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On the recursive sequence $x_{n+1} = \frac{x_{n-(k+1)}}{1+x_n x_{n-1} \ldots x_{n-k}}$

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Abstract. In this paper a solution of the following difference equation was investigated

$$x_{n+1} = \frac{x_{n-(k+1)}}{1 + x_n x_{n-1} \ldots x_{n-k}}, \; n = 0, 1, 2, \ldots$$

where $x_{-(k+1)}, x_{-k}, \ldots, x_{-1}, x_0 \in (0, \infty)$ and $k = 0, 1, 2, \ldots$.

Key words and phrases. Difference equation, period $k+2$ solution.

1. Introduction

The study of difference equations has been growing continuously for the last decade. This is largely due to the fact that difference equations manifest themselves as mathematical models describing real life situations in probability theory, queuing theory, statistical problems, stochastic time series, combinatorial analysis, number theory, geometry, electrical network, quanta in radiation, genetics in biology, economics, psychology, sociology, etc. In fact, now it occupies a central position in applicable analysis and will no doubt continue to play an important role in mathematics as a whole. Recently there has been a lot of interest in studying the periodic nature of nonlinear difference equations. For some recent results, concerning among other problems, the periodic nature of scalar nonlinear difference equations see, for example, [1–28].

Cinar [2, 3, 4] has studied the following problems with positive initial values

$$x_{n+1} = \frac{x_{n-1}}{1 + a x_n x_{n-1}},$$

$$x_{n+1} = \frac{x_{n-1}}{-1 + a x_n x_{n-1}},$$

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$$x_{n+1} = \frac{ax_{n-1}}{1 + bx_n x_{n-1}}$$

for $n = 0, 1, 2, ...,$ respectively.

In [18] Stevic solved the following problem

$$x_{n+1} = \frac{x_{n-1}}{1 + x_n} \text{ for } n = 0, 1, 2, \dots,$$

where $x_{-1}, x_0 \in (0, \infty)$. Also, this result was generalized to the equation of the following form:

$$x_{n+1} = \frac{x_{n-1}}{g(x_n)} \text{ for } n = 0, 1, 2, \dots,$$

where $x_{-1}, x_0 \in (0, \infty)$.

Simsek et al. [19, 20, 21, 24], studied the following problems with positive initial values

$$x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}},$$

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}},$$

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-1} x_{n-3}},$$

$$x_{n+1} = \frac{x_{n-3}}{1 + x_n x_{n-1} x_{n-2}},$$

for $n = 0, 1, 2, \dots,$ respectively.

In this paper we investigate the following nonlinear difference equation

$$x_{n+1} = \frac{x_{n-(k+1)}}{1 + x_n x_{n-1} ... x_{n-k}} \text{ for } n = 0, 1, 2, \dots \tag{1.1}$$

where $x_{-(k+1)}, x_{-k}, ..., x_{-1}, x_0 \in (0, \infty)$ and $k = 0, 1, 2, \dots$.

2. Main result

Theorem 1. *Consider the difference equation* (1.1). *Then the following statements are true.*

a) *The sequences* $(x_{(k+2)n-(k+1)}), (x_{(k+2)n-k}), \dots, (x_{(k+2)n})$ *are decreasing and there exist* $a_1, a_2, \dots, a_{k+2} \geqslant 0$ *such that*

$$\lim_{n\to\infty} x_{(k+2)n-(k+1)} = a_1, \ \lim_{n\to\infty} x_{(k+2)n-k} = a_2, \dots, \ \lim_{n\to\infty} x_{(k+2)n} = a_{k+2}.$$

b) $(a_1, a_2, \ldots, a_{k+2}, a_1, a_2, \ldots, a_{k+2}, \ldots)$ *is a solution of equation* (1.1) *of period k+2.*

c) $a_1 \times a_2 \times \ldots \times a_{k+2} = 0$.
d) *If there exists* $n_0 \in \mathbb{N}$ *such that* $x_{n-k} \geq x_{n+1}$ *for all* $n \geqslant n_0$, *then*

$$\lim_{n\to\infty} x_n = 0.$$

e) *The following formulas*

$$x_{(k+2)n+1} = x_{-(k+1)} \left(1 - \tfrac{x_0 x_{-1} \ldots x_{-k}}{1+x_0 x_{-1} \ldots x_{-k}} \sum_{j=0}^{n} \prod_{i=1}^{(k+2)j} \tfrac{1}{1+x_i x_{i-1} \ldots x_{i-k}}\right),$$

$$\vdots$$

$$x_{(k+2)n+k+2} = x_0 \left(1 - \tfrac{x_{-1} x_{-2} \ldots x_{-(k+1)}}{1+x_0 x_{-1} \ldots x_{-k}} \sum_{j=0}^{n} \prod_{i=1}^{(k+2)j+(k+1)} \tfrac{1}{1+x_i x_{i-1} \ldots x_{i-k}}\right)$$

holds.

f) *If* $x_{(k+2)n+1} \to a_1 \neq 0$, $x_{(k+2)n+2} \to a_2 \neq 0$,..., $x_{(k+2)n+k+1} \to a_{k+1} \neq 0$ *then* $x_{(k+2)n+k+2} \to 0$ *as* $n \to \infty$.

Proof. **a)** Firstly, from the equation (1.1), we obtain

$$x_{n+1}(1 + x_n x_{n-1} \ldots x_{n-k}) = x_{n-(k+1)}.$$

If $x_n, x_{n-1}, \ldots, x_{n-k} \in (0, +\infty)$, then $(1 + x_n x_{n-1} \ldots x_{n-k}) \in (1, +\infty)$. Since $x_{n+1} < x_{n-(k+1)}$, $n \in N$, we obtain that $\lim_{n\to\infty} x_{(k+2)n-(k+1)} = a_1$, $\lim_{n\to\infty} x_{(k+2)n-(k)} = a_2$, ..., $\lim_{n\to\infty} x_{(k+2)n} = a_{k+2}$.

b) $(a_1, a_2, \ldots, a_{k+2}, a_1, a_2, \ldots, a_{k+2}, \ldots)$ is a solution of equation (1.1) of period $k+2$.

c) In view of the equation (1.1), we obtain:

$$x_{(k+2)n+1} = \frac{x_{(k+2)n-(k+1)}}{1 + x_{(k+2)n} \ldots x_{(k+2)n-k}}.$$

Taking limit as $n \to \infty$ on both sides of the above equality, we get:

$$\lim_{n\to\infty} x_{(k+2)n+1} = \lim_{n\to\infty} \frac{x_{(k+2)n-(k+1)}}{1 + x_{(k+2)n} \ldots x_{(k+2)n-k}},$$

$$\left(\lim_{n\to\infty} x_{(k+2)n-(k+1)}\right) \times \left(\lim_{n\to\infty} x_{(k+2)n-(k)}\right) \times ... \times \left(\lim_{n\to\infty} x_{(k+2)n}\right) = 0.$$

Then $a_1 \times a_2 \times ... \times a_{k+2} = 0$.

d) If there exists $n_0 \in \mathbb{N}$ such that $x_{n-k} \geq x_{n+1}$ for all $n \geqslant n_0$, then $a_1 \leq a_2 \leq ... \leq a_{k+2} \leq a_1$. Since $a_1 \times a_2 \times ... \times a_{k+2} = 0$, we obtain the result.

e) Subracting $x_{n-(k+1)}$ from the left and right-hand sides of equation (1.1), we obtain:

$$x_{n+1} - x_{n-(k+1)} = \frac{1}{1 + x_n x_{-1} ... x_{n-k}} (x_n - x_{n-(k+2)})$$

and for $n \geqslant 1$ the following formula

$$x_n - x_{n-(k+2)} = (x_1 - x_{-(k+1)}) \prod_{i=1}^{n-1} \frac{1}{1 + x_i x_{i-1} ... x_{i-k}} \tag{2.1}$$

holds. Replacing n by $(k+2)j$ in (2.1) and summing from $j = 0$ to $j = n$, we obtain:

$$x_{(k+2)n+1} - x_{-(k+1)} = (x_1 - x_{-(k+1)}) \sum_{j=0}^{n} \prod_{i=1}^{(k+2)j} \frac{1}{1 + x_i x_{i-1} ... x_{i-k}}, \quad (n = 0, 1, 2, ...), \quad . \quad . \quad . \tag{2.2}$$

Also, replacing n by $(k+2)j + (k+1)$ in (2.1) and summing from $j = 0$ to $j = n$, we obtain:

$$x_{(k+2)n+k+2} - x_0 = (x_{k+1} - x_0) \sum_{j=0}^{n} \prod_{i=1}^{(k+2)j+(k+1)} \frac{1}{1 + x_i x_{i-1} ... x_{i-k}},$$

$$(n = 0, 1, 2, ...).$$

From the formulas above, we obtain:

$$x_{(k+2)n+1} = x_{-(k+1)}\left(1 - \frac{x_0x_{-1}...x_{-k}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

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$$x_{(k+2)n+k+2} = x_0\left(1 - \frac{x_{-1}x_{-2}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j+(k+1)}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right)... \quad (2.3)$$

f) Suppose that $a_1 = a_2 = ... = a_{k+2} = 0$. By (e) we have:

$$\lim_{n\to\infty} x_{(k+2)n+1} = \lim_{n\to\infty} x_{-(k+1)}\left(1 - \frac{x_0x_{-1}...x_{-k}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_1 = x_{-(k+1)}\left(1 - \frac{x_0x_{-1}...x_{-k}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_1 = 0 \Rightarrow \frac{1+x_0x_{-1}...x_{-k}}{x_0x_{-1}...x_{-k}} = \sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j}\frac{1}{1+x_ix_{i-1}...x_{i-k}}. \quad (2.4)$$

Similarly,

$$\lim_{n\to\infty} x_{(k+2)n+2} = \lim_{n\to\infty} x_{-(k)}\left(1 - \frac{x_0x_{-1}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j+1}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_2 = x_{-(k)}\left(1 - \frac{x_0x_{-1}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+1}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_2 = 0 \Rightarrow \frac{1+x_0x_{-1}...x_{-k}}{x_0x_{-1}...x_{-(k+1)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+1}\frac{1}{1+x_ix_{i-1}...x_{i-k}}, \quad (2.5)$$

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Similarly,

$$\lim_{n\to\infty} x_{(k+2)n+k+1} = \lim_{n\to\infty} x_{-1}\left(1 - \frac{x_0x_{-2}x_{-3}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j+k}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_{k+1} = x_{-1}\left(1 - \frac{x_0x_{-2}x_{-3}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+k}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_{k+1} = 0 \Rightarrow \frac{1+x_0x_{-1}...x_{-k}}{x_{-2}x_{-3}...x_{-(k+1)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+k}\frac{1}{1+x_ix_{i-1}...x_{i-k}}. \quad (2.6)$$

$$\lim_{n\to\infty} x_{(k+2)n+k+2} = \lim_{n\to\infty} x_0\left(1 - \frac{x_{-1}x_{-2}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{n}\prod_{i=1}^{(k+2)j+(k+1)}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_{k+2} = x_0\left(1 - \frac{x_{-1}x_{-2}...x_{-(k+1)}}{1+x_0x_{-1}...x_{-k}}\sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+(k+1)}\frac{1}{1+x_ix_{i-1}...x_{i-k}}\right),$$

$$a_{k+2} = 0 \Rightarrow \frac{1+x_0x_{-1}...x_{-k}}{x_{-1}x_{-2}...x_{-(k+1)}} = \sum_{j=0}^{\infty}\prod_{i=1}^{(k+2)j+(k+1)}\frac{1}{1+x_ix_{i-1}...x_{i-k}}. \quad (2.7)$$

From (2.4) and (2.5), we get:

$$\frac{1+x_0x_{-1}...x_{-k}}{x_0x_{-1}...x_{-k}} > \frac{1+x_0x_{-1}...x_{-k}}{x_0x_{-1}...x_{-(k+1)}},$$

thus $x_{-(k+1)} > x_{-k}$, and

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Therefore, from (2.6) and (2.7), we have:

$$\frac{1+x_0x_{-1}...x_{-k}}{x_0x_{-2}...x_{-(k+1)}} > \frac{1+x_0x_{-1}...x_{-k}}{x_{-1}x_{-2}...x_{-(k+2)}},$$

thus, $x_{-1} > x_0$.

From here we obtain $x_{-(k+1)} > x_{-k} > ... > x_{-1} > x_0$. We arrive at a contradiction which completes the proof of theorem. □

3. Examples

Consider the following equation $x_{n+1} = \frac{x_{n-3}}{1+x_n x_{n-1} x_{n-2}}$ which is a special case of (1.1) for $k = 2$.

Example 1. *If the initial conditions are selected as follows:* $x[-3] = 0.99999$; $x[-2] = 0.99998$; $x[-1] = 0.99997$; $x[0] = 0.99996$. *The following solutions are obtained:*

$$x(n) = \{0.500018, 0.666661, 0.74998, 0.799968, 0.357163, 0.549016, 0.648287, 0.709744, 0.285135, 0.485341, 0.590307, 0.656143, 0.240015, 0.44406, 0.551724, 0.619703, 0.208378, 0.414527, 0.523691, 0.592883, 0.184617, 0.392054, 0.502143, 0.572091, 0.165929, 0.374216, 0.484917, 0.555368, 0.150738, 0.359617, 0.470745, 0.541549, 0.138079, 0.347389, 0.458826, 0.529887, 0.127325, 0.336958, 0.448627, 0.519881, 0.118048, 0.327929, 0.439777, 0.511178, 0.109943, 0.32002, 0.432007, 0.503525, 0.102788, 0.313021, 0.42512, 0.49673, 0.0964144, 0.306775, 0.418964, 0.49065, 0.090695, 0.30116, 0.413424, 0.485172, 0.0855285, 0.296081, 0.408406, 0.480205, 0.0808346, 0.291461, 0.403837, 0.475679, 0.0765488, 0.287237, 0.399657, 0.471536, 0.0726179, 0.283359, 0.395817, 0.467726, 0.0689983, 0.279785, 0.392275, 0.464211, 0.0656534, 0.27648, 0.388997, 0.460956, 0.0625523, 0.273413, 0.385954, 0.457933, 0.0596689, 0.27056, 0.383122, 0.455118, 0.0569808, 0.267898, 0.380478, 0.45249, 0.0544686, 0.265409, 0.378006, 0.450031, 0.0521156, 0.263077, 0.375688, 0.447725, 0.0499071, 0.260887, 0.37351, 0.445558, 0.0478305, 0.258826, 0.371461, 0.443519, 0.0458743, 0.256885, 0.36953, 0.441596, 0.0440287, 0.255052, 0.367707, 0.43978, 0.0422847, 0.25332, 0.365983, 0.438062, 0.0406344, 0.251681, 0.36435, 0.436436, 0.0390707, 0.250127, 0.362803, 0.434894, 0.0375873, 0.248652, 0.361334, 0.43343, \ldots\}.$$

The limits of solutions approaching zero due to the fact that the initial condition is selected. The graph of the solutions is given below:

Example 2. *If the initial conditions are selected as follows:* $x[-3] = 0.1$; $x[-2] = 0.09$; $x[-1] = 0.08$; $x[0] = 0.07$; *The following solutions*

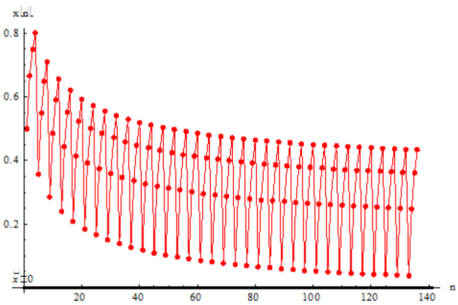

Figure 3.1. x(n) graph of the solutions.

are obtained:

$x(n) = \{$0.0999496, 0.0899497, 0.0799497, 0.0699497, 0.0998994, 0.0898994, 0.0798995, 0.0698996, 0.0998492, 0.0898493, 0.0798494, 0.0698495, 0.0997992, 0.0897993, 0.0797995, 0.0697996, 0.0997493, 0.0897495, 0.0797496, 0.0697498, 0.0996996, 0.0896997, 0.0796999, 0.0697001, 0.0996499, 0.0896501, 0.0796503, 0.0696506, 0.0996004, 0.0896006, 0.0796008, 0.0696011, 0.099551, 0.0895512, 0.0795515, 0.0695518, 0.0995017, 0.0895019, 0.0795022, 0.0695026, 0.0994525, 0.0894528, 0.0794531, 0.0694535, 0.0994034, 0.0894037, 0.0794041, 0.0694045, 0.0993544, 0.0893548, 0.0793552, 0.0693557, 0.0993056, 0.089306, 0.0793064, 0.0693069, 0.0992569, 0.0892573, 0.0792578, 0.0692583, 0.0992083, 0.0892087, 0.0792092, 0.0692098, 0.0991598, 0.0891602, 0.0791608, 0.0691614, 0.0991114, 0.0891119, 0.0791124, 0.0691131, 0.0990631, 0.0890637, 0.0790642, 0.0690649, 0.099015, 0.0890155, 0.0790161, 0.0690168, 0.0989669, 0.0889675, 0.0789681, 0.0689689, 0.098919, 0.0889196, 0.0789203, 0.068921, 0.0988712, 0.0888718, 0.0788725, 0.0688733, 0.0988235, 0.0888241, 0.0788248, 0.0688257, 0.0987759, 0.0887766, 0.0787773, 0.0687782, 0.0987284, 0.0887291, 0.0787299, 0.0687308, 0.098681, 0.0886817, 0.0786825, 0.0686835, 0.0986337, 0.0886345, 0.0786353, 0.0686363, 0.0985866, 0.0885874, 0.0785882, 0.0685892, 0.0985395, 0.0885403, 0.0785412, 0.0685422, 0.0984926, 0.0884934, 0.0784943, 0.0684954, 0.0984457, 0.0884466, 0.0784475, 0.0684486, 0.098399, 0.0883999, 0.0784009, 0.068402, 0.0983524, 0.0883533, 0.0783543, 0.0683554, $\ldots\}$.

In this case the limits of solutions are not approaching zero, since the initial condition are hasn't satisfied. The graph of the solutions is given below:

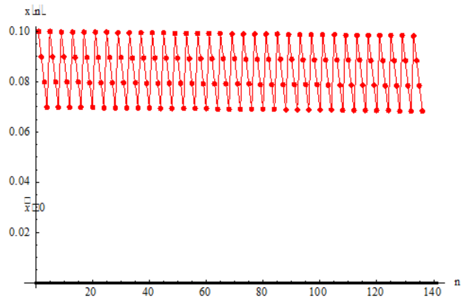

Figure 3.2. $x(n)$ graph of the solutions.

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