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UDC 539

# DESCRIPTION OF THE DEFORMATION PROPERTIES OF EVEN-EVEN $^{102-106}$Pd ISOTOPES

*A description of the even-even Pd isotopes for A = 102 to 106 in the framework of the Interacting Boson Model (IBM) are carried out. The low-lying positive parity yrast bands, as well as the associated $B(E2)$ values, have been calculated in good agreement with the experimental data. The contour plots of the potential energy surface are studied, by using the simplified form of IBM with an intrinsic coherent state. The critical points have been determined for $^{102}$Pd isotope, and O(6) symmetry is substantiated for $^{104,106}$Pd.*

*K e y w o r d s*: interacting boson model, Pd isotopes, energy levels, $B(E2)$, PES.

## 1. Introduction

Many attempts have been made to explore the factors responsible for the onset of large deformations in nuclei of the mass region $A \cong 102$ to 106. The Interacting Boson Model (IBM-1) is one of those attempts that have been successful in describing the low-lying nuclear collective motion in medium- and heavy-mass nuclei [1–4]. The IBM-1 [3] was used to describe the quadrupole collective states of the medium and heavy nuclei and provides a unified description of collective nuclear states in terms of a system of interacting bosons. Different reductions of the unitary group U(6) give three dynamical symmetry limits known as the symmetries of a harmonic oscillator, deformed rotor, and asymmetric deformed rotor, which are labeled as U(5), SU(3), and O(6), respectively [4–6]. Another phenomenological study indicated that nuclei might have an intermediate structure of the U(5)–SU(3), U(5)–O(6), and SU(3)–O(6) limits [7–9]. The neutron-rich palladium (Pd) isotopes span an interesting section of the nuclear chart close to the well-known transitional region from spherical to deformed rare-earth isotopes. In the case of Pd, $Z = 56$, there is an interesting region toward the closed proton shell at $Z = 50$. The study of these isotopes provides us with an insight into the effect that this might have on the transition from spherical to deformed isotopes[10]. Many experimental and theoretical studies on the structure of energy levels and electromagnetic transition properties of the even-even rare-earth isotopes had been investigated in [10–15].

The aim of the present work is to describe the energy levels and $B(E2)$ values by IBM-1. Furthermore, we will calculate the potential energy surface.

## 2. Method of Calculations

The IBM originally invented by Arima and Iachello [2, 3] (which carries no label to distinguish neutrons from protons) embodies a full group of symmetries and regularities of the low-lying quadrupole collective states of medium-heavy and heavy nuclei, which are characterized by six dimensions and are described by the U(6) group, to which most realistic nuclei belong.

In the IBM-1, the Hamiltonian is expressed in the second quantization formalism, which includes the creation operators, $s^\dagger$ and $d^\dagger$, and the annihilation operators, $\tilde{s}$ and $\tilde{d}$, for the bosons with $L = 0$ and 2. In second quantization form, the most general one-

and two-body rotationally invariant Hamiltonian that conserves the number of bosons is given by [3, 16, 17]

$$H = \varepsilon_s(s^\dagger\,\tilde{s}) + \varepsilon_d(d^\dagger\,\tilde{d}) + \\ + \sum_{L=0,2,4}\frac{1}{2}(2L+1)^{1/2}C_L\left[[d^\dagger\times d^\dagger]^{(L)}\times[\tilde{d}\times\tilde{d}]^{(l)}\right]^{(0)} + \\ + \frac{1}{\sqrt{2}}\upsilon_2\Big[[d^\dagger\times d^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)} + \\ + [d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{d}]^{(2)}\Big]^{(0)} + \\ + \frac{1}{2}\upsilon_0\Big[[d^\dagger\times d^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)} + \\ + [s^\dagger\times s^\dagger]^{(0)}\times[\tilde{d}\times\tilde{d}]^{(0)}\Big]^{(0)} + \\ + \frac{1}{2}u_0\Big[[s^\dagger\times s^\dagger]^{(0)}\times[\tilde{s}\times\tilde{s}]^{(0)}\Big]^{(0)} + \\ + u_2\Big[[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d}\times\tilde{s}]^{(2)}\Big]^{(0)}. \quad (1)$$

The dots indicate scalar products, and the crosses stand for tensor products, while $\varepsilon_s$ and $\varepsilon_d$ are the single-boson energies (one-body interactions), and $c_L$ ($L$ = 0, 2, 4), $v_L$ ($L$ = 0, 2), and $u_L$ ($L$ = 0, 2) describe the two-boson interactions. The number of bosons $N_b$ determined by the total number of active proton and neutron pairs is calculated from the nearest closed shell.

In the multipole form with six terms, the IBM-1 Hamiltonian can be written as [17]

$$\hat{H}=\varepsilon\hat{n}_d+a_0\hat{P}\hat{P}+a_1\hat{L}\hat{L}+a_2\hat{Q}\hat{Q}+a_3\hat{T}_3\hat{T}_3+a_4\hat{T}_4\hat{T}_4. \quad (2)$$

Here, the first term represents the boson energy operator ($\hat{n}_d = (d^\dagger.\ \tilde{d})$), the second term gives the pairing operator interaction ($\hat{p} = 1/2[(\tilde{d}.\tilde{d}) - (\tilde{s}.\ \tilde{s})]$). The third term is the O(3) angular momentum contribution ($\hat{L} = \sqrt{10}[d^\dagger\times\tilde{d}]^1$), and the fourth term represents the quadrupole interaction of the $L = 2$ $d$-bosons. The last two terms describe the octupole ($r = 3$) and hexadecapole ($r = 4$) operator interactions ($\hat{T}_r = [d^\dagger\times\tilde{d}]^{(r)}$).

*Table 1.* **The ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$ for $^{102-106}$Pd nuclei [21–24]**

| Nucleus | $^{102}$Pd | $^{104}$Pd | $^{106}$Pd |
|---|---|---|---|
| $R4/2$ | 2.29 | 2.38 | 2.41 |

The quadrupole operator is given by [17, 18]

$$\hat{Q} = [d^\dagger\times\tilde{s} + s^\dagger\times\tilde{d}]^{(2)} + \chi[d^\dagger\times\tilde{d}]^{(2)}, \quad (3)$$

where $\chi$ is the quadrupole structure parameter and takes the values 0 and $\pm\frac{\sqrt{7}}{2}$ [5, 17–19]). Since nuclear states have good angular momentum, the angular momentum group O(3) should be included in all subgroups of U(6). Under this restriction, there are three possible chains [3, 16, 17]:

$$\mathrm{U(6)} \supset \left\{\begin{matrix}\mathrm{U(5)}\supset\mathrm{O(5)}\\ \mathrm{SU(3)}\\ \mathrm{O(6)}\supset\mathrm{O(5)}\end{matrix}\right\} \supset \mathrm{O(3)}. \quad (4)$$

The corresponding dynamical symmetries are usually referred to as the U(5) (the effective parameter is $\varepsilon$), SU(3) (the effective parameter is $a_2$), and O(6) limits (the effective parameter is $a_0$), respectively.

## 3. Results and Discussions

### *3.1. Energy levels*

The Pd nuclei with the proton number $Z = 46$ have neutron numbers $N = 56$ to 60. In the framework of IBM-1, the total boson numbers $N_b$ rise with $N$ from 5 to 7 for even-even $^{102-106}$Pd isotopes. The degree (and type) of collectivity can be expressed in terms of the energy ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$, which is used as a starting point and is a good indicator of the shape deformation of the nucleus. Its value is 2.0 [17] for a spherical vibrator, 2.5 [17] for a $\gamma$-unstable rotor and 10/3 for a good rotor [17], 2.2 for the analytically solvable symmetry E(5) [7] on the U(5) to O(6) path, and 2.9 for the approximate X(5) symmetry on the U(5)–SU(3) path [20].

Table 1 shows experimental values of $R_{4/2} = E_{4_1^+}/E_{2_1^+}$ of those nuclei. In even-even $^{104,106}$Pd isotopes, $R_{4/2}$ attains the O(6) value of ∼2.5, while $^{102}$Pd lies close to the E(5) symmetry.

In the present work, the $\gamma$-unstable limit of the IBM-1 has been applied to even-even $^{104,106}$Pd. The calculations have been performed, by using IBM with PHINT code [25] for the sum of the proton bosons of the close shells (28 and 50) and the neutron bosons of the close shells (50 and 82). The number of bosons and the parameters of the IBM-1 Hamiltonian (2),

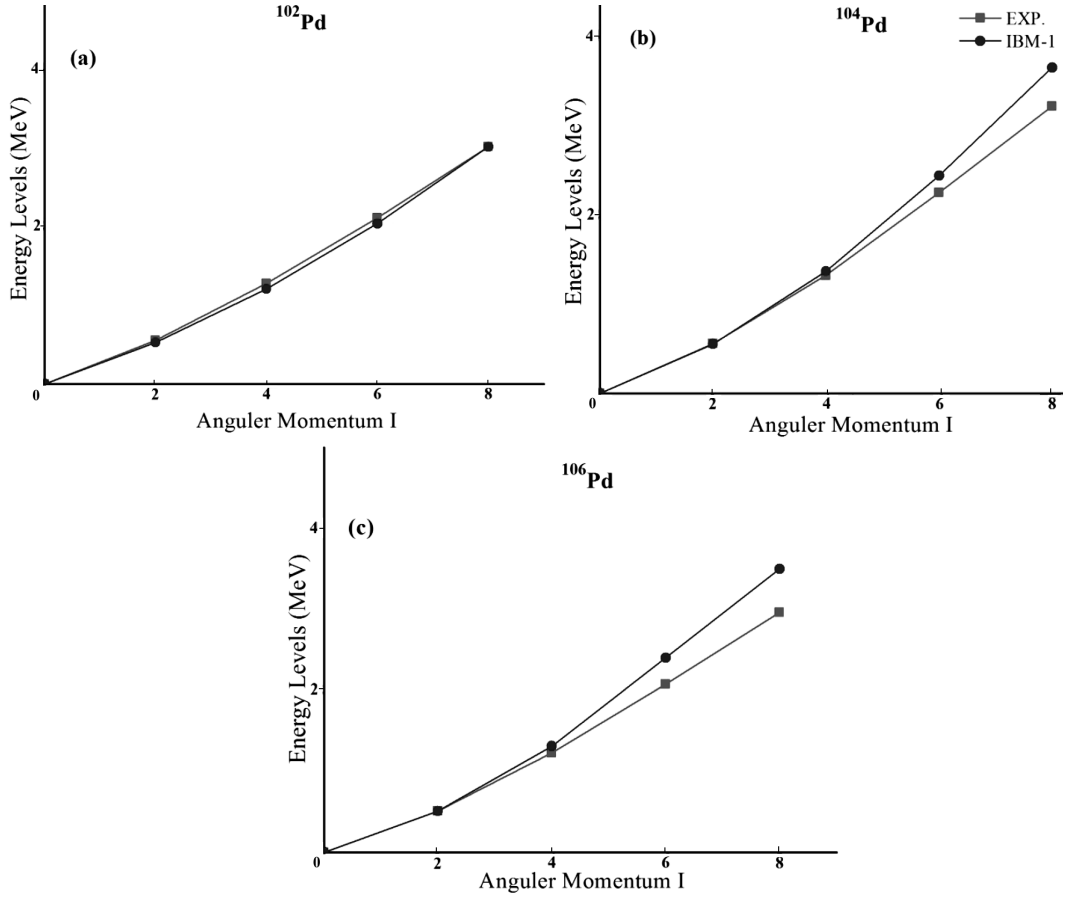


***Fig. 1.*** Comparison of the IBM-1 energy level calculations (circle point) with the available experimental data (square point) [21–24] in the ground band of even-even $^{102}$Pd ($a$), $^{104}$Pd ($b$), and $^{106}$Pd ($c$) isotopes

which give the best fit between theoretical and experimental energy levels [21–24] of the above isotopes, are shown in Table 2.

The IBM-1 energy level calculations in the Ground State Band (GSB) are plotted in Fig. 1, *a, b* and *c* as a function of the angular momentum I and show that the energy levels in GSB rise continuously with I with $Z = 46$ for $^{102-106}$Pd isotopes.

### *3.2. B(E2) transition*

The reduced matrix elements of the $E2$ operator $T^{E2}$ has the form [16, 17, 26]

$$T^{E2} = \alpha_2[d^\dagger s + s^\dagger d]^{(2)} + \beta_2[d^\dagger d]^{(2)}, \qquad (5)$$

where ($s^\dagger$, $d^\dagger$), and ($s$, $d$) are creation and annihilation operators for $s$ and $d$ bosons, respectively, while $\alpha_2$ and $\beta_2$ are two parameters, and ($\beta_2 = \chi\alpha_2$,

*Table 2.* **Adopted values for the parameters used in the IBM-1 calculations. All parameters are given in MeV, except for $N_b$**

| Isotopes | $N_b$ | EPS | PAIR | ELL | OCT |
|---|---|---|---|---|---|
| $^{102}$Pd | 5 | 0.55 | 0.0331 | 0.0297 | – |
| $^{104}$Pd | 6 | – | 0.0952 | 0.0484 | 0.0562 |
| $^{106}$Pd | 7 | – | 0.076 | 0.0611 | 0.0467 |

(EPS= $\varepsilon$, PAIR = $a_0/2$, ELL = $2a_1$, OCT = $a_3/5$) [17].

*Table 3.* **Parameters (in eb) used to reproduce $B(E2)$ values for $^{108-112}$Pd isotopes**

| Isotopes | $B_b$ | $e_B$ |
|---|---|---|
| $^{102}$Pd | 5 | 0.1004 |
| $^{104}$Pd | 6 | 0.0935 |
| $^{106}$Pd | 7 | 0.0926 |

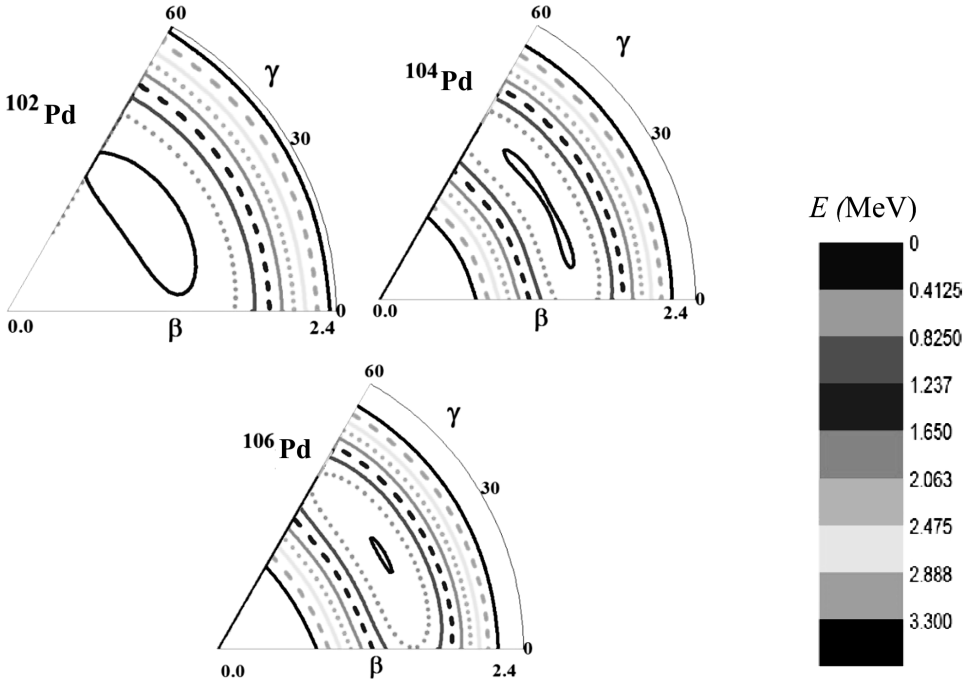


***Fig. 2.*** Potential energy surface in the $\gamma - \beta$-plane of even-even $^{102-106}$Pd nuclei

*Table 4.* **IBM-1 and experimental [21–24] values of $B(E2)$ (in $e^2\ b^2$) and the ratio $R((E2;\ 4_1^+ \to \to\ 2_1^+)/(E2;\ 2_1^+ \to 0_1^+)$ in the low-lying states for Pd isotopes**

| $A$ | $N_b$ | $2_1^+ \to 0_1^+$ | $4_1^+ \to 2_1^+$ | $6_1^+ \to 4_1^+$ | $8_1^+ \to 6_1^+$ | ratio $R$ |
|---|---|---|---|---|---|---|
| $^{102}$Pd | Exp. | 0.069 | 0.121 | – | – | 1.754 |
| | IBM-1 | 0.062 | 0.109 | 0.052 | 0.151 | 0.758 |
| $^{104}$Pd | Exp. | 0.105 | 0.140 | – | – | 1.333 |
| | IBM-1 | 0.104 | 0.137 | 0.139 | 0.124 | 1.317 |
| $^{106}$Pd | Exp. | 0.129 | 0.193 | 0.235 | 0.244 | 1.496 |
| | IBM-1 | 0.132 | 0.176 | 0.185 | 0.174 | 1.333 |

$\alpha_2 = e_B$ (effective charge of boson)). The reduced transition rates can be obtained from the matrix elements of the $T^{E2}$ operator as [3, 17, 27]

$$B((E2)I_i \to I_f) = \frac{1}{2L_i + 1} \left| \langle I_f \parallel T^{(E2)} \parallel I_i \rangle \right|^2. \quad (6)$$

The effective charge, $e_B = \alpha_2$, was determined by normalizing the predictions to the experimental $B((E2);\ 2_1^+ \to 0_1^+)$ values and tabulated in Table 3. The calculated $B(E2)$ values are compared with the experimental data [21–24] for all isotopes under study and, as given in Table 4, show that, in general, most of the calculated results in IBM-1 are reasonably consistent with the available experimental data.

Table 4 shows the $B(E2)$ in the low lying states and the ratio $R((E2;\ 4_1^+ \to 2_1^+)/(E2;\ 2_1^+ \to 0_1^+)$, which is equal to 2 in U(5) and to 1.4 in O(6) [3, 28, 29] for Pd isotopes with the proton number $Z = 46$. It is shown there is good agreement between $B(E2)$ calculated with the experimental data on Pd isotopes, except for $^{102,104}$Pd isotopes, because there are no sufficient experimental data [21–24]. The ratio $R$ is almost constant, as the neutron number increases toward the shell $N = 82$. In other words, the nuclei get O(6) symmetry in $^{104,106}$Pd, while $^{102}$Pd isotope is close to E(5) symmetry.

### *3.3. Potential energy surface*

In IBM-1, the geometrical interpretation of Hamiltonian (1) can be derived, by using the intrinsic condensed state ($|N,\beta,\gamma\rangle$) defined by [30, 31] as

$$(|N,\beta,\gamma\rangle) = 1/\sqrt{N!}(b_c^\dagger)^N|0\rangle, \tag{7}$$

where $|0\rangle$ denotes the boson vacuum, and

$$b_c^\dagger = (1+\beta^2)^{-1/2}\Big\{s^\dagger + \beta[\cos\gamma(d_0^\dagger) + \\ + \sqrt{1/2}\,\sin\gamma(d_2^\dagger + d_{-2}^\dagger)]\Big\}, \tag{8}$$

where $\beta$ is a measure of the total deformation of a nucleus. The shape is spherical for $\beta = 0$ and is distorted, when $\beta \neq 0$. The quantity $\gamma$ is the amount of deviation from the focus symmetry and correlates with the nucleus. If $\gamma = 0$, the shape is prolate, if $\gamma = \pi/3$, the shape becomes oblate, and, for $0 \leq \gamma \leq \pi/3$, the shape is triaxial (triaxial deformation helps one to understand the prolate-to-oblate shape transition) [33]. The calculated potential energy surfaces are shown in Fig. 2 for the even-even $^{102-104}$Pd. From this figure, the shape phase transition from rotational U(5) to $\gamma$-unstable symmetry O(6) in $^{102}$Pd is seen, while $^{104,106}$Pd nuclei are deformed and have $\gamma$-unstable-like characters $\left(\gamma \approx \frac{\pi}{6}\right)$.

## 4. Conclusion

In the present article, the nuclear structure of even-even $^{102-106}$Pd is investigated within the IBM-1. The energy level of low-lying states has been calculated. The results show good agreement with published experimental data on some even-even palladium isotopes having the neutron numbers 56, 58, and 60. This study confirms that $^{102}$Pd isotope is deformed and is described by U(5)–O(6), while $^{104,106}$Pd isotopes have $\gamma$-unstable property according to the energy ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$, which is used as the starting point and is a good indicator of the shape deformation of a nucleus. The electric transition probabilities $B(E2)$ obtained for these isotopes within the IBM-1 are compared with the available experimental data. In the present work, the shape phase with O(6) and the transitional region with U(5)–O(6) in the space of two control parameters $\beta$ and $\gamma$ have been analyzed. The potential energy surfaces for Pd isotopes show that $^{102}$Pd isotope is deformed and is characterized by U(5)–O(6) symmetry, while the $^{104,106}$Pd nuclei are deformed and have $\gamma$-unstable-like characters.

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ОПИС ВЛАСТИВОСТЕЙ ДЕФОРМАЦІЇ ПАРНО-ПАРНИХ ІЗОТОПІВ $^{102-106}$Pd

Р е з ю м е

Дано опис парно-парних ізотопів паладію для $A$ від 102 до 106 в моделі взаємодіючих бозонів (МВБ). Розраховані низьколежачі іраст-смуги з позитивною парністю і відповідні величини $B(E2)$, що добре узгоджуються з експериментом. З використанням спрощеного варіанта МВБ з власним когерентним станом вивчені контурні графіки поверхні потенціальної енергії. Знайдено критичні точки для $^{102}$Pd ізотопу і обґрунтовано O(6) симетрію для $^{104,106}$Pd.