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NONEXISTENCE RESULTS FOR A SYSTEM OF NONLINEAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

РЕЗУЛЬТАТИ ЩОДО НЕІСНУВАННЯ РОЗВ'ЯЗКІВ СИСТЕМИ НЕЛІНІЙНИХ ДРОБОВИХ ІНТЕГРО-ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ

We investigate the nonexistence of (nontrivial) global solutions for a system of nonlinear fractional equations. Each equation involves n fractional derivatives, a subfirst-order ordinary derivative, and a nonlinear source term. The fractional derivatives are of the Caputo type of order between 0 and 1. The nonlinear sources have the form of the convolution of a function of state with (possibly singular) kernel. We generalize the results available in the literature, in particular, the results obtained by Mennouni and Youkana [Electron. J. Different. Equat., **152**, 1 – 15 (2017)] and Ahmad and Tatar [Turkish J. Math., **43**, 2715 – 2730 (2019)].

Досліджено випадок неіснування (нетривіальних) глобальних розв'язків системи нелінійних дробових рівнянь. Кожне рівняння містить n дробових похідних, звичайну похідну підпершого порядку та нелінійний член, що відповідає джерелу. Дробові похідні мають порядок типу Капуто між 0 та 1. Нелінійні джерела мають форму згортки функції стану з (можливо, сингулярним) ядром. У цій статті узагальнено деякі відомі з літератури результати, зокрема результати Меннуні й Юкани [Electron. J. Different. Equat., **152**, 1 – 15 (2017)] та Ахмада і Татара [Turkish J. Math., **43**, 2715 – 2730 (2019)].

1. Introduction. In this paper, we consider the following Cauchy problem of fractional integro-differential equations:

$$\begin{gathered}\left({}^C D^{\alpha}_{0+} u\right)(t) + \sum_{i=1}^{n} a_i \left({}^C D^{\alpha_i}_{0+} u\right)(t) = \int_0^t k(t-\tau) f(u(\tau), v(\tau))\, d\tau, \quad t > 0,\\ \left({}^C D^{\beta}_{0+} v\right)(t) + \sum_{i=1}^{n} b_i \left({}^C D^{\beta_i}_{0+} v\right)(t) = \int_0^t h(t-\tau) g(u(\tau), v(\tau))\, d\tau, \quad t > 0, \qquad (1)\\ u(0) = u_0, \quad v(0) = v_0, \quad u_0, v_0 \in \mathbb{R},\end{gathered}$$

where $0 < \alpha_i < \alpha \leq 1,\ 0 < \beta_i < \beta \leq 1,\ a_i, b_i,\ i = 1, \ldots, n,$ are positive real numbers, the fractional derivative ${}^C D^{\rho}_{0+}$ is of Caputo type with order ρ. The two functions f and g are assumed to be real continuous differentiable functions defined on $\mathbb{R} \times \mathbb{R}$. The kernels k and h defined on $[0, \infty)$ are locally integrable functions different from zero almost everywhere.

We prove the nonglobal existence of nontrivial solutions to system (1) under some certain conditions on the functions f and g, the kernels k and h, the parameters $\alpha,\ \beta,\ \alpha_i,\ \beta_i,\ i = 1, 2, \ldots, n,$ and the initial conditions. The proof is based on the weak formulation of the problem with the test function method, used in [18], with some suitable estimation inequalities.

Before stating and proving our result, let us have a glance at the existing literature. The local and global existence of solutions for several classes of fractional differential equations have been

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studied in many papers, we refer to [1, 6, 9, 17] and the references therein. For results on blow-up and nonexistence of solutions for fractional differential equations and inequalities, we may mention the works in [3 – 5, 7, 8, 10, 11, 13 – 17].

It has been shown in [8] that the system

$$\begin{aligned} &u'(t) + \left({}^C D^{\alpha}_{0+} u\right)(t) = |v(t)|^p, \quad t > 0, \quad 0 < \alpha < 1, \quad p > 1, \\ &v'(t) + \left({}^C D^{\beta}_{0+} v\right)(t) = |u(t)|^q, \quad t > 0, \quad 0 < \beta < 1, \quad q > 1, \\ &u(0) = u_0, \quad v(0) = v_0, \end{aligned} \tag{2}$$

admits no global solutions when $u_0 > 0$ and $v_0 > 0$ with $1 - \dfrac{1}{pq} \le \beta + \dfrac{\alpha}{q}$ or $1 - \dfrac{1}{pq} \le \alpha + \dfrac{\beta}{p}$.

In [15], the authors presented estimates for the blowing-up solutions of the system (2) by matching them to the solutions of the ordinary system

$$\begin{aligned} &u'(t) = a|v(t)|^p, \quad t > 0, \quad p > 1, \\ &v'(t) = a|u(t)|^q, \quad t > 0, \quad q > 1, \end{aligned}$$

and the fractional system

$$\begin{aligned} &\left({}^C D^{\alpha}_{0+} u\right)(t) = a|v(t)|^p, \quad t > 0, \quad 0 < \alpha < 1, \quad p > 1, \\ &\left({}^C D^{\beta}_{0+} v\right)(t) = a|u(t)|^q, \quad t > 0, \quad 0 < \beta < 1, \quad q > 1, \end{aligned}$$

with either $a = \dfrac{1}{2}$ or $a = 1$.

It has been proved in [4], that the positive solution (u, v) of the system

$$\begin{aligned} &u'(t) - \left({}^C D^{\alpha}_{0+} u\right)(t) = u^p(t) v^q(t), \quad t > 0, \quad 0 < \alpha < 1, \\ &v'(t) - \left({}^C D^{\beta}_{0+} v\right)(t) = u^r(t) v^\tau(t), \quad t > 0, \quad 0 < \beta < 1, \\ &u(0) = u_0 > 0, \quad v(0) = v_0 > 0, \end{aligned}$$

with $0 < p < 1,\ 0 < \tau < 1,\ r > 1 - p,\ q > 1 - \tau$, blows up in finite time if $1 - \dfrac{1}{p'q'} \le \beta + \dfrac{\alpha}{q'}$ or $1 - \dfrac{1}{p'q'} \le \alpha + \dfrac{\beta}{p'}$, where $p' = \dfrac{r}{1-p}$ and $q' = \dfrac{q}{1-\tau}$.

The present authors studied, in [3], the nonexistence of (nontrivial) global solutions for the fractional integro-differential problem

$$\begin{aligned} &(D^{\alpha}_{0+} u)(t) + \lambda\left(D^{\beta}_{0+} u\right)(t) \ge \int\limits_0^t k(t-\tau)|u(\tau)|^p \, d\tau, \quad t > 0, \quad p > 1, \\ &\left(I^{1-\alpha} u\right)\left(0^+\right) = b, \quad b \in \mathbb{R}, \end{aligned} \tag{3}$$

where D^{α}_{0+} and D^{β}_{0+}, $0 < \beta < \alpha \leq 1$, are fractional derivatives of Riemann–Liouville type with orders α and β, respectively, $\lambda = 0, 1$ and the kernel function k is nonnegative different from zero almost everywhere. It has been shown that if $\left(t^{-\alpha p'} + \lambda^{p'} t^{-\beta p'}\right) k^{1-p'}(t) \in L^1_{\mathrm{loc}}[0, \infty)$ and

$$\lim_{T\to\infty} T^{1-p'} \left(\int\limits_0^T t^{-\alpha p'} k^{1-p'}(t)\, dt + \lambda^{p'} \int\limits_0^T t^{-\beta p'} k^{1-p'}(t)\, dt \right) = 0,$$

where $p' = \dfrac{p}{p-1}$, then the problem (3) does not have any nontrivial global solution when $b \geq 0$.

In [2], the following system of ordinary fractional differential equations has been considered:

$$\left({}^C D^{\alpha}_{0+} u\right)(t) + a_1 \left({}^C D^{\alpha_1}_{0+} u\right)(t) = \int\limits_0^t k(t-\tau) |v(\tau)|^{q_1}\, d\tau, \quad t > 0,$$

$$\left({}^C D^{\beta}_{0+} v\right)(t) + b_1 \left({}^C D^{\beta_1}_{0+} v\right)(t) = \int\limits_0^t h(t-\tau) |u(\tau)|^{q_2}\, d\tau, \quad t > 0,$$

$$u(0) = u_0, \quad v(0) = v_0, \quad u_0, v_0 \in \mathbb{R},$$

where $0 < \alpha_1 < \alpha \leq 1$, $0 < \beta_1 < \beta \leq 1$, $q_1 > 1, q_2 > 1$, and a_1, b_1 are either 0 or 1.

The authors of [17], studied the system

$$u'(t) + \sum_{i=1}^n a_i \left({}^C D^{\alpha_i}_{0+} u\right)(t) = \int\limits_0^t \frac{(t-\tau)^{-\gamma_1}}{\Gamma(1-\gamma_1)} f(u(\tau), v(\tau))\, d\tau, \quad t > 0,$$

$$v'(t) + \sum_{i=1}^n b_i \left({}^C D^{\beta_i}_{0+} v\right)(t) = \int\limits_0^t \frac{(t-\tau)^{-\gamma_2}}{\Gamma(1-\gamma_2)} g(u(\tau), v(\tau))\, d\tau, \quad t > 0,$$

$$u(0) = u_0 > 0, \quad v(0) = v_0 > 0,$$

where $0 < \alpha_i < 1$, $0 < \beta_i < 1$, $i = 1, \ldots, n$, and $0 < \gamma_j < 1$, $j = 1, 2$.

System (1) is a generalization of several systems of ordinary and fractional equations. In particular, it generalizes the systems that have been discussed in [2, 17].

When $f(u, v) = |v|^{q_1}$ and $g(u, v) = |u|^{q_2}$, $q_1 > 1$, $q_2 > 1$ and $i = 1$, we have the problem considered in [2] (see Theorems 3.3 and 3.4).

When $k(t) = \dfrac{1}{\Gamma(1-\gamma_1)} t^{-\gamma_1}$ and $h(t) = \dfrac{1}{\Gamma(1-\gamma_2)} t^{-\gamma_2}$, $t > 0$, $0 < \gamma_j < 1$, $j = 1, 2$ and $\alpha = \beta = 1$, the result of [17] (Theorem 4.1) follows as a special case, see Section 3, Corollary 5.

This paper is organized as follows. In Section 2, we recall briefly some preliminary definitions, notions, and properties from fractional calculus that we use in this paper. In Section 3, we state and prove our results and give some examples and special cases.

2. Preliminaries. For the convenience of the reader, we recall the fractional integrals and derivatives that are used in this paper. Some results concerning their properties will be mentioned.

The left-hand sided and right-hand sided Riemann–Liouville fractional integrals of $\alpha > 0,$ are defined by

$$(I^{\alpha}_{0+}y)(t) = \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-\tau)^{\alpha-1} y(\tau)\, d\tau, \quad t > 0,$$

$$(I^{\alpha}_{T-}y)(t) = \frac{1}{\Gamma(\alpha)} \int\limits_t^T (\tau - t)^{\alpha-1} y(\tau)\, d\tau, \quad t < T,$$

respectively, for all $y(t) \in L^q(0,T),\ T > 0,\ 1 \le q \le \infty,$ where $\Gamma(\alpha)$ is the Euler Gamma function. We define $I^0_{0+}y = I^0_{T-}y = y.$

The left-hand sided and right-hand sided Riemann–Liouville fractional derivatives of order $\alpha,$ $0 < \alpha < 1,$ are defined by

$$(D^{\alpha}_{0+}y)(t) = \frac{d}{dt}\big(I^{1-\alpha}_{0+}y\big)(t), \quad t > 0,$$

$$(D^{\alpha}_{T-}y)(t) = -\frac{d}{dt}\big(I^{1-\alpha}_{T-}y\big)(t), \quad t < T,$$

respectively, for all $y \in C^1[0,T].$ In particular, $D^1_{0+}y = \dfrac{dy}{dt},\ D^1_{T-}y = -\dfrac{dy}{dt}.$

It is shown in [12] that, for $\alpha \ge 0,\ \beta > 0,$

$$\Big(I^{\alpha}_{T-}(T-\tau)^{\beta-1}\Big)(t) = \frac{\Gamma(\beta)}{\Gamma(\beta+\alpha)}(T-t)^{\beta+\alpha-1},$$

$$\Big(D^{\alpha}_{T-}(T-\tau)^{\beta-1}\Big)(t) = \frac{\Gamma(\beta)}{\Gamma(\beta-\alpha)}(T-t)^{\beta-\alpha-1}.$$

The fractional integration by parts formula

$$\int\limits_0^T y_1(t)(I^{\alpha}_{0+}y_2)(t)\, dt = \int\limits_0^T y_2(t)(I^{\alpha}_{T-}y_1)(t)\, dt$$

is proved in [19], for $y_1 \in L^p(0,T)$ and $y_2 \in L^q(0,T),\ p \ge 1,\ q \ge 1$ and $\dfrac{1}{p} + \dfrac{1}{q} \le 1 + \alpha$ $\Big(p \ne 1$ and $q \ne 1$ in the case when $\dfrac{1}{p} + \dfrac{1}{q} = 1 + \alpha\Big),\ \alpha \ge 0.$

The left-hand sided and right-hand sided Caputo fractional derivatives of order $\alpha,\ 0 < \alpha < 1,$ are defined by

$$\big({}^C D^{\alpha}_{0+}y\big)(t) = (D^{\alpha}_{0+}(y(\tau) - y(0)))(t),$$

$$\big({}^C D^{\alpha}_{T-}y\big)(t) = (D^{\alpha}_{T-}(y(\tau) - y(T)))(t),$$

respectively, for all $y \in C^1[0,T]$. In particular, ${}^C D^1_{0+} y = \dfrac{dy}{dt} = y'$, ${}^C D^1_{T-} y = -\dfrac{dy}{dt} = -y'$.

Note that if $y(0) = 0$, then ${}^C D^\alpha_{0+} y = D^\alpha_{0+} y$, and if $y(T) = 0$, then ${}^C D^\alpha_{T-} y = D^\alpha_{T-} y$. For more details on these fractional integrals and derivatives and other fractional operators, the reader is advised to see the books [12, 19].

If $y \in AC[0,T]$, that is, y is absolutely continuous function, then ${}^C D^\alpha_{0+} y$ and ${}^C D^\alpha_{T-} y$ exist almost everywhere on $[0,T]$ and are given by

$$\big({}^C D^\alpha_{0+} y\big)(t) = \big(I^{1-\alpha}_{0+} y'\big)(t),$$
$$\big({}^C D^\alpha_{T-} y\big)(t) = -\big(I^{1-\alpha}_{T-} y'\big)(t).$$

To prove our main results in the next section, the following test function is considered:

$$\varkappa(t) := \begin{cases} T^{-\sigma}(T-t)^\sigma, & 0 \le t \le T, \quad \sigma >> 1, \\ 0, & t > T, \end{cases} \tag{4}$$

that has been used in [8]. For $\sigma > np - 1$, $n = 0, 1, 2, \ldots$ and $p > 1$, it is shown in [2] that

$$\int\limits_0^T \varkappa^{1-p}(t) |D^n \varkappa(t)|^p \, dt = C_{n,p} T^{1-np}, \quad T > 0,$$

where $C_{n,p} = \dfrac{\Gamma^p(\sigma+1)}{(\sigma - np + 1)\Gamma^p(\sigma - n + 1)}$.

3. Main results. To obtain main results in this paper, we need to start with the following lemmas.

Lemma 1 [2]. *Let $0 < \alpha \le 1$ and $\varkappa$ be as in* (4). *Suppose that $\omega \in AC[0,T]$. Then*

$$\int\limits_0^T \varkappa(t)\big({}^C D^\alpha_{0+} \omega\big)(t)\, dt = \int\limits_0^T \omega(t)(D^\alpha_{T-}\varkappa)(t)\, dt - G_\alpha T^{1-\alpha}\omega(0),$$

where $G_\alpha = \dfrac{\Gamma(\sigma+1)}{\Gamma(\sigma + 2 - \alpha)}$.

Lemma 2 [2]. *Let $0 < \gamma \le 1$, $p > 1$ and $\varkappa$ be as in* (4) *with $\sigma > p - 1$. Suppose that h is a function that is different from zero almost everywhere, nonnegative and $t^{-p\gamma} h^{1-p}(t) \in L^1_{\mathrm{loc}}[0, +\infty)$. Then, for any $T > 0$,*

$$\int\limits_0^T \big(D^\gamma_{T-}\varkappa\big)^p(t) \left(\int\limits_t^T h(\tau - t)\varkappa(\tau)\, d\tau\right)^{1-p} dt \le \Omega_{\gamma,p} T^{1-p} \int\limits_0^T t^{-p\gamma} h^{1-p}(t)\, dt,$$

where $\Omega_{\gamma,p} = \dfrac{\sigma^p}{(\sigma - p + 1)\Gamma^p(2-\gamma)}$.

In the sequel, we assume that the following hypotheses on the functions f and g, the kernels k and h, the parameters α, β, α_i, β_i, $i = 1, 2, \ldots, n$:

(H$_1$) The functions f and g satisfy the following growth condition:

$$f(x,y) \ge b|y|^{q_1} \quad \text{for all} \quad x, y \in \mathbb{R},$$

$$g(x,y) \geq a|x|^{q_2} \quad \text{for all} \quad x,y \in \mathbb{R}$$

for some constants a and b, where $q_1 > 1$ and $q_2 > 1$.

(H$_2$) The functions k and h are nonnegative functions different from zero almost everywhere with $t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)$, $t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)$, $t^{\frac{-\beta q_1}{q_1-1}} h^{\frac{-1}{q_1-1}}(t)$ and $t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t) \in L^1_{\text{loc}}[0,+\infty)$, $i=1,2,\ldots,n$, such that

$$\lim_{T\to\infty} T^{\frac{-q_1-1}{q_1q_2-1}} \left(\left(\int_0^T t^{\frac{-\beta q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)dt \right)^{\frac{q_1-1}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)dt \right)^{\frac{q_1-1}{q_1q_2-1}} \right)$$

$$\times \left(\left(\int_0^T t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt \right)^{\frac{q_1(q_2-1)}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt \right)^{\frac{q_1(q_2-1)}{q_1q_2-1}} \right) = 0$$

or

$$\lim_{T\to\infty} T^{\frac{-q_2-1}{q_1q_2-1}} \left(\left(t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt \right)^{\frac{q_2-1}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt \right)^{\frac{q_2-1}{q_1q_2-1}} \right)$$

$$\times \left(\left(\int_0^T t^{\frac{-\beta q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\,dt \right)^{\frac{q_2(q_1-1)}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\,dt \right)^{\frac{q_2(q_1-1)}{q_1q_2-1}} \right) = 0.$$

An example about functions k and h satisfying the above hypothesis is given at the end of this section in Corollary 1.

Theorem 1. *Assume that the functions f and g and kernels k and h satisfy* **(H$_1$)** *and* **(H$_2$)**. *Then problem* (1) *does not admit any nontrivial global solution when $u_0 \geq 0$ and $v_0 \geq 0$.*

Proof. We prove by contradiction. We assume that there exists a nontrivial solution (u,v) for all $T>0$. Multiply both sides of each equation in (1) by $\varkappa$ and integrating from 0 to T, we obtain

$$\int_0^T \varkappa(t)\big({}^C D^{\alpha}_{0+}u\big)(t)\,dt + \int_0^T \varkappa(t)\sum_{i=1}^n a_i\big({}^C D^{\alpha_i}_{0+}u\big)(t)\,dt = J_1,$$

$$\int_0^T \varkappa(t)\Big({}^C D^{\beta}_{0+}v\Big)(t)\,dt + \int_0^T \varkappa(t)\sum_{i=1}^n b_i\Big({}^C D^{\beta_i}_{0+}v\Big)(t)\,dt = J_2,$$

where

$$J_1 := \int_0^T \varkappa(t)\left(\int_0^t k(t-\tau)f(u(\tau),v(\tau))\,d\tau\right)dt,$$

$$J_2 := \int_0^T \varkappa(t) \left(\int_0^t h(t-\tau) g(u(\tau), v(\tau))\, d\tau \right) dt.$$

These integrals can be rewritten in the following forms:

$$J_1 = \int_0^T f(u(\tau), v(\tau)) \left(\int_\tau^T k(t-\tau)\varkappa(t)\, dt \right) d\tau = \int_0^T f(u(\tau), v(\tau)) K(\tau)\, d\tau$$

and

$$J_2 = \int_0^T g(u(\tau), v(\tau)) \left(\int_\tau^T h(t-\tau)\varkappa(t)\, dt \right) d\tau = \int_0^T g(u(\tau), v(\tau)) H(\tau)\, d\tau,$$

where

$$K(\tau) := \int_\tau^T k(t-\tau)\varkappa(t)\, dt,$$

$$H(\tau) := \int_\tau^T h(t-\tau)\varkappa(t)\, dt, \quad 0 \le \tau < t \le T.$$

It follows from Lemma 1 that

$$J_1 + u_0 \left(G_\alpha T^{1-\alpha} + \sum_{i=1}^n G_{\alpha_i} T^{1-\alpha_i} \right) = \int_0^T u(t) (D^\alpha_{T-}\varkappa)(t)\, dt + \int_0^T u(t) \sum_{i=1}^n a_i \big(D^{\alpha_i}_{T-}\varkappa\big)(t)\, dt \tag{5}$$

and

$$J_2 + v_0 \left(G_\beta T^{1-\beta} + \sum_{i=1}^n G_{\beta_i} T^{1-\beta_i} \right) = \int_0^T v(t) \Big(D^\beta_{T-}\varkappa\Big)(t)\, dt + \int_0^T v(t) \sum_{i=1}^n b_i \Big(D^{\beta_i}_{T-}\varkappa\Big)(t)\, dt. \tag{6}$$

The integrals on the left-hand sides of (5) and (6) can be expressed as

$$\int_0^T u(t) (D^\alpha_{T-}\varkappa)(t)\, dt + \int_0^T u(t) \sum_{i=1}^n a_i \big(D^{\alpha_i}_{T-}\varkappa\big)(t)\, dt$$

$$\le \left(\int_0^T |u(t)|^{q_2} H(t)\, dt \right)^{\frac{1}{q_2}} \left(\int_0^T H^{-\frac{q_2'}{q_2}}(t) (D^\alpha_{T-}\varkappa)^{q_2'}(t)\, dt \right)^{\frac{1}{q_2'}}$$

$$+ \left(\int_0^T |u(t)|^{q_2} H(t)\, dt \right)^{\frac{1}{q_2}} \sum_{i=1}^n a_i \left(\int_0^T H^{-\frac{q_2'}{q_2}}(t) \big(D^{\alpha_i}_{T-}\varkappa\big)^{q_2'}(t)\, dt \right)^{\frac{1}{q_2'}} \tag{7}$$

and

$$\int_0^T v(t)\Big(D^{\beta}_{T^-}\varkappa\Big)(t)\,dt+\int_0^T v(t)\sum_{i=1}^n b_i\Big(D^{\beta_i}_{T^-}\varkappa\Big)(t)\,dt$$

$$\leq\left(\int_0^T |v(t)|^{q_1}K(t)\,dt\right)^{\frac{1}{q_1}}\left(\int_0^T K^{-\frac{q_1'}{q_1}}(t)\Big(D^{\beta}_{T^-}\varkappa\Big)^{q_1'}(t)\,dt\right)^{\frac{1}{q_1'}}$$

$$+\left(\int_0^T |v(t)|^{q_1}K(t)\,dt\right)^{\frac{1}{q_1}}\sum_{i=1}^n b_i\left(\int_0^T K^{-\frac{q_1'}{q_1}}(t)\Big(D^{\beta_i}_{T^-}\varkappa\Big)^{q_1'}(t)\,dt\right)^{\frac{1}{q_1'}}, \tag{8}$$

where q_j' are the conjugates of $q_j,\ j=1,2.$

Denote

$$\begin{aligned} A &:= \int_0^T H^{-\frac{q_2'}{q_2}}(t)(D^{\alpha}_{T^-}\varkappa)^{q_2'}(t)\,dt, \quad A_i := \int_0^T H^{-\frac{q_2'}{q_2}}(t)\big(D^{\alpha_i}_{T^-}\varkappa\big)^{q_2'}(t)\,dt,\\ B &:= \int_0^T K^{-\frac{q_1'}{q_1}}(t)\Big(D^{\beta}_{T^-}\varkappa\Big)^{q_1'}(t)\,dt, \quad B_i := \int_0^T K^{-\frac{q_1'}{q_1}}(t)\Big(D^{\beta_i}_{T^-}\varkappa\Big)^{q_1'}(t)\,dt, \end{aligned} \tag{9}$$

then, with the help of (7), (8) and the growth condition in ($\mathbf{H}_1$), the relations (5) and (6) reduce to

$$J_1+u_0\left(G_\alpha T^{1-\alpha}+\sum_{i=1}^n G_{\alpha_i}T^{1-\alpha_i}\right)\leq a^{-\frac{1}{q_2}}J_2^{\frac{1}{q_2}}\left(A^{\frac{1}{q_2'}}+\sum_{i=1}^n a_iA_i^{\frac{1}{q_2'}}\right),$$

$$J_2+v_0\left(G_\beta T^{1-\beta}+\sum_{i=1}^n G_{\beta_i}T^{1-\beta_i}\right)\leq b^{-\frac{1}{q_1}}J_1^{\frac{1}{q_1}}\left(B^{\frac{1}{q_1'}}+\sum_{i=1}^n b_iB_i^{\frac{1}{q_1'}}\right).$$

Since $u_0,\ v_0,\ G_\alpha,\ G_\beta,\ G_{\alpha_i}$ and G_{β_i} are nonnegative, we deduce that

$$J_1\leq a^{-\frac{1}{q_2}}J_2^{\frac{1}{q_2}}\left(A^{\frac{1}{q_2'}}+\sum_{i=1}^n a_iA_i^{\frac{1}{q_2'}}\right)\quad\text{and}\quad J_2\leq b^{-\frac{1}{q_1}}J_1^{\frac{1}{q_1}}\left(B^{\frac{1}{q_1'}}+\sum_{i=1}^n b_iB_i^{\frac{1}{q_1'}}\right).$$

Consequently,

$$J_1\leq a^{-\frac{1}{q_2}}b^{-\frac{1}{q_1q_2}}J_1^{\frac{1}{q_1q_2}}\left(B^{\frac{1}{q_1'}}+\sum_{i=1}^n b_iB_i^{\frac{1}{q_1'}}\right)^{\frac{1}{q_2}}\left(A^{\frac{1}{q_2'}}+\sum_{i=1}^n a_iA_i^{\frac{1}{q_2'}}\right),$$

$$J_2\leq b^{-\frac{1}{q_1}}a^{-\frac{1}{q_1q_2}}J_2^{\frac{1}{q_1q_2}}\left(A^{\frac{1}{q_2'}}+\sum_{i=1}^n a_iA_i^{\frac{1}{q_2'}}\right)^{\frac{1}{q_1}}\left(B^{\frac{1}{q_1'}}+\sum_{i=1}^n b_iB_i^{\frac{1}{q_1'}}\right).$$

Furthermore,

$$J_1^{\frac{q_1q_2-1}{q_1q_2}} \le a^{\frac{-1}{q_2}} b^{\frac{-1}{q_1q_2}} \left(B^{\frac{1}{q_1'}} + \sum_{i=1}^{n} b_i B_i^{\frac{1}{q_1'}} \right)^{\frac{1}{q_2}} \left(A^{\frac{1}{q_2'}} + \sum_{i=1}^{n} a_i A_i^{\frac{1}{q_2'}} \right)$$

$$\le a^{\frac{-1}{q_2}} b^{\frac{-1}{q_1q_2}} \left(B^{\frac{1}{q_1'q_2}} + \sum_{i=1}^{n} b_i^{\frac{1}{q_2}} B_i^{\frac{1}{q_1'q_2}} \right) \left(A^{\frac{1}{q_2'}} + \sum_{i=1}^{n} a_i A_i^{\frac{1}{q_2'}} \right),$$

$$J_2^{\frac{q_1q_2-1}{q_1q_2}} \le a^{\frac{-1}{q_1q_2}} b^{\frac{-1}{q_1}} \left(A^{\frac{1}{q_2'}} + \sum_{i=1}^{n} a_i A_i^{\frac{1}{q_2'}} \right)^{\frac{1}{q_1}} \left(B^{\frac{1}{q_1'}} + \sum_{i=1}^{n} b_i B_i^{\frac{1}{q_1'}} \right)$$

$$\le a^{\frac{-1}{q_1q_2}} b^{\frac{-1}{q_1}} \left(A^{\frac{1}{q_1q_2'}} + \sum_{i=1}^{n} a_i^{\frac{1}{q_1}} A_i^{\frac{1}{q_1q_2'}} \right) \left(B^{\frac{1}{q_1'}} + \sum_{i=1}^{n} b_i B_i^{\frac{1}{q_1'}} \right).$$

Using basic power inequalities we end up with

$$J_1 \le a^{\frac{-q_1}{q_1q_2-1}} b^{\frac{-1}{q_1q_2-1}} \left(B^{\frac{1}{q_1'q_2}} + \sum_{i=1}^{n} b_i^{\frac{1}{q_2}} B_i^{\frac{1}{q_1'q_2}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \left(A^{\frac{1}{q_2'}} + \sum_{i=1}^{n} a_i A_i^{\frac{1}{q_2'}} \right)^{\frac{q_1q_2}{q_1q_2-1}}$$

$$\le (a^{q_1} b)^{\frac{-1}{q_1q_2-1}} 4^{\frac{1}{q_1q_2-1}} \left(B^{\frac{q_1}{q_1'(q_1q_2-1)}} + \left(\sum_{i=1}^{n} b_i^{\frac{1}{q_2}} B_i^{\frac{1}{q_1'q_2}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \right)$$

$$\times \left(A^{\frac{q_1q_2}{q_2'(q_1q_2-1)}} + \left(\sum_{i=1}^{n} a_i A_i^{\frac{1}{q_2'}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \right)$$

$$\le \left(\frac{4}{a^{q_1} b} \right)^{\frac{1}{q_1q_2-1}} \left(B^{\frac{q_1-1}{q_1q_2-1}} + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} b_i^{\frac{q_1}{q_1q_2-1}} B_i^{\frac{q_1-1}{q_1q_2-1}} \right)$$

$$\times \left(A^{\frac{q_1(q_2-1)}{q_1q_2-1}} + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} a_i^{\frac{q_1q_2}{q_1q_2-1}} A_i^{\frac{q_1(q_2-1)}{q_1q_2-1}} \right). \tag{10}$$

Analogously, we have

$$J_2 \le a^{\frac{-1}{q_1q_2-1}} b^{\frac{-q_2}{q_1q_2-1}} \left(A^{\frac{1}{q_1q_2'}} + \sum_{i=1}^{n} a_i^{\frac{1}{q_1}} A_i^{\frac{1}{q_1q_2'}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \left(B^{\frac{1}{q_1'}} + \sum_{i=1}^{n} b_i B_i^{\frac{1}{q_1'}} \right)^{\frac{q_1q_2}{q_1q_2-1}}$$

$$\le (ab^{q_2})^{\frac{-1}{q_1q_2-1}} 4^{\frac{1}{q_1q_2-1}} \left(A^{\frac{1}{q_1q_2'}} + \left(\sum_{i=1}^{n} a_i^{\frac{1}{q_1}} A_i^{\frac{1}{q_1q_2'}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \right)$$

$$\times \left(B^{\frac{q_1q_2}{q_1'(q_1q_2-1)}} + \left(\sum_{i=1}^{n} b_i B_i^{\frac{1}{q_1'}} \right)^{\frac{q_1q_2}{q_1q_2-1}} \right)$$

$$\le \left(\frac{4}{ab^{q_2}} \right)^{\frac{1}{q_1q_2-1}} \left(A^{\frac{q_2-1}{q_1q_2-1}} + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} a_i^{\frac{q_2}{q_1q_2-1}} A_i^{\frac{q_2-1}{q_1q_2-1}} \right)$$

$$\times\left(B^{\frac{q_2(q_1-1)}{q_1q_2-1}} + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} b_i^{\frac{q_1q_2}{q_1q_2-1}} B_i^{\frac{q_2(q_1-1)}{q_1q_2-1}}\right). \tag{11}$$

Now, the integrals $A,\ B,\ A_i$ and B_i defined in (9) can be estimated using Lemma 2 as follows:

$$\begin{aligned} A &= \int_0^T \left(\int_t^T h(\tau-t)\varkappa(\tau)\,d\tau\right)^{1-q_2'} (D^{\alpha}_{T^-}\varkappa(t))^{q_2'}\,dt \le \Omega_{\alpha,q_2'} T^{1-q_2'} \int_0^T t^{-\alpha q_2'} h^{1-q_2'}(t)\,dt, \\ B &= \int_0^T \left(\int_t^T k(\tau-t)\varkappa(\tau)\,d\tau\right)^{1-q_1'} \left(D^{\beta}_{T^-}\varkappa\right)^{q_1'}(t)\,dt \le \Omega_{\beta,q_1'} T^{1-q_1'} \int_0^T t^{-\beta q_1'} k^{1-q_1'}(t)\,dt, \end{aligned} \tag{12}$$

$$\begin{aligned} A_i &\le \Omega_{\alpha_i,q_2'} T^{1-q_2'} \int_0^T t^{-\alpha_i q_2'} h^{1-q_2'}(t)\,dt, \\ B_i &\le \Omega_{\beta_i,q_1'} T^{1-q_1'} \int_0^T t^{-\beta_i q_1'} k^{1-q_1'}(t)\,dt. \end{aligned} \tag{13}$$

Substituting (12) and (13) in (10) and (11), we obtain

$$\begin{aligned} J_1 \le \left(\frac{4}{a^{q_1}b}\right)^{\frac{1}{q_1q_2-1}} &\left(\left(\Omega_{\beta,q_1'} T^{1-q_1'} \int_0^T t^{-\beta q_1'} k^{1-q_1'}(t)\,dt\right)^{\frac{q_1-1}{q_1q_2-1}}\right. \\ &\left. + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} b_i^{\frac{q_1}{q_1q_2-1}} \left(\Omega_{\beta_i,q_1'} T^{1-q_1'} \int_0^T t^{-\beta_i q_1'} k^{1-q_1'}(t)\,dt\right)^{\frac{q_1-1}{q_1q_2-1}}\right) \\ &\times\left(\left(\Omega_{\alpha,q_2'} T^{1-q_2'} \int_0^T t^{-\alpha q_2'} h^{1-q_2'}(t)\,dt\right)^{\frac{q_1(q_2-1)}{q_1q_2-1}}\right. \\ &\left. + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} a_i^{\frac{q_1q_2}{q_1q_2-1}} \left(\Omega_{\alpha_i,q_2'} T^{1-q_2'} \int_0^T t^{-\alpha_i q_2'} h^{1-q_2'}(t)\,dt\right)^{\frac{q_1(q_2-1)}{q_1q_2-1}}\right) \end{aligned}$$

and

$$J_2 \le \left(\frac{4}{ab^{q_2}}\right)^{\frac{1}{q_1q_2-1}} \left(\left(\Omega_{\alpha,q_2'} T^{1-q_2'} \int_0^T t^{-\alpha q_2'} h^{1-q_2'}(t)\,dt\right)^{\frac{q_2-1}{q_1q_2-1}}\right.$$

$$+ n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} a_i^{\frac{q_2}{q_1q_2-1}} \left(\Omega_{\alpha_i,q_2'} T^{1-q_2'} \int\limits_0^T t^{-\alpha_i q_2'} h^{1-q_2'}(t)\, dt \right)^{\frac{q_2-1}{q_1q_2-1}} \Bigg)$$

$$\times \left(\left(\Omega_{\beta,q_1'} T^{1-q_1'} \int\limits_0^T t^{-\beta q_1'} k^{1-q_1'}(t)\, dt \right)^{\frac{q_2(q_1-1)}{q_1q_2-1}} \right.$$

$$\left. + n^{\frac{1}{q_1q_2-1}} \sum_{i=1}^{n} b_i^{\frac{q_1q_2}{q_1q_2-1}} \left(\Omega_{\beta_i,q_1'} T^{1-q_1'} \int\limits_0^T t^{-\beta_i q_1'} k^{1-q_1'}(t)\, dt \right)^{\frac{q_2(q_1-1)}{q_1q_2-1}} \right).$$

This leads to a contradiction, in the light of the condition ($\mathbf{H}_2$), as the solution is assumed to be nontrivial.

Theorem 1 is proved.

The next results can be considered as special cases of Theorem 1.

Theorem 2. *Let k and h be nonnegative functions which are different from zero almost everywhere. For any $T > 0$, suppose that there are some positive constants c, c_i, e, e_i, λ, η, η_i and λ_i, $i = 1, 2, \ldots, n$, with*

$$\lambda(q_1-1) + \eta q_1(q_2-1) < q_1+1, \quad \lambda(q_1-1) + \eta_i q_1(q_2-1) < q_1+1,$$
$$\lambda_i(q_1-1) + \eta q_1(q_2-1) < q_1+1, \quad \lambda_i(q_1-1) + \eta_i q_1(q_2-1) < q_1+1, \tag{14}$$

or

$$\eta(q_2-1) + \lambda q_2(q_1-1) < q_2+1, \quad \eta(q_2-1) + \lambda_i q_2(q_1-1) < q_2+1,$$
$$\eta_i(q_2-1) + \lambda q_2(q_1-1) < q_2+1, \quad \eta_i(q_2-1) + \lambda_i q_2(q_1-1) < q_2+1,$$

such that

$$\int\limits_0^T t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\, dt \le cT^{\eta}, \quad \int\limits_0^T t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\, dt \le c_i T^{\eta_i},$$
$$\int\limits_0^T t^{\frac{-\beta q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\, dt \le eT^{\lambda}, \quad \int\limits_0^T t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\, dt \le e_i T^{\lambda_i}. \tag{15}$$

Then problem (1) *does not admit any nontrivial global solution when $u_0 \ge 0$ and $v_0 \ge 0$.*

Proof. It is clear that (15) implies that

$$0 \le T^{\frac{-q_1-1}{q_1q_2-1}} \left(\left(\int\limits_0^T t^{\frac{-\beta q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\, dt \right)^{\frac{q_1-1}{q_1q_2-1}} + \sum_{i=1}^{n} \left(\int\limits_0^T t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\, dt \right)^{\frac{q_1-1}{q_1q_2-1}} \right)$$

$$\times\left(\left(\int_0^T t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt\right)^{\frac{q_1(q_2-1)}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt\right)^{\frac{q_1(q_2-1)}{q_1q_2-1}}\right)$$

$$\le M_1 T^{\frac{-q_1-1}{q_1q_2-1}}\left(T^{\frac{\lambda(q_1-1)}{q_1q_2-1}} + \sum_{i=1}^n T^{\frac{\lambda_i(q_1-1)}{q_1q_2-1}}\right)\left(T^{\frac{\eta q_1(q_2-1)}{q_1q_2-1}} + \sum_{i=1}^n T^{\frac{\eta_i q_1(q_2-1)}{q_1q_2-1}}\right)$$

$$\le M_1\left(T^{\frac{\lambda(q_1-1)+\eta q_1(q_2-1)-q_1-1}{q_1q_2-1}} + \sum_{i=1}^n T^{\frac{\lambda(q_1-1)+\lambda_i(q_1-1)-q_1-1}{q_1q_2-1}}\right.$$

$$\left. + \sum_{i=1}^n T^{\frac{\eta q_1(q_2-1)+\lambda_i(q_1-1)-q_1-1}{q_1q_2-1}} + \sum_{i=1}^n\sum_{j=1}^n T^{\frac{\lambda_i(q_1-1)+\eta_j q_1(q_2-1)-q_1-1}{q_1q_2-1}}\right)$$

and

$$0 \le T^{\frac{-q_2-1}{q_1q_2-1}}\left(\left(t^{\frac{-\alpha q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt\right)^{\frac{q_2-1}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\alpha_i q_2}{q_2-1}} h^{\frac{-1}{q_2-1}}(t)\,dt\right)^{\frac{q_2-1}{q_1q_2-1}}\right)$$

$$\times\left(\left(\int_0^T t^{\frac{-\beta q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\,dt\right)^{\frac{q_2(q_1-1)}{q_1q_2-1}} + \sum_{i=1}^n \left(\int_0^T t^{\frac{-\beta_i q_1}{q_1-1}} k^{\frac{-1}{q_1-1}}(t)\,dt\right)^{\frac{q_2(q_1-1)}{q_1q_2-1}}\right)$$

$$\le M_2\left(T^{\frac{\eta(q_2-1)+\lambda q_2(q_1-1)-q_2-1}{q_1q_2-1}} + \sum_{i=1}^n T^{\frac{\eta(q_2-1)+\lambda_i q_2(q_1-1)-q_2-1}{q_1q_2-1}}\right.$$

$$\left. + \sum_{i=1}^n T^{\frac{\lambda q_2(q_1-1)+\eta_i(q_2-1)-q_2-1}{q_1q_2-1}} + \sum_{i=1}^n\sum_{j=1}^n T^{\frac{\eta_i(q_2-1)+\lambda_j q_2(q_1-1)-q_2-1}{q_1q_2-1}}\right),$$

where

$$M_1 = \max\left\{c^{\frac{q_1(q_2-1)}{q_1q_2-1}}, c_i^{\frac{q_1(q_2-1)}{q_1q_2-1}}, e^{\frac{q_1-1}{q_1q_2-1}}, e_i^{\frac{q_1-1}{q_1q_2-1}}\right\},$$

$$M_2 = \max\left\{c^{\frac{q_2-1}{q_1q_2-1}}, c_i^{\frac{q_2-1}{q_1q_2-1}}, e^{\frac{q_2(q_1-1)}{q_1q_2-1}}, e_i^{\frac{q_2(q_1-1)}{q_1q_2-1}}\right\}, \quad i = 1, 2, \ldots, n.$$

Therefore, the hypothesis ($\mathbf{H}_2$) of Theorem 1, follows from (14).

Theorem 2 is proved.

Similarly, the following special case of Theorem 1 can be easily proved by just verifying the condition ($\mathbf{H}_2$). Also, it can be considered as a special case of Theorem 2.

***Corollary* 1.** *Let* $0 \le \alpha_j < \alpha < 1$, $0 \le \beta_i < \beta < 1$, $\gamma_1 > 1 - q_1(1-\beta)$, $\gamma_2 > 1 - q_2(1-\alpha)$ *and* $q_1, q_2 > 1$. *Suppose that* $k(t) \ge r_1 t^{-\gamma_1}$, $h(t) \ge r_2 t^{-\gamma_2}$, $t > 0$, *for some constants* $r_1, r_2 > 0$, *and*

$$\gamma_1 + q_1\gamma_2 < q_1(\beta_i + (\alpha_j - 1)q_2) + q_1 + 2$$

or

$$\gamma_2 + q_2\gamma_1 < q_2(\alpha_j + (\beta_i - 1)q_1) + q_2 + 2$$

for each $i = 1, 2, \ldots, n$ *and* $j = 1, 2, \ldots, n$. *Then problem* (1) *does not admit any nontrivial global solution when* $u_0 \geq 0$ *and* $v_0 \geq 0$.

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References

1. A. Aghajani, Y. Jalilian, J. J. Trujillo, *On the existence of solutions of fractional integro-differential equations*, Fract. Calc. Appl. and Anal., **15**, № 1, 44 – 69 (2012).
2. A. M. Ahmad, N.-E. Tatar, *Nonexistence of global solutions for a fractional system of strongly coupled integro-differential equations*, Turkish J. Math., **43**, 2715 – 2730 (2019).
3. A. M. Ahmad, K. M. Furati , N.-E. Tatar, *On the nonexistence of global solutions for a class of fractional integro-differential problems*, Adv. Different. Equat., № 1 (2017).
4. A. Alsaedi, B. Ahmad, M. Kirane, F. Al Musalhi, F. Alzahrani, *Blowing-up solutions for a nonlinear time-fractional system*, Bull. Math. Sci., **7**, № 2, 201 – 210 (2017).
5. Z. Bai, Y. Chen, H. Lian, S. Sun, *On the existence of blow up solutions for a class of fractional differential equations*, Fract. Calc. and Appl. Anal., **17**, № 4, 1175 – 1187 (2014).
6. J. Deng, L. Ma, *Existence and uniqueness of solutions of initial value problems for nonlinear fractional differential equations*, Appl. Math. Lett., **23**, Issue 6, 676 – 680 (2010).
7. J. Henderson, R. Luca, *Nonexistence of positive solutions for a system of coupled fractional boundary value problems*, Bound. Value Probl., **2015**, № 1 (2015).
8. K. Furati, M. Kirane, *Necessary conditions for the existence of global solutions to systems of fractional differential equations*, Fract. Calc. and Appl. Anal., **11**, 281 – 298 (2008).
9. J. Jiang, L. Liu, *Existence of solutions for a sequential fractional differential system with coupled boundary conditions*, Bound. Value Probl., **2016**, № 1 (2016).
10. M. Jleli, B. Samet, *Nonexistence results for some classes of nonlinear fractional differential inequalities*, J. Funct. Spaces, **2020**, Article ID 8814785 (2020).
11. A. Kadem, M. Kirane, C. M. Kirk, W. E. Olmstead, *Blowing-up solutions to systems of fractional differential and integral equations with exponential non-linearities*, IMA J. Appl. Math., **79**, № 6, 1077 – 1088 (2014).
12. A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and applications of fractional differential equations*, Elsevier B. V., Amsterdam, Netherlands (2006).
13. M. Kirane, B. Ahmad, A. Alsaedi, M. Al-Yami, *Non-existence of global solutions to a system of fractional diffusion equations*, Acta Appl. Math., **133**, № 1, 235 – 248 (2014).
14. M. Kirane, M. Medved, N. E. Tatar, *On the nonexistence of blowing-up solutions to a fractional functional differential equations*, Georgian Math. J., **19**, 127 – 144 (2012).
15. M. Kirane, S. A. Malik, *The profile of blowing-up solutions to a nonlinear system of fractional differential equations*, Nonlinear Anal., **73**, № 12, 3723 – 3736 (2010).
16. Y. Laskri, N.-E. Tatar, *The critical exponent for an ordinary fractional differential problem*, Comput. Math. Appl., **59**, № 3, 1266 – 1270 (2010).
17. A. Mennouni, A. Youkana, *Finite time blow-up of solutions for a nonlinear system of fractional differential equations*, Electron. J. Different. Equat., **152**, 1 – 15 (2017).
18. E. Mitidieri, S. I. Pohozaev, *A priori estimates and blow-up of solutions to nonlinear partial differential equations and inequalities*, Proc. Steklov Inst. Math., **234**, 1 – 383 (2001).
19. S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional integrals and derivatives: theory and applications*, Gordon and Breach (1987).

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