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A NEW APPROACH TO THE APPROXIMATION BY POSITIVE LINEAR OPERATORS IN WEIGHTED SPACES

НОВИЙ ПІДХІД ДО АПРОКСИМАЦІЇ ПОЗИТИВНИМИ ЛІНІЙНИМИ ОПЕРАТОРАМИ У ВАГОВИХ ПРОСТОРАХ

In the present paper, we deal with the problem of approximating a function by positive linear operators in weighted spaces. In this case, our main tool is the P_p-statistical convergence recently defined by [M. Ünver, C. Orhan, Numer. Funct. Anal. and Optim., **40**, 535 – 547 (2019)]. It is worth noting that the P_p-statistical convergence and the statistical convergence do not imply each other.

Розглянуто проблему наближення функції додатними лінійними операторами у вагових просторах. У цьому випадку наш основний інструмент — це P_p-статистична збіжність, нещодавно введена в [M. Ünver, C. Orhan, Numer. Funct. Anal. and Optim., **40**, 535 – 547 (2019)]. Варто зазначити, що з P_p-статистичної збіжності не випливає статистична збіжність, і навпаки.

1. Introduction. The classical Korovkin theorem obtains the uniform convergence of the sequence of positive linear operators to the identity operator in the space of real valued continuous functions, $C[0,1]$, by using only three functions. That is, these type of theorems exhibit a variety of test functions, which assume the approximation property holds on the whole space if it holds for them. Such a property was discovered by Korovkin in 1953 for the functions 1, x and x^2 in $C[0,1]$ [15]. Since the simplicity and efficiency of these theorems, Korovkin-type approximation theory is a popular and well studied area in approximation theory.

Using various types of convergence or by changing the test functions, many mathematicians have investigated the Korovkin-type approximation theorems for a sequence of positive linear operators defined on different spaces [1, 3, 5, 8, 10, 14, 17]. In 2002, Gadjiev and Orhan have given a Korovkin-type approximation theorem by using the concept of statistical convergence. This concept of convergence is really efficient to use since it makes a nonconvergent sequence to converge. Later this process has been developed by Duman et al. (see [7 – 10]) and have entered the literature with the keyword statistical approximation. Recent results about the statistical approximation may be found in the monograph by Anastassiou and Duman [2]. Various convergence methods have also been applied for the approximation in weighted spaces [4]. Recently, Ünver and Orhan [18] have defined P_p-statistical convergence and have proved a Korovkin-type theorem for a sequence of positive linear operators defined on $C[0,1]$ using this convergence. It is important to mention that P_p-statistical convergence do not imply each other. In this paper we develop Korovkin-type approximation theorem

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for a sequence of positive linear operators acting from a weighted space C_{ϱ_1} into a weighted space B_{ϱ_2} with the use of the P_p-statistical convergence. Then we display an application which shows that our new result is stronger than its classical version. We also obtain the rate of convergence of these operators.

Now we recall the concepts of weighted spaces considered in [4, 12]. As usual, a weight function $\varrho\colon \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$, nonincreasing on $(-\infty, 0)$, nondecreasing on $(0, \infty)$, $\varrho(0) = 1$ and $\lim_{|x|\to\infty} \varrho(x) = \infty$.

Then the corresponding weighted spaces are defined as follows:

$$B_\varrho := \big\{ f : \mathbb{R} \to \mathbb{R} : |f| \le \varrho M_f \text{ on } \mathbb{R} \text{ for some } M_f > 0 \big\},$$

$$C_\varrho := \big\{ f : \mathbb{R} \to \mathbb{R} : f \in B_\varrho \text{ and } f \text{ is continuous on } \mathbb{R} \big\}.$$

It is well-known that the spaces B_ϱ and C_ϱ are Banach spaces with the norm

$$|f|_\varrho = \sup_{x\in\mathbb{R}} \frac{|f(x)|}{\varrho(x)}.$$

Now, let ϱ_1 and ϱ_2 be two weight functions. Assume also that the condition

$$\lim_{|x|\to\infty} \frac{\varrho_1(x)}{\varrho_2(x)} = 0 \tag{1}$$

holds. Then one can easily notice that $C_{\varrho_1} \subset C_{\varrho_2}$ and $B_{\varrho_1} \subset B_{\varrho_2}$. If T is a positive linear operator from C_{ϱ_1} into B_{ϱ_2}, then the operator norm is given by

$$\|T\|_{C_{\varrho_1}\to B_{\varrho_2}} := \sup_{|f|_{\varrho_1}=1} \|Tf\|_{\varrho_2} = \big\|T(\varrho_1)\big\|_{\varrho_2}.$$

The following approximation theorem for a sequence of positive linear operators acting from the weighted space C_{ϱ_1} into B_{ϱ_2} may be found in [12, 13].

Theorem 1. *Let ϱ_1 and ϱ_2 be two weight functions satisfying* (1). *Assume that $\{T_n\}$ is a sequence of positive linear operators acting from C_{ϱ_1} into B_{ϱ_2}. If*

$$\lim_n \|T_n(F_v) - F_v\|_{\varrho_1} = 0,$$

where $F_v(x) = \dfrac{x^v \varrho_1(x)}{1+x^2}$, $v = 0, 1, 2$, *then, for all* $f \in C_{\varrho_1}$, *we have*

$$\lim_n \big\|T_n(f) - f\big\|_{\varrho_2} = 0.$$

Theorem 1 has been studied in [12, 13] and its statistical version has been obtained in [4, 8, 9]. Now let us recall the concept of P_p-statistical convergence which is the primary concept in the paper.

Let (p_j) be real sequence with $p_0 > 0$ and $p_1, p_2, \ldots \ge 0$ and such that the corresponding power series $p(t) := \sum_{j=0}^{\infty} p_j t^j$ has radius of convergence R with $0 < R \le \infty$. If the limit

$$\lim_{t\to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} x_j p_j t^j = L$$

exists, then we say that $x = (x_j)$ is convergent in the sense of power series method [6, 16]. The power series method is regular if and only if

$$\lim_{t\to R^-} \frac{p_j t^j}{p(t)} = \quad \text{for each} \quad j \in \mathbb{N}_0$$

holds [6].

Let P_p be a regular power series method and $E \subset \mathbb{N}_0$. If the limit

$$\delta_{P_p}(E) := \lim_{t\to R^-} \frac{1}{p(t)} \sum_{j\in E} p_j t^j$$

exists, then $\delta_{P_p}(E)$ is called the P_p-density of E.

The sequence $x = (x_j)$ of real numbers is said to be P_p-statistically convergent to L if for every $\varepsilon > 0$, $\delta_{P_p}(E) = 0$, that is, for every $\varepsilon > 0$,

$$\lim_{t\to R^-} \frac{1}{p(t)} \sum_{j\in E_\varepsilon} p_j t^j = 0,$$

where $E_\varepsilon = \big\{ j \in \mathbb{N}_0 \colon |x_j - L| \geq \varepsilon \big\}$.

It is already shown that statistical convergence and P_p-statistical convergence do not imply each other [18].

2. Korovkin theorem in weighted spaces. In this section we give a Korovkin-type approximation of a function f by means of positive linear operators from a weighted space C_{ϱ_1} into a weighted space B_{ϱ_2} with the use of P_p-statistical convergence. We mainly motivated by the paper [4].

We first prove the following lemma which is needed for the proof of our main theorem.

Lemma 1. *Let ϱ_1 and ϱ_2 be two weight functions satisfying* (1). *Assume that $\{T_n\}$ is a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2}. If*

$$st_{P_p} - \lim \big\| T_n(F_v) - F_v \big\|_{\varrho_1} = 0, \tag{2}$$

where $F_v(x) = \dfrac{x^v \varrho_1(x)}{1+x^2}$, $v = 0, 1, 2,$ *then, for any* $s > 0$ *and for all* $f \in C_{\varrho_1}$, *we have*

$$st_{P_p} - \lim \big\| T_n(f) - f \big\|_{\varrho_2, [-s,s]} = 0.$$

Proof. By using the same procedure as in the proof of Lemma 2.3 in [4], we can obtain that, for a given $f \in C_{\varrho_1}$,

$$\big\| T_n(f) - f \big\|_{\varrho_2, [-s,s]} \leq M \left(\varepsilon + \sum_{v=0}^{2} \big\| T_n(F_v) - F_v \big\|_{\varrho_1} \right)$$

holds for all $n \in \mathbb{N}$ and for some $M > 0$ independent of n and x. Now, for a given $r > 0$, choose $\varepsilon > 0$ such that $M\varepsilon < r$. Then setting

$$D = \left\{ n \in \mathbb{N} \colon \sum_{v=0}^{2} \big\| T_n(F_v) - F_v \big\|_{\varrho_1} \geq \frac{r - M\varepsilon}{M} \right\},$$

$$D_0 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_0) - F_0\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\},$$

$$D_1 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_1) - F_1\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\},$$

$$D_2 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_2) - F_2\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\}$$

it is easy to see that $D \subset D_0 \cup D_1 \cup D_2$. This implies that

$$0 \leq \delta_{P_p}\Big(\Big\{n \in \mathbb{N} \colon \left\|T_n(f) - f\right\|_{\varrho_2,[-s,s]} \geq r\Big\}\Big) \leq \sum_{i=0}^{2} \delta_{P_p}(D_i).$$

Then, using the hypothesis (2), we get

$$\delta_{P_p}\Big(\Big\{n \in \mathbb{N} \colon \left\|T_n(f) - f\right\|_{\varrho_2,[-s,s]} \geq r\Big\}\Big) = 0.$$

Lemma 1 is proved.

Now we can give our main theorem which is a new approach to Korovkin-type approximation in weighted spaces.

Theorem 2. *Let ϱ_1 and ϱ_2 be as in Lemma* 1. *Suppose that $\{T_n\}$ is a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2}. Then, for all $f \in C_{\varrho_1}$, we have*

$$st_{P_p} - \lim \left\|T_n(f) - f\right\|_{\varrho_2} = 0$$

provided that

$$st_{P_p} - \lim \left\|T_n(F_v) - F_v\right\|_{\varrho_1} = 0, \tag{3}$$

where $F_v(x) = \dfrac{x^v \varrho_1(x)}{1 + x^2}$, $v = 0, 1, 2.$

Proof. By noticing $T_n(F_v) = T_n(F_v) - F_v + F_v$ and $F_v \in B_{\varrho_1}$, $v = 0, 1, 2$, (3) implies that $T_n(F_v) - F_v \in B_{\varrho_1}$ and this gives $T_n(F_v) \in B_{\varrho_1}$, $v = 0, 1, 2$. Since $\varrho_1 = F_0 + F_2$, we get $T_n(\varrho_1) \in B_{\varrho_1}$. Also $\|T_n\|_{C_{\varrho_1} \to B_{\varrho_1}} = \|T_n(\varrho_1)\|_{\varrho_1} \leq M_1 < \infty$ holds, which means that $\{T_n\}$ is uniformly bounded from C_{ϱ_1} into B_{ϱ_1}. For a given $f \in C_{\varrho_1}$, we have

$$\|T_n(f)\|_{\varrho_1} \leq \|T_n\|_{C_{\varrho_1} \to B_{\varrho_1}} \|f\|_{\varrho_1} \leq M_1 |f|_{\varrho_1},$$

which implies $T_n(f) \in B_{\varrho_1}$. Therefore, we obtain $T_n(C_{\varrho_1}) \subset B_{\varrho_1}$. Finally, from (1), (3) and the fact that $\|T_n\|_{C_{\varrho_1} \to B_{\varrho_2}} \leq M_2 < \infty$, we obtain that the sequence $\{T_n\}$ is uniformly bounded from C_{ϱ_1} into B_{ϱ_2}.

Now, for a given $\varepsilon > 0$, pick an $s_0 > 0$ such that $\varrho_1(x) \leq \varepsilon \varrho_2(x)$ for all $|x| \geq s_0$. We may write, for $f \in C_{\varrho_1}$,

$$\left\|T_n(f) - f\right\|_{\varrho_2} \leq \sup_{|x| \leq s_0} \frac{|T_n(f;x) - f(x)|}{\varrho_2(x)} + \sup_{|x| > s_0} \frac{|T_n(f;x) - f(x)|}{\varrho_2(x)} \leq$$

$$\leq \left\|T_n(f) - f\right\|_{\varrho_2,[-s_0,s_0]} + \varepsilon \left\|T_n(f) - f\right\|_{\varrho_1} \leq$$

$$\leq \left\|T_n(f) - f\right\|_{\varrho_2,[-s_0,s_0]} + \varepsilon(\|T_n(f)\|_{\varrho_1} + |f|_{\varrho_1}).$$

Hence, by using the Lemma 1, we complete the proof.

3. Rate of convergence. Throughout this section, we assume that $\varrho_1(x) = 1 + x^2$ on $\mathbb{R}$ and we obtain the rate of convergence, by using the following weighted modulus of continuity:

$$\omega_{\varrho_1}(f;\delta) = \sup_{\substack{c|x-t|\leq\delta \\ x,t\in\mathbb{R}}} \frac{\left|f(x) - f(t)\right|}{\varrho_1(x) + \varrho_1(t)}, \quad \delta > 0, \quad f \in C_{\varrho_1}.$$

It can be easily seen that, for any $\delta > 0$,

$$\left|f(x) - f(t)\right| \leq \left[\varrho_1(x) + \varrho_1(t)\right]\left\{2 + \frac{|t-x|}{\delta}\right\}\omega_{\varrho_1}(f;\delta)$$

which implies

$$\left|f(x) - f(t)\right| \leq 4\varrho_1(x).\varrho_1(t)\omega_{\varrho_1}(f;\delta)\left(1 + \frac{(t-x)^2}{\delta^2}\right).$$

By using similar operations which have already been used in [4], we can write, for any $\delta > 0$ and all $f \in C_{\varrho_1}$, that

$$\left|T_n\big(f(t);x\big) - f(x)\right| \leq$$

$$\leq 4\varrho_1(x)\omega_{\varrho_1}(f;\delta)\left\{\left|T_n(\varrho_1(t);x) - \varrho_1(x)\right| + \varrho_1(x) + \frac{1}{\delta^2}T_n(\varrho_1(t)\phi_x(t);x)\right\}+$$

$$+|f(x)|\left|T_n(F_0(t);x) - F_0(x)\right|,$$

where $\phi_x(t) = (t-x)^2$, and we obtain

$$\left\|T_n(f) - f\right\|_{\varrho_2^2} \leq 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\left\{\left\|T_n(\varrho_1) - \varrho_1\right\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + \frac{1}{\delta^2}\left\|T_n(\varrho_1\phi_x)\right\|_{\varrho_2}\right\}+$$

$$+\|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\left\|T_n(F_0) - F_0\right\|_{\varrho_1}$$

provided that $T_n(\varrho_1\phi_x) \in B_{\varrho_2}$. For example, by considering $T_n : C_{\varrho_2} \to B_{\varrho_2}$ and assuming $\varrho_1\phi_x \in$ $\in C_{\varrho_2}$, then one can see that $T_n(\varrho_1\phi_x) \in B_{\varrho_2}$. In this case, taking $\delta := \delta_n = \sqrt{\left\|T_n(\varrho_1\phi_x)\right\|_{\varrho_2}}$ and combining the above inequalities, we conclude that

$$\left\|T_n(f) - f\right\|_{\varrho_2^2} \leq 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\left\{\left\|T_n(\varrho_1) - \varrho_1\right\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + 1\right\}+$$

$$+\|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\left\|T_n(F_0) - F_0\right\|_{\varrho_1}.$$

We now introduce P_p-statistical rate of convergence under the light of [7, 11].

Definition 1. *Let (a_n) be a positive nonincreasing sequence of real numbers and let P_p be a regular power series method. A sequence $x = (x_n)$ is P_p-statistically convergent to the number L with rate $o(a_n)$ if, for every $\varepsilon > 0$,*

$$\lim_{0<t\to R^-}\left[\frac{1}{p(t)}\sum_{n\,:\,|x_n-L|\geq\varepsilon a_n} p_n t^n\right] = 0.$$

In this case, we write $x_n - L = st_{P_p} - o(a_n)$ as $n \to \infty$.

Now we will find P_p-statistical rate of convergence of the sequence of $\{T_n\}$ in Theorem 2.

***Conclusion* 1.** *Let ϱ_2 and T_n be as in Theorem 2 and $T_n\big(\varrho_1(t-x)^2\big) \in B_{\varrho_2}$. Assume that (a_n), (b_n) and (c_n) are any nonincreasing sequences of positive real numbers. If the conditions*

(i) $\big\|T_n(F_0) - F_0\big\|_{\varrho_1} = st_{P_p} - o(a_n),\ n \to \infty,$

(ii) $\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} = st_{P_p} - o(b_n),\ n \to \infty,$

(iii) $\omega_{\varrho_1}(f;\delta_n) = st_{P_p} - o(c_n),\ n \to \infty$

hold, then, for all $f \in C_{\varrho_1}$,

$$\big\|T_n(f) - f\big\|_{\varrho_2^2} = st_{P_p} - o(d_n), \quad n \to \infty,$$

where $\delta_n = \sqrt{\|T_n(\varrho_1\phi_x)\|_{\varrho_2}}$, $d_n = \max\big\{a_n, c_n, b_n c_n\big\}$.

Proof. By using the similar idea in Lemma 4 in [7] and taking care of the right-hand side of the equality

$$\big\|T_n(f) - f\big\|_{\varrho_2^2} \le 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\Big\{\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + 1\Big\} +$$

$$+\|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\big\|T_n(F_0) - F_0\big\|_{\varrho_1},$$

one can easily see that $d_n = \max\{a_n, c_n, c_n b_n\}$.

4. Applications. In this section, we provide an example of sequence of positive linear operators illustrating that Theorem 2 is stronger than Theorem 1.

***Example* 1.** Let us consider

$$p_n = \begin{cases} 1, & n = 2k, \\ 0, & n = 2k+1, \end{cases} \qquad s_n = \begin{cases} 0, & n = 2k, \\ 1, & n = 2k+1. \end{cases}$$

One can easily see that P_p is a regular power series method. For every $\varepsilon > 0$, since $E_\varepsilon = \big\{n \in \mathbb{N}_0 : |s_n - 0| \ge \varepsilon\big\} = \big\{n \in \mathbb{N}_0 : n = 2k+1\big\}$, we have $\delta_{P_p}(E_\varepsilon) = 0$ that is the sequence (s_n) is P_p-statistical convergent to 0. Let $\varrho_1(x) = 1 + x^2$ and $\varrho_2(x) = 1 + x^4$. In this case, the test functions F_v become $F_v(x) = x^v$. Consider the following Gauss–Weierstrass operators defined by

$$W_n\big(f(t);x\big) = \sqrt{\frac{n}{2\pi}} \int\limits_{-\infty}^{+\infty} f(t) e^{-\frac{1}{2}n(t-x)^2} dt.$$

Then observe that, for each $n \in \mathbb{N}$, W_n is a positive linear operator acting from C_{ϱ_1} into B_{ϱ_2}. We also get

$$W_n(F_0(t);x) = 1, \qquad W_n(F_1(t);x) = x, \qquad W_n(F_2(t);x) = x^2 + \frac{1}{n}.$$

Furthermore, since

$$W_n\big(\varrho_1(t);x\big) = x^2 + 1 + \frac{1}{n} \le x^2 + 2 \le 2\varrho_1(x) \le 4\varrho_2(x),$$

$\{W_n\}$ is a uniformly bounded sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_1} (or, into B_{ϱ_2}). Define

$$T_n(f(t);x) = (1 + s_n)W_n(f(t);x).$$

Then one can immediately show that $\{T_n\}$ is a uniformly bounded sequence of positive linear operators since its construction. By using the test functions $F_v(x) = x^v,\ v = 0, 1, 2,$ we obtain that

$$T_n(F_0(t); x) = 1 + s_n,$$

$$T_n(F_1(t); x) = (1 + s_n)x,$$

$$T_n(F_2(t); x) = (1 + s_n)\left(x^2 + \frac{1}{n}\right),$$

which implies that the sequence $\{T_n\}$ satisfies the conditions of main theorem and we conclude that, for all $f \in C_{\varrho_1}$,

$$st_{P_p} - \lim \left\|T_n(f) - f\right\|_{\varrho_2} = 0.$$

However, since (s_n) is not convergent to zero and not statistically convergent, $\{T_n\}$ satisfies neither Theorem 1 nor the statistical version.

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