DOI: 10.37863/umzh.v75i2.6680

UDC 512.5

M. Sahai[1], **S. F. Ansari** (University of Lucknow, India)

GROUP OF UNITS OF FINITE GROUP ALGEBRAS OF GROUPS OF ORDER 24

ГРУПА ОДИНИЦЬ СКІНЧЕННОГРУПОВИХ АЛГЕБР ГРУП 24-ГО ПОРЯДКУ

Let F be a finite field of characteristic p. The structures of the unit groups of group algebras over F of the three groups $D_{24},$ S_4 and $SL(2,\mathbb{Z}_3)$ of order 24 are completely described in numerous works. We present the unit groups of the group algebras over F for the remaining groups of order 24, namely, $C_{24},$ $C_{12}\times C_2,$ $C_2^3\times C_3,$ $C_3\rtimes C_8,$ $C_3\rtimes Q_8,$ $D_6\times C_4,$ $C_6\rtimes C_4,$ $C_3\rtimes D_8,$ $C_3\times D_8,$ $C_3\times Q_8,$ $A_4\times C_2,$ and $D_{12}\times C_2.$

Нехай F — скінченне поле характеристики $p.$ Структури груп одиниць групових алгебр над F для трьох груп $D_{24},$ S_4 та $SL(2,\mathbb{Z}_3)$ 24-го порядку повністю описано в багатьох роботах. Ми наводимо групи одиниць групових алгебр над F для решти груп 24-порядку, а саме $C_{24},$ $C_{12}\times C_2,$ $C_2^3\times C_3,$ $C_3\rtimes C_8,$ $C_3\rtimes Q_8,$ $D_6\times C_4,$ $C_6\rtimes C_4,$ $C_3\rtimes D_8,$ $C_3\times D_8,$ $C_3\times Q_8,$ $A_4\times C_2$ та $D_{12}\times C_2.$

1. Introduction. Let FG be the group algebra of a finite group G over a finite field F of characteristic p containing $q=p^k$ elements and let $U(FG)$ be the group of units in FG. Explicit calculations in $U(FG)$ are usually difficult, even when $|G|$ is very small. So to find the structure of $U(FG)$ is an interesting as well as challenging problem. In this paper, we study $U(FG)$, where G is a group of order 24. We denote the Jacobson radical of FG by $J(FG)$ and $V=1+J(FG)$.

Let $D_n,$ $Q_n,$ and C_n be the dihedral group, the quaternion group and the cyclic group of order n, respectively. The fifteen nonisomorphic groups of order 24 are $C_{24},$ $C_{12}\times C_2,$ $C_6\times C_2^2,$ $C_3\rtimes C_8,$ $SL(2,\mathbb{Z}_3),$ $C_3\rtimes Q_8,$ $S_4,$ $C_4\times D_6,$ $D_{24},$ $C_6\rtimes C_4,$ $C_3\rtimes D_8,$ $C_3\times D_8,$ $C_3\times Q_8,$ $C_2\times A_4$ and $D_{12}\times C_2$. The structures of $U(FS_4)$ and $U(FSL(2,\mathbb{Z}_3))$ are given in [5, 7], respectively. Also, the structure of $U(FD_{24})$ for $p\leq 3$ is given in [7, 9] and for $p>3$ in [12]. For $p=2$, Maheshwari [7], obtained the structure of $U(FG)$, where G is a non-Abelian group of order 24. For $p=3$, Monaghan [9], studied the structure of $U(FG)$ where G is a non-Abelian group of 24 such that G has a normal subgroup of order 3. Here, we completely describe the structure of $U(FG)$ in the remaining cases.

The group algebra FG and its Wedderburn decomposition have applications in coding theory. Cyclic codes can be realized as ideals of group algebras over cyclic groups as well as many other codes appear as ideals of group algebras of noncyclic groups (see [10, 11]). A good description of the structure of $U(FG)$ has applications in group ring cryptography, for investigation of the Lie properties of group rings, isomorphism problem and other open questions in this area (see [1, 4]). Our notations are same as in [12, 14].

The number of simple components of $FG/J(FG)$ is given in [2]. Let $m,$ η and T be as in [2]. We restate here that for a p-regular element $g\in G,$ γ_g is the sum of all the conjugates of g and the cyclotomic F-class of γ_g is $S_F(\gamma_g)=\{\gamma_{g^t}\mid t\in T\}$.

Lemma 1.1 [2, Proposition 1.2]. *The number of simple components of $FG/J(FG)$ is equal to the number of cyclotomic F-classes in G.*

[1] Corresponding author, e-mail: meena_sahai@hotmail.com.

Lemma 1.2 [2, Theorem 1.3]. *Suppose that $Gal(F(\eta)/F)$ is cyclic. Let w be the number of cyclotomic F-classes in G. If $K_1,\ K_2,\ldots,\ K_w$ are the simple components of $Z(FG/J(FG))$ and $S_1,\ S_2,\ \ldots,\ S_w$ are the cyclotomic F-classes of G, then with a suitable reordering of indices, $|S_i| = [K_i : F]$.*

2. Main results. Let $l \in \{3, 6, 12, 24\}$. If $q \equiv \pm 5, \pm 7, \pm 11 \mod l$, then $q^2 \equiv 1 \mod l$. So $|T| = 2$ and $|S_F(\gamma_g)| \leq 2$. Let r and s be the numbers of elements $g \in G$ such that $|S_F(\gamma_g)| = 1$ and $|S_F(\gamma_g)| = 2$, respectively. Then, for $p > 3$, by Lemmas 1.1 and 1.2,

$$Z(FG) \cong F^r \oplus F_2^s.$$

As $\dim_F(Z(FG)) = c =$ the number of conjugacy classes in G, so $s = (c - r)/2$.

Theorem 2.1. *Let F be a finite field of characteristic p with $|F| = q = p^k$.*

(i) *If $p = 2$, then*

$$U(FC_{24}) \cong \begin{cases} C_8^{3k} \times C_4^{3k} \times C_2^{6k} \times C_{2^k-1}^3, & \text{if } q \equiv 1 \bmod 3, \\ C_8^{3k} \times C_4^{3k} \times C_2^{6k} \times C_{2^k-1} \times C_{2^{2k}-1}, & \text{if } q \equiv -1 \bmod 3. \end{cases}$$

(ii) *If $p = 3$, then*

$$U(FC_{24}) \cong \begin{cases} C_3^{16k} \times C_{3^k-1}^8, & \text{if } q \equiv 1 \bmod 8, \\ C_3^{16k} \times C_{3^k-1}^2 \times C_{3^{2k}-1}^3, & \text{if } q \equiv -1, 3 \bmod 8, \\ C_3^{16k} \times C_{3^k-1}^4 \times C_{3^{2k}-1}^2, & \text{if } q \equiv -3 \bmod 8. \end{cases}$$

(iii) *If $p > 3$, then*

$$U(FC_{24}) \cong \begin{cases} C_{p^k-1}^{24}, & \text{if } q \equiv 1 \bmod 24, \\ C_{p^k-1}^2 \times C_{p^{2k}-1}^{11}, & \text{if } q \equiv -1, 11 \bmod 24, \\ C_{p^k-1}^4 \times C_{p^{2k}-1}^{10}, & \text{if } q \equiv 5 \bmod 24, \\ C_{p^k-1}^6 \times C_{p^{2k}-1}^9, & \text{if } q \equiv -5, 7 \bmod 24, \\ C_{p^k-1}^8 \times C_{p^{2k}-1}^8, & \text{if } q \equiv -7 \bmod 24, \\ C_{p^k-1}^{12} \times C_{p^{2k}-1}^6, & \text{if } q \equiv -11 \bmod 24. \end{cases}$$

Proof. Let $C_{24} = \langle x \rangle$.

(i) If $p = 2$, then $|C_{24} : C_8| \neq 0$ in F. So $J(FC_{24}) = \omega(C_8)$ and $FC_{24}/J(FC_{24}) \cong FC_3$. Thus,

$$U(FC_{24}) \cong V \times U(FC_3).$$

Now, $\alpha = \sum_{i=0}^{23} a_i x^i \in \omega(C_8)$ if and only if $\sum_{j=0}^{7} a_{3j+i} = 0,\ i = 0, 1, 2$. Also, $\alpha^2 = \sum_{j=0}^{11} \sum_{i=0}^{1} a_{12i+j}^2 x^{2j},\ \alpha^4 = \sum_{j=0}^{5} \sum_{i=0}^{3} a_{6i+j}^4 x^{4j}$ and $\alpha^8 = \sum_{j=0}^{2} \sum_{i=0}^{7} a_{3i+j}^8 x^{8j} = 0$. Since $\dim_F(J(FG)) = 21,\ |V| = 2^{21k}$. Let $V \cong C_2^{l_1} \times C_4^{l_2} \times C_8^{l_3}$, so that $2^{21k} = 2^{l_1+2l_2+3l_3}$. It is easy to see that

$$S = \{\alpha \in \omega(C_8) \mid \alpha^2 = 0 \text{ and } \alpha = \beta^4 \text{ for some } \beta \in \omega(C_8)\} =$$
$$= \left\{ \sum_{i=0}^{2} a_{4i} x^{4i}(1 + x^{12}) \mid a_{4i} \in F \right\}.$$

Thus, $|S| = 2^{3k}$ and $l_3 = 3k$. Also,

$$S_1 = \{\alpha \in \omega(C_8) \mid \alpha^2 = 0 \text{ and } \alpha = \beta^2 \text{ for some } \beta \in \omega(C_8)\} =$$
$$= \left\{ \sum_{i=0}^{5} a_{2i} x^{2i}(1 + x^{12}) \mid a_{2i} \in F \right\}.$$

Hence, $|S_1| = 2^{6k}$, $l_2 = 3k$ and $l_1 = 6k$ which leads to $V \cong C_2^{6k} \times C_4^{3k} \times C_8^{3k}$.

For $U(FC_3)$, see [13, Theorem 2.2].

(ii) If $p = 3$, then $|C_{24} : C_3| = 8 \neq 0$ in F, $J(FC_{24}) = \omega(C_3)$ and $FC_{24}/J(FC_{24}) \cong FC_8$. Thus,

$$U(FG) \cong V \times U(FC_8).$$

Since $\dim_F J(FC_{24}) = 16$ and $J(FC_{24})^3 = 0$, $V \cong C_3^{16k}$. Hence,

$$U(FC_{24}) \cong C_3^{16k} \times U(FC_8).$$

The structure of $U(FC_8)$ is given in [13, Theorem 3.3].

(iii) If $p > 3$, then $m = 24$.

If $q \equiv \pm 1 \bmod 24$, then we have [13, Lemma 2.2].

If $q \equiv 5 \bmod 24$, then $T = \{1, 5\}$ and $|S_F(\gamma_g)| = 1$ for $g = 1$, $x^{\pm 6}$ and x^{12}. So $r = 4$, $s = 10$ and

$$FC_{24} \cong F^4 \oplus F_2^{10}.$$

If $q \equiv -5, 7 \bmod 24$, then $|S_F(\gamma_g)| = 1$ for $g = 1$, $x^{\pm 4}$, $x^{\pm 8}$, x^{12}. So $r = 6$, $s = 9$ and

$$FC_{24} \cong F^6 \oplus F_2^9.$$

If $q \equiv -7 \bmod 24$, then $T = \{1, 17\} \bmod 24$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1$, $x^{\pm 3}$, $x^{\pm 6}$, $x^{\pm 9}$, x^{12}. So $r = s = 8$ and

$$FC_{24} \cong F^8 \oplus F_2^8.$$

If $q \equiv 11 \bmod 24$, then $T = \{1, 11\} \bmod 24$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1$, x^{12}. So $r = 2$, $s = 11$ and

$$FC_{24} \cong F^2 \oplus F_2^{11}.$$

If $q \equiv -11 \bmod 24$, then $T = \{1, 13\} \bmod 24$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1$, $x^{\pm 2}$, $x^{\pm 4}$, $x^{\pm 6}$, $x^{\pm 8}$, $x^{\pm 10}$, x^{12}. So $r = 12$, $s = 6$ and

$$FC_{24} \cong F^{12} \oplus F_2^6.$$

Theorem 2.1 is proved.

Theorem 2.2. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_2 \times C_{12}$.*

(i) *If* $p = 2$, *then*

$$U(FG) \cong \begin{cases} C_2^{9k} \times C_4^{6k} \times C_{2^k-1}^3, & \textit{if } q \equiv 1 \bmod 3, \\ C_2^{9k} \times C_4^{6k} \times C_{2^k-1} \times C_{2^{2k}-1}, & \textit{if } q \equiv -1 \bmod 3. \end{cases}$$

(ii) *If* $p = 3$, *then*

$$U(FG) \cong \begin{cases} C_3^{16k} \times C_{3^k-1}^8, & \textit{if } q \equiv 1 \bmod 4, \\ C_3^{16k} \times C_{3^k-1}^4 \times C_{3^{2k}-1}^2, & \textit{if } q \equiv -1 \bmod 4. \end{cases}$$

(iii) *If* $p > 3$, *then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^{24}, & \textit{if } q \equiv 1 \bmod 12, \\ C_{p^k-1}^4 \times C_{p^{2k}-1}^{10}, & \textit{if } q \equiv -1 \bmod 12, \\ C_{p^k-1}^8 \times C_{p^{2k}-1}^8, & \textit{if } q \equiv 5 \bmod 12, \\ C_{p^k-1}^{12} \times C_{p^{2k}-1}^6, & \textit{if } q \equiv -5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y \mid x^{12} = y^2 = (x, y) = 1 \rangle$.

(i) Let $H = \langle x^3 \rangle \times \langle y \rangle$. If $p = 2$, then $|G : H| \neq 0$ in F, $J(FG) = \omega(H)$ and $FG/J(FG) \cong$ $\cong FC_3$. Thus,

$$U(FG) \cong V \times U(FC_3).$$

Now, let $\alpha = \sum_{j=0}^{11} \sum_{i=0}^{11} a_{12j+i} x^i y^j$. Then $\alpha \in \omega(H)$ if and only if $\sum_{i=0}^{7} a_{3i+j} = 0$ for $j = 0, 1, 2$. Also, $\alpha^2 = \sum_{j=0}^{5} \sum_{i=0}^{3} a_{6i+j}^2 x^{2j}$ and $\alpha^4 = \sum_{j=0}^{2} \sum_{i=0}^{7} a_{3i+j}^4 x^{4j} = 0$. Since $\dim_F(J(FG)) = 21$, $|V| = 2^{21k}$. Let $V \cong C_2^{l_1} \times C_4^{l_2}$, so that $2^{21k} = 2^{l_1 + 2l_2}$. Clearly

$$S = \{\alpha \in \omega(H) \mid \alpha^2 = 0 \text{ and } \alpha = \beta^2 \text{ for some } \beta \in \omega(H)\} =$$

$$= \left\{ \sum_{i=0}^{2} a_{2i} x^{2i} (1 + x^6) \mid a_{2i} \in F \right\}.$$

Thus, $|S| = 2^{3k}$ and $l_2 = 3k$. So $V \cong C_2^{15k} \times C_4^{3k}$.

(ii) Let $K = \langle x^4 \rangle$. If $p = 3$, then $|G : K| \neq 0$ in F, $J(FG) = \omega(K)$ and $FG/J(FG) \cong$ $\cong F(C_2 \times C_4)$. Hence,

$$U(FG) \cong V \times U(F(C_2 \times C_4)).$$

Since $\dim_F J(FG) = 16$ and $J(FG)^3 = 0$, $V \cong C_3^{16k}$.

The rest follows by [13, Theorem 3.4].

(iii) If $p > 3$, then $m = 12$.

If $q \equiv 1 \bmod 12$, then $T = \{1\} \bmod 12$. Thus, $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by Lemmas 1.1 and 1.2,

$$FG \cong F^{24}.$$

If $q \equiv -1 \bmod 12$, then $T = \{1, 11\} \bmod 12$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1,\ x^6,\ y,\ x^6y$. So $r = 4,\ s = 10$ and

$$FG \cong F^4 \oplus F_2^{10}.$$

If $q \equiv 5 \bmod 12$, then $T = \{1, 5\} \bmod 12$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1,\ x^{\pm 3},\ x^6,\ y,\ x^{\pm 3}y,\ x^6y$. So $r = s = 8$ and

$$FG \cong F^8 \oplus F_2^8.$$

If $q \equiv -5 \bmod 12$, then $T = \{1, 7\} \bmod 12$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1,\ x^{\pm 2},\ x^{\pm 4},\ x^6,\ y,\ x^{\pm 2}y,\ x^{\pm 4}y,\ x^6y$. So $r = 12,\ s = 6$ and

$$FG \cong F^{12} \oplus F_2^6.$$

Theorem 2.2 is proved.

Theorem 2.3. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_2^3 \times C_3$.*

(i) *If $p = 2$, then*

$$U(FG) \cong \begin{cases} C_2^{21k} \times C_{2^k-1}^3, & \text{if } q \equiv 1 \bmod 3, \\ C_2^{21k} \times C_{2^k-1} \times C_{2^{2k}-1}, & \text{if } q \equiv -1 \bmod 3. \end{cases}$$

(ii) *If $p = 3$, then*

$$U(FG) \cong C_3^{16k} \times C_{3^k-1}^8.$$

(iii) *If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^{24}, & \text{if } q \equiv 1 \bmod 6, \\ C_{p^k-1}^8 \times C_{p^{2k}-1}^8, & \text{if } q \equiv -1 \bmod 6. \end{cases}$$

Proof. Let $C_2^3 \times C_3 = \langle x, y, z, w \mid x^3 = 1, y^2 = z^2 = w^2 = 1, xy = yx, xz = zx, xw = wx, yz = zy, yw = wy, zw = wz \rangle$.

(i) Let $H = \langle y \rangle \times \langle z \rangle \times \langle w \rangle$. If $p = 2$, then $|G : H| \neq 0$ in F, $J(FG) = \omega(H)$ and $FG/J(FG) \cong FC_3$. Hence,

$$U(FG) \cong V \times U(FC_3).$$

Since $\dim_F J(FG) = 21$ and $\alpha^2 = 0$ for all $\alpha \in \omega(H)$, $V \cong C_2^{21k}$.

(ii) Let $K = \langle x \rangle$. If $p = 3$, then $|G : K| \neq 0$ in F, $J(FG) = \omega(K)$ and $FG/J(FG) \cong FC_2^3$. Hence,

$$U(FG) \cong V \times U(FC_2^3).$$

Since $\dim_F J(FG) = 16$ and $J(FG)^3 = 0$, $V \cong C_3^{16k}$.

$U(FC_2^3)$ is given in [13, Theorem 3.5].

(iii) If $p > 3$, then $m = 6$. If $q \equiv 1 \bmod 6$, then $T = \{1\} \bmod 6$. Thus, $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by Lemmas 1.1 and 1.2,

$$FG \cong F^{24}.$$

If $q \equiv -1 \bmod 6$, then $T = \{1, 5\} \bmod 6$. Thus, $|S_F(\gamma_g)| = 1$ for $g = 1,\ y,\ z,\ w,\ yz,\ yw,\ zw,\ yzw$. So $r = s = 8$ and

$$FG \cong F^8 \oplus F_2^8.$$

Theorem 2.3 is proved.

Now, we discuss non-Abelian groups of order 24. Since the case $p = 2$ is dealt with in [7], we consider $p \geq 3$ only.

Table 1. Conjugacy classes in $C_2 \times A_4$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x\}$	2
$[y]$	$\{y, yz, yw, yzw\}$	3
$[z]$	$\{z, w, zw\}$	2
$[xy]$	$\{xy, xyz, xyw, xyzw\}$	6
$[xz]$	$\{xz, xw, xzw\}$	2
$[y^{-1}]$	$\{y^{-1}, y^{-1}z, y^{-1}w, y^{-1}zw\}$	3
$[xy^{-1}]$	$\{xy^{-1}, xy^{-1}z, xy^{-1}w, xy^{-1}zw\}$	6

Theorem 2.4. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_2 \times A_4$.*
(i) *If $p = 3$, then*

$$U(FG) \cong C_3^{4k} \rtimes (C_{3^k-1}^2 \times GL(3, F)^2).$$

(ii) *If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^6 \times GL(3, F)^2, & \text{if } q \equiv 1 \bmod 6, \\ C_{p^k-1}^2 \times C_{p^{2k}-1}^2 \times GL(3, F)^2, & \text{if } q \equiv -1 \bmod 6. \end{cases}$$

Proof. Let $G = \langle x, y, z, w \mid x^2 = y^3 = z^2 = w^2 = 1, xyx = y, xz = zx, xw = wx, zw = = wz, wy = ywz, zy = yw \rangle$.

(i) Let $p = 3$. Clearly, $\widehat{T_3} = 1 + (y + y^{-1})(1 + z)(1 + w)$. Let $\alpha = a_0 + a_1 z + a_2 w + a_3 zw + + a_4 x + a_5 xz + a_6 xw + a_7 xzw + a_8 y + a_9 yz + a_{10} yw + a_{11} yzw + a_{12} y^{-1} + a_{13} y^{-1} z + a_{14} y^{-1} w + + a_{15} y^{-1} zw + a_{16} xy + a_{17} xyz + a_{18} xyw + a_{19} xyzw + a_{20} xy^{-1} + a_{21} xy^{-1} z + a_{22} xy^{-1} w + a_{23} xy^{-1} zw$.

If $\alpha \widehat{T_3} = 0$, then, for $j = 0,\ 1,\ 2,\ 3$, we have

$$a_j + \sum_{i=0}^{3} (a_{8+i} + a_{12+i}) = 0, \quad a_{12+j} + \sum_{i=0}^{3} (a_i + a_{8+i}) = 0,$$

$$a_{4+j} + \sum_{i=0}^{3} (a_{16+i} + a_{20+i}) = 0, \quad a_{16+j} + \sum_{i=0}^{3} (a_{4+i} + a_{20+i}) = 0,$$

$$a_{8+j} + \sum_{i=0}^{3} (a_i + a_{12+i}) = 0, \quad a_{20+j} + \sum_{i=0}^{3} (a_{4+i} + a_{16+i}) = 0.$$

After solving these equations, we get $a_{4i} = a_{k+4i}$ for $k = 1, 2, 3$ and $i = 0, 1, 2, 3, 4, 5$. Also, $a_0 + a_8 + a_{12} = 0$ and $a_4 + a_{16} + a_{20} = 0$. Hence, $\mathrm{Ann}\,(\widehat{T_3}) = \{((b_0 + b_1 x)(1 - y^{-1}) + (b_2 + + b_3 x)(y - y^{-1}))(1 + z + w + zw) \mid b_i \in F\}$.

Since x and $(1+z)(1+w) \in Z(FG)$, $\mathrm{Ann}\,(\widehat{T_3})^3 = 0$. Thus, $\mathrm{Ann}\,(\widehat{T_3}) \subseteq J(FG)$. By [16, Lemma 2.2], $J(FG) = \mathrm{Ann}\,(\widehat{T_3})$ and $\dim_F(J(FG)) = 4$. Hence, $V \cong C_3^{4k}$.

As $[1]$, $[x]$, $[z]$ and $[xz]$ are the 3-regular conjugacy classes, so $m = 2$. Thus, $q \equiv 1 \bmod 2$ and $S_F(\gamma_g) = \{\gamma_g\}$ for $g = 1,\ x,\ z,\ xz$. Therefore, by [2, Theorem 1.3], this yield four components in the Wedderburn decomposition of $FG/J(FG)$. Since $\dim_F(FG/J(FG)) = 20$, we have

$$FG/J(FG) \cong F^2 \oplus M(3,F)^2.$$

Hence,

$$U(FG) \cong C_3^{4k} \rtimes (C_{3^k-1}^2 \times GL(3,F)^2).$$

(ii) If $p > 3$, then $m = 6$. Since $G/G' \cong C_6$, thus, by [8, Proposition 3.6.11],

$$FG \cong FC_6 \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i)\right). \tag{2.1}$$

Since $\dim_F(Z(FG)) = 8$, $l \leq 2$.

If $q \equiv 1 \bmod 6$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.1), [13, Theorem 4.1], Lemmas 1.1 and 1.2,

$$FG \cong F^6 \oplus \left(\bigoplus_{i=1}^{2} M(n_i, F)\right).$$

Now $\sum_{i=1}^{2} n_i^2 = 18$ gives $n_i = 2$ for $i = 1, 2$. Hence,

$$FG \cong F^6 \oplus M(3,F)^2.$$

If $q \equiv -1 \bmod 6$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ z,\ xz$. So $r = 4$, $s = 2$ and

$$FG \cong F^2 \oplus F_2^2 \oplus M(3,F)^2.$$

Theorem 2.4 is proved.

All the remaining groups of order 24 contain a normal subgroup of order 3 and in view of [9], we need to consider only the semisimple case $p > 3$.

Table 2. Conjugacy classes in $C_3 \rtimes C_8$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, x^{-1}\}$	3
$[y]$	$\{y, xy, x^{-1}y\}$	8
$[y^2]$	$\{y^2\}$	4
$[y^3]$	$\{y^3, xy^3, x^{-1}y^3\}$	8
$[y^4]$	$\{y^4\}$	2
$[y^{-1}]$	$\{y^{-1}, xy^{-1}, x^{-1}y^{-1}\}$	8
$[y^{-2}]$	$\{y^{-2}\}$	4
$[y^{-3}]$	$\{y^{-3}, xy^{-3}, x^{-1}y^{-3}\}$	8
$[xy^2]$	$\{xy^2, x^{-1}y^2\}$	4
$[xy^{-2}]$	$\{xy^{-2}, x^{-1}y^{-2}\}$	4
$[xy^4]$	$\{xy^4, x^{-1}y^4\}$	2

Theorem 2.5. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_3 \rtimes C_8$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^8 \times GL(2,F)^4, & \text{if } q \equiv 1, -7 \bmod 24, \\ C_{p^k-1}^2 \times C_{p^{2k}-1}^3 \times GL(2,F)^2 \times GL(2,F_2), & \text{if } q \equiv -1, -5, 7, 11 \bmod 24, \\ C_{p^k-1}^4 \times C_{p^{2k}-1}^2 \times GL(2,F)^4, & \text{if } q \equiv 5, -11 \bmod 24. \end{cases}$$

Proof. Let $G = \langle x, y \mid x^3 = y^8 = 1, yxy^{-1} = x^{-1} \rangle$.

FG is semisimple, so all the conjugacy classes of G are p-regular and $m = 24$. Since $G/G' \cong C_8$, thus, by [8, Proposition 3.6.11],

$$FG \cong FC_8 \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right), \tag{2.2}$$

where each F_i is a finite extension of F. Since $\dim_F(Z(FG)) = 12$, $l \leq 4$.

If $q \equiv 1,\ -7 \bmod 24$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.2), [13, Theorem 3.3], Lemmas 1.1 and 1.2,

$$FG \cong F^8 \oplus \left(\bigoplus_{i=1}^{4} M(n_i, F) \right).$$

Now $\dim_F(FG) = 24$, gives $n_i = 2$ for $i = 1, 2, 3, 4$. Hence,

$$FG \cong F^8 \oplus M(2, F)^4.$$

If $q \equiv 5,\ -11 \bmod 24$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ y^{\pm 2},\ y^4,\ xy^{\pm 2},\ xy^4$. So $r = 8,\ s = 2$ and

$$FG \cong F^4 \oplus F_2^2 \oplus M(2, F)^4.$$

If $q \equiv -1,\ -5,\ 7,\ 11 \bmod 24$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ y^4,\ xy^4$. So $r = s = 4$ and

$$FG \cong F^2 \oplus F_2^3 \oplus M(2, F)^2 \oplus M(2, F_2).$$

Theorem 2.5 is proved.

Table 3. Conjugacy classes in $C_3 \rtimes Q_8$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, x^{-1}\}$	12
$[x^2]$	$\{x^2, x^{-2}\}$	6
$[x^3]$	$\{x^3, x^{-3}\}$	4
$[x^4]$	$\{x^4, x^{-4}\}$	3
$[x^5]$	$\{x^5, x^{-5}\}$	12
$[x^6]$	$\{x^6\}$	2
$[y]$	$\{y, x^{\pm 2}y, x^{\pm 4}y, x^6 y\}$	4
$[xy]$	$\{x^{\pm 1}y, x^{\pm 3}y, x^{\pm 5}y\}$	4

Theorem 2.6. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_3 \rtimes Q_8$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C^4_{p^k-1} \times GL(2,F)^5, & \text{if } q \equiv \pm 1 \bmod 12, \\ C^4_{p^k-1} \times GL(2,F)^3 \times GL(2,F_2), & \text{if } q \equiv \pm 5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y \mid x^{12} = 1, x^6 = y^2, yxy^{-1} = x^{-1} \rangle$.
Since $p > 3$ and $G/G' \cong C_2^2$, thus, by [8, Proposition 3.6.11],

$$FG \cong FC_2^2 \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right). \tag{2.3}$$

Since $\dim_F(Z(FG)) = 9$, $l \leq 5$.

If $q \equiv \pm 1 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.3), [13, Theorem 3.2], Lemmas 1.1 and 1.2,

$$FG \cong F^4 \oplus \left(\bigoplus_{i=1}^{5} M(n_i, F) \right).$$

Now $\dim_F(FG) = 24$ gives $n_i = 2$ for $i = 1, 2, 3, 4, 5$. Hence,

$$FG \cong F^4 \oplus M(2,F)^5.$$

If $q \equiv \pm 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x^2,\ x^3,\ x^4,\ x^6,\ y,\ xy$. So $r = 7,\ s = 1$ and

$$FG \cong F^4 \oplus M(2,F)^3 \oplus M(2,F_2).$$

Theorem 2.6 is proved.

Table 4. Conjugacy classes in $C_4 \times D_6$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, x^{-1}\}$	3
$[y]$	$\{y, xy, x^{-1}y\}$	2
$[z]$	$\{z\}$	4
$[z^2]$	$\{z^2\}$	2
$[z^{-1}]$	$\{z^{-1}\}$	4
$[xz]$	$\{xz, x^{-1}z\}$	12
$[xz^2]$	$\{xz^2, x^{-1}z^2\}$	6
$[xz^{-1}]$	$\{xz^{-1}, x^{-1}z^{-1}\}$	12
$[yz]$	$\{yz, xyz, x^{-1}yz\}$	4
$[yz^2]$	$\{yz^2, xyz^2, x^{-1}yz^2\}$	2
$[yz^{-1}]$	$\{yz^{-1}, xyz^{-1}, x^{-1}yz^{-1}\}$	4

Theorem 2.7. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_4 \times D_6$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^8 \times GL(2,F)^4, & \text{if } q \equiv 1, 5 \bmod 12, \\ C_{p^k-1}^4 \times C_{p^{2k}-1}^2 \times GL(2,F)^2 \times GL(2,F_2), & \text{if } q \equiv -1, -5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y, z \mid x^3 = y^2 = z^4 = xyxy = 1, xz = zx, yz = zy \rangle$.
Since $G/G' \cong (C_2 \times C_4)$, thus, by [8, Proposition 3.6.11],

$$FG \cong F(C_2 \times C_4) \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right). \tag{2.4}$$

Since $\dim_F(Z(FG)) = 12$, $l \leq 4$.

If $q \equiv 1,\ 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.4), [13, Theorem 3.4], Lemmas 1.1 and 1.2,

$$FG \cong F^8 \oplus \left(\bigoplus_{i=1}^{4} M(n_i, F) \right).$$

Now $\sum_{i=1}^{4} n_i^2 = 16$ gives $n_i = 2$ for $i = 1, 2, 3, 4$. Hence,

$$FG \cong F^8 \oplus M(2, F)^4.$$

If $q \equiv -1,\ -5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ y,\ z^2,\ xz^2,\ yz^2$. So $r = 6$, $s = 3$ and

$$FG \cong F^4 \oplus F_2^2 \oplus M(2, F)^2 \oplus M(2, F_2).$$

Theorem 2.7 is proved.

Table 5. Conjugacy classes in $C_6 \rtimes C_4$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, xy^2, xy^{-2}\}$	4
$[x^2]$	$\{x^2\}$	2
$[x^{-1}]$	$\{x^{-1}, x^{-1}y^{-2}, x^{-1}y^2\}$	4
$[y]$	$\{y, y^{-1}\}$	6
$[y^2]$	$\{y^2, y^{-2}\}$	3
$[y^3]$	$\{y^3\}$	2
$[xy]$	$\{xy, xy^{-1}, xy^3\}$	4
$[x^2y]$	$\{x^2y, x^2y^{-1}\}$	6
$[x^{-1}y]$	$\{x^{-1}y, x^{-1}y^{-1}, x^{-1}y^3\}$	4
$[x^2y^2]$	$\{x^2y^2, x^2y^{-2}\}$	4
$[x^2y^3]$	$\{x^2y^3\}$	2

Theorem 2.8. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_6 \rtimes C_4$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^8 \times GL(2,F)^4, & \text{if } q \equiv 1, 5 \bmod 12, \\ C_{p^k-1}^4 \times C_{p^{2k}-1}^2 \times GL(2,F)^4, & \text{if } q \equiv -1, -5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y \mid x^4 = y^6 = 1, yxy = x \rangle$.
Since $G/G' \cong (C_2 \times C_4)$, thus, by [8, Proposition 3.6.11],

$$FG \cong F(C_2 \times C_4) \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right). \tag{2.5}$$

Since $\dim_F(Z(FG)) = 12$, $l \le 4$.

If $q \equiv 1,\ 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.5), [13, Theorem 3.4], Lemmas 1.1 and 1.2,

$$FG \cong F^8 \oplus \left(\bigoplus_{i=1}^{4} M(n_i, F) \right).$$

Now $\sum_{i=1}^{4} n_i^2 = 16$ gives $n_i = 2$ for $i = 1, 2, 3, 4$. Hence,

$$FG \cong F^8 \oplus M(2, F)^4.$$

If $q \equiv -1,\ -5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x^2,\ y,\ y^2,\ y^3,\ x^2y,\ x^2y^2,\ x^2y^3$. So $r = 8$, $s = 2$ and

$$FG \cong F^4 \oplus F_2^2 \oplus M(2, F)^4.$$

Theorem 2.8 is proved.

Table 6. Conjugacy classes in $C_3 \rtimes D_8$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x^{\pm 1}\}$	3
$[y]$	$\{y^{\pm 1}, xy^{\pm 1}, x^{-1}y^{\pm 1}\}$	4
$[z]$	$\{z, y^2z\}$	2
$[y^2]$	$\{y^2\}$	2
$[yz]$	$\{y^{\pm 1}z, xy^{\pm 1}z, x^{-1}y^{\pm 1}z\}$	2
$[xz]$	$\{xz, x^{-1}y^2z\}$	6
$[x^{-1}z]$	$\{x^{-1}z, xy^2z\}$	6
$[xy^2]$	$\{x^{\pm 1}y^2\}$	6

Theorem 2.9. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_3 \rtimes D_8$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^4 \times GL(2,F)^5, & \text{if } q \equiv 1, 5 \bmod 12, \\ C_{p^k-1}^4 \times GL(2,F)^3 \times GL(2,F_2), & \text{if } q \equiv -1, -5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y, z \mid x^3 = y^4 = z^2 = yzyz = 1, xyx = y, xz = zx \rangle$.
Since $G/G' \cong C_2^2$, thus, by [8, Proposition 3.6.11],

$$FG \cong FC_2^2 \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right). \tag{2.6}$$

Since $\dim_F(Z(FG)) = 9$, $l \leq 5$.

If $q \equiv 1,\ 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.6), [13, Theorem 3.2], Lemmas 1.1 and 1.2,

$$FG \cong F^4 \oplus \left(\bigoplus_{i=1}^{5} M(n_i, F) \right).$$

Now $\sum_{i=1}^{5} n_i^2 = 20$ gives $n_i = 2$ for $i = 1, 2, 3, 4, 5$. Hence,

$$FG \cong F^4 \oplus M(2, F)^5.$$

If $q \equiv -1,\ -5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ y,\ z,\ y^2,\ yz,\ xy^2$. So $r = 7,\ s = 1$ and

$$FG \cong F^4 \oplus M(2, F)^3 \oplus M(2, F_2).$$

Theorem 2.9 is proved.

Table 7. Conjugacy classes in $C_3 \times D_8$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x\}$	3
$[x^{-1}]$	$\{x^{-1}\}$	3
$[y]$	$\{y, y^{-1}\}$	4
$[y^2]$	$\{y^2\}$	2
$[z]$	$\{z, y^2z\}$	2
$[xy]$	$\{xy, xy^{-1}\}$	12
$[x^{-1}y]$	$\{x^{-1}y, x^{-1}y^{-1}\}$	12
$[xy^2]$	$\{xy^2\}$	6
$[x^{-1}y^2]$	$\{x^{-1}y^2\}$	6
$[xz]$	$\{xz, xy^2z\}$	6
$[x^{-1}z]$	$\{x^{-1}z, x^{-1}y^2z\}$	6
$[yz]$	$\{yz, y^{-1}z\}$	2
$[xyz]$	$\{xyz, xy^{-1}z\}$	6
$[x^{-1}yz]$	$\{x^{-1}yz, x^{-1}y^{-1}z\}$	6

Theorem 2.10. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_3 \times D_8$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^{12} \times GL(2, F)^3, & \text{if } q \equiv 1, -5 \bmod 12, \\ C_{p^k-1}^{4} \times C_{p^{2k}-1}^{4} \times GL(2, F) \times GL(2, F_2), & \text{if } q \equiv -1, 5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y \mid x^3 = y^4 = z^2 = yzyz = 1, xy = yx, xz = zx\rangle$.
Since $G/G' \cong (C_2 \times C_6)$, thus, by [8, Proposition 3.6.11],

$$FG \cong F(C_2 \times C_6) \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i)\right). \tag{2.7}$$

Since $\dim_F(Z(FG)) = 15$, $l \leq 3$.

If $q \equiv 1, -5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.7), [15, Theorem 3.5], Lemmas 1.1 and 1.2,

$$FG \cong F^{12} \oplus \left(\bigoplus_{i=1}^{3} M(n_i, F)\right).$$

Now $\sum_{i=1}^{3} n_i^2 = 12$ gives $n_i = 2$ for $i = 1, 2, 3$. Hence,

$$FG \cong F^{12} \oplus M(2, F)^3.$$

If $q \equiv -1,\ 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ y,\ y^2,\ z,\ yz$. So $r = s = 5$ and

$$FG \cong F^4 \oplus F_2^4 \oplus M(2, F) \oplus M(2, F_2).$$

Theorem 2.10 is proved.

Table 8. Conjugacy classes in $C_3 \times Q_8$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, x^{-1}\}$	4
$[x^2]$	$\{x^2\}$	2
$[y]$	$\{y, y^{-1}\}$	4
$[xy]$	$\{xy, x^{-1}y\}$	4
$[z]$	$\{z\}$	3
$[z^{-1}]$	$\{z^{-1}\}$	3
$[xz]$	$\{xz, x^{-1}z\}$	12
$[x^2z]$	$\{x^2z\}$	6
$[xz^{-1}]$	$\{xz^{-1}, x^{-1}z^{-1}\}$	12
$[x^2z^{-1}]$	$\{x^2z^{-1}\}$	6
$[yz]$	$\{yz, y^{-1}z\}$	12
$[xyz]$	$\{xyz, x^{-1}yz\}$	12
$[xyz^{-1}]$	$\{xyz^{-1}, x^{-1}yz^{-1}\}$	12
$[yz^{-1}]$	$\{yz^{-1}, y^{-1}z^{-1}\}$	12

Theorem 2.11. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = C_3 \times Q_8$. If $p > 3$, then*

$$U(FG) \cong \begin{cases} C_{p^k-1}^{12} \times GL(2, F)^3, & \text{if } q \equiv 1, -5 \bmod 12, \\ C_{p^k-1}^{4} \times C_{p^{2k}-1}^{4} \times GL(2, F) \times GL(2, F_2), & \text{if } q \equiv -1, 5 \bmod 12. \end{cases}$$

Proof. Let $G = \langle x, y, z \mid x^4 = z^3 = 1, x^2 = y^2, yxy^{-1} = x^{-1}, xz = zx, yz = zy\rangle$.
Since $G/G' \cong (C_2 \times C_6)$, thus, by [8, Proposition 3.6.11],

$$FG \cong F(C_2 \times C_6) \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i)\right). \tag{2.8}$$

Since $\dim_F(Z(FG)) = 15,\ l \leq 3$.

If $q \equiv 1,\ -5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.8), [15, Theorem 3.5], Lemmas 1.1 and 1.2,

$$FG \cong F^{12} \oplus \left(\bigoplus_{i=1}^{3} M(n_i, F)\right).$$

Now $\sum_{i=0}^{3} n_i^2 = 12$ gives $n_i = 2$ for $i = 1, 2, 3$. Hence,

$$FG \cong F^{12} \oplus M(2, F)^3.$$

If $q \equiv -1, 5 \bmod 12$, then $|S_F(\gamma_g)| = 1$ for $g = 1,\ x,\ x^2,\ y,\ xy$. So $r = s = 5$ and

$$FG \cong F^4 \oplus F_2^4 \oplus M(2, F) \oplus M(2, F_2).$$

Theorem 2.11 is proved.

Table 9. Conjugacy classes in $D_{12} \times C_2$

Representative	Elements in the class	Order of element
$[1]$	$\{1\}$	1
$[x]$	$\{x, x^{-1}\}$	6
$[x^2]$	$\{x^2, x^{-2}\}$	3
$[x^3]$	$\{x^3\}$	2
$[y]$	$\{y, x^2y, x^{-2}y,\}$	2
$[xy]$	$\{xy, x^{-1}y, x^3y\}$	2
$[z]$	$\{z\}$	2
$[xz]$	$\{xz, x^{-1}z\}$	6
$[x^2z]$	$\{x^2z, x^{-2}z\}$	6
$[x^3z]$	$\{x^3z\}$	2
$[yz]$	$\{yz, x^2yz, x^{-2}yz\}$	2
$[xyz]$	$\{xyz, x^{-1}yz, x^3yz\}$	2

Theorem 2.12. *Let F be a finite field of characteristic p with $|F| = q = p^k$ and let $G = D_{12} \times C_2$. If $p > 3$, then*

$$U(FG) \cong C_{p^k-1}^8 \times GL(2, F)^4, \quad \textit{if}\ \ q \equiv \pm 1\ \textit{mod}\ 6.$$

Proof. Let $G = \langle x, y \mid x^6 = y^2 = z^2 = xyxy = 1, xz = zx, yz = zy\rangle$.
Since $G/G' \cong C_2^3$, thus, by [8, Proposition 3.6.11],

$$FG \cong FC_2^3 \oplus \left(\bigoplus_{i=1}^{l} M(n_i, F_i) \right). \tag{2.9}$$

Since $\dim_F(Z(FG)) = 12$, $l \leq 4$.

If $q \equiv \pm 1 \mod 6$, then $|S_F(\gamma_g)| = 1$ for all $g \in G$. Therefore, by (2.9), [13, Theorem 3.5], Lemmas 1.1 and 1.2,

$$FG \cong F^8 \oplus \left(\bigoplus_{i=1}^{4} M(n_i, F) \right).$$

Now $\sum_{i=1}^{4} n_i^2 = 16$ gives $n_i = 2$ for $i = 1, 2, 3, 4$. Hence,

$$FG \cong F^8 \oplus M(2, F)^4.$$

Theorem 2.12 is proved.

Acknowledgment. The financial assistance provided to the second author in the form of a Senior Research Fellowship from University Grants Commission, India is gratefully acknowledged.

References

1. A. A. Bovdi, J. Kurdics, *Lie properties of the group algebra and the nilpotency class of the group of units*, J. Algebra, **212**, № 1, 28 – 64 (1999).
2. R. A. Ferraz, *Simple components of the center of $FG/J(FG)$*, Commun. Algebra, **36**, № 9, 3191 – 3199 (2008).
3. The GAP Groups. GAP-Groups, Algorithms and Programming, Version 4.7.8 (2015); http://www.gap-system.org.
4. T. Hurley, *Group rings for communications*, Int. J. Group Theory, **4**, 1 – 23 (2015).
5. M. Khan, R. K. Sharma, J. B. Srivastava, *The unit group of FS_4*, Acta Math. Hungar., **118**, № 1-2, 105 – 113 (2008).
6. S. Maheshwari, *The unit group of group algebras $F_qSL(2, \mathbb{Z}_3)$*, J. Algebra Comb. Discrete Struct. and Appl., **3**, № 4, 1 – 6 (2016).
7. S. Maheshwari, R. K. Sharma, *A note on units in $F_qSL(2, \mathbb{Z}_3)$*, Ukr. Math. J., **73**, № 8, 1331 – 1337 (2022); DOI: 10.1007/s11253-022-01994-7.
8. C. P. Milies, S. K. Sehgal, *An introduction to group ring*, Algebra and Appl., vol. 1, Kluwer Acad. Publ., Dordrecht (2002).
9. F. Monaghan, *Units of some group algebras of non-Abelian groups of order 24 over any finite field of characteristic 3*, Int. Electron. J. Algebra, **12**, 133 – 161 (2012).
10. V. S. Pless, W. C. Huffman, *Handbook of coding theory*, Elsevier, New York (1998).
11. R. E. Sabin, S. J. Lomonaco, *Metacyclic error correcting codes*, Appl. Algebra Engrg. Comm. and Comput., **6**, № 3, 191 – 210 (1995).
12. M. Sahai, S. F. Ansari, *Unit groups of group algebras of certain dihedral groups-II*, Asian-Eur. J. Math., **12**, № 4 (2018).
13. M. Sahai, S. F. Ansari, *Unit groups of finite group algebras of Abelian groups of order at most 16*, Asian-Eur. J. Math., **14**, № 3 (2021).
14. M. Sahai, S. F. Ansari, *Unit groups of group algebras of groups of order 18*, Comm. Algebra, **49**, № 8, 3273 – 3282 (2021).
15. G. Tang, G. Yanyan, *The unit group of FG of group with order 12*, Int. J. Pure and Appl. Math., **73**, № 2, 143 – 158 (2011).
16. G. Tang, Y. Wei, Y. Li, *Unit groups of group algebras of some small groups*, Czechoslovak Math. J., **64**, № 1, 149 – 157 (2014).

Received 09.04.21