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ENERGY LEVELS OF ISOBARIC NUCLEI (^{16}N, ^{16}F) WITHIN THE MODIFIED SURFACE DELTA-INTERACTION MODEL

UDC 539

The modified surface delta-interaction model is applied to calculate the energy levels of ^{16}N and ^{16}F nuclei. The good agreement between theoretical and experimental data is attained for excitation energies. This indicates that the shell model describes properly the structure of these nuclei.

K e y w o r d s: energy levels, isobaric nuclei, shell model, delta-interaction model.

1. Introduction

The nuclear shell model (NSM) has a very long history, during which much efforts were directed to the improvement of the formal and technical aspects of the model applied to the explanation of nuclear properties [1]. It was introduced some (55) years ago to clarify the regularities of the nuclear properties related to the magic number [2]. The shell model has been successful in the characterization of configurations with a few nucleons outside "closed shells" or missing from them [3]. Having a simple shell model in mind, Talmi used the surface delta-interaction to evaluate properties of nuclear states with few "nucleons" on a magic core [4]. Likewise, the shell model has been successful in describing the configurations with a few nucleons outside closed shells or missing from them [3]. It employs the following assumption: first, there exists an inert core model of a close shell, which acts with central forces upon valence nucleons; second, there exists a residual interaction caused by two-body forces acting between the valence nucleons [5]. Here, this work presents a modified surface delta-interaction (SDI) method and will apply it to calculate the energy levels of (^{16}N, ^{16}F) nuclei, which contain one particle and one hole outside of the closed shell ^{16}O.

2. Particle-Hole Formalism

By knowing the nuclear potentials for all nucleons of a nucleus, one can calculate the energy levels [6] with the normal assumption of two-body forces defined by a potential V. The Hamiltonian can be written in the form [7]

$$H = \sum_{\alpha\beta} \langle \alpha | E | \beta \rangle \eta_\alpha^+ \eta_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle \eta_\alpha^+ \eta_\beta^+ \eta_\delta \eta_\gamma, \tag{1}$$

where η_α^+ and η_α denote the (creation and annihilation) operators, respectively, of a nucleon in the state $|\alpha\rangle$, E is the kinetic energy operator, and $\langle \alpha\beta | V | \delta\gamma \rangle$ idealizes the normalized antisymmetrized matrix element of the nucleon-nucleon interaction. The wave functions in the (one-particle)–(one-hole) configuration space are represented by [7, 8]

$$|0; ph^{-1}\rangle = a_h a_p^+ |0\rangle. \tag{2}$$

We denote particle states by the labels $p1$, $p2$, and $p3$ and hole states by labels $h1$, $h2$, and $h3$. The NSM Hamiltonian reads

$$H_{s.p}|0;ph^{-1}\rangle=(e_p-e_h)a_h a_p^+|0\rangle=(e_p-e_h)|0;ph^{-1}\rangle, \quad (3)$$

where e_p represents the single-particle (s.p.) energy for a particle, and e_h symbolizes the single-particle energy for a hole. Consequently, we can construct a basis of one-particle (one-hole) states with the injection energy as given in Eq. (3). For the description of excited states in nuclei, one still has to perform the angular momentum coupling. Once a sufficiently large set of basis functions is obtained, the coupled (particle-hole) states are given by

$$0;ph^{-1};|JM\rangle=\sum_{m_p m_h}(-1)^{j_h-m_h}\langle j_p m_p j_h m_h|JM\rangle\times$$
$$\times\, a_{j_h-m_h}a^+_{j_h-m_h}|0\rangle. \quad (4)$$

The ground state is taken to be a closed configuration, and the excited state is a mixed (one particle, one hole) configuration. This approach is usually referred to as the Tamm–Dancoff approximation (TDA).

3. Surface Delta-Interaction

The residual nucleon-nucleon interaction is that part of the interaction, which is not involved in the central average potential. As a consequence of the Pauli principle, most inelastic collisions inside the nucleus are prohibited. Therefore, the nucleons move almost freely in the nuclear matter, and the (effective interaction) between the nucleons occurs mainly on the nuclear surface. Using this assumption, Moszkowski and co-workers [5] have proposed a simple model able to describe the interaction between the valence nucleons. This model of interaction assumes the following.

I. The residual interaction $V_{1,2}$ between particles 1 and 2 takes place at the nuclear surface only.

II. The two-body forces are delta-forces; i.e., the interaction takes place only if the two nucleons are at the same place [9, 10]:

$$V_{1,2}=-4\pi A'_T\delta(\Omega_{1,2}), \quad (5)$$

where A'_T represents the interaction strength, and $\Omega_{1,2}$ is the angular distance between the interacting particles. The factor is introduced for the normalization purposes.

III. The radial one-particle wave functions of the active shell have the same absolute value at the nuclear surface. The interaction defined in this way possesses all the features of pairing interaction. It is short-ranging and allows only symmetric spatial states. It should be pointed out that SDI exists not only between particles coupled to ($J=0$, $T=l$), but also between particles coupled to ($J\neq 0$). Unlike the pairing, SDI acts also in states with $T=0$. Therefore, SDI should be a better approximation, than a pure pairing one. Following the work by Glaudemans *et al.* [11], where the isospin dependence of the interaction was taken into account, the "interaction" may be written in the form [7–10]

$$V_{1,2}=-4\pi A^{'}_T\delta(\Omega_{1,2})\delta(\hat{r_1}-R_0)\delta(\hat{r_2}-R_0)+B_{\tau_1\cdot\tau_2}, \quad (6)$$

where $\hat{r}_1$, $\hat{r}_2$ are the position vectors of interacting particles, R_0 is the nuclear radius [9–12, 13], A'_T is the strength of interaction, and τ_n are the isospin operators ($n=1,2$). $T=t_1+t_2$ depends on the isospin of interacting particles. The correction term $B_{\tau_1\cdot\tau_2}$ is introduced to account for the splitting between the groups of levels with different isospins. The eigenvalue of the operator product $\tau_1\cdot\tau_2$ is found from the relation $T_2=t_1^2+t_2^2+2t_1^2t_2^2$. Taking into account that ($t=1/2\tau$ and $t_1=t_2=1/2$, we get

$$t_1t_2=\frac{T(T+1)-t_1(t_1+1)-t_2(t_2+1)}{2}=$$
$$=\frac{2T(T+1)-3}{4}, \quad (7)$$

$$\tau_1\cdot\tau_2=2T(T+1)-3. \quad (8)$$

Such a form of the interaction is called a modified surface delta-interaction (MSDI). In the simple case of two valence nucleons outside of the closed shell, the Hamiltonian is

$$H=H_{\rm Core}+E_{j_1}+E_{j_2}+\langle j_1j_2|V_{(1,2)}|j_3j_4\rangle, \quad (9)$$

where $E(j_{1,2})$ are the single-particle energies. The antisymmetrized matrix element of $V_{1,2}$ is given by [14, 15]

$$\langle j_1j_2|V|j_3j_4\rangle=-\frac{A_T}{2(2J+1)}\times$$
$$\times\{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)/2(2J+1)\times$$
$$\times(1+\delta_{1,2})\}^{1/2}[(-1)^{l_1+l_2+j_3+j_4}]\times$$

$$\times\left[\begin{array}{ccc} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{array}\right]\left[\begin{array}{ccc} j_3 & j_4 & J \\ 1/2 & -1/2 & 0 \end{array}\right][1-(-1)^{l_3+l_4+J+T}] -$$
$$-\left[\begin{array}{ccc} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{array}\right]\left[\begin{array}{ccc} j_3 & j_4 & J \\ 1/2 & -1/2 & 0 \end{array}\right]\times$$
$$\times[1+(-1)^T]+[2T(T|+1)-3]B\delta_{1,3}\delta_{2,4}, \qquad (10)$$

where $\left[\begin{array}{ccc} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{array}\right]$ and $\left[\begin{array}{ccc} j_3 & j_4 & J \\ 1/2 & -1/2 & 0 \end{array}\right]$ are the Clebsch–Gordan coefficients [7–13] and j_1, j_2, j_3, and j_4 are the spin states of particles. Correspondingly, J and T indicate the spin and isospin of a two-particle state. The diagonal matrix elements with $j_1 = j_2$ and $j_3 = j_4$ correspond to pure states. In the case of pure states, the excitation energies are given by the difference between the binding energies of the level and the ground state. When the configuration mixing is considered, the energies are obtained by carrying out the diagonalization of energy matrices. The behavior of two-hole nuclei is almost the same as that of two-particle nuclei except for that the single-particle energy changes the sign [14–16]: $e_{h(\text{hole})} = -e_{p(\text{particle})}$. The residual interaction is given by the MSDI defined in Eq. (6), and the matrix of the Hamiltonian is [7, 8]

$$\langle p_1 h_1^{-1}|H|p_2 h_2^{-1}\rangle_\Gamma =$$
$$= (e_{p_1} - e_{h_1})\delta_{p1p2}\delta_{h1h2} + \langle p_1 h_1^{-1}|V|p_2 h_2^{-1}\rangle. \qquad (11)$$

Then

$$\langle p_1 h_1^{-1}|V|p_2 h_2^{-1}\rangle =$$
$$= \frac{1}{4}\sqrt{(2j_{p_1}+1)(2j_{p_2}+1)(2j_{h_1}+1)(2j_{h_2}+1)}\times$$
$$\times(-1)^Q[A_0[1+2(-1)^{l_{h2}+l_{p2}+J}]+A_1[1+2(-1)^T]\times$$
$$\times\left[\begin{array}{ccc} j_{p2} & j_{h1} & J \\ 1/2 & -1/2 & 0 \end{array}\right]\left[\begin{array}{ccc} j_{h2} & j_{p2} & J \\ 1/2 & -1/2 & 0 \end{array}\right]\times$$
$$\times(-1)^Z[A_0 - A_1[1+2(-1)^T]\left[\begin{array}{ccc} j_{p1} & j_{h1} & J \\ 1/2 & 1/2 & -1 \end{array}\right]\times$$
$$\times\left[\begin{array}{ccc} j_{h2} & j_{p2} & J \\ 1/2 & 1/2 & -1 \end{array}\right]\delta^{\text{even}}_{l_{p1}+l_{p2}l_{h1}+l_{h2}} -$$
$$-C+B[1+2(-1)^T]]\delta_{p1p2}\delta_{h1h2}, \qquad (12)$$

where $\Gamma = TJ$, $Q = j_{h1} + j_{h2} + l_{p1} + l_{h1} + J + T$, and $Z = j_{h1} + j_{p2} + l_{p2} + l_{h1}$, J is the total angular momentum, and values of A_0, A_1, B, and C as functions of the mass number A [7, 14] are obtained from fits to experimental data in the varied mass region, where $A_0 \simeq A_1 \simeq B \simeq \frac{25}{A}$. For a two-particle or two-hole nucleus [10] in the states j_1 and j_2 $(j_1 j_2)$, the allowable angular momentum values are

$$J = j_1 + j_2, j_1 + j_2 - 1, j_1 + j_2 - 2, j_1 + j_2 - 3, |j_1 - j_2|. \qquad (13)$$

4. Results and Discussion

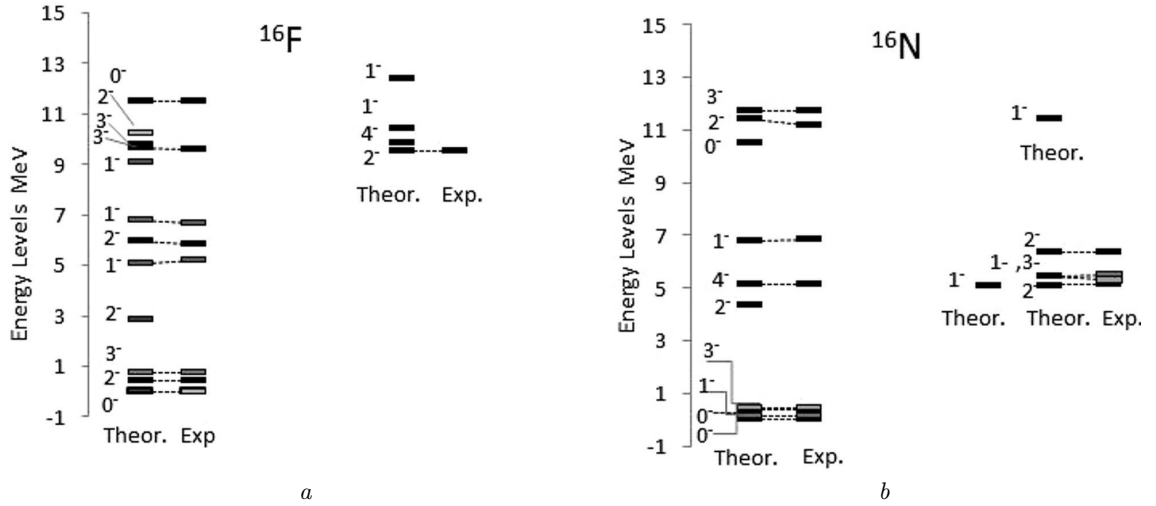
The MSDI discussed in the previous section has been used extensively as the effective two-body interaction in many regions in the Periodic table [14, 15]. Nuclear properties of many nuclei were calculated within various models and were compared with experimental data [19, 20]. In this work, we present the results of calculations of the energy levels of ^{16}N and ^{16}F nuclei.

4.1. ^{16}F *nucleus*

The model space includes $0p_{3/2}$ and $0p_{1/2}$ hole orbits and ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$ particle orbits in ^{16}F nucleus. It is one-proton (particle) outside the inert core ^{16}O and one neutron (hole). The energy levels can be obtained, by using the single-particle energy [17, 18], where $e_{P(\text{particle})} = e_{0d_{5/2}} = 0.6$ MeV, $e_{1s_{1/2}} = 0.105$ MeV, $e_{0d_{3/2}} = -4.499$ MeV, $e_{H(\text{hole})} = e_{0p_{1/2}} = 15.663$ MeV, $e_{0p_{3/2}} = 21.84$ MeV. The parameters $A_0 = 2.1$, $A_1 = 1.99$, $B = 0.9$ and $C = 0.0$. Table 2 and Figure, *a* show the comparison between theoretical and experimental values for ^{16}F nucleus [19]. The MSDI calculations of the energies and parity are in good agreement with the experimental values. In the "first" sequences for the energy spectra with MSDI, we predicted a good correspon-

Table 1. **Matrix element values X_F and X_N (in MeV) for ^{16}N and ^{16}F nuclei, respectively. $X = \langle j_{p_1} j_{h_1^{-1}}|V|j_{p_2} j_{h_2^{-1}}\rangle$, $M = (^{16}N, ^{16}F)$**

j_{p1} (M)	$j_{h_1^{-1}}$ (M)	j_{p2} (M)	$j_{h_2^{-1}}$ (M)	J (M)	X_F	X_N
5/2	1/2	5/2	1/2	3	1.865	1.202
5/2	1/2	5/2	1/2	2	1.615	0.897
1/2	1/2	1/2	1/2	0	3.499	0.809
1/2	1/2	1/2	1/2	1	4.389	1.759
3/2	1/2	3/2	1/2	1	1.662	1.352
3/2	1/2	3/2	1/2	2	2.530	0.883
5/2	3/2	5/2	1/2	1	1.341	1.233
5/2	3/2	5/2	3/2	2	2.556	0.885
5/2	3/2	5/2	3/2	3	1.939	1.197
5/2	3/2	5/2	3/2	4	2.129	0.913
1/2	3/2	1/2	3/2	1	4.951	1.706
1/2	3/2	1/2	3/2	2	1	1.001
3/2	3/2	3/2	3/2	0	2.767	0.934
3/2	3/2	3/2	3/2	1	13.214	1.251
3/2	3/2	3/2	3/2	2	3.191	0.865
3/2	3/2	3/2	3/2	3	3.488	1.321



a *b*

Comparison the MSDI calculations with the available experimental data [19, 20]

dence for states 1^-_1, 2^-_1, and 3^-_1 with empirical values. The new level 4^-_1 is expected, though it did not appear in the experiment. But, in the "second" sequences of energy levels, we expect that the states 0^-_2, 1^-_2, and 3^-_2 are rather close to the experimental data. The new level 2^-_2 was expected, but it was not yet revealed in the experiment. The "third" sequences of energy levels, the energy of 2^-_3 is rather close to the experimental data. In "other" sequences, the state 2^-_4 well corresponds to the experiment. We have also calculated new levels such as 1^-_3, 3^-_3, 1^-_4, 2^-_5, and 1^-_5.

Table 2. **Comparison between theoretical and experimental values**

J^π	Energy (MeV)	Energy (MeV)	J^π	Energy (MeV)	Energy (MeV)
J^- ^{16}F	Theor.	Exp. [19]	J^- ^{16}N	Theor.	Exp. [20]
0_1	0	0	2_1	0	0
2_1	0.424	0.424	1_1	0.39	0.39
3_1	0.721	0.721	0_1	0.122	0.1204
2_2	2.83	–	3_1	0.456	–
1_1	5.07	5.2	2_2	4.35	–
2_3	5.939	5.856	1_2	5.056	–
1_2	6.788	6.679	3_2	5.106	5.129
1_3	9.07	–	4_1	5.134	5.1507
2_4	9.55	9.5	1_3	5.418	5.318
3_2	9.67	9.6	3_2	5.418	5.521
3_3	9.801	–	2_4	6.341	6.374
4_1	9.864	–	1_4	6.811	6.84
2_5	10.24	–	0_2	10.48	–
1_4	10.44	–	1_5	11.42	–
0_2	11.513	11.5	2_5	11.442	11.16
1_5	12.403	–	3_3	11.747	11.701

4.2. Nucleus ^{16}N

In the frame of the shell model, we have considered ^{16}N nucleus. In this case, there are one proton (hole) and one neutron (particle) outside of the inert core ^{16}O. One proton occupies the model space $0p_{3/2}$, $0p_{1/2}$, and one neutron occupies the model space ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$). The spectrum of this nucleus was calculated with the use of Eqs. (11) and (12). The single-particle energies [17, 18] are as follows: $e_{P(\text{particle})} = e_{0d_{5/2}} = 4.14$ MeV, $e_{1s_{1/2}} = 3.272$ MeV, $e_{0d_{3/2}} = -0.942$ MeV, $e_{H(\text{hole})} = e_{0p_{1/2}} = 12.127$ MeV, and $e_{0p_{3/2}} = 18.451$ MeV. The parameters $A_0 = 0.16$, $A_1 = 0.13$, $B = 1.4$, and $C = 0.0$. Table 2 and Figure, *b* show the comparison between theoretical and experiment values for ^{16}N nucleus. The MSDI calculations of the energies and parity are in good agreement with the experimental values [20]. In the "first" sequences for energy spectra, we have obtained good correspondence for the states $1^-_1, 0^-_1$, and 4^-_1 with empirical values. In the "second" sequences of energy levels, we expect that the state 3^-_2 is rather close to the experimental data. In the third sequence, the energy levels 1^-_3 and 3^-_3 well corrospond to the experimental data. The same is true for the states 1^-_4, 2^-_4, and 2^-_5. But the expected new levels 2^-_1, 3^-_1, 0^-_2, 1^-_2, and 1^-_5 are not yet found in the experiment.

5. Conclusion

We have attained a good agreement between theoretical and experimental data for excitation energies. There are many unconfirmed experimental energy levels revealed in our calculations. We have also predicted some new values for energy levels which were not yet specified in the experiment. The choice of model MSDI effective interactions is suitable in this mass region. The theoretical calculations within the nuclear shell model with MSDI quite well agree with the experimental data. This indicates that the shell model is very good to illustrate the structure of ^{16}F and ^{16}N nuclei.

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РІВНІ ЕНЕРГІЇ ІЗОБАРИЧНИХ ЯДЕР ^{16}N І ^{16}F В МОДИФІКОВАНІЙ МОДЕЛІ З ПОВЕРХНЕВОЮ ДЕЛЬТА-ВЗАЄМОДІЄЮ

Р е з ю м е

Модифіковану модель з поверхневою дельта-взаємодією застосовано для розрахунку рівнів енергії ядер ^{16}N і ^{16}F. Отримано добре узгодження теоретичних і експериментальних даних для енергій збудження. Це свідчить про те, що оболонкова модель правильно описує структуру цих ядер.