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APPLICATION OF THE GENERALIZED ABSORPTANCE FOR ACCOUNTING THE RECOIL AND DOPPLER EFFECTS

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A method of calculation of the absorption coefficient in the terms of quantum optics with regard for the quantization of the electromagnetic field and the Doppler effect is presented. It is shown that the local value of the absorption coefficient non-linearly depends on the atomic density and initial intensity. The analytically derived results are demonstrated in graphs for the local absorption coefficient as a function of the frequency. The relatively strong dependence of the absorptance on the path length of an optical light beam is caused by the interatomic coupling through the intermediary of an electromagnetic field. The splitting of the absorption line induced by the Doppler effect in the system placed between mirrors is demonstrated.

Keywords: absorption coefficient, quantum optics, Doppler effect, commutation relation, approximate evaluation.

1. Introduction

We present an example of the analytical evaluation of the quantum optically defined absorption coefficient with regard for the Doppler effect. As was shown in the previous work [1], the absorption coefficient can be defined by the commutation relation between the model Hamiltonian and the operator of intensity. The estimation technique for the averaged commutators is demonstrated below.

In comparison with other models (e.g., in works [2–5]), we use a time-independent model Hamiltonian formally allowing us to represent the general solution in a fairly compact form without discussing the chronology of events in the system. In addition, the “single-atom” methods such as the equation of motion for the one-particle population matrix applied to find the number of quanta imbibed by atoms or liberated into the environment per unit time are not used here. On certain assumptions, it is

enough to know the information about the distribution of states of the system at the given initial moment of time for the calculation of the coefficient of absorption.

The presented calculations will demonstrate the alternative method of estimation for the line broadening arising from the photon exchange between atoms ([6–12]) and the ensemble statistic properties in comparison with the commonly used method of solving the corresponding integro-differential form of the Maxwell–Bloch equations (see, e.g., [13]). The developed method can be used in quantum nonlinear optics (e.g., [14]) for the description of the spectral distributions of the transmitted and reflected signals modified by the dynamical Lorentz shift (see [13]) and the non-stationary pump-probe interaction between short laser pulses propagating in a resonant optically dense coherent medium ([15]), when nonlinear collective response is formed by light wave packets of different group velocities under the interatomic interaction, etc.

2. Model

Let, during the infinitesimal time intervals, the state function of N atoms occupying the volume V acquire the infinitesimal change determined by the following Hamiltonian:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}^A + \hat{\mathcal{H}}^F + \hat{\mathcal{H}}^{AF}. \quad (1)$$

Here, the first term acting on the eigenvector $|\varepsilon, \mathbf{r}, \alpha\rangle_i$ gives the sum of translational kinetic ε_i and intrinsic $\hbar\omega_{\alpha_i}$ energies for the i -th atom. The quantum of energy equals the energy of a valent electron (electrons) on the energy level indexed by the symbol α_i , $i = 1, \dots, N$ ($\alpha = b$ or a). The second term represents the energy of the free electromagnetic field in the volume V_{el} . The third term is the energy of interaction between the atoms and the electromagnetic field expressed by the standard dipole-field coupling

$$\hat{\mathcal{H}}^{AF} = - \sum_{i=1}^N \hat{d}_i \cdot \mathcal{E}(\mathbf{r}_i),$$

where the dipole moment operator for the i -th atom is denoted by \hat{d}_i .

The electromagnetic field $\mathcal{E}(\mathbf{r}_i)$ can be defined through the superposition of its modes, q , determined by the corresponding Fock states $|n_q\rangle$. The modes of the electromagnetic field correspond to Fock states (or number states with a well-defined number of quanta) formally obtained through the standard procedure of quantization in the volume $V_{\text{el}} > V$. Then the total “free” field energy can be represented through the sum of corresponding quanta of energy. Note that the symbol q denotes the mode of the electromagnetic field with the propagation wave vector \mathbf{k} and the j -th polarization with $j = 1, 2$ (see details, e.g., in [16]). Accordingly, the Fock states denoted by $|n_q\rangle$ are the eigenvectors of the corresponding number operators. Recall that the state $|n_q\rangle$ determines the number of photons n corresponding to the mode q .

Therefore, we can introduce a pure quantum state of the system as a superposition of the basis states $\prod_{i=1}^N |\varepsilon, \mathbf{r}, \alpha\rangle_i \prod_q |n_q\rangle$ with $q \equiv (\mathbf{k}, j)$, and $\{\} \equiv \{\varepsilon\}, \{\alpha\}, \{n_q\}$. Note that the number of Fock states (photons) indexed by n_q is not limited above.

In the case of the Hamiltonian independent of time, in accordance with the accepted model of a device counting photons (see, e.g., [16] and [17]), the local

absorption coefficient can be defined as follows:

$$\alpha_{\text{tot}} \approx \frac{\delta}{\delta z} \ln \sum_{\Psi} \langle \Psi |_0 \left(\hat{\mathcal{I}} + \frac{i}{\hbar} \tau \left[\hat{\mathcal{H}}, \hat{\mathcal{I}} \right] + \frac{1}{2} \left(\frac{i}{\hbar} \tau \right)^2 \left[\hat{\mathcal{H}}, \left[\hat{\mathcal{H}}, \hat{\mathcal{I}} \right] \right] \right) \hat{\rho}_{\Psi} | \Psi \rangle_0, \quad (2)$$

where $\hat{\mathcal{I}} = \mathcal{E}_+ \mathcal{E}_-$ with $\mathcal{E}_+(\mathbf{r}) = \sum_q \hat{e}_q \mathcal{E}_q e^{-i\mathbf{k}_q \cdot \mathbf{r}} a_q^\dagger$ and $\mathcal{E}_-(\mathbf{r}) = \sum_{q'} \hat{e}_{q'} \mathcal{E}_{q'} e^{i\mathbf{k}_{q'} \cdot \mathbf{r}} a_{q'}$. It is assumed that the optical pathway inside the sample is parallel to the Z -axis; therefore, $r = z$. The statistical distribution over the pure states of the system $|\Psi\rangle$ at the initial moment of time $t = 0$ is given by the operator $\hat{\rho}_{\Psi}$. The symbol $|\Psi\rangle_0$ implies the value of state function at the initial moment of time. Note that the statistical operator $\hat{\rho}_{\Psi}$ can be put, in general, in front of the exponential function $e^{-\frac{i}{\hbar} \hat{\mathcal{H}} t}$ and, therefore, can be discussed as a function of the time t . Then expression (2) has to be considered as approximate over a given interval of time τ in comparison with the commonly used representation of the operator of statistical distribution.

3. Example of Evaluation for the Absorption Coefficient

In what follows, we discuss the averaged commutator of the operators associated with the atom-field interaction and the intensity. After the substitution of the value of state function $|\Psi\rangle$ at the initial moment of time, we have

$$\sum_{\Psi} \langle \Psi |_0 \left[\hat{\mathcal{H}}'^{AF}, \hat{\mathcal{I}} \right] \hat{\rho}_{\Psi} | \Psi \rangle_0 \rightarrow \langle \bar{\Psi} | \hat{Y} | \bar{\Psi} \rangle, \quad (3)$$

where $\langle \bar{\Psi} | = \sum_{\{\}} C_{\{\}}^* (0) \prod_{i'=1}^N \langle \varepsilon, \mathbf{r}, \alpha' |_{i'} \prod_{q'} \langle n_{q'} |$, and

$$\hat{Y} = \sum_{i=1}^N \mathcal{F} \hat{\mathbf{S}}_i \cdot \sum_{q, \bar{q}} \hat{e}_q \mathcal{E}_q^2 (\hat{e}_q \cdot \hat{e}_{\bar{q}}) \mathcal{E}_{\bar{q}} \times \left(e^{i(\mathbf{k}_{\bar{q}} - \mathbf{k}_q) \cdot \mathbf{r}} e^{i\mathbf{k}_q \cdot \mathbf{r}_i} a_{\bar{q}} - A d j \right). \quad (4)$$

Here, the symbol $\mathcal{F} \hat{\mathbf{S}}_i$ redefines the dipole moment operator \hat{d}_i indicating the necessity to consider the corresponding Doppler frequency shift in the dipole-field coupling.

We now make certain guesses. At the initial moment of time, the system states are assumed to be thermally distributed. Therefore, the following approximations take place. First, the splitting

$$C_{\{\}}(0) = C_{\{\varepsilon\}, \{\alpha\}}(0) C_{\{n_q\}}(0); \quad C_{\{n_q\}} = C_{n_q} \dots C_{n'_q} \dots (5)$$

with the initial thermal distribution of photons in the volume V_{el} given by

$$\mathcal{C}_{n_q}^*(0)\mathcal{C}_{n_q}(0) = \rho_q \left[1 - e^{-\hbar\nu_k/k_B T_s} \right] e^{-n\hbar\nu_k/k_B T_s}, \quad (6)$$

where ρ_q is some distribution of frequencies modeling the density of states for a light beam passed through a monochromator. Note that, in our case, the distribution describes the light beam with a quite narrow frequency band and the linear polarization. Accordingly, we accept that the introduced “weighted” coefficients \mathcal{C}_{n_q} are real. Hence,

$$\mathcal{C}_{n_q}(0) = \left(\mathcal{C}_{n_q}^*(0)\mathcal{C}_{n_q}(0) \right)^{1/2}. \quad (7)$$

Neglecting the possibility of the existence of atomic bounded states and the difference between particles, the initial distribution of atoms over kinetic energies takes the form of the Maxwell–Boltzmann distribution

$$\sum'_{\{\varepsilon\},\{\alpha\}} \sum_{i=1}^N \mathcal{C}_{\{\varepsilon\},\{\alpha\}}^*(0)\mathcal{C}_{\varepsilon,\{\alpha\}}(0) \rightarrow N\mathcal{C}_\varepsilon^*(0)\mathcal{C}_\varepsilon(0), \quad (8)$$

where $\mathcal{C}_\varepsilon^*(0)\mathcal{C}_\varepsilon(0) \sim \exp(-\varepsilon/k_B T)$, and the prime sign above the summation \sum' means that the summation is made over all states having energy ε for a particle. In other words, the summation is performed excluding, for instance, the j -th atom

$$\sum'_{\{\varepsilon\}} = \sum_{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{j-1}, \varepsilon_{j+1}, \dots, \varepsilon_N}.$$

Accordingly, this approximation assigns that $\mathcal{C}_\varepsilon(0) = \left(\frac{1}{Z}\right)^{1/2} \exp(-\varepsilon/2k_B T)$. Here, in the case of transition from the “discrete” summation to the integration over the space of translational momenta (formally, the states with the energy $\varepsilon = \mathbf{p}^2/(2M)$ are degenerate with respect to the direction of the momentum \mathbf{p}), the normalization factor Z is set to be $\frac{V}{(2\pi\hbar)^3} (2\pi M k_B T)^{3/2}$.

In addition, such splitting as $\mathcal{C}_{\varepsilon,\alpha} = \mathcal{C}_\varepsilon \mathcal{C}_\alpha$ is valid. Moreover, the following approximation holds for such values of the source temperature T_S that $k_B T_S \leq 1.2\hbar\omega_0$ with maximum relative error less than 0.25 at $k_B T_S = 1.2\hbar\omega_0$:

$$\mathcal{C}_a^* \mathcal{C}_b = e^{-\frac{\hbar\omega_0}{2k_B T_S}} \sqrt{1 - e^{-\frac{\hbar\omega_0}{k_B T_S}}} \approx e^{-\frac{\hbar\omega_0}{2k_B T_S}}. \quad (9)$$

Suppose now that the initial state of the field is completely defined by the wave vector \mathbf{k} directed along the symmetry axis of the volume filled by atoms. A correlation between the field and atoms can be neglected at the initial moment of time.

Recall that, in our notation, the introduced state vectors such as $|\mathbf{r}\rangle_i$ are the eigenvectors of the free particle Hamiltonian operator (do not confuse with the eigenvectors for the coordinate operator $\hat{\mathbf{r}}$) and, therefore, can be represented through the corresponding wavefunction in the form $|\mathbf{r}\rangle_i \stackrel{\text{def}}{=} \frac{1}{\sqrt{V}} e^{\frac{i}{\hbar} \mathbf{p}_i \cdot \mathbf{r}_i}$. Then, in the calculation of the average quantity of the commutator, we use the following property:

$$\frac{1}{V} \int d\mathbf{r}_i e^{i\mathbf{k}_q \cdot \mathbf{r}_i} e^{\frac{i}{\hbar} \mathbf{p}_i \cdot \mathbf{r}_i} e^{-\frac{i}{\hbar} \mathbf{p}'_i \cdot \mathbf{r}_i} = \delta_{\mathbf{p}'_i, \mathbf{p}_i + \hbar\mathbf{k}}. \quad (10)$$

In the volume V , the momentum of the i -th particle can have the values $\mathbf{p}_i = \mathbf{p}_n = \frac{2\pi\hbar}{\sqrt{V}^{1/3}} \mathbf{n}$, where \mathbf{n} is the collection of three natural numbers $\mathbf{n} = (n_1, n_2, n_3)$. In the below-given expression, it is assumed for simplicity that the dipole matrix elements introduced in the operator \hat{S}_i for $i = 1, \dots, N$ during the transitions between the two energy levels b and a (or vice versa) follow the direction parallel to the polarization vector \hat{e}_q of the emitted or absorbed quantum of energy. This is in compliance with the classical sense of the irradiation by a uniform linear antenna (and the law of conservation of the angular momentum). In view of the indistinguishability of the particles of one kind (in unbound states or bound states), the notation $|S|$ is therefore used for the inner product $\hat{S}_i \cdot \hat{e}_q$ in the representation of averages below.

Thus, using the approximations defined in (5) - (9), the averaged commutator (3) can be represented in the following way:

$$\sum_{\Psi} \langle \Psi | (0) \left[\hat{\mathcal{H}}'^{AF}, \hat{\mathcal{I}} \right] \hat{\rho}_{\Psi} | \Psi \rangle (0) \cong N (I_1 I_2 - I_{1-} I_{2-}). \quad (11)$$

Here,

$$I_1 = |S| \sum_{\varepsilon', \varepsilon, \omega} \delta_{\varepsilon', \varepsilon + \Delta\varepsilon} \delta_{\omega, \omega_0 + \Delta\omega} \delta_{\mathbf{p}', \mathbf{p} + \hbar\mathbf{k}_\omega} \times \mathcal{C}_{\varepsilon', a}^*(0) \mathcal{C}_{\varepsilon, b}(0) \mathcal{E}_\omega^2 e^{-i\mathbf{k}_\omega \cdot \mathbf{r}}; \quad (12)$$

$$I_{1-} = |S| \sum_{\varepsilon', \varepsilon, \omega} \delta_{\varepsilon', \varepsilon - \Delta\varepsilon} \delta_{\omega, \omega_0 + \Delta\omega} \delta_{\mathbf{p}', \mathbf{p} - \hbar\mathbf{k}_\omega} \times \mathcal{C}_{\varepsilon', b}^*(0) \mathcal{C}_{\varepsilon, a}(0) \mathcal{E}_\omega^2 e^{i\mathbf{k}_\omega \cdot \mathbf{r}}, \quad (13)$$

where $\delta_{\varepsilon', \varepsilon + \Delta\varepsilon}$, $\delta_{\omega, \omega_0 + \Delta\omega}$ are the Kronecker deltas with the defined variables: $\mathbf{p} \pm \hbar\mathbf{k} = \mathbf{p}'$ with plus sign for absorption and minus for emission, $\Delta\varepsilon = \hbar\omega_0 \frac{\hat{\mathbf{k}} \cdot \mathbf{p}}{Mc}$, $\Delta\omega \simeq \left(2 \frac{\hat{\mathbf{k}} \cdot \mathbf{p}}{Mc}\right) \omega_0$;

$$I_2 = \sum_{n_q} C_{n_q-1}^*(0) C_{n_q}(0) \mathcal{E}_q \sqrt{n_q} e^{i\mathbf{k}_q \cdot \mathbf{r}}, \quad (14)$$

$$I_{2-} = \sum_{n_q} C_{n_q+1}^*(0) C_{n_q}(0) \mathcal{E}_q \sqrt{n_q + 1} e^{-i\mathbf{k}_q \cdot \mathbf{r}}. \quad (15)$$

Let us start with the evaluation of the first kind of sums, I_1 and I_{1-} . After the limit transition from the summation to the integration over the energies or momenta, we derive the following expression $I_1 \equiv I_1(\mathcal{K}, \hat{\mathbf{k}})$:

$$I_1(\mathcal{K}, \hat{\mathbf{k}}) \approx K_1 \frac{1}{(\pi a)^{3/2}} e^{-\frac{\hbar\omega_0}{2k_B T_s}} e^{-i\frac{\omega_0}{c} \mathbf{r} \cdot \hat{\mathbf{k}}} \times \\ \times \int d\mathbf{p} \omega_0 \left(1 + 2 \frac{\hat{\mathbf{k}} \cdot \mathbf{p}}{Mc}\right) e^{-\frac{p^2}{a}} e^{-\mathcal{K} \mathbf{p} \cdot \hat{\mathbf{k}}}, \quad (16)$$

where $K_1 = \frac{\hbar|S|}{2\varepsilon_0 V_{\text{el}}}$, $a = 2Mk_B T$, $\mathcal{K} = \frac{\hbar\omega_0}{Mc} b + 2i\omega_0 \frac{(\mathbf{r} \cdot \hat{\mathbf{k}})}{Mc^2}$, and $\mathcal{K}_- = \frac{\hbar\omega_0}{Mc} b_- - 2i\omega_0 \frac{(\mathbf{r} \cdot \hat{\mathbf{k}})}{Mc^2}$ with $b = \frac{1}{2k_B T} + 2 \frac{1}{2k_B T_s}$ and $b_- = -\frac{1}{2k_B T} + 2 \frac{1}{2k_B T_s}$.

It can be shown that the above-introduced integral I_1 has the following value:

$$I_1 = K_1 \omega_0 \left[1 - \mathcal{K} \frac{a}{Mc}\right] e^{-\frac{\hbar\omega_0}{2k_B T_s}} e^{-i\frac{\omega_0}{c} \mathbf{r} \cdot \hat{\mathbf{k}}} e^{\frac{a}{4} \mathcal{K}^2}; \quad (17)$$

$$I_{1-} \equiv I_1(\mathcal{K}_-, -\hat{\mathbf{k}}).$$

The second-kind integrals appearing in presentation (3) are denoted by I_2 and I_{2-} . Below, we demonstrate how the values of the integrals can be estimated. First, we make the limit transition from summation to integration:

$$I_2 \simeq \frac{\pi^{1/2}}{2} \frac{V_{\text{el}}}{(2\pi\hbar)^3} \left(\frac{1}{2\varepsilon_0 V_{\text{el}}}\right)^{1/2} \times \\ \times \int d\mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} (\hbar\nu)^{\frac{1}{2}} \left\{1 - e^{-\frac{\hbar\nu}{k_B T_s}}\right\} \left(\frac{k_B T_s}{\hbar\nu}\right)^{3/2} \rho(\mathbf{k}). \quad (18)$$

Suppose that, for a defined direction of propagation $\hat{\mathbf{k}}$, the initial distribution of the modes over their directions of propagation and frequencies can be **interpolated** by the normal (or Gaussian) distribution \mathcal{N} with the mean $\bar{\nu}$ and its variance $\Delta\nu^2$:

$$\rho(\mathbf{k}) \equiv \text{Norm} \mathcal{N}(\bar{\nu}, \Delta\nu^2) \delta(\hat{\sigma} - \hat{\mathbf{k}}), \quad (19)$$

where $\hat{\sigma}$ is a unit vector, $\delta(\hat{\sigma} - \hat{\mathbf{k}})$ is the Dirac delta function for the solid angle (being a certain limit of the corresponding Gaussian distribution). By definition, we have

$$\mathcal{N}(\bar{\nu}, \Delta\nu^2) = \frac{1}{\sqrt{2\pi\Delta\nu^2}} e^{-\frac{(\nu - \bar{\nu})^2}{2\Delta\nu^2}}. \quad (20)$$

The constant Norm is defined to normalize the transition from the summation over modes q and quantum numbers n_q to the integration over frequencies and numbers for the probability to find the system in any state $|n_q\rangle$: $\sum_{n_q} C_{n_q}^*(0) C_{n_q}(0) \rightarrow \int d\nu dn \dots = 1$. So that

$$\text{Norm} = \frac{(2\pi\hbar)^3}{V_{\text{el}}} \frac{c^3}{\bar{\nu}^2 + \Delta\nu^2}. \quad (21)$$

Note that if $\left(\frac{\hbar\nu}{k_B T_s} < 1\right)$, then $1 - e^{-\frac{\hbar\nu}{k_B T_s}} \approx \frac{\hbar\nu}{k_B T_s}$ with an accuracy of $\frac{1}{2} \left(\frac{\hbar\nu}{k_B T_s}\right)^2$ as for the alternating-sign Taylor series. Let the frequency distribution at the initial time moment be quite narrow, so that $\bar{\nu}/\sqrt{2\pi\Delta\nu^2} \ll 10^{-5}$ for the beam with the mean frequency satisfying $\left(\frac{\hbar\nu}{k_B T_s} < 1\right)$. Then this approximation can be used in the integral over the frequency below in the text.

Hence, expression (14) can be represented or, in other words, interpolated by the integration over the continuum in the following way:

$$I_2 \approx K_2 \int_0^\infty d\nu \nu^2 e^{i\nu \hat{\mathbf{k}} \cdot \frac{\mathbf{r}}{c}} \mathcal{N}(\bar{\nu}, \Delta\nu^2). \quad (22)$$

Here,

$$K_2 = \frac{1}{\bar{\nu}^2 + \Delta\nu^2} \left(\frac{\pi k_B T_s}{8\varepsilon_0 V_{\text{el}}}\right)^{1/2}. \quad (23)$$

For the condition $\frac{(\nu - \bar{\nu})^2}{2\Delta\nu^2} \ll 1$ that is valid for a monochromator, we approximate the above expression in such a way that

$$\int_0^\infty d\nu \nu^2 e^{i\nu \hat{\mathbf{k}} \cdot \frac{\mathbf{r}}{c}} \mathcal{N}(\bar{\nu}, \Delta\nu^2) \simeq \frac{\partial^2}{\partial(\hat{\mathbf{k}} \cdot \frac{\mathbf{r}}{c})^2} F(\mathcal{N}(\bar{\nu}, \Delta\nu^2)), \quad (24)$$

where $F(\mathcal{N}(\bar{\nu}, \Delta\nu^2))$ denotes the one-dimensional Fourier transform of the Gaussian distribution with

the parameter $-\hat{k} \cdot \frac{\mathbf{r}}{c}$. Using the known Fourier transform of the normal distribution, we have

$$F(\mathcal{N}(\bar{\nu}, \Delta\nu^2)) = e^{i\hat{k} \cdot \frac{\mathbf{r}}{c} \bar{\nu}} e^{-\frac{\Delta\nu^2 (\hat{k} \cdot \frac{\mathbf{r}}{c})^2}{2}}. \quad (25)$$

Thus,

$$I_2 = \Phi \left[- \left(-i\bar{\nu} + \Delta\nu^2 \left(\hat{k} \cdot \frac{\mathbf{r}}{c} \right)^2 + \Delta\nu^2 \right), \quad (26)$$

where $\Phi = K_2 e^{i\hat{k} \cdot \frac{\mathbf{r}}{c} \bar{\nu}} e^{-\frac{\Delta\nu^2 (\hat{k} \cdot \frac{\mathbf{r}}{c})^2}{2}}$.

Inasmuch as, in our approximation, the coefficients \mathcal{C}_{nq} are real, expression (15) can therefore be defined through expression (22) with the parameter of negative sign: $I_{2-} = I_2^*$.

Having the estimated commutator (3), the other commutators can also be found. For example, by analogy with the provided above, we have

$$\begin{aligned} \sum_{\Psi} \langle \Psi | (0) \left[\hat{\mathcal{H}}'^{AF}, \left[\hat{\mathcal{H}}'^{AF}, \hat{\mathcal{I}} \right] \right] \hat{\rho}_{\Psi}(0) | \Psi \rangle &\approx \\ \approx -N I_3 - 2N(N-1) I_1 I_{1-}, &\quad (27) \end{aligned}$$

where the first sum in (27) can be represented in the limit of continuous distribution as follows:

$$I_3 = \left(\frac{\hbar\omega_0}{2\varepsilon_0 V} \right)^2 |S|^2 \left(1 + \frac{2}{M^2 c^2 a} \right). \quad (28)$$

Then

$$\begin{aligned} \sum_{\Psi} \langle \Psi | (0) \left[\hat{\mathcal{H}}^F, \left[\hat{\mathcal{H}}'^{AF}, \hat{\mathcal{I}} \right] \right] \hat{\rho}_{\Psi}(0) | \Psi \rangle &\approx \\ \approx -N(I_1 I_2' + I_{1-} I_{2-}'), &\quad (29) \end{aligned}$$

where, by definition, $I_{2-}' = (I_2')^*$ and

$$\begin{aligned} I_2' &= -i\hbar\Phi \left[-i\bar{\nu} + \Delta\nu^2 \left(\hat{k} \cdot \frac{\mathbf{r}}{c} \right)^2 \right] \times \\ &\times \left\{ \left[-i\bar{\nu} + \Delta\nu^2 \left(\hat{k} \cdot \frac{\mathbf{r}}{c} \right)^2 \right]^2 - 3\Delta\nu^2 \right\}. \quad (30) \end{aligned}$$

Thus, substituting all above-found averaged commutators into the averaged series (2) representing the expression under the sign of logarithm in (2), the total absorption coefficient takes the following approximate form:

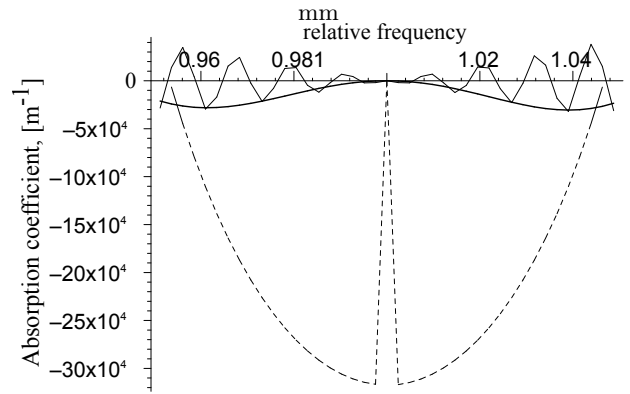
$$\alpha_{\text{tot}} \approx \Re \left\{ \frac{\partial}{\partial z} \ln \left(I(0) + \frac{i}{\hbar} \tau N I^{(1)}(\mathbf{r}) - \right. \right.$$

$$\left. \left. - \frac{1}{2} \left(\frac{i}{\hbar} \tau \right)^2 N I^{(2)}(\mathbf{r}) \right) \right\}, \quad (31)$$

where \Re denotes the real part of a quantity. The initial intensity $I(0)$ is determined by expression (2) for $t = 0$. Note that, at $t = 0$, average (2) can be expressed in terms of I_2 and I_{2-} as $I(0) \propto \langle \Psi | (0) \hat{\mathcal{I}} \hat{\rho}_{\Psi} | \Psi \rangle (0) = I_2 I_{2-} \approx \left(\frac{\pi k_B T_s}{8 \varepsilon_0 V_{\text{el}}} \right)$. Moreover, $I^{(1)}(\mathbf{r}) = I_1 I_2 - I_{1-} I_{2-}$ and $I^{(2)}(\mathbf{r}) = I_3 + 2(N-1) I_1 I_{1-} + I_1 I_2' + I_{1-} I_{2-}'$.

The real and imaginary parts of the derived expression under the sign of \Re in (31) can formally be related to a complex refractive index \underline{n} (see, e.g., [18] and [19]). To demonstrate certain features that can be deduced from (31) and can be proper for local absorption/reemission processes, we built the line shapes in Figure.

The following values of parameters of the system were chosen in the drawing of the curves below. The accuracy of the graphs is set $0.024 \times 10^{-1} \omega_0$, $\Delta\nu^2 = 10^8 \text{ s}^{-2}$. The values of volumes are set as $V_{\text{el}} = 0.1 \text{ m}^3$ and $V = 0.001 \text{ m}^3$. The atoms are initially in the thermodynamic equilibrium with the walls having the temperature $T = 500 \text{ K}$, while the source of light has the equivalent temperature



Absorption coefficient as a function of the relative average (mean) frequency of a light beam $\frac{\bar{\nu}}{\omega_0}$. The dashed curve is constructed for the space displacement $z = 7 \times 10^{-3} \text{ m}$, by neglecting the terms of the second order (27) and (29) in (31) in the time interval τ . The thin line is built for a vicinity of $z = 5 \times 10^{-5} \text{ m}$. The bold line describes the response of the system in the case of $z = 5 \times 10^{-6} \text{ m}$. The splitting of the dashed line is induced by the Doppler effect. The source of light corresponds to a "weak" laser beam with the equivalent temperature 9000 K. The temperature of atomic system is $T = 500 \text{ K}$. The time parameter is $\tau = 10^8 \frac{2\pi}{\omega_0} \text{ s}$

around 9000 K (a weak laser beam). The number of atoms in unit volume is equal to $6.022 \times 10^{23} / (5 \times 10^5)$ atoms per cubic meter. The time interval is $\tau = 10^8 \frac{2\pi}{\omega_0}$ s, and the resonant (cyclic) frequency is $\omega_0 \approx 0.102 \times 10^{16} \times \pi$ Hz. The atomic mass is approximately equal to the sodium atomic mass: $M = m_{\text{Na}} \approx 0.382 \times 10^{-25}$ kg. In this case, the transition dipole matrix element is defined by the value $|S| \approx 1.725 \times 10^{-29}$ C m, corresponding to the sodium vapor D_2 line induced by a π -polarized laser beam. The approximate values of physical constants are $\epsilon_0 \approx 8.854 \times 10^{-12}$ A \times s/(V \times m), $k_B \approx 1.381 \times 10^{-23}$ J \times K $^{-1}$, and $c \approx 299792458$ m/s.

The absorption coefficient represented by expression (31) as a function of the relative frequency $\frac{\nu}{\omega_0}$ is shown in Figure for different space phases in different approximations. The dashed curve is constructed for the space displacement $z = 7 \times 10^{-3}$ m, by neglecting the second-order terms (27) and (29) in the time interval τ in (31). The thin line is built for a vicinity of $z = 5 \times 10^{-5}$ m. The bold line describes the response of the system in the case of $z = 5 \times 10^{-6}$ m. The splitting of the dashed line is induced by the Doppler effect.

It is worth noting the following. On the definite assumptions, as in the above-given example, the decay phenomena giving a finite absorption line width in the dipole approximation can be implicitly included in the dynamics of the system. Furthermore, some non-ideality is already imposed by the applied averaging procedure (see the discussion, e.g., in [20] and [21]) in expression (2) in comparison with the commonly used introduction of an explicit phenomenological non-Hermitian term (see, e.g., [22]). As can be seen from the calculations in this work, the source of non-ideality is the model statistical distribution for atomic states. Mainly, the introduced integrals I_1 and I_{1-} , not being the complex conjugate, generate the complex values. Physically, it can be expressed as follows. Inasmuch as the system is initially out of the thermodynamic (and dynamic) equilibrium with the electromagnetic field, the time evolution has to be expected to proceed in the direction of a thermal or dynamical balance.

4. Conclusion

As can be seen from the above-given example, the formation of the common line shape can be due to the

coupling non-linear in atomic number (27). The non-linearity can be not pronounced for a quite wide set of initial conditions, corresponding to the so-called non-coherent (collective) state of the system. It is not in contradiction with the widely used Fermi golden rule regarding the proportionality of the transition rate to the square of the transition dipole matrix elements.

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ЗАСТОСУВАННЯ УЗАГАЛЬНЕНОГО КОЕФІЦІЄНТА ПОГЛИНАННЯ ДЛЯ ВРАХУВАННЯ ЕФЕКТІВ ДОПЛЛЕРА ТА ВІДДАЧІ

Резюме

Подано приклад розрахунку коефіцієнта поглинання у термінах квантової оптики, враховуючи ефект Доплера. Для локального коефіцієнта поглинання отримано вираз, що базується на комутаційному співвідношенні між операторами електричного поля та інтенсивності і нелінійно залежить від атомної густини та початкової інтенсивності. Застосування аналітичних результатів продемонстровано графіками залежності коефіцієнта поглинання від частоти скануючого поля. Відносно сильна залежність поглинальної здатності від довжини шляху променя світла спричинена міжатомною взаємодією за посередництвом електромагнітного поля. Продемонстровано розщеплення поглинальної лінії завдяки ефекту Доплера у системі, розміщеній між двома дзеркалами.