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LOCAL DISTANCE ANTIMAGIC CHROMATIC NUMBER FOR THE UNION OF STAR AND DOUBLE STAR GRAPHS

АНТИМАГІЧНЕ ХРОМАТИЧНЕ ЧИСЛО ЛОКАЛЬНОЇ ВІДСТАНІ ДЛЯ ОБ'ЄДНАННЯ ЗІРКОВИХ І ПОДВІЙНИХ ЗІРКОВИХ ГРАФІВ

Let $G = (V, E)$ be a graph on p vertices with no isolated vertices. A bijection f from V to $\{1, 2, 3, \ldots, p\}$ is called a local distance antimagic labeling if, for any two adjacent vertices u and v, we receive distinct weights (colors), where a vertex x has the weight $w(x) = \sum_{v \epsilon N(x)} f(v)$. The local distance antimagic chromatic number $\chi_{lda}(G)$ is defined as the least number of colors used in any local distance antimagic labeling of G. We determine the local distance antimagic chromatic number for the disjoint union of t copies of stars and double stars.

Нехай $G = (V, E)$ — граф на p вершинах без ізольованих вершин. Бієкція f з V на $\{1, 2, 3, \ldots, p\}$ називається локальним дистанційним антимагічним маркуванням, якщо для будь-яких двох суміжних вершин u і v отримано різні ваги (кольори), де вершина x має вагу $w(x) = \sum_{v \epsilon N(x)} f(v)$. Антимагічне хроматичне число локальної відстані $\chi_{lda}(G)$ визначається, як найменша кількість кольорів, що використовуються в будь-якому локальному дистанційному антимагічному маркуванні G. Отримано антимагічне хроматичне число локальної відстані для об'єднання t копій зірок та подвійних зірок, що не перетинаються.

1. Introduction. Let $G = (V, E)$ be a simple graph. Then the number of vertices of G is denoted by p. For graph-theoretic terms, we refer to Chartrand and Lesniak [6].

Hartsfield and Ringel [12] introduced antimagic labeling, which is defined as a bijection $f : E \rightarrow \{1, 2, \ldots, |E|\}$, for each vertex $u \in V(G)$, the weight $w(u) = \sum_{e \in E(u)} f(e)$, where $E(u)$ is the set of edges incident to u. If $w(u) \neq w(v)$ for any two distinct vertices u and $v \in V(G)$, then f is called antimagic labeling of G. A graph G is called antimagic if G has antimagic labeling. Hartsfield and Ringel's [12] conjectured that every connected graph with at least three vertices admits antimagic labeling. They also made a weak conjecture that every tree with at least three vertices admits antimagic labeling. These two conjectures were partially shown to be accurate by several authors, but they are still unsolved. For a detailed and interesting review of these conjectures, one can see chapter 6 of [11] and in [4, 7, 19].

Arumugam et al. [3] posed a new definition as a relaxation of the notion of antimagic labeling. They called a bijection $f : E \rightarrow \{1, 2, \ldots, |E|\}$ is *local antimagic labeling* (LOCAL) of G if for any two adjacent vertices u and v in $V(G)$, the condition $w(u) \neq w(v)$ holds.

Any local antimagic labeling induces a proper vertex coloring of G where the vertex v is assigned the color $w(v)$. Also, they proposed a new graph coloring parameter *local antimagic chromatic number* (LACN) is defined as the minimum number of colors taken over all colorings of G induced by LOCALs of G, denotes $\chi_{la}(G)$.

They conjectured that any connected graph with at least three vertices admits LOCAL. This was independently conjectured in "On combination of the 1-2-3 conjecture and the antimagic labeling

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conjecture”, DMTCS 2017, by Bensmail et al. [5] and that the conjecture was solves in “Proof of a local antimagic conjecture” DMTCS 2018 by Haslegrave [15].

Several authors have recently studied and investigated the LACN for several family graphs. For further study, see in [11, 23, 24].

In 2012, Arumugam and Kamatchi [16] introduced distance antimagic ((a, d)-distance antimagic) labeling, and they obtained some basic results. For further study see in [1, 2, 8, 9, 11, 13, 14, 16, 19, 21, 22, 25].

The notion of local antimagic labeling is motivated to introduce new labeling as local distance antimagic labeling and a local parameter distance antimagic chromatic number studied by Priyadharshini and Nalliah [20]. This was independently studied by Divya and Devi [10].

Let G be a graph of order p and size q having no isolated vertices. A bijection $f: V \to \{1, 2, 3, \ldots, p\}$ is called *local distance antimagic labeling* (LDAL), if, for any two adjacent vertices u and v, we have $w(u) \neq w(v)$, where $w(u) = \sum_{x\in N(u)} f(x)$. A graph G is called local distance antimagic if G has LDAL.

The local distance antimagic chromatic number (LDACN) $\chi_{lda}(G)$ is defined to be the minimum number of colors taken over all colorings of G induced by local distance antimagic labelings of G and denotes $\chi_{lda}(G)$.

They obtained LDACN for some classes of graphs. Recently, in [20], the authors obtained some class of graphs with LDACN 2 and also they proved that let T be a tree on $n \geq 2$ vertices with k leaves and let $\mathcal{L} = \{N(l), \text{where } l \text{ is a leaf}\}$ be the set of vertices of T. Let $|\mathcal{L}| = t$. Then $\chi_{lda}(T) \geq t+1$. This result only holds for $n \geq 3$, since for the 2-vertex tree we have $k = \chi_{lda}(T) = 2$. Thus, this result is restated as follows.

Theorem 1.1 [20]. *Let T be a tree on $n \geq 3$ vertices with k leaves and let $\mathcal{L} = \{N(l)$, where l is a leaf$\}$. Let $|\mathcal{L}| = t$. Then $\chi_{lda}(T) \geq t+1$.*

In 2021, Bacă et al. [18] first investigated the LACN for disconnected graphs. This motivates us to study the LDACN for disconnected graphs is an interesting problem.

In this paper, we obtain the LDACN of the disjoint union of t-copies of star and double stars.

2. The LDACN of the disjoint union of t-copies of stars. Consider the disconnected graph $tK_{1,r}$ is the disjoint union of t copies of stars.

Theorem 2.1. *Let $tK_{1,1}, t \geq 1$, be the disjoint union of t copies of stars. Then $\chi_{lda}(tK_{1,1}) = 2t$.*

Proof. Let $tK_{1,1} = tK_2$ be the disjoint union of t copies of stars. Let $V(tK_{1,1}) = \{c_j, v_j, 1 \leq j \leq t\}$. Suppose that f is LDAL of $tK_{1,1}$. Then every component $tK_{1,1}$ central vertex and its leaf has received distinct weights. Hence $\chi_{lda}(tK_{1,1}) \geq 2t$. Define a labeling f by $f(c_j) = j$ and $f(v_j) = t + j$. Then the vertex weights are $w(c_j) = t + j$ and $w(v_j) = j, 1 \leq j \leq t$. Hence $\chi_{lda}(tK_{1,1}) \geq 2t$. Thus, $\chi_{lda}(tK_{1,1}) = 2t$.

Theorem 2.2. *Let $tK_{1,2}$ be the disjoint union of t copies of stars. Then*

$$\chi_{lda}(tK_{1,2}) = \begin{cases} t+1, & t = 1, 2, \\ t, & t \geq 3. \end{cases}$$

Proof. Let $V(tK_{1,2}) = \{c_j, u_1^j, u_2^j, 1 \leq j \leq t\}$ and $E(tK_{1,2}) = \{c_j u_1^j, c_j u_2^j, 1 \leq j \leq t\}$. Then $|V(tK_{1,2})| = 3t$. Suppose that f is a LDAL of the graph $tK_{1,2}$.

Case 1: $t = 1, 2$. For $t = 1$, clearly $\chi_{lda}(tK_{1,2}) \geq 2$. Define a labeling $f : V(K_{1,2}) \to \{1, 2, 3\}$ by $f(c_1) = 2,\ f(u_1^1) = 1, f(u_2^1) = 3$. Then the vertex weights are $w(c_1) = 4,\ w(u_1^1) = 2 = w(u_2^1)$. Hence $\chi_{lda}(K_{1,2}) \leq 2$. Thus, $\chi_{lda}(K_{1,2}) = 2$.

For $t = 2$, suppose that $\chi_{lda}(2K_{1,2}) = 2$. Then there exists LDAL f with two colors w_1 and w_2. Since the leaves $u_1^1,\ u_2^1,\ u_1^2,\ u_2^2$ to receive colors w_1 and w_2, it follows, we get $w_1 = f(c_1)$ and $w_2 = f(c_2)$ and hence the vertices c_1 and c_2 must receive the color w_2 and w_1. The minimum and maximum possible weight of the central $c_1(c_2)$ is 3 and 11. Therefore, we obtain $3 \leq w_1,\ w_2 \leq 6$.

Label 6 received possible vertices are either leaf or central vertex. If the leaf u_1^1 with $f(u_1^1) = 6$, then the central vertex c_2 has weight $w(c_1) > 6$, and hence the central vertex c_1 must receive a new color w_3. If $f(c_1) = 6$, then the vertex c_2 must receive a weight 6, $w(c_2) = 6 = w_1$ and hence we get $(f(v_1^2), f(v_2^2)) \in \{(1, 5), (2, 4)\}$.

If $(f(v_1^2), f(v_2^2)) = (1, 5)$, then the weight of the central vertex c_1 weight $w(c_1) \in \{2, 3, 4\}$. The leaves u_1^1, u_2^1 are must receive the labels $(f(u_1^1), f(u_1^2)) \in \{(1, 2), (1, 3)\}$ and hence we obtain $f(v_1^2) = f(u_1^1)$, which is a contradiction. Therefore, the central vertex c_1 must receive a new color w_3.

If $(f(v_1^2), f(v_2^2)) = (2, 4)$, then the weight of the central vertex $c_1,\ w(c_1) \in \{1, 3, 5\}$. The leaves u_1^1, u_2^1 are must receive the labels $(f(u_1^1), f(u_1^2)) \in \{(2, 1), (2, 3), (1, 4)\}$ and hence we get $f(v_1^2) = f(u_1^1)$ or $f(v_2^2) = f(u_1^2)$, which is a contradiction. Therefore, the central vertex c_1 must receive a new color w_3. Hence, $\chi_{lda}(2K_{1,2}) \geq 3$.

Define $f : V(2K_{1,2}) \to \{1, 2, 3, 4, 5, 6\}$ by $f(c_1) = 5,\ f(c_2) = 6, f(u_1^1) = 2, f(u_1^2) = 4$, $f(u_2^1) = 1, f(u_2^2) = 3$. Then the vertex weights are $w(c_2) = 4,\ w(u_1^1) = w(u_2^1) = 5,\ w(c_1) = w(u_1^2) = w(u_2^2) = 6$. Hence $\chi_{lda}(2K_{1,2}) \leq 3$. Thus, $\chi_{lda}(2K_{1,2}) = 3$.

Case 2: $t \geq 3$. Since the pendant vertices $u_1^j,\ u_2^j,\ 1 \leq j \leq t$, are received the colors $c_1,\ c_2,\ c_3, \ldots, c_t$, we get $\chi_{lda}(tK_{1,2}) \geq t$.

Define a labeling $f : V(tK_{1,2}) \to \{1, 2, 3, \ldots, 3t\}$ by $f(c_1) = 3t, f(c_2) = t + 2,\ f(c_j) = t + j, 3 \leq j \leq t - 1,\ f(c_t) = 3t - 1,\ f(u_1^1) = 1, f(u_1^2) = t + 1,\ f(u_1^j) = j, 2 \leq j \leq t$, $f(u_2^j) = 3t - j, 2 \leq j \leq t$. Then the vertex weights are $w(c_1) = w(u_1^2) = w(u_2^2) = t + 2, w(u_1^1) = w(u_1^2) = w(c_j) = 3t,\ 2 \leq j \leq t,\ w(u_1^j) = w(u_2^j) = t + j,\ 3 \leq j \leq t - 1,\ w(u_1^t) = w(u_2^t) = 3t - 1$. Thus, $\chi_{lda}(tK_{1,2}) \leq t$. Hence, $\chi_{lda}(tK_{1,2}) = t$.

***Example* 2.1.** The LDAL for the graph $5K_{1,2}$ with 5-colors is given in Fig. 1, and the colors are 7, 8, 9,14 and 15.

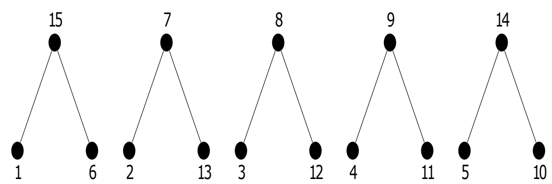

Fig. 1. $\chi_{lda}(5K_{1,2}) = 5$.

Theorem 2.3. *Let* $tK_{1,r},\ r \geq 3$, *be the disjoint union of* t *copies of stars. Then* $\chi_{lda}(tK_{1,r}) = t + 1$.

Proof. Suppose that $\chi_{lda}(tK_{1,r}) = t$. Then there exists an LDAL f with t colors. Since the leaves of each component are the same weight, these t weights are distinct, and each is at most

$p = t(r+1)$. The sum of weights of central vertices is at least $\sum_{i=1}^{tr} i = \frac{t^2r^2+tr}{2}$. Thus, at least one of these weights is at least $\frac{tr^2+r}{2} > \frac{tr^2}{2} > t(r+1)$, if $r \geq 3$, so distinct from the r weights used on the leaves. Therefore, at least one of the central vertices has received a new color $t+1$. Hence, $\chi_{lda}(tK_{1,r}) \geq t+1$.

For $r \geq 3, t \geq 1$, define a labeling $f : V(tK_{1,r}) \to \{1,2,3,\ldots,3t\}$ by the following cases.

Case (i): $r \geq 3$ is odd and $t \geq 1$. Then

$$f(c_j) = 2j-1, \quad 1 \leq j \leq t,$$

$$f(u_i^j) = \begin{cases} 2t+2-2j & \text{for } i=1 \text{ and } 1 \leq j \leq t, \\ 2t+j & \text{for } i=2 \text{ and } 1 \leq j \leq t, \\ ti+j & \text{for } i=3 \text{ or } i \text{ is even } 4 \leq i \leq r \text{ and } 1 \leq j \leq t, \\ t(i+1)+1-j & \text{for } i \text{ is odd } 5 \leq i \leq r \text{ and } 1 \leq j \leq t. \end{cases}$$

This labeling is also shown in Table 1 for easy reading, and the corresponding row sum obtains the last column values.

Table 1. Use integers in $[1,(r+1)t] - \{1,3,5,7,\ldots,2t-3,2t-1\}$

j	u_1^j	u_2^j	u_3^j	$\ldots$	u_{r-1}^j	u_r^j	weight
1	$2t$	$2t+1$	$3t+1$	$\cdots$	$(r-1)t+1$	$(r+1)t$	$w(c_1)$
2	$2t-2$	$2t+2$	$3t+2$	$\cdots$	$(r-1)t+2$	$(r+1)t-1$	$w(c_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$	$\vdots$	$\vdots$
$t-1$	4	$3m-1$	$4m-1$	$\cdots$	$(n-1)m+(m-1)$	$(n-1)m+(m+2)$	$w(c_{t-1})$
t	2	$3t$	$4t$	$\cdots$	$(r-1)t+t$	$(r-1)t+(t+1)$	$w(c_t)$

Then the vertex weights are

$$w(c_j) = t(r+2) + \frac{r+1}{2} + \frac{t(r^2-5)}{2}, \quad 1 \leq j \leq t,$$

$$w(u_i^j) = tr + j, \quad 1 \leq j \leq t, \quad 1 \leq i \leq r.$$

Note that the weight of the central vertex is greater than the maximum weight of a leaf, so f admits a LDAL.

Case (ii): $r \geq 3$ is even and $t \geq 1$. Then

$$f(c_j) = tr + j, \quad 1 \leq j \leq t,$$

$$f(u_i^j) = \begin{cases} t(i-1)+j & \text{for } i \text{ is odd } 1 \leq i \leq r \text{ and } 1 \leq j \leq t, \\ ti+1-j & \text{for } i \text{ is even } 2 \leq i \leq r \text{ and } 1 \leq j \leq t. \end{cases}$$

This labeling is also shown in Table 2 for easy reading, and the corresponding row sum obtains the last column values.

Table 2. Use integers in $[1, rt]$

j	u_1^j	u_2^j	u_3^j	$\dots$	u_{r-1}^j	u_r^j	weight
1	1	$2t$	$2t+1$	$\cdots$	$(r-1)t+1$	tr	$w(c_1)$
2	2	$2t-1$	$2t+2$	$\cdots$	$(r-1)t+2$	$tr-1$	$w(c_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$	$\vdots$
$t-1$	$t-1$	$t+2$	$3t-1$	$\cdots$	$(r-1)t+(t-1)$	$tr-(t-2)$	$w(c_{t-1})$
t	t	$t+1$	$3t$	$\cdots$	$(r-1)t+t$	$tr-(t-1)$	$w(c_t)$

Then the vertex weights are

$$w(c_j) = \frac{r(tr+1)}{2}, \quad 1 \le j \le t,$$

$$w(u_i^j) = tr + j, \quad 1 \le j \le t, \quad 1 \le i \le r.$$

Note that the weight of the central vertex is greater than the maximum weight of a leaf, so f admits a LDAL. Therefore, f is admitting a LDAL of $tK_{1,r}$ that induces a local distance antimagic vertex coloring using exactly $t+1$ colors. Hence $\chi_{lda}(tK_{1,r}) \le t+1$. Thus, $\chi_{lda}(tK_{1,r}) = t+1$.

***Example* 2.2.** The LDAL for the graph $4K_{1,5}$ with 5-colors is given in Fig. 2, and the colors are 1, 3, 5, 7 and 71.

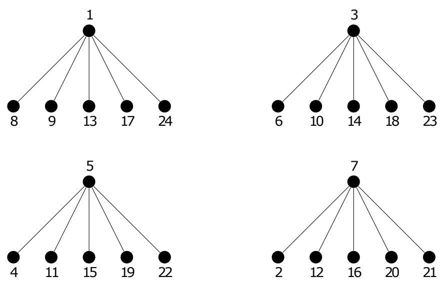

Fig. 2. $\chi_{lda}(4K_{1,5}) = 5$.

***Example* 2.3.** The LDAL for the graph $4K_{1,4}$ with 5-colors is given in Fig. 3, and the colors are 17, 18, 19, 20 and 34.

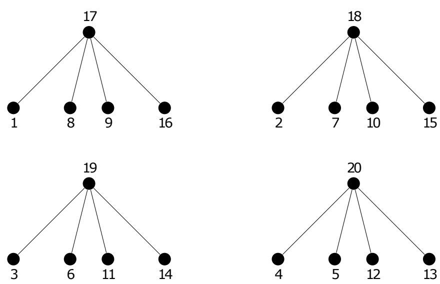

Fig. 3. $\chi_{lda}(4K_{1,4}) = 5$.

From Theorems 2.1, 2.2 and 2.3, we get the following theorem.

Theorem 2.4. *Let $tK_{1,r}$ be the disjoint union of t copies of stars. Then*

$$\chi_{lda}(tK_{1,r}) = \begin{cases} t & for \quad r=2 \quad and \quad t \geq 3, \\ t+1 & for \quad r=2 \quad and \quad t=1,2, \\ t+1 & for \quad r \geq 3 \quad and \quad t \geq 1, \\ 2t & for \quad r=1 \quad and \quad t \geq 1. \end{cases}$$

3. The LDACN of disjoint union of *t* copies of double stars. This section investigates the LDACN of the disjoint union of t copies double stars.

Let $tS_{m.n}$, $m \leq n$, be the disjoint union of t copies double stars. Let $V(tS_{m,n}) = \{x_1^j, x_2^j, u_i^j, v_h^j\}$ for $1 \leq i \leq m, 1 \leq h \leq n$ and $1 \leq j \leq t$. Let u_i^j be the ith vertex of jth copy of a double star. Then $|V(tS_{m,n})| = p = t(m+n+2)$.

Lemma 3.1. *Let $tS_{m,n}$ be the disjoint union of t copies double stars, where $m \leq n$. Then $\chi_{lda}(tS_{m,n}) \geq 2t+1$.*

Proof. Suppose that the graph $tS_{m,n}, m \leq n$ admits the LDAL f. Since the leaves of each component of $tS_{m,n}$ are the same weight, these 2t weights are distinct, it follows we get $\chi_{lda}(tS_{m,n}) \geq 2t$. If a leaf receives a label $p = t(m+n+2)$, then its adjacent central vertex received a weight more than p, and hence we obtain $\chi_{lda}(tS_{m,n}) \geq 2t+1$. If the central vertex receives a label p, then its adjacent other central vertex weight is more than p, and hence we have $\chi_{lda}(tS_{m,n}) \geq 2t+1$. Thus, $\chi_{lda}(tS_{m,n}) \geq 2t+1$.

Theorem 3.1. *Let $tS_{m,n}, m < n$, be the disjoint union of t copies double stars, where $m \geq 3$ is odd and $n = \dfrac{m^2+1}{2} + 2k$, $k \geq 0$, $t \geq \dfrac{m+3}{2}$. Then $\chi_{lda}(tS_{m,n}) = 2t+1$.*

Proof. Let $V(tS_{m,n}) = \{x_1^j, x_2^j, u_i^j, w_h^j\}$ for $1 \leq i \leq m$ and $1 \leq h \leq n$ and $1 \leq j \leq t$. We define a labeling function f. For ease of exposition, we initially we use dummy vertices $v_1^j, v_2^j, \ldots, v_{n+1}^j$ and then assign one of their labels to x_1^j and the other n labels to $w_1^j, w_2^j, \ldots, w_n^j$. Then

$$f(x_2^j) = j, \quad 1 \leq j \leq t,$$

$$f(u_{i-1}^j) = \begin{cases} t(i-1)+j & \text{for } i \text{ is odd, } 3 \leq i \leq m \text{ and } 1 \leq j \leq t, \\ ti+1-j & \text{for } i \text{ is even, } 2 \leq i \leq m-1 \text{ and } 1 \leq j \leq t, \\ tm+j & \text{for } i = m+1 \text{ and } 1 \leq j \leq t. \end{cases}$$

For easy reading the above function as follows (see Table 3):

Table 3. Use integers in $[1, t(m+1)]$

$j \downarrow$	x_2^j	u_1^j	u_2^j	$\ldots$	u_{m-1}^j	u_m^j	$w(x_1^j)$ is the row sum
1	1	$2t$	$2t+1$	$\cdots$	$(m-1)t+1$	$tm+1$	a
2	2	$2t-1$	$2t+2$	$\cdots$	$(m-1)t+2$	$tm+2$	$a+2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$t-1$	$t-1$	$t+2$	$3t-1$	$\cdots$	$(m-1)t+(t-1)$	$tm+(t-1)$	$a+2(t-2)$
t	t	$t+1$	$3t$	$\cdots$	$(m-1)t+t$	$tm+t$	$a+2(t-1)$

In Table 3 $a = t(m+1) + \dfrac{t(m^2-3)}{2} + \dfrac{m+3}{2}$.

Case 1: Let $\alpha = t - \dfrac{m+1}{2}$ and $\alpha = 1$ or 2. Then

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - 2(j-1) & \text{for } i \text{ is odd}, \ 1 \le i \le n+1, \text{and } j = 1,2, \\ t(m+i) + \dfrac{m-1}{2} + 2(j-1) & \text{for } i \text{ is even}, \ 1 \le i \le n+1, \\ & i \ne \dfrac{m^2+1}{2} + 1, \text{and } j = 1; \\ & \text{for } i \text{ is even}, \ 1 \le i \le n+1, \\ & i \ne \dfrac{m^2+1}{2} - 1, \text{and } j = 2. \end{cases}$$

If $\alpha = 1$, then

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} & \\ \quad - 2\left(t - \dfrac{m+1}{2}\right) + 1 + 2(j-3) & \text{for } i \text{ is odd}, \ 1 \le i \le n+1, i \ne \dfrac{m^2+1}{2}, \\ & \text{and } 3 \le j \le \left\lfloor \dfrac{t}{2} \right\rfloor + 2, \\ t(m+i) + \dfrac{m-1}{2} & \\ \quad + 2\left(t - \dfrac{m+1}{2}\right) - 1 - 2(j-3) & \text{for } i \text{ is even}, \ 1 \le i \le n+1, \\ & \text{and } 3 \le j \le \left\lfloor \dfrac{t}{2} \right\rfloor + 2, \end{cases}$$

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - 2 & \\ \quad + 2\left(\left\lceil \dfrac{t}{2} \right\rceil - 1\right) - 2\left(j - \left(\left\lfloor \dfrac{t}{2} \right\rfloor + 3\right)\right) & \text{for } i \text{ is odd}, \ 1 \le i \le n+1, \\ & \text{and } \left\lceil \dfrac{t}{2} \right\rceil + 3 \le j \le t, \\ t(m+i) - \dfrac{m-3}{2} + 1 & \\ \quad + 2\left(\left\lfloor \dfrac{t}{2} \right\rfloor - 1\right) + 2\left(j - \left(\left\lfloor \dfrac{t}{2} \right\rfloor + 3\right)\right) & \text{for } i \text{ is even}, \ 1 \le i \le n+1, \\ & i \ne \dfrac{m^2+1}{2} + 1, \text{and } \left\lfloor \dfrac{t}{2} \right\rfloor + 3 \le j \le t. \end{cases}$$

If $\alpha = 2$, then

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} \\ \quad -2\left(t - \dfrac{m+1}{2}\right) + 1 + 2(j-3) & \text{for } i \text{ is odd}, \ 1 \le i \le n+1, i \ne \dfrac{m^2+1}{2}, \\ & \text{and } 3 \le j \le \left\lceil \dfrac{t}{2} \right\rceil + 2, \\ t(m+i) + \dfrac{m-1}{2} \\ \quad +2\left(t - \dfrac{m+1}{2}\right) - 1 - 2(j-3) & \text{for } i \text{ is even}, \ 1 \le i \le n+1, \\ & \text{and } 3 \le j \le \left\lceil \dfrac{t}{2} \right\rceil + 2, \end{cases}$$

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - 2 \\ +2\left(\left\lceil \dfrac{t}{2} \right\rceil - 1\right) - 2\left(j - \left(\left\lfloor \dfrac{t}{2} \right\rfloor + 3\right)\right) & \text{for } i \text{ is odd}, \ 1 \le i \le n+1, \\ & \text{and } \left\lfloor \dfrac{t}{2} \right\rfloor + 3 \le j \le t, \\ t(m+i) - \dfrac{m-3}{2} - 1 \\ +2\left(\left\lceil \dfrac{t}{2} \right\rceil - 1\right) + 2\left(j - \left(\left\lfloor \dfrac{t}{2} \right\rfloor + 3\right)\right) & \text{for } i \text{ is even}, \ 1 \le i \le n+1, i \ne \dfrac{m^2+1}{2} + 1, \\ & \text{and } \left\lfloor \dfrac{t}{2} \right\rfloor + 3 \le j \le t. \end{cases}$$

Now, we define the central vertices x_1^j, $1 \le j \le t$, labels are as follows:

$$f(x_1^j) = \begin{cases} f(v_i^j) & \text{for } i = \dfrac{m^2+1}{2} + 1 \ \text{ and } \ j = 1, \\ f(v_i^j) & \text{for } i = \dfrac{m^2+1}{2} - 1 \ \text{ and } \ j = 2, \\ f(v_i^j) & \text{for } i = \dfrac{m^2+1}{2} \ \text{ and } \ 3 \le j \le \left\lfloor \dfrac{t}{2} \right\rfloor + 2, \\ f(v_i^j) & \text{for } i = \dfrac{m^2+1}{2} + 1 \ \text{ and } \ \left\lfloor \dfrac{t}{2} \right\rfloor + 3 \le j \le t. \end{cases}$$

Note that the central vertices x_1^j, $1 \le j \le t$, labels forms an arithmetic progression with common difference 2 and an initial term $a = t(m+1) + \dfrac{t(m^2-3)}{2} + \dfrac{m+3}{2}$. Then the vertex weights are

$$w(x_1^j) = a + 2(j-1), \quad 1 \le j \le t,$$

$$w(x_2^j) = \frac{(n+1)[t(n+2m+3)+1]}{2}, \quad 1 \le j \le t,$$

$$w(v_h^j) = j, \quad 1 \le j \le t, \quad 1 \le h \le n+1,$$

$$w(u_i^j) = \begin{cases} f(x_1^j) & \text{for } 1 \le i \le m+1 \text{ and } j = 1; \\ & \text{this corresponding set is } \{a + 2(t-1)\}, \\ f(x_1^j) & \text{for } 2 \le j \le t \text{ and } 1 \le i \le m+1; \\ & \text{this corresponding set is } \{a, a+2, a+4, \ldots, a+2(t-2)\}. \end{cases}$$

Therefore, f is a LDAL of $tS_{m,n}$ that induces a local distance antimagic vertex coloring using exactly $2t+1$ colors. Hence, $\chi_{lda}(tS_{m,n}) \le 2t+1$.

Case 2: Let $\alpha = t - \dfrac{m+1}{2}$ and $3 \le \alpha \le t-2$. Note that, for $m \ge 3$, we get

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - 2(j-1) & \text{for } i \text{ is odd, } 1 \le i \le n+1, \\ & \text{and } 1 \le j \le \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 2, \\ t(m+i) + \dfrac{m-1}{2} + 2(j-1) & \text{for } i \text{ is even, } 1 \le i \le n+1,\ i \ne \dfrac{m^2+1}{2} - 1, \\ & \text{and } 1 \le j \le \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 2. \end{cases}$$

If $\alpha = t - \dfrac{m+1}{2}$ is even, then

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - \dfrac{2t-m+1}{2} & \\ +2\Big(j - \Big[\left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3\Big]\Big) & \text{for } i \text{ is odd, } 1 \le i \le n+1,\ i \ne \dfrac{m^2+1}{2}, \\ & \text{and } \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3 \le j \le \left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 2, \\ t(m+i) + \dfrac{m-1}{2} + \dfrac{2t-m+1}{2} & \\ -2\Big(j - \Big[\left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3\Big]\Big) & \text{for } i \text{ is even, } 1 \le i \le n+1, \\ & \text{and } \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3 \le j \le \left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 2, \end{cases}$$

$$f(v_i^j) = \begin{cases} t(m+1+i) - \dfrac{m-3}{2} - \dfrac{2t-m+1}{2} + 2\Big(\left\lfloor \dfrac{t}{2} \right\rfloor - 1\Big) & \\ +1 - 2\Big(j - \Big(\left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3\Big)\Big) & \text{for } i \text{ is odd, } 1 \le i \le n+1, \\ & \text{and } \left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3 \le j \le t, \\ t(m+i) + \dfrac{m-1}{2} + \dfrac{2t-m+1}{2} - 2\Big(\left\lfloor \dfrac{t}{2} \right\rfloor - 1\Big) & \\ -1 + 2\Big(j - \Big(\left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3\Big)\Big) & \text{for } i \text{ is even, } 1 \le i \le n+1, \\ & i \ne \dfrac{m^2+1}{2} + 1, \\ & \text{and } \left\lceil \dfrac{t}{2} \right\rceil + \left\lfloor \dfrac{\alpha}{2} \right\rfloor + 3 \le j \le t. \end{cases}$$

If $\alpha = t - \dfrac{m+1}{2}$ is odd, then

$$f(v_i^j)=\begin{cases} t(m+1+i)-\dfrac{m-3}{2}-\dfrac{2t-m+1}{2}+1 & \\ +2\Big(j-\Big[\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big]\Big) & \text{for } i \text{ is odd}, \ 1\le i\le n+1, \ i\ne\dfrac{m^2+1}{2}, \\ & \text{and } \Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+2, \\ t(m+i)+\dfrac{m-1}{2}+\dfrac{2t-m+1}{2}-1 & \\ -2\Big(j-\Big[\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big]\Big) & \text{for } i \text{ is even}, \ 1\le i\le n+1, \\ & \text{and } \Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+2, \end{cases}$$

$$f(v_i^j)=\begin{cases} t(m+1+i)-\dfrac{m-3}{2}-\dfrac{2t-m+1}{2}+2 & \\ +2\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor-1\Big)-2\Big(j-\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big)\Big) & \text{for } i \text{ is odd}, \ 1\le i\le n+1, \\ & \text{and } t \text{ is odd}, \ \Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le t, \\ t(m+i)+\dfrac{m-1}{2}+\dfrac{2t-m+1}{2}-2 & \\ -2\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor-1\Big)+2\Big(j-\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big)\Big) & \text{for } i \text{ is even}, \ 1\le i\le n+1, \\ & i\ne\dfrac{m^2+1}{2}+1, \\ & \text{and } t \text{ is odd}, \ \Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le t, \\ t(m+1+i)-\dfrac{m-3}{2}-\dfrac{2t-m+1}{2} & \\ +2\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor-1\Big)-2\Big(j-\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big)\Big) & \text{for } i \text{ is odd}, \ 1\le i\le n+1, \\ & \text{and } t \text{ is even}, \ \Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le t, \\ t(m+i)+\dfrac{m-1}{2}+\dfrac{2t-m+1}{2} & \\ -2\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor-1\Big)+2\Big(j-\Big(\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\Big)\Big) & \text{for } i \text{ is even}, \ 1\le i\le n+1, \\ & i\ne\dfrac{m^2+1}{2}+1, \\ & \text{and } t \text{ is even}, \ \Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le t. \end{cases}$$

Now, we define the central vertices x_1^j, $1\le j\le t$, labels are as follows:

$$f(x_1^j)=\begin{cases} f(v_i^j) & \text{for } i=\dfrac{m^2+1}{2}-1 \text{ and } 1\le j\le\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+2, \\ f(v_i^j) & \text{for } i=\dfrac{m^2+1}{2} \text{ and } \Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le\Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+2, \\ f(v_i^j) & \text{for } i=\dfrac{m^2+1}{2}+1 \text{ and } \Big\lfloor\dfrac{t}{2}\Big\rfloor+\Big\lfloor\dfrac{\alpha}{2}\Big\rfloor+3\le j\le t. \end{cases}$$

Note that the central vertices x_1^j, $1 \le j \le t$, labels forms an arithmetic progression with common difference 2 and an initial term $a = t(m+1) + \dfrac{t(m^2-3)}{2} + \dfrac{m+3}{2}$. Then the vertex weights are

$$w(x_1^j) = a + 2(j-1), \quad 1 \le j \le t,$$

$$w(x_2^j) = \frac{(n+1)[t(n+2m+3)+1]}{2}, \quad 1 \le j \le t,$$

$$w(v_h^j) = j, \quad 1 \le j \le t, \quad 1 \le h \le n+1,$$

$$w(u_i^j) = \begin{cases} f(x_1^j) & \text{for } 1 \le i \le m+1 \text{ and } j = 1; \\ & \text{this corresponding set is } \{a + 2(t-1)\}, \\ f(x_1^j) & \text{for } 2 \le j \le t \text{ and } 1 \le i \le m+1; \\ & \text{this corresponding set is } \{a, a+2, a+4, \ldots, a+2(t-2)\}. \end{cases}$$

Therefore, f is a LDAL of $tS_{m,n}$ that induces a local distance antimagic vertex coloring using exactly $2t+1$ colors. Hence, $\chi_{lda}(tS_{m,n}) \le 2t+1$ and, by Lemma 3.1, we get $\chi_{lda}(S_{m,n}) = 2t+1$.

***Example* 3.1.** The LDAL for the graph $7S_{3,7}$ with 15-colors is given in the Tables 4 and 5, and the colors are $[1,7] \cup \{52, 54, 56, 58, 60, 62, 64, 452\}$.

Table 4. Use integers in $[1, 28]$

j	x_2^j	u_1^j	u_2^j	u_3^j	$w(x_1^j)$
1	1	14	15	22	52
2	2	13	16	23	54
3	3	12	17	24	56
4	4	11	18	25	58
5	5	10	19	26	60
6	6	9	20	27	62
7	7	8	21	28	64

Table 5. Use integers in $[29, 84]$

j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	v_6^j	v_7^j	v_8^j	$w(x_2^j)$
1	35	36	49	50	63	64	77	78	452
2	33	38	47	52	61	66	75	80	452
3	31	40	45	54	59	68	73	82	452
4	29	42	43	56	57	70	71	84	452
5	30	41	44	55	58	69	72	83	452
6	32	39	46	53	60	67	74	81	452
7	34	37	48	51	62	65	76	79	452

***Example* 3.2.** The LDAL for the graph $8S_{5,13}$ with 17-colors is given in the Tables 6 and 7, and the colors are $[1,8] \cup \{140, 142, 144, 146, 148, 150, 152, 154, 1463\}$.

Table 6. Use integers in $[1,48]$

j	x_2^j	u_1^j	u_2^j	u_3^j	u_4^j	u_5^j	$w(x_1^j)$
1	1	16	17	25	40	41	140
2	2	15	18	26	39	42	142
3	3	14	19	27	38	43	144
4	4	13	20	28	37	44	146
5	5	12	21	29	36	45	148
6	6	11	22	30	35	46	150
7	7	10	23	31	34	47	152
8	8	9	24	32	33	48	154

Table 7. Use integers in $[49,160]$

j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	v_6^j	v_7^j	v_8^j	v_9^j	v_{10}^j	v_{11}^j	v_{12}^j	v_{13}^j	v_{14}^j	$w(x_2^j)$
1	55	58	71	74	87	90	103	106	119	122	135	138	151	154	1463
2	53	60	69	76	85	92	101	108	117	124	133	140	149	156	1463
3	51	62	67	78	83	94	99	110	115	126	131	142	147	158	1463
4	49	64	65	80	81	96	97	112	113	128	129	144	145	160	1463
5	50	63	66	79	82	95	98	111	114	127	130	143	146	159	1463
6	52	61	68	77	84	93	100	109	116	125	132	141	148	157	1463
7	54	59	70	75	86	91	102	107	118	123	134	139	150	155	1463
8	56	57	72	73	88	89	104	105	120	121	136	137	152	153	1463

***Example* 3.3.** The LDAL for the graph $7S_{11,61}$ with 15-colors is given in the Tables 8 and 9, and the colors are $[1,7]\cup\{504,506,508,510,512,514,516,18693\}$.

Table 8. Use integers in $[1,84]$

j	x_2^j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	v_6^j	v_7^j	v_8^j	v_9^j	v_{10}^j	v_{11}^j	$w(x_2^j)$
1	1	14	15	28	29	42	43	56	57	70	71	78	504
2	2	13	16	27	30	41	44	55	58	69	72	79	506
3	3	12	17	26	31	40	45	54	59	68	73	80	508
4	4	11	18	25	32	39	46	53	60	67	74	81	510
5	5	10	19	24	33	38	47	52	61	66	75	82	512
6	6	9	20	23	34	37	48	51	62	65	76	83	514
7	7	8	21	22	35	36	49	50	63	64	77	84	516

Table 9. Use integers in $[85, 518]$

j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	v_6^j	v_7^j	v_8^j	$\ldots$	v_{60}^j	v_{61}^j	v_{62}^j	$w(x_2^j)$
1	87	96	101	110	115	124	129	138	$\ldots$	502	507	516	18693
2	85	98	99	112	113	126	127	140	$\ldots$	504	505	518	18693
3	86	97	100	111	114	125	128	139	$\ldots$	503	506	517	18693
4	88	95	102	109	116	123	130	137	$\ldots$	501	508	515	18693
5	90	93	104	107	118	121	132	135	$\ldots$	499	510	513	18693
6	91	92	105	106	119	120	133	134	$\ldots$	498	511	512	18693
7	89	94	103	108	117	122	131	136	$\ldots$	500	509	514	18693

***Example* 3.4.** The LDAL for the graph $10S_{15,113}$ with 21-colors is given in the Tables 10 and 11, and the colors are $[1, 10] \cup \{1279, 1281, 1283, 1285, 1287, 1289, 1291, 1293, 1295, 1297, 83277\}$.

Table 10. Use integers in $[1, 160]$

j	x_2^j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	v_6^j	v_7^j	v_8^j	v_9^j	v_{10}^j	v_{11}^j	v_{12}^j	v_{13}^j	v_{14}^j	v_{15}^j	$w(x_2^j)$
1	1	20	21	40	41	60	61	80	81	100	101	120	121	140	141	151	1279
2	2	19	22	39	42	59	62	79	82	99	102	119	122	139	142	152	1281
3	3	18	23	38	43	58	63	78	83	98	103	118	123	138	143	153	1283
4	4	17	24	37	44	57	64	77	84	97	104	117	124	137	144	154	1285
5	5	16	25	36	45	56	65	76	85	96	105	116	125	136	145	155	1287
6	6	15	26	35	46	55	66	75	86	95	106	115	126	135	146	156	1289
7	7	14	27	34	47	54	67	74	87	94	107	114	127	134	147	157	1291
8	8	13	28	33	48	53	68	73	88	93	108	113	128	133	148	158	1293
9	9	12	29	32	49	52	69	72	89	92	109	112	129	132	149	159	1295
10	10	11	30	31	50	51	70	71	90	91	110	111	130	131	150	160	1297

Table 11. Use integers in $[161, 1300]$

j	v_1^j	v_2^j	v_3^j	v_4^j	v_5^j	$\ldots$	v_{111}^j	v_{112}^j	v_{113}^j	v_{114}^j	$w(x_2^j)$
1	164	177	184	197	204	$\ldots$	1264	1277	1284	1297	83277
2	162	179	182	199	202	$\ldots$	1262	1279	1282	1299	83277
3	161	180	181	200	201	$\ldots$	1261	1280	1281	1300	83277
4	163	178	183	198	203	$\ldots$	1263	1278	1283	1298	83277
5	165	176	185	196	205	$\ldots$	1265	1276	1285	1296	83277
6	167	174	187	194	207	$\ldots$	1267	1274	1287	1294	83277
7	169	172	189	192	209	$\ldots$	1269	1272	1289	1292	83277
8	170	171	190	191	210	$\ldots$	1270	1271	1290	1291	83277
9	168	173	188	193	208	$\ldots$	1268	1273	1288	1293	83277
10	166	175	186	195	206	$\ldots$	1266	1275	1286	1295	83277

Lemma 3.1 and Theorem 3.1 leads to the following problem.

***Problem* 3.1.** Determine the LDACN of the disjoint union of t copies double stars $tS_{m,n}, m < n,$ $\chi_{lda}(tS_{m,n}) = 2t + 1,$ except for the m and n values are given in Theorem 3.1.

4. Conclusion. This paper investigated the LDACN for the disjoint union of t copies of stars. We also obtained the LDACN of the disjoint union of t copies double stars is $2t + 1$ for certain m and n values.

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