{\phi^\pm(z)}{X^\pm(z)}, \quad z \in D^\pm, \tag{2.7}$$

we obtain

$$F^+(t) - F^-(t) = G(t)\overline{F^+(t)} + g(t), \quad t \in \Gamma. \tag{2.8}$$

The solution of the problem (2.8) we will seek in the following form:

$$F(z) = \frac{1}{2\pi i}\int\limits_\Gamma \frac{\mu(\tau)}{\tau - z}\,d\tau + P_{\kappa-1}(z), \quad z \in D^\pm, \tag{2.9}$$

where $\mu(\tau)$ is an unknown function satisfying the Hölder condition and $P_{\kappa-1}(z) = \gamma_0 + \gamma_1 z + \ldots$ $\ldots + \gamma_{\kappa-1}z^{\kappa-1}$ is a polynomial with arbitrary complex-valued coefficients $\gamma_0, \gamma_1, \ldots, \gamma_{\kappa-1}$.

Applying the Sokhotski – Plemelj formulae to the function $F(z)$, the problem (2.8) can be transformed to the following singular integral equation:

$$\mu(t) = G(t)\left(\frac{1}{2}\overline{\mu(t)} + \overline{\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu(\tau)}{\tau - t}\,d\tau}\right) + G(t)\overline{P_{\kappa-1}(t)} + g(t), \quad t \in \Gamma, \tag{2.10}$$

which can be solved by the method of successive approximations.

First, we establish the coefficients $\gamma_0, \gamma_1, \ldots, \gamma_{\kappa-1}$ of the polynomial $P_{\kappa-1}(z)$. They will be uniquely determined if we supplement (2.3) with new conditions

$$-\operatorname*{Res}_{z=\infty}\left(z^{j-1}\phi^-(z)\right) = A_j, \quad j = 1, 2, \ldots, \kappa, \tag{2.11}$$

where A_j are arbitrary constants (see [11]). In view of (2.7) and (2.9) we have

$$\phi^-(z) = X^-(z)\left(\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu(\tau)}{\tau - z}\,d\tau + P_{\kappa-1}(z)\right).$$

Thus, using the expansion

$$X^-(z) = \frac{1}{z^\kappa}\left(1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \ldots\right)$$

of a canonical function $X^-(z)$ in a neighborhood of $z = \infty$, from (2.11) we derive the system of equations

$$\begin{gathered}\gamma_{\kappa-1} = A_1,\\ \gamma_{\kappa-2} + p_1\gamma_{\kappa-1} = A_2,\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \gamma_0 + p_1\gamma_1 + \ldots + p_{\kappa-1}\gamma_{\kappa-1} = A_\kappa.\end{gathered}$$

The solution of this system can be given by a recurrent formulae of the form

$$\gamma_{\kappa-i} = A_i - \sum_{j=\kappa-i+1}^{\kappa-1} p_j\gamma_j, \quad i = 1, \ldots, \kappa. \tag{2.12}$$

Now let us solve equation (2.10) by successive approximation method taking the function

$$\mu_0(t) = g^*(t) = G(t)\overline{P_{\kappa-1}(t)} + g(t), \quad t \in \Gamma,$$

as an initial guess. The next approximations are given by the formula

$$\mu_{n+1}(t) = G(t)\left(\frac{1}{2}\overline{\mu_n(t)} + \overline{\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau - t}\,d\tau}\right) + g^*(t), \quad n = 0, 1, 2, \ldots. \tag{2.13}$$

Then the successive approximations for the problem (2.5), (2.11) have the form

$$\phi_n^\pm(z) = X^\pm(z)\left(\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau - z}\,d\tau + P_{\kappa-1}(z)\right), \quad z \in D^\pm, \quad n = 0, 1, 2, \ldots. \tag{2.14}$$

Hence, applying Sokhotski – Plemelj formulae to the above functions, by (2.1) it finally follows that approximate solution of the problem (1.1), (2.11) is given by the formula

$$\varphi_n(t) = X^-(t)\left(\frac{1}{2}(A(t)+1)\mu_n(t)+\right.$$

$$\left.+\frac{1}{2\pi i}(A(t)-1)\int\limits_\Gamma \frac{\mu_n(\tau)}{\tau - t}\,d\tau + (A(t)-1)P_{\kappa-1}(t)\right), \quad t\in\Gamma, \quad n = 0,1,2,\ldots,$$

where coefficients of the polynomial $P_{\kappa-1}$ satisfy relation (2.12).

Let us consider the case $\operatorname{Ind} A(t) = \kappa < 0$. As we have to assume $P_{\kappa-1}(z)\equiv 0$ in (2.9), we get

$$\phi^\pm(z) = X^\pm(z)\frac{1}{2\pi i}\int\limits_\Gamma \frac{\mu(\tau)}{\tau - z}\,d\tau, \quad z\in D^\pm.$$

Since in this case the order of the pool of the function $X^-(z)$ in a neighborhood of $z=\infty$ is $|\kappa|$, then the conditions

$$\frac{1}{2\pi i}\int\limits_\Gamma \mu(\tau)\tau^{j-1}\,d\tau = 0, \quad j = 1,2,\ldots,|\kappa|,$$

are necessary and sufficient to preserve the analyticity of the function $\phi^-(z)$, $\phi^-(\infty) = 0$, in a neighborhood of the point $z=\infty$. The above equalities follow from the expansion

$$\frac{1}{2\pi i}\int\limits_\Gamma \frac{\mu(\tau)}{\tau - z}\,d\tau = \sum_{k=1}^{\infty}\frac{c_k}{z^k}, \quad z\in D^-,$$

where

$$c_k = -\frac{1}{2\pi i}\int\limits_\Gamma \mu(\tau)\tau^{k-1}\,d\tau, \quad k=1,2,\ldots.$$

The approximation $\phi_n^\pm(z)$ with $\kappa<0$ can be determined from the formula

$$\phi_n^\pm(z) = X^\pm(z)\left(\frac{1}{2\pi i}\int\limits_\Gamma \frac{\mu_n(\tau)}{\tau - z}\,d\tau - \sum_{k=1}^{|\kappa|}\frac{c_k^{(n)}}{z^k}\right),$$

where

$$c_k^{(n)} = -\frac{1}{2\pi i}\int\limits_\Gamma \mu_n(\tau)\tau^{k-1}\,d\tau, \quad k=1,2,\ldots,|\kappa|.$$

Similarly like in the case of nonnegative index we apply Sokhotski – Plemelj formulae to the functions $\phi_n^\pm(z)$ and by (2.1) we get the solution of the problem (1.1)

$$\varphi_n(t) = X^-(t)\left(\frac{1}{2}(A(t)+1)\mu_n(t)+\right.$$

$$+\frac{1}{2\pi i}(A(t)-1)\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau-t}\,d\tau+(A(t)-1)\sum_{k=1}^{|\kappa|}\frac{c_k^{(n)}}{z^k}\Bigg),\quad t\in\Gamma,\quad n=0,1,2,\ldots .$$

In the case of $\kappa\geq 0$ we use Faber polynomial to find an approximate value of the expression

$$\frac{1}{2}\overline{\mu_n(t)}+\overline{\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau-t}\,d\tau},\quad n=1,2,\ldots,\quad \mu_0(t)=g^*(t),$$

occurring in (2.13). For this purpose we introduce the integral of Cauchy-type

$$F_n(z)=\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau-z}\,d\tau,\quad z\in D^+,\quad n=0,1,\ldots . \tag{2.15}$$

By [15, p. 215] we have

$$F_n(z)=\sum_{k=0}^{\infty}a_k^{(n)}\Phi_k(z),\quad z\in D^+, \tag{2.16}$$

where $\Phi_k(z),\ k=0,1,2,\ldots,$ are Faber polynomials of degree k and

$$a_k^{(n)}=\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\Phi^{k+1}(\tau)}\Phi'(\tau)\,d\tau=\frac{1}{2\pi i}\int\limits_{|t|=1}\frac{\mu_n(\Psi(t))}{t^{k+1}}\,dt. \tag{2.17}$$

The function $\Phi(z)$ transforms conforemely and univalently the area D^- in the coordinate system $\mathbb{C}_z$ to the exterior of the circle $|w|>1$ in the coordinate system $\mathbb{C}_w$ and satisfies the conditions $\Phi(\infty)=0,\ \Phi'(\infty)>0$, where $z=\Psi(w)$ is the inversion of the function $w=\Phi(z)$.

Using the Sokhotski – Plemelj formulae, from (2.15), (2.16) we obtain

$$\frac{1}{2}\mu_n(t)+\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau-t}\,d\tau=\sum_{k=0}^{\infty}a_k^{(n)}\Phi_k(t),\quad t\in\Gamma.$$

Then by (2.13) we get

$$\mu_{n+1}(t)=G(t)\sum_{k=0}^{\infty}\overline{a_k^{(n)}\Phi_k(t)}+g^*(t),\quad t\in\Gamma,\quad n=0,1,\ldots .$$

Finally, we have to find $\phi_n^{\pm}(z),\ n=0,1,\ldots .$ If $z\in D^+$, then by (2.14) and expansion (2.16) we obtain

$$\phi_n^+(z)=X^+(z)\sum_{k=0}^{\infty}a_k^{(n)}\Phi_k(z)+X^+(z)P_{\kappa-1}(z),\quad z\in D^+,$$

where $a_k^{(n)}$ are given by the equalities (2.17).

If $z\in D^-$ we find the corresponding formulae in a similar way. In this case we have

$$\phi_n^-(z)=X^-(z)\sum_{k=1}^{\infty}\frac{b_k^{(n)}}{\Phi^k(z)}+X^-(z)P_{\kappa-1}(z),\quad z\in D^-,\quad n=0,1,\ldots,$$

where coefficients $b_k^{(n)}$ are defined using the expansion

$$\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu_n(\tau)}{\tau - z}\,d\tau = \sum_{k=1}^{\infty}\frac{b_k^{(n)}}{\Phi^k(z)}, \quad z \in \overline{D^-}.$$

3. Approximation criterion. Now we will give the conditions for the convergence of a sequence of functions $\{\mu_n(t)\},\ n = 0, 1, \ldots,$ determined by (2.13) in the spaces $L_2(\Gamma)$. For this purpose we rewrite equation (2.10) in the form

$$\mu - T\mu = g^*,$$

where

$$T\mu \equiv G(t)\left(\frac{1}{2}\overline{\mu(t)} + \overline{\frac{1}{2\pi i}\int\limits_{\Gamma}\frac{\mu(\tau)}{\tau - t}\,d\tau}\right)$$

and

$$g^*(t) = G(t)\overline{P_{\kappa-1}(t)} + g(t).$$

As it was proved in [13], the following theorem is true.

Theorem 1. *Let $a_1(t),\ b_1(t),\ a_2(t),$ and $b_2(t)$ be given continuous functions defined on the closed Lyapunov curve Γ satisfying the condition*

$$a_1(t)\overline{a_2(t)} - \overline{b_1(t)}b_2(t) \neq 0 \quad \forall t \in \Gamma. \tag{3.1}$$

Moreover, let

$$\mathrm{Ind}A(t) = \mathrm{Ind}\left(\overline{b_1(t)}b_2(t) - a_1(t)\overline{a_2(t)}\right) = \kappa \geq 0.$$

If

$$q = \frac{1}{2}(1 + S_2)\max_{t\in\Gamma}\left|\frac{\overline{a_1(t)}b_2(t) - b_1(t)\overline{a_2(t)}}{a_1(t)\overline{a_2(t)} - \overline{b_1(t)}b_2(t)}\right| < 1, \tag{3.2}$$

where S_2 is a norm of the singular integral operator

$$S\mu = \frac{1}{\pi i}\int\limits_{\Gamma}\frac{\mu(\tau)}{\tau - t}\,d\tau$$

in the space L_2, then the boundary problem (2.5), (2.11) *has a unique solution for an arbitrary function $C(t) \in L_2(\Gamma)$. This solution can be found by the successive approximations method.*

In the case $\mathrm{Ind}\,A(t) = \kappa < 0$ *if the condition* (3.2) *is fulfilled and the necessary and sufficient conditions of the form*

$$\frac{1}{2\pi i}\int\limits_{\Gamma}(I - T)^{-1}g(\tau)\tau^{j-1}\,d\tau = 0, \quad j = 1, 2, \ldots, |\kappa|, \tag{3.3}$$

hold, where

$$g(t) = \frac{\overline{a_2(t)}c(t) - b_2(t)\overline{c(t)}}{X^+(t)},$$

then the boundary-value problem (2.5) *has a unique solution.*

Similarly, in the Banach space H^α, $0 < \alpha \leq 1$, consisting of functions determined on the curve Γ and belonging to the function class $H(\alpha)$, with the norm given by the formula

$$\|f\|_{H^\alpha} = \max_{t\in\Gamma} |f(t)| + \sup_{t_1,t_2\in\Gamma} \frac{|f(t_2) - f(t_1)|}{|t_2 - t_1|^\alpha},$$

we have the following theorem [13].

Theorem 2. *Let the functions* $a_1(t)$, $b_1(t)$, $a_2(t)$, *and* $b_2(t)$ *belong to the function class* $H(\alpha)$, $0 < \alpha < 1$, *and let the condition* (3.1) *be satisfied. If* $\mathrm{Ind} A(t) = \kappa \geq 0$ *and the condition*

$$\left\| \frac{\overline{a_1(t)} b_2(t) - b_1(t)\overline{a_2(t)}}{a_1(t)\overline{a_2(t)} - \overline{b_1(t)} b_2(t)} \frac{\overline{X^+(t)}}{X^+(t)} \right\|_{H^\alpha} M_\alpha < 1 \tag{3.4}$$

is fulfilled for some constant M_α *then the boundary-value problem* (2.5), (2.11) *is uniquely solvable. If* $\mathrm{Ind}\, A(t) < 0$ *and if the convergence condition* (3.4) *and the necessary and sufficient conditions* (3.3) *are satisfied, then the boundary-value problem* (2.5) *has a unique solution.*

References

1. I. Caraus, *The numerical solution for system of singular integro-differential equations by Faber – Laurent polynomials*, Numer. Anal. and Appl., **3401**, ser. Lect. Notes Comput. Sci., 219 – 223 (2005).
2. V. D. Didenko, B. Silbermann, S. Roch, *Some peculiarities of approximation methods for singular integral equations with conjugation*, Methods and Appl. Anal., **7**, № 4, 663 – 686 (2000).
3. R. Duduchava, *On general singular integral operators of the plane theory of elasticity*, Rend. Sem. Mat. Univ. Politec. Torino, **42**, № 3, 15 – 41 (1984).
4. R. Duduchava, *General singular integral equations and basic problems of planar elasticity theory*, Trudy Tbiliss. Mat. Inst. AN GSSR, **82**, 45 – 89 (1986).
5. S. W. Ellacott, *A survey of Faber methods in numerical approximation*, Comput. and Math. Appl., **12**, № 5, 1103 – 1107 (1986).
6. F. D. Gakhov, *On the Riemann boundary value problem*, Mat. Sb., **44**, № 4, 673 – 683 (1937).
7. F. D. Gakhov, *Boundary value problems*, Dover Publ., Mineloa (1990).
8. A. I. Kalandiya, *Mathematical methods of two-dimensional elasticity*, Mir, Moscow (1975).
9. E. Ladopoulos, G. Tsamasphyros, *Approximations of singular integral equations on Lyapunov contours in Banach spaces*, Comput. and Math. Appl., **50**, № 3-4, 567 – 573 (2005).
10. G. S. Litvinchiuk, *The boundary problems and singular equations with displacement*, Nauka, Moscow (1997).
11. N. I. Muskhelishvili, *Singular integral equations: boundary problems of function theory and their application to mathematical physics*, Dover Publ., Mineloa (2008).
12. N. I. Muskhelishvili, *Some basic problems of the mathematical theory of elasticity*, Springer Sci. & Business Media (2013).
13. M. A. Sheshko, P. Karczmarek, D. Pylak, P. Wójcik, *Application of Faber polynomials to the approximate solution of a generalized boundary value problem of linear conjugation in the theory of analytic functions*, Comput. Math. Appl., **67**, № 8, 1474 – 1481 (2014).
14. I. Spinei, V. Zolotarevschi, *Direct methods for solving singular integral equations with complex conjugation*, Comput. Sci., **6**, № 1, 83 – 91 (1998).
15. P. K. Suetin, *Series of Faber polynomials*, Gordon and Breach Sci. Publ., New York (1998).
16. I. N. Vekua, *On singular linear integral equations*, Dokl. Akad. Nauk SSSR, **26**, № 8, 335 – 338 (1940).
17. I. N. Vekua, *Generalized analytic functions*, Pergamon Press, Oxford (1962).
18. N. Vekua, *Systems of singular integral equations and certain boundary value problems*, Nauka, Moscow (1970).
19. P. Wójcik, M. A. Sheshko, S. M. Sheshko, *Application of Faber polynomials to the approximate solution of singular integral equations with the Cauchy kernel*, Different. Equat., **49**, № 2, 198 – 209 (2013).
20. V. Zolotarevskii, Z. Li, I. Caraus, *Approximate solution of singular integro-differential equations by reduction over Faber – Laurent polynomials*, Different. Equat., **40**, № 12, 1764 – 1769 (2004).

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