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## CENTRALITY DEPENDENCES OF CHARACTERISTICS OF NEGATIVE PIONS IN $\pi^- + {}^{12}\text{C}$ INTERACTIONS AT 40 GeV/c

UDC 539

*The collision centrality dependences of the mean multiplicities and kinematical characteristics of negative pions produced in  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c are investigated. It is obtained that the average values of the partial inelasticity coefficients of negative pions (without "leading" pions) do not depend on the  $\pi^- + {}^{12}\text{C}$  collision centrality. It is shown that the normalized transverse momentum distributions, as well as the average values of the transverse momentum of negative pions, do not depend within the uncertainties on the  $\pi^- + {}^{12}\text{C}$  collision centrality. The average value of the emission angle of the negative pions was found to increase with the  $\pi^- + {}^{12}\text{C}$  collision centrality. The comparison of the normalized emission angle distributions of the negative pions in the peripheral and central  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c supported the assumption that such an increase in the average emission angle is likely due to an increase in the number of binary collisions and multiple rescatterings with the  $\pi^- + {}^{12}\text{C}$  collision centrality. The obtained experimental findings are reproduced qualitatively well by the Modified FRITIOF model calculations.*

*Keywords:* pion production, collision centrality, relativistic nuclear collisions.

Pions are the most abundantly produced particles in hadron-nucleus and nucleus-nucleus collisions at high energies due to their low production threshold energy [1, 2]. The biggest fraction of the collision en-

ergy in relativistic nuclear collisions is spent on the pion production. Hence, the investigation of the collision centrality and incident energy dependences of the kinematical distributions and average kinematical characteristics of pions is important for the extraction of a valuable information on the dynamics of hadron-nucleus and nucleus-nucleus collisions at high energies [3–11].

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In the present work, we investigate the collision centrality dependences of the mean multiplicities per collision event and various kinematical characteristics of negative pions in  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c. Because we cannot measure the collision impact parameter in experiments, we used the number of participant protons in a collision event to define the collision centrality. We followed Refs. [12–14], where the number of participant protons was used to define the collision centrality, to divide the total set of  $\pi^- + {}^{12}\text{C}$ -collision events at 40 GeV/c into the groups of peripheral, semicentral, and central collisions. Therefore, to define the collision centrality, we used the average number per event (mean multiplicity),  $\langle \nu \rangle$ , of participant protons in  $\pi^- + {}^{12}\text{C}$ -collisions at 40 GeV/c and the number of participant protons,  $\nu$ , in each individual collision event. All protons, except those (with momentum  $p < 0.25$  GeV/c in the laboratory frame) “evaporated” from the target nucleus, were considered to be participant protons. The events with  $\nu \leq \langle \nu \rangle$ ,  $\langle \nu \rangle < \nu \leq 2\langle \nu \rangle$ , and with  $\nu > 2\langle \nu \rangle$  were selected to be peripheral, semicentral, and central collision events, respectively.

It should be noted that, in our experiment, we registered and measured all the directly produced charged pions and all those pions produced via the strong decay of excited baryon resonances, vector mesons (such as  $\rho^0$ ,  $w^0$ , and  $f^0$  mesons), *etc.* Pions produced via the secondary weak decays such as  $K^0 \rightarrow \pi^+\pi^-$  were not registered.

The experimental data were obtained using a 2-meter propane ( $\text{C}_3\text{H}_8$ ) bubble chamber of the Laboratory of High Energies (LHE) of the Joint Institute for Nuclear Research (JINR, Dubna, Russia) exposed to  $\pi^-$  mesons accelerated to a momentum of 40 GeV/c at the HEPI (High Energy Physics Institute, Russia) accelerator. The experimental data consist of 15 841  $\pi^- + {}^{12}\text{C}$  collision events with practically all the secondary charged particles of reactions measured over the full ( $4\pi$ ) solid angle. All singly charged negative particles were taken as the negative pions. The admixture of protons among  $\pi^+$  mesons with momenta  $p > 750$  MeV/c was estimated to be less than 11–12% [15]. The mean relative uncertainty  $\langle \Delta p/p \rangle$  of the momentum measurements of charged pions was 14 %. The average threshold for the registration and measurement of the momenta of charged pions was about 50 MeV/c, corresponding to a track length of around 5 cm in a propane chamber. The

so-called leading particles were excluded from the further analysis. The particle was considered to be a leading one, if, in a given collision event, it had the maximum momentum, whose value exceeds one third (1/3) of the initial (incident) momentum, i.e.,  $p > 13.33$  GeV/c.

In the experiment, the fast participant protons (in the momentum range of  $0.75 < p < 13.33$  GeV/c) were separated from singly charged positive particles as the ones having the maximal transverse momentum in a given event. Then the average number of participant protons per event, including protons with momenta in the range of  $0.25 < p < 0.75$  GeV/c, proved to be  $0.98 \pm 0.01$ .

Practically all the spectator nucleons of a target carbon nucleus have momenta  $p_n < p_{\text{max}}^{\text{F}}$ , where  $p_{\text{max}}^{\text{F}}$  is the maximum Fermi momentum in the nucleus rest frame. The maximum Fermi momentum is about 0.2–0.3 GeV/c for carbon nuclei, when making allowance for the average relative uncertainty of measurements of the proton momentum,  $\langle \frac{\Delta p}{p} \rangle \approx 14\%$ , in our experiment. The participant protons are those, which remain after excluding the spectator protons.

The other methodological procedures of the experiment are presented in Refs. [16, 17].

In order to estimate the possible systematic (methodological) uncertainties, the momenta of the protons that stopped in the chamber and have track length longer than 40 cm were measured three times, and their momenta were calculated with regard for their path length and trajectory curvature in the magnetic field in the chamber. The results obtained with the use of these two methods coincide within the uncertainties of less than 5%, which shows the practical absence of such systematic (methodological) uncertainties. In addition, the presence or absence of systematic (methodological) uncertainties in the experiment was checked by the reconstruction of  $\pi^0$  mass based on the measurements of the decay products of 2 gamma quanta. To estimate such systematic uncertainties, the momenta of the electrons and positrons produced from the chain reaction  $\pi^0 \rightarrow 2\gamma \rightarrow 2^*(\gamma \rightarrow e^+e^-)$  in the propane bubble chamber were measured. The measurements were made for 10 such reactions of  $\pi^0$  decay in the chamber, and the mass of a  $\pi^0$  meson was reconstructed from the momentum measurements of the electrons and positrons with the sufficiently long tracks in the

**Table 1. The values of the fraction of the events ( $\alpha$ ) with the various degrees of centrality of  $\pi^- + {}^{12}\text{C}$  collisions, the mean multiplicities ( $\langle n(\pi) \rangle$ ) per event, and the average partial inelasticity coefficients ( $\langle K(\pi) \rangle$ ) of the negative pions at different centralities of  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c. All the values are calculated for the negative pions with  $P_{\text{lab}} > 50$  MeV/c. Statistical errors are shown**

Quantity	Collision centrality					
	Peripheral		Semicentral		Central	
	Exp.	Model	Exp.	Model	Exp.	Model
$\alpha$ , %	$68 \pm 1$	$64 \pm 1$	$20 \pm 1$	$24 \pm 1$	$12 \pm 1$	$12 \pm 1$
$\langle n(\pi) \rangle$	$2.67 \pm 0.01$	$2.63 \pm 0.01$	$2.99 \pm 0.03$	$3.15 \pm 0.02$	$4.07 \pm 0.05$	$3.30 \pm 0.03$
$\langle K(\pi) \rangle$	$0.21 \pm 0.01$	$0.21 \pm 0.01$	$0.22 \pm 0.01$	$0.21 \pm 0.01$	$0.24 \pm 0.03$	$0.21 \pm 0.02$

**Table 2. The average values of the total momentum ( $P$ ), transverse momentum ( $P_t$ ), and emission angles ( $\Theta$ ) of the negative pions in the laboratory frame for the selected three groups of centralities of  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c. The average values are calculated for the negative pions with  $P_{\text{lab}} > 50$  MeV/c. Statistical errors are shown**

Quantity	Collision centrality					
	Peripheral		Semicentral		Central	
	Exp.	Model	Exp.	Model	Exp.	Model
$\langle P \rangle$ , GeV/c	$3.17 \pm 0.01$	$3.13 \pm 0.02$	$2.93 \pm 0.03$	$2.72 \pm 0.02$	$2.39 \pm 0.04$	$2.60 \pm 0.02$
$\langle P_t \rangle$ , GeV/c	$357 \pm 2$	$315 \pm 1$	$352 \pm 3$	$314 \pm 2$	$347 \pm 4$	$315 \pm 2$
$\langle \Theta \rangle$ , degrees	$19.5 \pm 0.2$	$17.1 \pm 0.1$	$22.4 \pm 0.3$	$20.0 \pm 0.2$	$25.9 \pm 0.4$	$21.0 \pm 0.5$

bubble chamber. In all such measurements, the reconstructed masses of  $\pi^0$  practically coincided with the known mass, 134.98 MeV, of  $\pi^0$  within the precision of less than 1%.

For the purpose of comparison with the experimental data, we simulated 30000 events using the Modified FRITIOF model [18–21] for  $\pi^- + {}^{12}\text{C}$  collision events at 40 GeV/c. The model parameters used for the simulation were the same as given in Ref. [19]. In the experiment, all the collision events regardless of the impact parameter of a collision were recorded. Therefore, collision events in the Modified FRITIOF model were simulated for minimum bias events without restrictions on the impact parameter of a collision. The average multiplicity per event of the participant protons in the Modified FRITIOF model proved to be  $0.99 \pm 0.01$ , which coincides within statistical errors with that obtained in the experiment.

It is worth mentioning that, both in the experiment and in the model, we considered only those negative

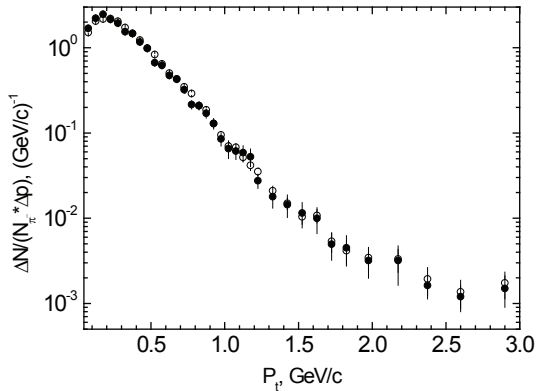
pions that had momenta in the laboratory frame exceeding 50 MeV/c, i.e., the detection threshold for pions in the propane bubble chamber.

In Table 1, the values of the fraction of the events ( $\alpha$ ) with the various degrees of centrality of  $\pi^- + {}^{12}\text{C}$  collisions, the mean multiplicities ( $\langle n(\pi) \rangle$ ) per event, and the average partial inelasticity coefficients ( $\langle K(\pi) \rangle$ ) of the negative pions at different centralities of  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c are presented. The average values of the full momentum ( $P$ ), transverse momentum ( $P_t$ ), and emission angles ( $\Theta$ ) of the negative pions in the laboratory frame for the considered three groups of centralities of  $\pi^- + {}^{12}\text{C}$  collisions are given in Table 2.

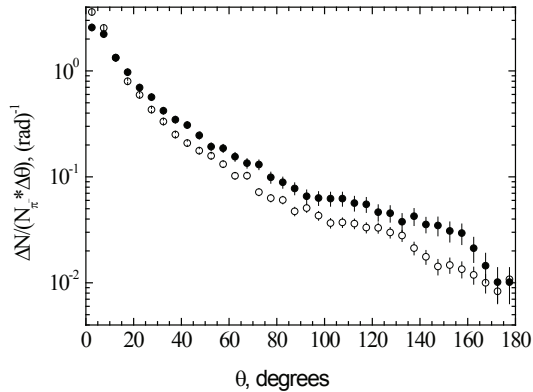
The partial inelasticity coefficient of the negative pions in each collision event was calculated using the formula

$$K(\pi) = \sum_i (E_i/E_0), \quad (1)$$

where  $E_i$  is the total energy of the  $i$ -th pion, and  $E_0$  is the total energy of an incident projectile pion.



**Fig. 1.** Experimental transverse momentum distributions of the negative pions in the peripheral (open circles) and central (closed circles)  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c. Distributions are normalized per one negative pion. Statistical errors are shown



**Fig. 2.** Experimental emission angle distributions of the negative pions in the peripheral (open circles) and central (closed circles)  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c. Distributions are normalized per one negative pion. Statistical errors are shown

As seen from Table 1, the main fraction of  $\pi^-+^{12}\text{C}$  collision events are the peripheral interactions, and the mean multiplicity of the negative pions increases with the  $\pi^-+^{12}\text{C}$  collision centrality. At the same time, as observed from Table 1, the average value of the partial inelasticity coefficients of the negative pions ( $\langle K(\pi) \rangle$ ) does not depend on the centrality of  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c, both in the experiment and model.

As seen from Table 2, the average value of the full momentum of the negative pions decreases noticeably with an increase in the centrality of  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c. The average value of the transverse momentum of the negative pions, as ob-

served from Table 2, remains practically constant for all the three groups of  $\pi^-+^{12}\text{C}$  collision centralities and, hence, does not depend on the collision centrality. The comparison of the experimental transverse momentum distributions of the negative pions, normalized per one negative pion, in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c is presented in Fig. 1. As observed from this figure, the normalized experimental transverse momentum distributions of the negative pions coincide within the uncertainties in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions and, hence, also do not depend on the collision centrality. We observed that the normalized transverse momentum distributions practically coincide for all the three analyzed groups of  $\pi^-+^{12}\text{C}$  collision centralities also in the model.

The independence of the shapes and the average values of the transverse momentum distributions of the negative pions of the  $\pi^-+^{12}\text{C}$  collision centrality could be explained by the smallness of the projectile pion and the relatively low mass of a target carbon nucleus resulting in a quite small number of secondary nucleon (binary) collisions in the target nucleus.

As seen from Table 2, the average value of the emission angles of the negative pions increases with the  $\pi^-+^{12}\text{C}$  collision centrality both in the experiment and model. This is likely due to an increase in the number of the binary collisions and multiple rescatterings on target carbon nucleons, as the  $\pi^-+^{12}\text{C}$  collision centrality increases. The comparison of the experimental emission angle distributions of the negative pions, normalized per one negative pion, in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c, presented in Fig. 2, supports the above assumption. Figure 2 shows the clear differences between the emission angle distributions of the negative pions produced in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c. As one can see from Fig. 2, the fraction of the negative pions with lower emission angles decreases and that with the larger emission angles increases noticeably in the central  $\pi^-+^{12}\text{C}$  collisions as compared to the peripheral ones.

The similar behavior was observed from comparison of the emission angle distributions of the negative pions, normalized per one negative pion, in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c in the model. The increase of the fraction of the negative pions with larger emission angles in the central collisions compared to the peripheral ones is obviously

due to the increase of the number of relatively slow pions resulting from secondary interactions (rescatterings) on the target carbon nucleons with increasing the collision centrality.

The experimental full momentum distributions of negative pions in the peripheral and central  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c are presented in Fig. 3. As seen from Fig. 3, on the whole, the fraction of the negative pions with lower momentum increases and that with the larger momentum decreases noticeably in the

Table 3. Experimental transverse momentum distributions of the negative pions

Peripheral (open circles) collisions			Central (closed circles) collisions		
X axis	Y axis	Error of Y	X axis	Y axis	Error of Y
0.075	1.50587	0.03255	0.075	1.68454	0.06478
0.125	2.05172	0.03799	0.125	2.21283	0.07424
0.175	2.17545	0.03912	0.175	2.47131	0.07846
0.225	2.19142	0.03926	0.225	2.14741	0.07314
0.275	2.04132	0.03789	0.275	1.94105	0.06954
0.325	1.72767	0.03486	0.325	1.5407	0.06195
0.375	1.47236	0.03218	0.375	1.4714	0.06054
0.425	1.23082	0.02942	0.425	1.16512	0.05387
0.475	0.98034	0.02626	0.475	0.99149	0.0497
0.525	0.83176	0.02419	0.525	0.66591	0.04073
0.575	0.64722	0.02134	0.575	0.62175	0.03935
0.625	0.5033	0.01882	0.625	0.4729	0.03432
0.675	0.43083	0.01741	0.675	0.42657	0.0326
0.725	0.34751	0.01563	0.725	0.32233	0.02833
0.775	0.29028	0.01429	0.775	0.21515	0.02315
0.825	0.20856	0.01211	0.825	0.20926	0.02283
0.875	0.18672	0.01146	0.875	0.1701	0.02059
0.925	0.12874	0.00952	0.925	0.12927	0.01794
0.975	0.09507	0.00818	0.975	0.08505	0.01455
1.025	0.07002	0.00702	1.025	0.06556	0.0154
1.075	0.06803	0.00692	1.075	0.06125	0.01235
1.125	0.05181	0.00604	1.125	0.05869	0.01209
1.175	0.0417	0.00542	1.175	0.05255	0.01235
1.225	0.03521	0.00352	1.225	0.02757	0.0052
1.325	0.02096	0.00272	1.325	0.01786	0.00472
1.425	0.01497	0.00229	1.425	0.01445	0.00424
1.525	0.0104	0.00191	1.525	0.01148	0.00378
1.625	0.01075	0.00194	1.625	0.00996	0.00334
1.725	0.00534	0.00137	1.725	0.00497	0.00175
1.825	0.00416	0.00127	1.825	0.0045	0.00175
1.975	0.00345	0.00111	1.975	0.00319	0.00121
2.175	0.00331	1E-3	2.175	0.00319	0.00155
2.375	0.00194	7E-4	2.375	0.00163	5E-4
2.6	0.00137	5E-4	2.6	0.0012	4E-4

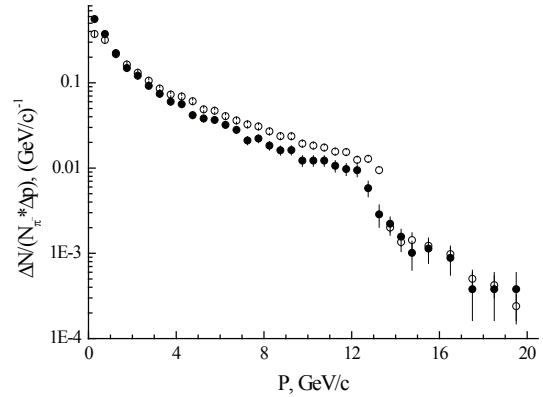


Fig. 3. Experimental total momentum distributions of the negative pions in the peripheral (open circles) and central (closed circles)  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c. Distributions are normalized per one negative pion. Statistical errors are shown

central  $\pi^- + {}^{12}\text{C}$  collisions as compared to the peripheral ones. The sharp drop in the momentum spectra of the negative pions in the peripheral and central collisions at the momentum range  $p > 12$  GeV/c in Fig. 3 is obviously due to the exclusion of the leading particles from the present analysis. The increase (decrease) of the fraction of the negative pions with the smaller (larger) momentum in the central collisions compared to the peripheral ones is likely due to an increase of the number of relatively slow pions resulting from secondary interactions (rescatterings) on the target carbon nucleons with increasing the collision centrality. This finding supports further the differences observed between the emission angle distributions of the negative pions in the peripheral and central  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c seen in Fig. 2.

The Modified FRITIOF model does not reproduce well the absolute values of the kinematical characteristics of the negative pions in  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c, as seen from Tables 1 and 2. However, this model describes qualitatively well the changes of the kinematical characteristics of negative pions with changing the centrality of  $\pi^- + {}^{12}\text{C}$  collisions.

In summary, the collision centrality dependences of the mean multiplicities and kinematical characteristics of the negative pions produced in  $\pi^- + {}^{12}\text{C}$  collisions at 40 GeV/c have been analyzed. The average values of the partial inelasticity coefficients of the negative pions (without “leading” pions) are found to be independent of the  $\pi^- + {}^{12}\text{C}$  collision centrality. The normalized transverse momentum distribu-

tions, as well as the average values of the transverse momentum of the negative pions, are shown to be independent within the uncertainties of the  $\pi^-+^{12}\text{C}$  collision centrality. This could be explained by the smallness of the impinging projectile pions and the relatively low mass of a target carbon nucleus resulting in a quite small number of secondary nucleon (binary) collisions in the target nucleus. The average

Table 4. Experimental emission angle distributions of the negative pions

Peripheral (open circles) collisions			Central (closed circles) collisions		
X axis	Y axis	Error of Y	X axis	Y axis	Error of Y
2.5	3.59667	0.03787	2.5	2.56826	0.06027
7.5	2.54641	0.03187	7.5	2.22358	0.05608
12.5	1.33252	0.02305	12.5	1.33182	0.0434
17.5	0.79777	0.01784	17.5	0.97265	0.03709
22.5	0.59425	0.01539	22.5	0.69614	0.03138
27.5	0.43103	0.01311	27.5	0.56581	0.02829
32.5	0.33319	0.01153	32.5	0.42113	0.02441
37.5	0.2511	0.01001	37.5	0.34704	0.02215
42.5	0.20824	0.00911	42.5	0.30811	0.02087
47.5	0.17657	0.00839	47.5	0.24627	0.01866
52.5	0.15781	0.00793	52.5	0.19272	0.01651
57.5	0.13176	0.00745	57.5	0.18628	0.01623
62.5	0.10242	0.00639	62.5	0.15502	0.01481
67.5	0.10246	0.00639	67.5	0.13473	0.0138
72.5	0.07179	0.00535	72.5	0.13075	0.0136
77.5	0.06305	0.00501	77.5	0.09901	0.01183
82.5	0.06052	0.00491	82.5	0.08888	0.01064
87.5	0.04728	0.00434	87.5	0.07777	0.01002
92.5	0.05075	0.0045	92.5	0.06557	0.00963
97.5	0.04306	0.00414	97.5	0.06311	0.00936
102.5	0.03666	0.00377	102.5	0.06231	0.00939
107.5	0.03714	0.00385	107.5	0.06231	0.00939
112.5	0.03621	0.0038	112.5	0.05651	0.00894
117.5	0.03322	0.00364	117.5	0.05506	0.00882
122.5	0.033	0.00363	122.5	0.04637	0.0081
127.5	0.02989	0.00343	127.5	0.04539	0.00801
132.5	0.02793	0.00334	132.5	0.03767	0.0073
137.5	0.02128	0.00291	137.5	0.04249	0.00775
142.5	0.01758	0.00265	142.5	0.03555	0.0059
147.5	0.01425	0.00238	147.5	0.03478	0.00701
152.5	0.01471	0.00242	152.5	0.0309	0.00661
157.5	0.01345	0.00232	157.5	0.02945	0.00645
162.5	0.01189	0.00218	162.5	0.02122	0.00566
167.5	0.01001	0.002	167.5	0.01449	0.00453
172.5	0.00832	0.00182	172.5	0.01014	0.00379
177.5	0.01078	0.00207	177.5	0.01014	0.00379

value of the emission angle (full momentum) of the negative pions is found to increase (decrease), as the  $\pi^-+^{12}\text{C}$  collision centrality increases. The comparison of the normalized emission angle (full momentum) distributions of the negative pions in the peripheral and central  $\pi^-+^{12}\text{C}$  collisions at 40 GeV/c supports the assumption that such an increase (decrease) of the average emission angle (average momentum) is likely due to an increase in the number of binary col-

Table 5. Experimental total momentum distributions of the negative pions

Peripheral (open circles) collisions			Central (closed circles) collisions		
X axis	Y axis	Error of Y	X axis	Y axis	Error of Y
0.275	0.37369	0.00485	0.275	0.55958	0.01116
0.75	0.31797	0.00471	0.75	0.37363	0.0096
1.25	0.21732	0.00389	1.25	0.22269	0.00741
1.75	0.16401	0.00338	1.75	0.14905	0.00606
2.25	0.13104	0.00302	2.25	0.12104	0.00546
2.75	0.10577	0.00272	2.75	0.09232	0.00477
3.25	0.08549	0.00244	3.25	0.07421	0.00428
3.75	0.07284	0.00225	3.75	0.06005	0.00385
4.25	0.06905	0.0022	4.25	0.05616	0.00372
4.75	0.06077	0.00206	4.75	0.04178	0.00321
5.25	0.04881	0.00185	5.25	0.03816	0.00307
5.75	0.04703	0.00181	5.75	0.03665	0.00301
6.25	0.0406	0.00168	6.25	0.0321	0.00281
6.75	0.03619	0.00159	6.75	0.02805	0.00263
7.25	0.03251	0.00151	7.25	0.02105	0.00228
7.75	0.03079	0.00147	7.75	0.02223	0.00234
8.25	0.02695	0.00137	8.25	0.01836	0.00213
8.75	0.02369	0.00129	8.75	0.01617	0.002
9.25	0.0236	0.00128	9.25	0.01626	0.002
9.75	0.01936	0.00116	9.75	0.01221	0.00174
10.25	0.01836	0.00113	10.25	0.01224	0.00172
10.75	0.01739	0.0011	10.75	0.01223	0.00175
11.25	0.0156	0.00104	11.25	0.01061	0.00162
11.75	0.01542	0.00104	11.75	0.00973	0.00155
12.25	0.01249	9.33855E-4	12.25	0.00943	0.00153
12.75	0.01284	9.46912E-4	12.75	0.00581	0.0012
13.25	0.00945	8.12088E-4	13.25	0.00286	8.40201E-4
13.75	0.00201	3.7431E-4	13.75	0.00222	4.49643E-4
14.25	0.00135	3.07422E-4	14.25	0.00157	3.49643E-4
14.75	0.00143	3.1573E-4	14.75	0.00101	3.76776E-4
15.5	0.00122	2.0674E-4	15.5	0.00114	3.74464E-4
16.5	9.76586E-4	1.84676E-4	16.5	8.84604E-4	3.30308E-4
17.5	5.02082E-4	1.32381E-4	17.5	3.79151E-4	2.16341E-4
18.5	4.23045E-4	1.21558E-4	18.5	3.79151E-4	2.16341E-4
19.5	2.40044E-4	9.15353E-5	19.5	3.79151E-4	2.16341E-4

lisions and multiple rescatterings with increasing the  $\pi^- + {}^{12}\text{C}$  collision centrality. The obtained experimental results were reproduced qualitatively well by the Modified FRITIOF model calculations.

## APPENDIXES

### Numerical Data for the figures

See Tables 3–5.

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ЗАЛЕЖНІСТЬ ХАРАКТЕРИСТИК НЕГАТИВНО  
ЗАРЯДЖЕНИХ ПІОНІВ, НАРОДЖЕНИХ В РЕАКЦІЇ  
 $\pi^- + {}^{12}\text{C}$  ПРИ 40 ГЕВ/С, ВІД ЦЕНТРАЛЬНОСТІ

Резюме

Досліджуються залежності середніх множин і кінематичних характеристик  $\pi^-$  мезонів, народжених в  $\pi^- + {}^{12}\text{C}$  зіткненнях при 40 ГЕВ/С від центральності зіткнень. Отримано, що середні значення коефіцієнтів парціальної непружності негативно заряджених піонів (без “лідуючих” піонів) не залежать від неї, як і нормовані розподіли поперечних імпульсів у межах невизначеностей. Знайдено, що середнє значення кута вильоту  $\pi^-$  мезонів росте зі збільшенням центральності  $\pi^- + {}^{12}\text{C}$  зіткнень. Порівняння нормованих розподілів кута вильоту  $\pi^-$  в периферичних і центральних зіткненнях підтримує припущення про те, що зростання середнього значення кута вильоту зобов’язане зростанню числа бінарних зіткнень і багаторазових перерозсіань зі збільшенням центральності. Отримані експериментальні результати якісно відтворені розрахунками по модифікованій моделі FRITIOF.