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UDC 539

BLACK HOLE TORSION EFFECT AND ITS RELATION TO INFORMATION

In order to study the effects of the torsion on the gravitation in space-time and its relation to information, we use the Schwarzschild metric, where the torsion is naturally introduced through the spin particle density. In the black hole scenario, we derive an analytic solution for the black hole gravitational radius with the spin included. Then we calculate its entropy in the cases of parallel and antiparallel spins. Moreover, four analytical solutions for the spin density as a function of the number of information are found. Using these solutions in the case of parallel spin, we obtain expressions for the Ricci scalar as a function of the information number N, and the cosmological constant λ is also revealed.

Keywords: gravitation, quantization, torsion, spin, black holes.

1. Introduction

A natural way to talk about spin effects in gravitation is through torsion. Its introduction becomes significant for the understanding of the last stage in the black hole evaporation. It could be the case of an evaporating black hole of mass M_H that disappears via an explosion burst, which can last for the time $t_p = 10^{-44}$ s, when it reaches a mass of the order of Planck's mass

$$m_p = \sqrt{\frac{\hbar c}{G}} = 10^{-15}\ \text{s}. \tag{1}$$

If this happens, there might be three distinct possibilities for the fate of the evaporating black hole [3]: The black hole may evaporate completely leaving no residue, in which case it would give rise to a serious problem of quantum consistency. If the final state of evaporation leaves a naked singularity behind, then it might violate the cosmic censorship at the quantum level. If a stable remnant of the residue with approximately Planck's mass remains, the emission process might stop.

If somebody tries to quantize the gravitational field, he must know that the quantization has to be directed with the unique structure of the space-time itself. The quantization will also imply that somebody might try to discretize the space and, probably, the time. Progress in this direction will also be related to the introduction of a spin in the theory of general relativity. The general relativity (GR) is the simplest theory of gravity which agrees with all present-day data. A major recent success is the detection of the lensed emission near the event horizon in the center of M-87 supergiant elliptic galaxy in the constellation Virgo. All the data obtained are consistent with the presence of a central Kerr black hole, as predicted by the general theory of relativity [1]. Somebody might want to formulate a generalized theory of general relativity to compare GR with various theories that explain other physical interactions. As an example, we say that the electromagnetic forces, strong interactions, and weak interactions are described with the help of quantum relativistic fields interacting in a flat Minkowski space. Furthermore, the fields that represent the interactions are defined over the space-time. But, at the same time, they are distinguished from the space-time which, we must say, is not affected by them. On the other hand, the gravitational interactions can modify the space-time geometry, but they are not represented by a new field. They are just represented by their effect on the geometry of the space itself. Thus, we can say that most parts of the modern physics are successful in being described in a flat rigid space-time geometry. But a small fraction of the remaining physics, i.e., macroscopic gravitational physics, requires the use of a curved dynamical geometric background. To overcome this difficulty, somebody might try to extend the geometric principles of GR into microphysics in order to establish a direct comparison and possibly some connection between gravity and other interactions. In GR theory, the matter is represented by the energy-momentum tensor, which essentially gives description of the mass density distribution in space-time. Therefore, the idea of mass-energy in GR is enough to define the properties of classical macroscopic bodies.

Looking at the microscopic level, we know that the matter is composed of elementary particles that obey the laws of special relativity and quantum mechanics. Each particle is characterized not only by its mass, but also by a spin measured in units of $\hbar$. At the microscopic level, the mass and the spin are two independent quantities. The mass distributions in space-time are described by the energy-momentum tensor, whereas the spin distribution is described, in field theory, by the spin density tensor. Inside any microscopic body, the spins of elementary particles are, in general, randomly oriented with the total average spin equal to zero. Therefore, the spin density tensor of a macroscopic body is zero. This explains why the energy-momentum tensor is adequate to dynamically characterize a macroscopic matter. Thus, the gravitational interactions can be sufficiently described by the Riemannian geometry. Another point that should be stressed is that the spin density tensor represents the intrinsic angular momentum of particles, and not the classical orbital angular momentum due to the macroscopic rotation. A fundamental difference is that the latter can be eliminated by an appropriate coordinate transformation. On the other hand, the spin density can be eliminated at a point only. The spin density tensor is a non-vanishing quantity, if the spins inside a body are oriented at least partially along a preferred direction and, at the same time, are not affected by the rotation of the macroscopic body. At the macroscopic level, the energy-momentum tensor is not enough to characterize the dynamics of the matter sources, because the spin density tensor is also needed, unless we are considering scalar fields that correspond to spineless particles. In the case where GR must be extended to include microphysics, the matter must be considered and described, by using the mass and the spin density. On the other hand, the mass is related to a curvature in a generalized theory of GR, and the spin should be related to the spin density tensor or, probably, to a different property of the space-time. The geometric property of the space-time in relation to spin in the U4 theory is the torsion.

The torsion, thus, can be described by the antisymmetric part of Christoffel symbols of the second kind. Therefore, the torsion tensor reads [5]:

$$Q^{\mu}_{\nu\lambda} = \frac{1}{2}\left(\Gamma^{\mu}_{\nu\lambda} - \Gamma^{\mu}_{\lambda\nu}\right) = \Gamma^{\mu}_{[\nu\lambda]}. \tag{2}$$

The torsion is characterized by a third-rank tensor that is antisymmetric in the first two indices and has 24 independent components. If the torsion does not vanish, the affine connection is not coincident with the Christoffel connection. Therefore, the geometry is not any longer the Riemannian, but rather Riemann–Cartan space-time with a non-symmetric connection. To introduce the torsion simply represents a very natural way of modifying GR. The relation of the torsion and the spin allows one to modify the GR theory and Riemannian geometry resulting in a more natural and complete description of the matter at the microscopical level as well. Finally, the early Universe is the place, where GR must be applied together with quantum theory. On the other hand, GR is a classical field theory. So far, the quantization of the gravity has been a problem in our effort to develop a consistent and coherent theory in understanding the physics of the early Universe.

In the presence of a torsion, the space-time is called a Riemann–Cartan manifold and is denoted by U4. When the torsion is taken into consideration, one can define distances in the following way. Supposing that we consider a small close circuit, we can write [5] the non-closure property given by the integral:

$$\ell^{\mu} = \oint Q^{\mu}_{\nu\lambda} dx^{\nu} \wedge dx^{\mu} \neq 0, \tag{3}$$

where $dx^{\nu}dx^{\mu}$ is the area element enclosed by the loop, ι^{μ} represents the so-called closure failure, and the torsion tensor $Q^{\mu}_{\nu\lambda}$ is a true tensorial quantity. In other words, the geometric meaning of the torsion can be represented by the failure of the loop closure. It has now the dimension of length, andn the torsion tensor itself has the dimension of L^{-1} .

2. Quantum Gravity and Torsion

The inclusion of the torsion into GR might constitute a way to the quantization of gravity, by considering the effect of the spin and connecting the torsion to the defects in the topology of space-time. For that, we can define a minimal unit of length l, as well as a minimal unit of time t. In GR and quantum field theory, there are now, indeed, difficulties due to the existence of infinities and singularities. One of the reasons is the consideration of point mass particles, which results in the divergence of the energy integrals going to infinity. In the case of collapsing bodies in GR, we have singularities. All these difficulties can disappear, if, together with the introduction of a torsion, we introduce the minimal time and length or, in other words, if we consider a discretized space-time. If we want to quantize the gravity, we cannot exactly follow the same procedure of quantization used in other fields. Indeed, the gravity is not a force, but the curvature and torsion of the space-time. The inclusion of the torsion in the space-time gives rise to space-time topology defects. The problem may be avoided, if the torsion is included. In this case, the asymmetric part of the connection $\Gamma^{\mu}_{[\nu\lambda]}$ or, in other words, the torsion tensor $Q^{\mu}_{\nu\lambda}$ is a true tensorial quantity. Since the torsion is related to the intrinsic spin, we see that the intrinsic spin $\hbar$ and, hence, the spin are quantized. We can conclude that the space-time defect in topology should occur in multiples of Planck's length $l_p = \sqrt{\frac{G\hbar}{c^3}}$. In other words, we can write [5]

$$\oint Q^{\mu}_{\nu\lambda} dx^{\nu} \wedge dx^{\lambda} = n\sqrt{\frac{\hbar G}{c^3}} n^{\mu}, \tag{4}$$

where n is an integer, and n^{μ} is a unit point vector. This is a relation analogous to the Bohr–Sommerfeld relation in quantum mechanics. The torsion tensor $Q^{\mu}_{\nu\lambda}$ plays the role of a field strength, which is analogous to that of the electromagnetic field tensor $F_{\mu\nu}$. Equation (4) defines the minimal fundamental length, a minimal length that enters the picture through the unit of action $\hbar$. In other words, $\hbar$ represents the intrinsic defect that is built in the torsion structure of space-time, in quantized units of $\hbar$ related to a quantized time like-vector with the dimension of length. This vector is related to the intrinsic geometric structure, when the torsion is considered. The intrinsic spin in units of $\hbar$ characterizes all the matter, and, therefore, the torsion is now entering the geometry. Thus, the Einstein–Cartan theory of gravitation can provide the corresponding quantum gravity effects. At the same time, we can also define the time at the quantum geometric level again through the torsion according to the equation:

$$t = \frac{1}{c} \oint Q^{\mu}_{\nu\lambda} dx^{\nu} \wedge dx^{\lambda} = n\sqrt{\frac{\hbar G}{c^5}}. \tag{5}$$

So, when the torsion is included, it is important that a minimal time interval given by Eq. (5) exists and is different from zero. This is the smallest unit of time $t_p = 5.391 \times 10^{-44}$ s. In the limit as $\hbar \to 0$, we recover the classical geometry of GR and, if $c \to \infty$, the Newtonian case. Finally, the geodesic equations in the case of a nonzero spin turn to

$$\frac{d^2 x^{\mu}}{dp^2} + \Gamma^{\mu}_{\nu\lambda} \frac{dx^{\mu}}{dp}\frac{dx^{\nu}}{dp} = -2Q^{\mu}_{\nu\lambda}\frac{dx^{\nu}}{dp}\frac{dx^{\lambda}}{dp}, \tag{6}$$

where p is an affine parameter. To understand the spin effects in gravitation, we can use the torsion. Consequently, let us first write a Schwarzschild metric that includes torsion effects [4]:

$$ds^2 = c^2\left(1 - \frac{2GM}{c^2 r} \pm \frac{3G^2 s^2}{2r^4 c^6}\right) dt^2 - \\ -\left(1 - \frac{2GM}{c^2 r} \pm \frac{3G^2 s^2}{2r^4 c^6}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{7}$$

where s is the torsion. We can write $s = \sigma r^3$, where σ is the spin density [4]. So, the Schwarzschild metric is modified by the inclusion of torsion effects. The torsion gives a natural way to understand the spin effects in gravitation. Making use of an expression that relates the torsion to the spin density, we can eliminate s and include σ in Eq. (7). Our primary goal is to establish a possible relation between the spin density σ and the information number N and between the Ricci scalar, as derived from Eq. (7), and information. This is an effort to understand why information plays an important role in the space-time structure in the case wherethe torsion effects are included in gravitation.

3. Analysis

Consider the case of a Schwarzschild metric with the torsion. Substituting $s = \sigma r^3$ [4], we get the gravitational radius:

$$\left(1-\frac{2GM}{c^2r}\pm\frac{3G^2\sigma^2}{2c^6}r^2\right)=0. \quad (8)$$

In the case of a spin parallel to the gravitation (plus sign), we have

$$\left(1-\frac{2GM}{c^2r}+\frac{3G^2\sigma^2}{2c^6}r^2\right)=0. \quad (9)$$

From whence, we obtain

$$r_{H_{\uparrow\downarrow}}=\frac{1}{3}\big[-(2^{2/3}c^6)/\big((9c^4G^5M\sigma^4+ + \sqrt{c^8G^6\sigma^6\left(2c^{10}+81G^4M^2\sigma^2\right)}\big)^{1/3}\big)\big]\pm \pm\frac{1}{3}\big[\big(2^{1/3}\big(9c^4G^5M\sigma^4+ + \sqrt{c^8G^6\sigma^6(2c^{10}+81G^4M^2\sigma^2)}^{1/3}\big)\big)/\big(G^2\sigma^2\big)\big], \quad (10)$$

where the plus sign in Eq. (10) corresponds to the plus sign of the second term in Eq. (8). The negative sign in Eq. (10) corresponds to the negative sign in the second term of Eq. (8). In other words, we deal with parallel and antiparallel spins. Let us write the entropy formula as [6]

$$S=\frac{k_{\rm B}}{4\ell_p^2}A_H, \quad (11)$$

where k_B is the Boltzmann constant, $l_p^2=\frac{G\hbar}{c^3}$ is Planck's length, and A_H is horizon area [2]. This is the Bekenstein–Hawking area-entropy law. This is a macroscopic formula, and it should be viewed in the same light as the classical macroscopic thermodynamic formulae. It describes how the properties of event horizons in general relativity change as their parameters are varied. Substituting Eq. (10) in Eq. (12), we obtain

$$S=\frac{\pi k_{\rm B}}{\ell_p^2}(r_{H_{\uparrow\downarrow}})^2= =\frac{\pi k_{\rm B}}{\ell_p^2}\left[\frac{1}{3}\left[-\left(2^{2/3}c^6\right)/\left(\left(9c^4G^5M\sigma^4+ + \sqrt{c^8G^6\sigma^6\left(2c^{10}+81G^4M^2\sigma^2\right)}\right)^{1/3}\right)\pm \pm\left(2^{1/3}\left(9c^4G^5M\sigma^4+ + \sqrt{c^8G^6\sigma^6(2c^{10}+81G^4M^2\sigma^2)}\right)^{1/3}\right)/(G^2\sigma^2)\right]\right]^2, \quad (12)$$

where the minus sign in the root stands for the parallel torsion and plus stands for the antiparallel one. We note that the information number in nats is given by [8]

$$N=\frac{S}{k_{\rm B}\ln 2}. \quad (13)$$

Using the positive sign, equating Eqs. (12) and (13), and solving for the spin density as a function of information in nat N, we obtain the following solutions:

$$\sigma_{1_\uparrow}=\sigma_{2_\uparrow}=\pm i\left(\frac{4c^4M}{G\ell_p^3N^{\frac{3}{2}}}\left(\frac{\pi}{\ln 2}\right)^{3/2}+\frac{\pi c^6}{G\ell_p^2N\ln 2}\right)^{1/2}, \quad (14)$$

$$\sigma_{3_\uparrow}=\sigma_{4_\uparrow}=\pm i\left(\frac{4c^4M}{G\ell_p^3N^{\frac{3}{2}}}\left(\frac{\pi}{\ln 2}\right)^{3/2}-\frac{\pi c^6}{G\ell_p^2N\ln 2}\right)^{1/2}. \quad (15)$$

Similarly, the negative sign (or antiparallel spin) gives the only real solution:

$$\sigma_{1_\downarrow}=\left[\frac{8\pi^3c^8M^2}{3G^2\ell_p^6\left(\Phi_0+\frac{6\sqrt{\Gamma_0}}{G^5\ell_p^9}\right)^{1/3}}+\frac{2\left(\Phi_0+\frac{6\sqrt{\Gamma_0}}{G^5\ell_p^9}\right)^{1/3}}{9N^3\ln 2^3}+ +\frac{4\pi c^6}{9G^2\ell_p^2N\ln 2}+\frac{2\pi^2c^{12}N\ln 2}{9G^4\ell_p^6(\Phi_0+\frac{6\sqrt{\Gamma_0}}{G^5\ell_p^9})^{1/3}}\right]^{1/2}, \quad (16)$$

where the quantities Γ_0 and Φ_0 are defined as follows:

$$\Gamma_0=-48\pi^9c^{24}\ln 2^9M^6N^9+ +24\pi^8c^{28}G^2\ell_p^2M^4\ln 2^{10}N^{10}+ +\pi^7c^{32}\ell_p^4M^2N^{11}\ln 2^{11}, \quad (17)$$

$$\Phi_0=\frac{36\pi^4c^{14}M^2N^5\ln 2^5}{G^4\ell_p^8}+\frac{\pi^3c^{18}N^6\ln 2^6}{G^6\ell_p^6}. \quad (18)$$

4. Calculation of the Ricci Scalar and Its Relation to Information

Next, we are going to calculate the Ricci scalar in the cases of parallel and antiparallel spins. So, we define the metric coefficients to be

$$A(r)=c^2\left[1-\frac{2GM}{rc^2}\pm\frac{3G^2\sigma^2}{2c^6}r^2\right], \quad (19)$$

and

$$B(r) = \left[1 - \frac{2GM}{rc^2} \pm \frac{3G^2\sigma^2}{2c^6}r^2\right]^{-1}. \quad (20)$$

The correspondent Ricci scalar is given by [9]

$$R = -\frac{2}{r^2B(r)}\left[1 - B(r) + \frac{r^2A''(r)}{2A(r)} + \right.$$

$$\left. + \frac{A'(r)}{A(r)}\left(r - \frac{r^2A'(r)}{4A(r)}\right) - \frac{B'(r)}{B(r)}\left(r + \frac{r^2A'(r)}{4A(r)}\right)\right]. \quad (21)$$

In the case of the torsion parallel to the gravity, we get

$$R = -\frac{18G^2\sigma^2}{c^6} = -\frac{9}{2}\left(\frac{R_{\rm Sch}}{M}\right)^2\left(\frac{\sigma}{c}\right)^2. \quad (22)$$

Similarly, in the case of the torsion antiparallel to the gravity, we obtain

$$R = \frac{18G^2\sigma^2}{c^6} = \frac{9}{2}\left(\frac{R_{\rm Sch}}{M}\right)^2\left(\frac{\sigma}{c}\right)^2. \quad (23)$$

Next, we proceed in writing the Ricci scalar as a function of the information number in nats N. In this calculation, we will only deal with a parallel spin. Therefore, we use Eqs. (22) and (15) and obtain

$$R\left(\sigma_1/\sigma_2\right)_\uparrow =$$

$$= \frac{18G^2}{c^6}\left[\left(\frac{\pi}{\ln 2}\right)^{3/2}\frac{4c^4M}{3G\ell_p^3N^{3/2}} + \frac{2\pi c^6}{3G^2\ell_p^2N\ln 2}\right]^2, \quad (24)$$

$$R\left(\sigma_3/\sigma_4\right)_\uparrow =$$

$$= -\frac{18G^2}{c^6}\left[\left(\frac{\pi}{\ln 2}\right)^{3/2}\frac{4c^4M}{3G\ell_p^3N^{\frac{3}{2}}} - \frac{2\pi c^6}{3G^2\ell_p^2N\ln 2}\right]^2, \quad (25)$$

which simplifies to

$$R\left(\sigma_1/\sigma_2\right)_\uparrow = 12\left(\frac{\pi}{\ln 2}\right)^{3/2}\left(\frac{R_{\rm Sch}}{\ell_p^3N^{\frac{3}{2}}}\right) + \frac{12\pi}{N\ell_p^2\ln 2}, \quad (26)$$

$$R\left(\sigma_3/\sigma_4\right)_\uparrow = 12\left(\frac{\pi}{\ln 2}\right)^{3/2}\left(\frac{R_{\rm Sch}}{\ell_p^3N^{\frac{3}{2}}}\right) + \frac{12\pi}{N\ell_p^2\ln 2}. \quad (27)$$

With reference to [6] and [7], we note that

$$\Lambda = \frac{3\pi}{N\ell_p^2\ln 2}. \quad (28)$$

Equation (28) gives the cosmological constant as a function of the information number N. Therefore, Eqs. (26) and (27) for the Ricci scalar become

$$R\left(\sigma_1/\sigma_2\right)_\uparrow = 12\left(\frac{\pi}{\ln 2}\right)^{3/2}\left(\frac{R_{\rm Sch}}{\ell_p^3N^{3/2}}\right) + 4\Lambda, \quad (29)$$

$$R\left(\sigma_3/\sigma_4\right)_\uparrow = 12\left(\frac{\pi}{\ln 2}\right)^{3/2}\left(\frac{R_{\rm Sch}}{\ell_p^3N^{3/2}}\right) + 4\Lambda. \quad (30)$$

5. Conclusion

We have examined the effect of a torsion in the Schwarzschild metric corrected for torsion effects and its relation to information. In this case, the torsion effects can be represented by the spin density. We start by calculating the entropy at the horizon of such a black hole, and then we equate the entropy to a known expression that gives the entropy in terms of the information number N. Thus, we obtain analytical expressions for the spin density as a function of the information number N. We obtain two spin density solutions. One of them is real, and another one is imaginary. Moreover, we have found that, for the spin density, both real and imaginary roots scale proportionally to the information number N according to the relation $\sigma \propto \frac{1}{N^{\frac{3}{2}}} - \frac{1}{N}$. In the case of parallel spin, we find that Ricci scalar also depends on the information number according to the relation $R \propto N^{\frac{3}{2}} + N^{-1}$ for both parallel and antiparallel spins. This comes from an extra term that is equal to the cosmological constant λ expressed as a function of the information number N adds the information dependence to the Ricci scalar via the cosmological constant λ. In this aspect, we can perceive the cosmological constant as a cosmological depository of information that affects the space-time structure or is included as an important parameter in the space-time structure and in the geometry of the Universe. Therefore, we conclude that information enters this torsion-corrected metric via the dependence of the spin density on the information number N, as well as the cosmological constant itself.

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Received 08.07.19

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ЕФЕКТ КРУЧЕННЯ ЧОРНОЇ ДІРИ
ТА ЇЇ ВІДНОШЕННЯ ДО ІНФОРМАЦІЇ

Р е з ю м е

Для вивчення впливу кручення на гравітацію в просторі-часі та його відношення до інформації ми користуємося метрикою Шварцшільда, де кручення природно вводиться через спінову щільність частинки. В сценарії чорної діри ми отримали аналітичний розв'язок для гравітаційного радіуса чорної діри з включенням спіну, звідки ми обчислили ентропію для випадків паралельних та антипаралельних спінів. Більше того, ми знайшли чотири аналітичні розв'язки для спінової щільності в залежності від числа інформації. Користуючись цими розв'язками, ми отримали вирази для коефіцієнтів Річчі як функції числа інформації N; отримано також значення для космологічної константи.