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DYNAMICAL BEHAVIOR OF RATIONAL DIFFERENCE EQUATION $x_{n+1} = \dfrac{x_{n-13}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}$

ДИНАМІЧНА ПОВЕДІНКА РАЦІОНАЛЬНОГО РІЗНИЦЕВОГО РІВНЯННЯ $x_{n+1} = \dfrac{x_{n-13}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}$

Discrete-time systems are sometimes used to explain natural phenomena encountered in nonlinear sciences. We study the periodicity, boundedness, oscillation, stability, and some exact solutions of nonlinear difference equations. Exact solutions are obtained by using the standard iteration method. Some well-known theorems are used to test the stability of the equilibrium points. Some numerical examples are also provided to confirm the validity of the theoretical results. The numerical component is implemented with the Wolfram Mathematica. The presented method may be simply applied to other rational recursive issues.

In this paper, we explore the dynamics of adhering to rational difference formula

$$x_{n+1} = \frac{x_{n-13}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}},$$

where the initials are arbitrary nonzero real numbers.

Системи з дискретним часом іноді використовуються для пояснення природних явищ, що зустрічаються в нелінійних науках. У цій статті ми вивчаємо періодичність, обмеженість, коливання, стійкість та деякі точні розв'язки нелінійних різницевих рівнянь. Точні розв'язки отримано з використанням стандартного ітераційного методу. Деякі відомі теореми використовуються для перевірки стійкості точок рівноваги. Також наведено кілька числових прикладів, які підтверджують достовірність теоретичних результатів. Числові розрахунки реалізовано за допомогою системи Wolfram Mathematica. Наведений метод може бути легко застосований до розв'язку інших раціональних рекурсивних задач.

У цій статті ми досліджуємо динаміку дотримання формули раціональних різниць

$$x_{n+1} = \frac{x_{n-13}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}},$$

де ініціали — довільні ненульові дійсні числа.

1. Introduction. The evolution of a certain natural phenomena is often described over a course of time using differential equations. However, many real life problems can be sometimes modeled using discrete time steps which lead to difference equations. Hence, recursive equations have an effective and powerful role in mathematics. They are successfully utilized to investigate some applications in engineering, physics, biology, economic and others. For instance, recursive equations have been well used in modeling some natural phenomena such as the size of a population, the Fibonacci sequence, the drug in the blood system, the transmission of information, the pricing of a certain commodity, the propagation of annual plants, and others [9]. In addition, some scholars have used difference equations to find the numerical solutions of some differential equations. More specifically, discretizing a given

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differential equation gives a difference equation. For example, Runge – Kutta scheme is obtained from discretizing a first order differential equation. This raises the question of the convergence of the difference scheme to the solution of a differential equation. Or, in a broader sense, the question of the correspondence between the properties of solutions of differential equations and their difference approximations. The work [17] is devoted to questions of conservation of a solution bounded on the entire axis in the transition from differential to difference equations and vice versa. In [18], similar questions were considered to preserve the oscillatory property of solutions to second-order equations. The development of technology has motivated the use of recurrence equations as approximations to partial differential equations. It is worth mentioning that fractional order difference equations are often utilized to investigate some real life phenomena emerging in nonlinear sciences.

Most properties of recursive expressions have been widely discussed by some researchers. For example, researchers have explored the stability, periodicity, boundedness and solutions of some recursive equations. We here present some published works. Alayachi et al. [4] analyzed the local and global attractivity, periodicity and the solutions of a sixth order difference equation. Some numerical examples have been also presented in [4]. In [22], Sanbo and Elsayed presented the periodicity, stability and some solutions of a fifth order recursive equation. Almatrafi and Alzubaidi [5] discussed the dynamical behaviors of an eighth order difference relation and showed some 2D figures for the obtained results. Moreover, Ahmed et al. [3], found new solutions and investigated the dynamical analysis for some nonlinear difference relations of fifteenth order. More discussions about nonlinear recursive problems can be seen in [1 – 29].

The motivation of writing this article arises from the investigation of eighteenth order difference equations given in [28]. We consider more sophisticated rational difference equations of fourteenth order. Therefore, this work aims to analyze some dynamical properties such as equilibrium points, local and global behaviors, boundedness, and analytic solutions of the nonlinear recursive equations

$$x_{n+1}=\frac{x_{n-13}}{\pm 1\pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}. \tag{1}$$

Here, the initial values $x_{-13},\ x_{-12},\ x_{-11},\ldots,\ x_{-2},\ x_{-1},\ x_0$, are arbitrary nonzero real numbers. In this work, we also illustrate some 2D figures with the help of Wolfram Mathematica to validate the obtained results.

Let I be some interval of real numbers and

$$f: I^{k+1}\to I$$

be a continuously differentiable function. Then, for every set of initial conditions

$$x_{-k},x_{-k+1},\ldots,x_0\in I,$$

the difference equation

$$x_{n+1}=f(x_n,x_{n-1},\ldots,x_{n-k}), \tag{2}$$

has a unique solution $\{x_n\}_{n=-k}^{\infty}$ [20]. A point $\overline{x}\in I$ is called an equilibrium point of equation (2) if

$$\overline{x}=f(\overline{x},\overline{x},\ldots,\overline{x}).$$

That is, $x_n=\overline{x}$ for $n\geq 0$ is a solution of equation (2) or, equivalently, $\overline{x}$ is a fixed point of f.

Definition 1 (stability). (i) *The equilibrium point* $\overline{x}$ *of equation* (2) *is called locally stable if, for every* $\epsilon > 0$, *there exists* $\delta > 0$ *such that, for all*

$$x_{-k},\ x_{-k+1}, \ldots,\ x_{-1},\ x_0 \in I,$$

with

$$|x_{-k} - \overline{x}| + |x_{-k+1} - \overline{x}| + \ldots + |x_0 - \overline{x}| < \delta,$$

we have

$$|x_n - \overline{x}| < \epsilon \quad \textit{for all} \quad n \geq k.$$

(ii) *The equilibrium point* $\overline{x}$ *of equation* (2) *is called locally asymptotically stable if* $\overline{x}$ *is a locally stable solution of equation* (2) *and there exists* $\gamma > 0$ *such that, for all*

$$x_{-k},\ x_{-k+1}, \ldots,\ x_{-1},\ x_0 \in I,$$

with

$$|x_{-k} - \overline{x}| + |x_{-k+1} - \overline{x}| + \ldots + |x_0 - \overline{x}| < \gamma,$$

we have

$$\lim_{n\to\infty} x_n = \overline{x}.$$

(iii) *The equilibrium point* $\overline{x}$ *of equation* (2) *is called a global attractor if, for all*

$$x_{-k},\ x_{-k+1}, \ldots,\ x_{-1},\ x_0 \in I,$$

we have

$$\lim_{n\to\infty} x_n = \overline{x}.$$

(iv) *The equilibrium point* $\overline{x}$ *of equation* (2) *is called a global asymptotically stable if* $\overline{x}$ *is locally stable and* $\overline{x}$ *is also a global attractor of equation* (2).

(v) *The equilibrium point* $\overline{x}$ *of equation* (2) *is called unstable if* $\overline{x}$ *is not locally stable. The linearized equation of equation* (2) *about the equilibrium* $\overline{x}$ *is the linear difference equation*

$$y_{n+1} = \sum_{i=0}^{k} \frac{\partial f(\overline{x}, \overline{x}, \ldots, \overline{x})}{\partial x_{n-i}} y_{n-i}.$$

Theorem 1 (see [19]). *Assume that* $p, q \in \mathbb{R}$ *and* $k \in \mathbb{N}_0$. *Then*

$$|p| + |q| < 1$$

is a sufficient condition for the asymptotic stability of the difference equation

$$x_{n+1} + p x_n + q x_{n-k} = 0, \quad n \in \mathbb{N}_0.$$

***Remark* 1.** Theorem 1 can be easily extended to general linear equations of the form

$$x_{n+k} + p_1 x_{n+k-1} + \ldots + p_k x_n = 0, \quad n \in \mathbb{N}_0, \tag{3}$$

where $p_1, p_2, \ldots, p_k \in \mathbb{R}$ and $k \in \mathbb{N}$. Then (3) is asymptotically stable provided that

$$\sum_{i=1}^{k} |p_i| < 1.$$

Definition 2 (periodicity). *A sequence* $\{x_n\}_{n=-k}^{\infty}$ *is said to be periodic with period* p *if* $x_{n+p} = x_n$ *for all* $n \geq -k$.

Definition 3. *The equilibrium point* $\overline{x}$ *is said to be hyperbolic if* $|f(\overline{x})| \neq 1$. *If* $|f(\overline{x})| = 1$, x *is non hyperbolic.*

2. Solution of the difference equation $\boldsymbol{x_{n+1} = \dfrac{x_{n-13}}{1+x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}}$**.** In this section, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers

$$x_{n+1} = \frac{x_{n-13}}{1 + x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}, \tag{4}$$

where

$$\begin{array}{lllll} x_{-13} = s, & x_{-12} = r, & x_{-11} = p, & x_{-10} = m, & x_{-9} = l, \\ x_{-8} = j, & x_{-7} = h, & x_{-6} = g, & x_{-5} = f, & x_{-4} = e, \\ x_{-3} = d, & x_{-2} = c, & x_{-1} = b, & x_0 = a. & \end{array} \tag{5}$$

Theorem 2. *Let* $\{x_n\}_{n=-13}^{\infty}$ *be a solution of* (4). *Then*

$$x_{14n-13} = \frac{s\prod_{i=0}^{n-1}(1+7ibdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+1)bdfhkps)}, \qquad x_{14n-12} = \frac{r\prod_{i=0}^{n-1}(1+7iacegjvr)}{\prod_{i=0}^{n-1}(1+(7i+1)acegjvr)},$$

$$x_{14n-11} = \frac{p\prod_{i=0}^{n-1}(1+(7i+1)bdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+2)bdfhkps)}, \qquad x_{14n-10} = \frac{v\prod_{i=0}^{n-1}(1+(7i+1)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+2)acegjvr)},$$

$$x_{14n-9} = \frac{k\prod_{i=0}^{n-1}(1+(7i+2)bdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+3)bdfhkps)}, \qquad x_{14n-8} = \frac{j\prod_{i=0}^{n-1}(1+(7i+2)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+3)acegjvr)},$$

$$x_{14n-7} = \frac{h\prod_{i=0}^{n-1}(1+(7i+3)bdfhkps)}{\prod_{i=0}^{n-1}(1+7i+4)bdfhkps)}, \qquad x_{14n-6} = \frac{g\prod_{i=0}^{n-1}(1+(7i+3)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+4)acegjvr)},$$

$$x_{14n-5} = \frac{f\prod_{i=0}^{n-1}(1+(7i+4)bdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+5)bdfhkps)}, \qquad x_{14n-4} = \frac{e\prod_{i=0}^{n-1}(1+(7i+4)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+5)acegjvr)},$$

$$x_{14n-3} = \frac{d\prod_{i=0}^{n-1}(1+(7i+5)bdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+6)bdfhkps)}, \qquad x_{14n-2} = \frac{c\prod_{i=0}^{n-1}(1+(7i+5)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+6)acegjvr)},$$

$$x_{14n-1} = \frac{b\prod_{i=0}^{n-1}(1+(7i+6)bdfhkps)}{\prod_{i=0}^{n-1}(1+(7i+7)bdfhkps)}, \qquad x_{14n} = \frac{a\prod_{i=0}^{n-1}(1+(7i+6)acegjvr)}{\prod_{i=0}^{n-1}(1+(7i+7)acegjvr)},$$

where $x_0, \ldots, x_{-13}$ *defines as in* (5).

Proof. The proof of each formula are carried out in similar way. So, we will demonstrate proof using one of the formula. We will employ the mathematical induction method. Suppose that $n > 0$ and our assumption holds for $n = 1$, that is,

$$x_{14n-27} = \frac{s\prod_{i=0}^{n-2}(1+7ibdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+1)bdfhkps)}, \qquad x_{14n-26} = \frac{r\prod_{i=0}^{n-2}(1+7iacegjvr)}{\prod_{i=0}^{n-2}(1+(7i+1)acegjvr)},$$

$$x_{14n-25} = \frac{p\prod_{i=0}^{n-2}(1+(7i+1)bdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+2)bdfhkps)}, \qquad x_{14n-24} = \frac{v\prod_{i=0}^{n-2}(1+(7i+1)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+2)acegjvr)},$$

$$x_{14n-23} = \frac{k\prod_{i=0}^{n-2}(1+(7i+2)bdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+3)bdfhkps)}, \qquad x_{14n-22} = \frac{j\prod_{i=0}^{n-2}(1+(7i+2)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+3)acegjvr)},$$

$$x_{14n-21} = \frac{h\prod_{i=0}^{n-2}(1+(7i+3)bdfhkps)}{\prod_{i=0}^{n-2}(1+7i+4)bdfhkps)}, \qquad x_{14n-20} = \frac{g\prod_{i=0}^{n-2}(1+(7i+3)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+4)acegjvr)},$$

$$x_{14n-19} = \frac{f\prod_{i=0}^{n-2}(1+(7i+4)bdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+5)bdfhkps)}, \qquad x_{14n-18} = \frac{e\prod_{i=0}^{n-2}(1+(7i+4)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+5)acegjvr)},$$

$$x_{14n-17} = \frac{d\prod_{i=0}^{n-2}(1+(7i+5)bdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+6)bdfhkps)}, \qquad x_{14n-16} = \frac{c\prod_{i=0}^{n-2}(1+(7i+5)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+6)acegjvr)},$$

$$x_{14n-15} = \frac{b\prod_{i=0}^{n-2}(1+(7i+6)bdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+7)bdfhkps)}, \qquad x_{14n-14} = \frac{a\prod_{i=0}^{n-2}(1+(7i+6)acegjvr)}{\prod_{i=0}^{n-2}(1+(7i+7)acegjvr)}.$$

Now, by using equation (4), one has

$$x_{14n-13} = \frac{x_{14n-27}}{1+x_{14n-15}x_{14n-17}x_{14n-19}x_{14n-21}x_{14n-23}x_{14n-25}x_{14n-27}}$$

$$= \frac{\dfrac{s\prod_{i=0}^{n-2}(1+7ibdfhkps)}{\prod_{i=0}^{n-2}(1+(7i+1)bdfhkps)}}{1+bdfhkps\prod_{i=0}^{n-2}\frac{1+(7i+6)bdfhkps}{1+(7i+7)bdfhkps}\frac{1+(7i+5)bdfhkps}{1+(7i+6)bdfhkps}\frac{1+(7i+4)bdfhkps}{1+(7i+5)bdfhkps}\cdots\frac{1+7ibdfhkps}{1+(7i+1)bdfhkps}}$$

$$= \frac{s\prod_{i=0}^{n-2}\dfrac{(1+7ibdfhkps)}{(1+(7i+1)bdfhkps)}}{1+bdfhkps\prod_{i=0}^{n-2}\dfrac{1+7ibdfhkps}{1+(7i+7)bdfhkps}}$$

$$= s\prod_{i=0}^{n-2}\frac{1+7ibdfhkps}{1+(7i+1)bdfhkps}\left(\frac{1}{1+\dfrac{bdfhkps}{1+(7i-7)bdfhkps}}\right)$$

$$= s\prod_{i=0}^{n-2}\frac{1+7ibdfhkps}{1+(7i+1)bdfhkps}\left(\frac{1+(7i-7)bdfhkps}{1+(7i-6)bdfhkps}\right).$$

Hence, we have

$$x_{14n-13} = s\prod_{i=0}^{n-1}\frac{1+7ibdfhkps}{1+(7i+1)bdfhkps}.$$

Similarly,

$$x_{14n-12} = \frac{x_{14n-26}}{1+x_{14n-14}x_{14n-16}x_{14n-18}x_{14n-20}x_{14n-22}x_{14n-24}x_{14n-26}}$$

$$= \frac{\prod_{i=0}^{n-2}\frac{r(1+7iacegjvr)}{1+(7i+1)acegjvr}}{1+acegjvr\prod_{i=0}^{n-2}\frac{1+(7i+6)acegjvr}{1+(7i+7)acegjvr}\frac{1+(7i+5)acegjvr}{1+(7i+6)acegjvr}\frac{1+(7i+4)acegjvr}{1+(7i+5)acegjvr}\cdots\frac{1+7iacegjvr}{1+(7i+1)acegjvr}}$$

$$= \frac{r\prod_{i=0}^{n-2}\frac{(1+7iacegjvr)}{(1+(7i+1)acegjvr)}}{1+acegjvr\prod_{i=0}^{n-2}\frac{1+7iacegjvr}{1+(7i+7)acegjvr}}$$

$$= r\prod_{i=0}^{n-2}\frac{1+7iacegjvr}{1+(7i+1)acegjvr}\left(\frac{1}{1+\frac{acegjvr}{1+(7i-7)acegjvr}}\right)$$

$$= r\prod_{i=0}^{n-2}\frac{1+7iacegjvr}{1+(7i+1)acegjvr}\left(\frac{1+(7i-7)acegjvr}{1+(7i-6)acegjvr}\right).$$

Therefore, we obtain

$$x_{14n-12} = r\prod_{i=0}^{n-1}\frac{1+7iacegjvr}{1+(7i+1)acegjvr}.$$

Other relations can also be obtained in a similar way, and thus the proof is complete.

Theorem 3. *Equation* (4) *has unique equilibrium point which is the number zero and this equilibrium is not locally asymptotically stable. Also*, $\overline{x}$ *is non hyperbolic.*

Proof. For the equilibriums of equation (4), we get

$$\overline{x} = \frac{\overline{x}}{1+\overline{x}^7}.$$

Then

$$\overline{x}+\overline{x}^8 = \overline{x}, \quad \overline{x}^8 = 0.$$

Thus, the equilibrium point of (4) is $\overline{x} = 0$.

Let $f\colon (0,\infty)^7 \to (0,\infty)$ be the function defined by

$$f(l,o,t,w,\alpha,\beta,\gamma) = \frac{l}{1+lotw\alpha\beta\gamma}.$$

Therefore, it follows that

$$f_l(l,o,t,w,\alpha,\beta,\gamma) = \frac{1}{(1+lotw\alpha\beta\gamma)^2}, \qquad f_o(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2tw\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2},$$
$$f_t(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ow\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2}, \qquad f_w(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha\beta\gamma}{(1+lotw\alpha\beta\gamma)^2},$$
$$f_\alpha(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2owt\beta\gamma}{(1+lotw\alpha\beta\gamma)^2}, \qquad f_\beta(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha w\gamma}{(1+lotw\alpha\beta\gamma)^2},$$
$$f_\gamma(l,o,t,w,\alpha,\beta,\gamma) = \frac{-l^2ot\alpha w\beta}{(1+lotw\alpha\beta\gamma)^2}.$$

We see that

$$f_l(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 1, \qquad f_o(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, \qquad f_t(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0,$$
$$f_w(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, \qquad f_\alpha(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0, \qquad f_\beta(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0,$$
$$f_\gamma(\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x},\overline{x}) = 0.$$

The proof now follows by using Theorem 2.

To illustrate the results of this section, we now consider numerical examples which represent different types of solutions to equation (4).

***Example* 1.** Assume that

$$x_{-13} = 2.8, \quad x_{-12} = 2.6, \quad x_{-11} = 2.5, \quad x_{-10} = 2.55, \quad x_{-9} = 2.35,$$
$$x_{-8} = 2.25, \quad x_{-7} = 2.15, \quad x_{-6} = 2.05, \quad x_{-5} = 2.13, \quad x_{-4} = 2.65,$$
$$x_{-3} = 2.57, \quad x_{-2} = 2.42, \quad x_{-1} = 2.36, \quad x_0 = 2.33.$$

According to the above initial conditions, Fig. 1 was obtained. Figure 1 shows the dynamic behavior of equation (4).

***Example* 2.** Suppose that

$$x_{-13} = 3.8, \quad x_{-12} = 2.6, \quad x_{-11} = 2.5, \quad x_{-10} = 3.55, \quad x_{-9} = 2.35,$$
$$x_{-8} = 2.25, \quad x_{-7} = 2.15, \quad x_{-6} = 3.29, \quad x_{-5} = 2.13, \quad x_{-4} = 3.65,$$
$$x_{-3} = 2.57, \quad x_{-2} = 2.42, \quad x_{-1} = 2.36, \quad x_0 = 2.33.$$

According to the above initial conditions, Fig. 2 was obtained. Figure 2 shows the dynamic behavior of equation (4).

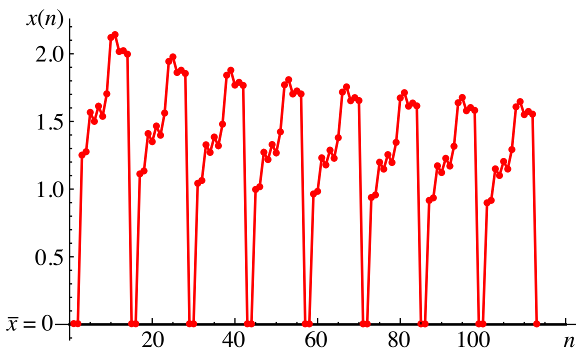

Fig. 1

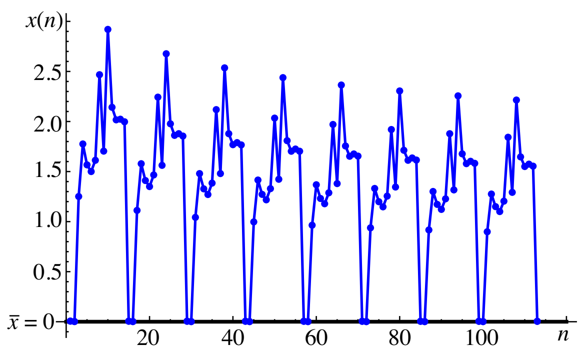

Fig. 2

3. Solution the difference equation $x_{n+1} = \dfrac{x_{n-13}}{1 - x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}$.
In this section, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1} = \frac{x_{n-13}}{1 - x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}. \tag{6}$$

Theorem 4. *Let $\{x_n\}_{n=-13}^{\infty}$ be a solution of equation* (6). *Then*

$$x_{14n-13} = \frac{s\prod_{i=0}^{n-1}(1 - 7ibdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+1)bdfhkps)}, \qquad x_{14n-12} = \frac{r\prod_{i=0}^{n-1}(1 - 7iacegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+1)acegjvr)},$$

$$x_{14n-11} = \frac{p\prod_{i=0}^{n-1}(1 - (7i+1)bdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+2)bdfhkps)}, \qquad x_{14n-10} = \frac{v\prod_{i=0}^{n-1}(1 - (7i+1)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+2)acegjvr)},$$

$$x_{14n-9} = \frac{k\prod_{i=0}^{n-1}(1 - (7i+2)bdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+3)bdfhkps)}, \qquad x_{14n-8} = \frac{j\prod_{i=0}^{n-1}(1 - (7i+2)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+3)acegjvr)},$$

$$x_{14n-7} = \frac{h\prod_{i=0}^{n-1}(1 - (7i+3)bdfhkps)}{\prod_{i=0}^{n-1}(1 - 7i+4)bdfhkps)}, \qquad x_{14n-6} = \frac{g\prod_{i=0}^{n-1}(1 - (7i+3)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+4)acegjvr)},$$

$$x_{14n-5} = \frac{f\prod_{i=0}^{n-1}(1 - (7i+4)bdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+5)bdfhkps)}, \qquad x_{14n-4} = \frac{e\prod_{i=0}^{n-1}(1 - (7i+4)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+5)acegjvr)},$$

$$x_{14n-3} = \frac{d\prod_{i=0}^{n-1}(1 - (7i+5)bdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+6)bdfhkps)}, \qquad x_{14n-2} = \frac{c\prod_{i=0}^{n-1}(1 - (7i+5)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+6)acegjvr)},$$

$$x_{14n-1} = \frac{b\prod_{i=0}^{n-1}(1 - (7i+6)bdfhkps)}{\prod_{i=0}^{n-1}(1 - (7i+7)bdfhkps)}, \qquad x_{14n} = \frac{a\prod_{i=0}^{n-1}(1 - (7i+6)acegjvr)}{\prod_{i=0}^{n-1}(1 - (7i+7)acegjvr)}$$

holds.

Proof. Suppose that $n > 0$ and our assumption holds for $n = 1$. That is,

$$x_{14n-27} = \frac{s\prod_{i=0}^{n-2}(1 - 7ibdfhkps)}{\prod_{i=0}^{n-2}(1 - (7i+1)bdfhkps)}, \qquad x_{14n-26} = \frac{r\prod_{i=0}^{n-2}(1 - 7iacegjvr)}{\prod_{i=0}^{n-2}(1 - (7i+1)acegjvr)},$$

$$x_{14n-25} = \frac{p\prod_{i=0}^{n-2}(1 - (7i+1)bdfhkps)}{\prod_{i=0}^{n-2}(1 - (7i+2)bdfhkps)}, \qquad x_{14n-24} = \frac{v\prod_{i=0}^{n-2}(1 - (7i+1)acegjvr)}{\prod_{i=0}^{n-2}(1 - (7i+2)acegjvr)},$$

$$x_{14n-23} = \frac{k\prod_{i=0}^{n-2}(1 - (7i+2)bdfhkps)}{\prod_{i=0}^{n-2}(1 - (7i+3)bdfhkps)}, \qquad x_{14n-22} = \frac{j\prod_{i=0}^{n-2}(1 - (7i+2)acegjvr)}{\prod_{i=0}^{n-2}(1 - (7i+3)acegjvr)},$$

$$x_{14n-21}=\frac{h\prod_{i=0}^{n-2}(1-(7i+3)bdfhkps)}{\prod_{i=0}^{n-2}(1-7i+4)bdfhkps)},\qquad x_{14n-20}=\frac{g\prod_{i=0}^{n-2}(1-(7i+3)acegjvr)}{\prod_{i=0}^{n-2}(1-(7i+4)acegjvr)},$$

$$x_{14n-19}=\frac{f\prod_{i=0}^{n-2}(1-(7i+4)bdfhkps)}{\prod_{i=0}^{n-2}(1-(7i+5)bdfhkps)},\qquad x_{14n-18}=\frac{e\prod_{i=0}^{n-2}(1-(7i+4)acegjvr)}{\prod_{i=0}^{n-2}(1-(7i+5)acegjvr)},$$

$$x_{14n-17}=\frac{d\prod_{i=0}^{n-2}(1-(7i+5)bdfhkps)}{\prod_{i=0}^{n-2}(1-(7i+6)bdfhkps)},\qquad x_{14n-16}=\frac{c\prod_{i=0}^{n-2}(1-(7i+5)acegjvr)}{\prod_{i=0}^{n-2}(1-(7i+6)acegjvr)},$$

$$x_{14n-15}=\frac{b\prod_{i=0}^{n-2}(1-(7i+6)bdfhkps)}{\prod_{i=0}^{n-2}(1-(7i+7)bdfhkps)},\qquad x_{14n-14}=\frac{a\prod_{i=0}^{n-2}(1-(7i+6)acegjvr)}{\prod_{i=0}^{n-2}(1-(7i+7)acegjvr)}.$$

Now, by using the main equation (6), one has

$$\begin{aligned}
x_{14n-13}&=\frac{x_{14n-27}}{1-x_{14n-15}x_{14n-17}x_{14n-19}x_{14n-21}x_{14n-23}x_{14n-25}x_{14n-27}}\\
&=\frac{\dfrac{s\prod_{i=0}^{n-2}(1-7ibdfhkps)}{\prod_{i=0}^{n-2}(1-(7i+1)bdfhkps)}}{1-bdfhkps\prod_{i=0}^{n-2}\frac{1-(7i+6)bdfhkps}{1-(7i+7)bdfhkps}\frac{1-(7i+5)bdfhkps}{1-(7i+6)bdfhkps}\frac{1-(7i+4)bdfhkps}{1-(7i+5)bdfhkps}\cdots\frac{1-7ibdfhkps}{1-(7i+1)bdfhkps}}\\
&=\frac{s\prod_{i=0}^{n-2}\dfrac{(1-7ibdfhkps)}{(1-(7i+1)bdfhkps)}}{1-bdfhkps\prod_{i=0}^{n-2}\dfrac{1-7ibdfhkps}{1-(7i+7)bdfhkps}}\\
&=s\prod_{i=0}^{n-2}\frac{1-7ibdfhkps}{1-(7i+1)bdfhkps}\left(\frac{1}{1-\dfrac{bdfhkps}{1-(7i-7)bdfhkps}}\right)\\
&=s\prod_{i=0}^{n-2}\frac{1-7ibdfhkps}{1-(7i+1)bdfhkps}\left(\frac{1-(7i-7)bdfhkps}{1-(7i-6)bdfhkps}\right).
\end{aligned}$$

Hence, we have

$$x_{14n-13}=s\prod_{i=0}^{n-1}\frac{1-7ibdfhkps}{1-(7i+1)bdfhkps}.$$

Similarly,

$$x_{14n-12}=\frac{x_{14n-26}}{1+x_{14n-14}x_{14n-16}x_{14n-18}x_{14n-20}x_{14n-22}x_{14n-24}x_{14n-26}}$$

$$= \frac{\prod_{i=0}^{n-2} \frac{r(1-7iacegjvr)}{1-(7i+1)acegjvr}}{1-acegjvr\prod_{i=0}^{n-2} \frac{1-(7i+6)acegjvr}{1-(7i+7)acegjvr}\frac{1-(7i+5)acegjvr}{1-(7i+6)acegjvr}\frac{1-(7i+4)acegjvr}{1-(7i+5)acegjvr}\cdots\frac{1-7iacegjvr}{1-(7i+1)acegjvr}}$$

$$= \frac{r\prod_{i=0}^{n-2} \frac{(1-7iacegjvr)}{(1-(7i+1)acegjvr)}}{1-acegjvr\prod_{i=0}^{n-2} \frac{1-7iacegjvr}{1-(7i+7)acegjvr}}$$

$$= r\prod_{i=0}^{n-2} \frac{1-7iacegjvr}{1-(7i+1)acegjvr}\left(\frac{1}{1-\dfrac{acegjvr}{1-(7i-7)acegjvr}}\right)$$

$$= r\prod_{i=0}^{n-2} \frac{1-7iacegjvr}{1-(7i+1)acegjvr}\left(\frac{1-(7i-7)acegjvr}{1-(7i-6)acegjvr}\right).$$

Therefore, we obtain

$$x_{14n-12} = r\prod_{i=0}^{n-1} \frac{1-7iacegjvr}{1-(7i+1)acegjvr}.$$

Similarly, it is easily obtained in other relationships.

Theorem 5. *Equation* (6) *has a unique equilibrium point* $\overline{x} = 0$, *which is not locally asymptotically stable*.

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted.

For confirming the outcomes of this section, we take into consideration mathematical instances which stand for various kind of solutions to equation (6).

***Example* 3.** We consider

$$x_{-13} = 3.1, \quad x_{-12} = 2.6, \quad x_{-11} = 2.5, \quad x_{-10} = 3.55, \quad x_{-9} = 2.35,$$

$$x_{-8} = 2.25, \quad x_{-7} = 2.15, \quad x_{-6} = 3.29, \quad x_{-5} = 2.13, \quad x_{-4} = 1.65,$$

$$x_{-3} = 2.57, \quad x_{-2} = 2.42, \quad x_{-1} = 2.21, \quad x_0 = 2.33.$$

According to the above initial conditions, Fig. 3 was obtained. Figure 3 shows the dynamic behavior of equation (6).

***Example* 4.** Suppose that

$$x_{-13} = 5.1, \quad x_{-12} = 2.6, \quad x_{-11} = 2.5, \quad x_{-10} = 3.55, \quad x_{-9} = 5.35,$$

$$x_{-8} = 2.25, \quad x_{-7} = 2.15, \quad x_{-6} = 3.29, \quad x_{-5} = 5.13, \quad x_{-4} = 0.3,$$

$$x_{-3} = 2.57, \quad x_{-2} = 2.42, \quad x_{-1} = 2.21, \quad x_0 = 2.33.$$

According to the above initial conditions, Fig. 4 was obtained. Figure 4 shows the dynamic behavior of equation (6).

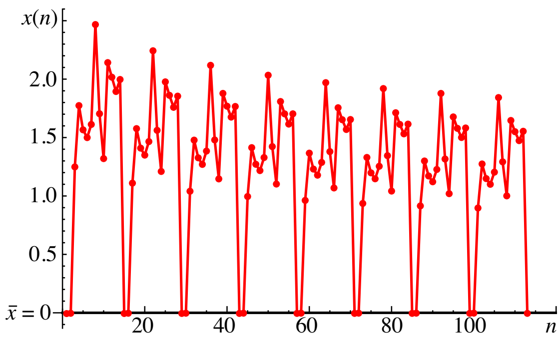

Fig. 3

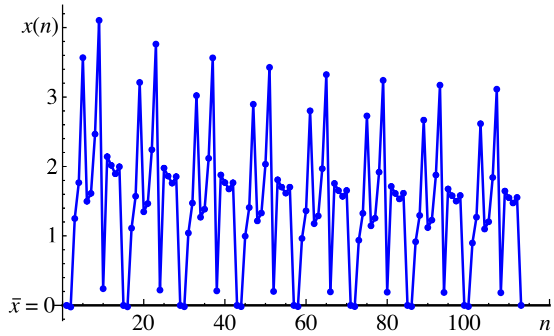

Fig. 4

4. Solution the difference equation $x_{n+1} = \dfrac{x_{n-13}}{-1+x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}$. In this case, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers

$$x_{n+1} = \frac{x_{n-13}}{-1 + x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}, \tag{7}$$

where $x_0, \ldots, x_{-13}$ defines as in (5) with

$$x_{-1}x_{-3}x_{-5}x_{-7}x_{-9}x_{-11}x_{-13} \neq 1, \qquad x_0x_{-2}x_{-4}x_{-6}x_{-8}x_{-10}x_{-12} \neq 1.$$

Theorem 6. *Every solution $\{x_n\}_{n=-13}^{\infty}$ of equation* (7) *is periodic with period twenty eight and is of the form*

$$x_{28n-27} = \frac{s}{-1+bdfhkps}, \qquad x_{28n-26} = \frac{r}{-1+acegjvr},$$

$$x_{28n-25} = p(-1+bdfhkps), \qquad x_{28n-24} = v(-1+acegjvr),$$

$$x_{28n-23} = \frac{k}{-1+bdfhkps}, \qquad x_{28n-22} = \frac{j}{-1+acegjvr},$$

$$x_{28n-21} = h(-1+bdfhkps), \qquad x_{28n-20} = g(-1+acegjvr),$$

$$x_{28n-19} = \frac{f}{-1+bdfhkps}, \qquad x_{28n-18} = \frac{e}{-1+acegjvr},$$

$$x_{28n-17} = d(-1+bdfhkps), \qquad x_{28n-16} = c(-1+acegjvr),$$

$$x_{28n-15} = \frac{b}{-1+bdfhkps}, \qquad x_{28n-14} = \frac{a}{-1+acegjvr},$$

$$x_{28n-13} = s, \qquad x_{28n-12} = r, \qquad x_{28n-11} = p,$$

$$x_{28n-10} = v, \qquad x_{28n-9} = k, \qquad x_{28n-8} = j,$$

$$x_{28n-7} = h, \qquad x_{28n-6} = g, \qquad x_{28n-5} = f,$$

$$x_{28n-4} = e, \qquad x_{28n-3} = d, \qquad x_{28n-2} = c,$$

$$x_{28n-1} = b, \qquad x_{28n} = a.$$

The solutions consist of 28 *periods.*

Proof. Suppose that

$$x_{28n-55}=\frac{s}{-1+bdfhkps},\qquad x_{28n-54}=\frac{r}{-1+acegjvr},\qquad x_{28n-53}=p(-1+bdfhkps),$$

$$x_{28n-52}=v(-1+acegjvr),\qquad x_{28n-51}=\frac{k}{-1+bdfhkps},\qquad x_{28n-50}=\frac{j}{-1+acegjvr},$$

$$x_{28n-49}=h(-1+bdfhkps),\qquad x_{28n-48}=g(-1+acegjvr),\qquad x_{28n-47}=\frac{f}{-1+bdfhkps},$$

$$x_{28n-46}=\frac{e}{-1+acegjvr},\qquad x_{28n-45}=d(-1+bdfhkps),\qquad x_{28n-44}=c(-1+acegjvr),$$

$$x_{28n-43}=\frac{b}{-1+bdfhkps},\qquad x_{28n-42}=\frac{a}{-1+acegjvr},$$

$$x_{28n-41}=s,\qquad x_{28n-40}=r,\qquad x_{28n-39}=p,$$

$$x_{28n-38}=v,\qquad x_{28n-37}=k,\qquad x_{28n-36}=j,$$

$$x_{28n-35}=h,\qquad x_{28n-34}=g,\qquad x_{28n-33}=f,$$

$$x_{28n-32}=e,\qquad x_{28n-31}=d,\qquad x_{28n-30}=c,$$

$$x_{28n-29}=b,\qquad x_{28n-28}=a.$$

Now, it follows from equation (7) that

$$x_{28n-27}=\frac{x_{28n-41}}{-1+x_{28n-29}x_{28n-31}x_{28n-33}x_{28n-35}x_{28n-37}x_{28n-39}x_{28n-41}}=\frac{s}{-1+bdfhkps}.$$

Then we have

$$x_{28n-27}=\frac{s}{bdfhkps-1}.$$

Other relation can be given by the same way.

Theorem 7. *Equation* (7) *has three equilibrium points which are* $0,\ \pm\sqrt[7]{2}$, *and these equilibrium points are not locally asymptotically stable.*

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted.

***Example* 5.** Assume that

$$x_{-13}=1.1,\quad x_{-12}=1.12,\quad x_{-11}=1.13,\quad x_{-10}=1.14,\quad x_{-9}=1.15,$$

$$x_{-8}=1.16,\quad x_{-7}=1.17,\quad x_{-6}=1.18,\quad x_{-5}=1.19,\quad x_{-4}=1.2,$$

$$x_{-3}=1.21,\quad x_{-2}=1.22,\quad x_{-1}=1.23,\quad x_{0}=1.24.$$

According to the above initial conditions, Fig. 5 was obtained. Figure 5 shows the dynamic behavior of equation (7).

***Example* 6.** Suppose that

$$x_{-13}=1.1,\quad x_{-12}=1.12,\quad x_{-11}=1.13,\quad x_{-10}=1.4,\quad x_{-9}=1.15,$$

$$x_{-8}=1.16,\quad x_{-7}=1.17,\quad x_{-6}=1.3,\quad x_{-5}=1.19,\quad x_{-4}=1.2,$$

$$x_{-3} = 1.21, \qquad x_{-2} = 1.22, \qquad x_{-1} = 1.23, \qquad x_0 = 1.5.$$

According to the above initial conditions, Fig. 6 was obtained. Figure 6 shows the dynamic behavior of equation (7).

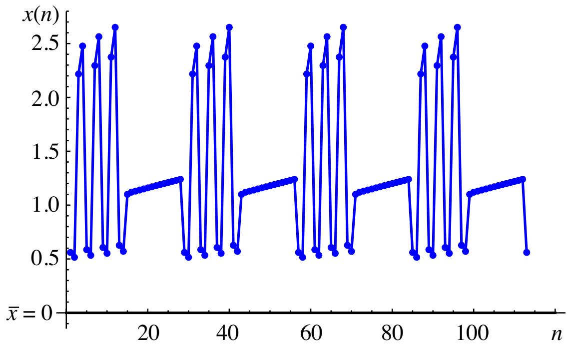

Fig. 5

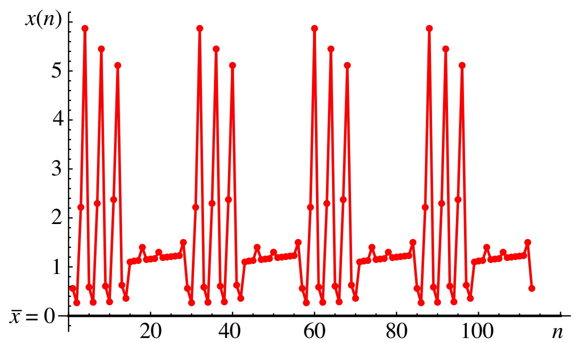

Fig. 6

5. Solution the difference equation $x_{n+1} = \dfrac{x_{n-13}}{-1 - x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}$. In this section, we give a specific form of the solutions of the difference equation below, provided that the initial conditions are arbitrary real numbers,

$$x_{n+1} = \frac{x_{n-13}}{-1 - x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}}, \tag{8}$$

where, $x_0, \ldots, x_{-13}$ defines as in (5) with

$$x_{-1}x_{-3}x_{-5}x_{-7}x_{-9}x_{-11}x_{-13} \neq -1, \qquad x_0x_{-2}x_{-4}x_{-6}x_{-8}x_{-10}x_{-12} \neq -1.$$

Theorem 8. *Every solution $\{x_n\}_{n=-13}^{\infty}$ of equation* (8) *is periodic with period twenty eight and is of the form*

$$x_{28n-27} = \frac{-s}{1 + bdfhkps}, \qquad x_{28n-26} = \frac{-r}{1 + acegjvr}, \qquad x_{28n-25} = -p(1 + bdfhkps),$$

$$x_{28n-24} = -v(1 + acegjvr), \qquad x_{28n-23} = \frac{-k}{1 + bdfhkps}, \qquad x_{28n-22} = \frac{-j}{1 + acegjvr},$$

$$x_{28n-21} = h(1 + bdfhkps), \qquad x_{28n-20} = g(1 + acegjvr), \qquad x_{28n-19} = \frac{-f}{1 + bdfhkps},$$

$$x_{28n-18} = \frac{-e}{1 + acegjvr}, \qquad x_{28n-17} = d(1 + bdfhkps), \qquad x_{28n-16} = c(1 + acegjvr),$$

$$x_{28n-15} = \frac{-b}{1 + bdfhkps}, \qquad x_{28n-14} = \frac{-a}{1 + acegjvr},$$

$$x_{28n-13} = s, \qquad x_{28n-12} = r, \qquad x_{28n-11} = p,$$

$$x_{28n-10} = v, \qquad x_{28n-9} = k, \qquad x_{28n-8} = j,$$

$$x_{28n-7} = h, \qquad x_{28n-6} = g, \qquad x_{28n-5} = f,$$

$$x_{28n-4} = e, \qquad x_{28n-3} = d, \qquad x_{28n-2} = c,$$

$$x_{28n-1} = b, \qquad x_{28n} = a.$$

The solutions consist of 28 *periods*.

Proof. Suppose that

$$x_{28n-55} = \frac{-s}{1+bdfhkps}, \qquad x_{28n-54} = \frac{-r}{1+acegjvr}, \qquad x_{28n-53} = -p(1+bdfhkps),$$

$$x_{28n-52} = -v(1+acegjvr), \qquad x_{28n-51} = \frac{-k}{1+bdfhkps}, \qquad x_{28n-50} = \frac{-j}{1+acegjvr},$$

$$x_{28n-49} = h(1+bdfhkps), \qquad x_{28n-48} = g(1+acegjvr), \qquad x_{28n-47} = \frac{-f}{1+bdfhkps},$$

$$x_{28n-46} = \frac{-e}{1+acegjvr}, \qquad x_{28n-45} = d(1+bdfhkps), \qquad x_{28n-44} = c(1+acegjvr),$$

$$x_{28n-43} = \frac{-b}{1+bdfhkps}, \qquad x_{28n-42} = \frac{-a}{1+acegjvr},$$

$$x_{28n-41} = s, \qquad x_{28n-40} = r, \qquad x_{28n-39} = p,$$

$$x_{28n-38} = v, \qquad x_{28n-37} = k, \qquad x_{28n-36} = j,$$

$$x_{28n-35} = h, \qquad x_{28n-34} = g, \qquad x_{28n-33} = f,$$

$$x_{28n-32} = e, \qquad x_{28n-31} = d, \qquad x_{28n-30} = c,$$

$$x_{28n-29} = b, \qquad x_{28n-28} = a.$$

Now, it follows from equation (8) that

$$\begin{aligned} x_{28n-27} &= \frac{x_{28n-41}}{-1 - x_{28n-29}x_{28n-31}x_{28n-33}x_{28n-35}x_{28n-37}x_{28n-39}x_{28n-41}} \\ &= \frac{s}{-1-bdfhkps}. \end{aligned}$$

Then we have

$$x_{28n-27} = \frac{-s}{bdfhkps+1}.$$

Other relations can be given by the same way.

Theorem 9. *Equation* (8) *has three equilibrium point which are* $0,\ \pm\sqrt[7]{-2}$ *and this equilibrium points is not locally asymptotically stable*.

Proof. The proof is the same as the proof of Theorem 3 and hence is omitted.

***Example* 7.** We consider

$$x_{-13} = 1.1, \quad x_{-12} = 1.12, \quad x_{-11} = 1.13, \quad x_{-10} = 1.4, \quad x_{-9} = 1.15,$$

$$x_{-8} = 1.16, \quad x_{-7} = 1.17, \quad x_{-6} = 1.3, \quad x_{-5} = 1.19, \quad x_{-4} = 1.2,$$

$$x_{-3} = 1.45, \quad x_{-2} = 1.56, \quad x_{-1} = 1.6, \quad x_0 = 1.5.$$

According to the above initial conditions, Fig. 7 was obtained. Figure 7 shows the dynamic behavior of equation (8).

***Example* 8.** Suppose that

$$
\begin{array}{lllll}
x_{-13} = 1.26, & x_{-12} = 1.32, & x_{-11} = 1.38, & x_{-10} = 1.4, & x_{-9} = 1.15, \\
x_{-8} = 1.16, & x_{-7} = 1.17, & x_{-6} = 1.3, & x_{-5} = 1.19, & x_{-4} = 1.25, \\
x_{-3} = 1.45, & x_{-2} = 1.56, & x_{-1} = 1.6, & x_0 = 1.5. &
\end{array}
$$

According to the above initial conditions, Fig. 8 was obtained. Figure 8 shows the dynamic behavior of equation (8).

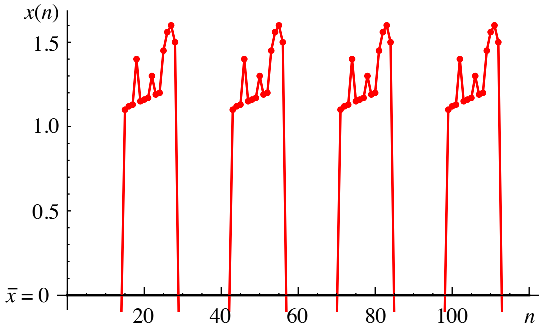

Fig. 7

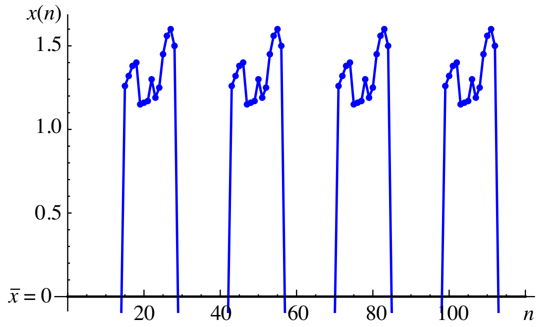

Fig. 8

6. Conclusion. We study the behavior of the difference equation

$$x_{n+1} = \frac{x_{n-13}}{\pm 1 \pm x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}x_{n-11}x_{n-13}},$$

where the initials are positive real numbers. Local stability is discussed. Moreover, we get the solution of some special cases. Finally, some numerical examples are given.

Conflict of interest. The authors declare that they have no potential conflict of interest in relation to the study in this paper.

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Author contributions. All authors have contributed equally to the work.

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