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SPIN-POLARIZED CURRENT-DRIVEN FERROMAGNETIC DOMAIN WALL MOTION WITH A SKYRMION-LIKE BUILDING BLOCK

The purpose of the research is the construction of an analytic model for the description of a spin-polarized current-driven ferromagnetic domain wall motion with a skyrmion-like building block. The motion velocity of the ferromagnetic domain wall with a skyrmion-like building block is found as a function of the driving torques and an external magnetic field strength.

K e y w o r d s: ferromagnet, domain wall, skyrmion-like building block, spin-polarized current.

1. Introduction

Recently, domain walls in ferromagnetic nanosized samples have been an urgent object of researches as promising carriers of information bits for applications in magnetic memory devices [1]. Moreover, the domain wall in a ferromagnet can have the simplest defect-free structure such as a "transverse" wall or include vortices and other topological defects such as a "vortex" wall. Among a wide variety of magnetic topological objects, the following are most distinguished as "building blocks" in the internal structure of domain walls: vortex, antivortex, bimeron, Bloch line, and Bloch point.

In magnets, the Bloch lines divide the surface of a domain wall into two subdomains and significantly affect the properties of domain walls. Numerous studies have been devoted to the construction of magnetic memory devices based on Bloch lines [2, 3]. To date, the Bloch lines in ferromagnets with high uniaxial anisotropy are the most studied [4]. The corresponding studies began much earlier in weakly anisotropic films [5], and modern achievements in this direction are described in [6]. The Bloch lines are observed regardless of the sign of the magnetic anisotropy constant in cubic ferromagnets [7,8]. A theoretical model of Bloch lines in weak ferromagnets was proposed in [9]. Local bends were observed in the Bloch lines moving at high speeds in yttrium orthoferrite, and they

were associated with the movement of vortices along the domain wall [10].

The Bloch point is one of the examples of point topological defects in domain walls and was first proposed in [11, 12]. The defining property of the Bloch point is that it represents the topological singularity of the magnetization field, and one can find all possible directions of the magnetization vector on the sphere with infinitely small radius centered at the Bloch point. Unlike other topological spin textures such as magnetic skyrmions and vortices, the Bloch points have a unique feature – the local magnetization at the Bloch point completely disappears. This was experimentally confirmed in yttrium ferrite garnet crystals, micron-thick garnet films, and magnetic cylindrical wire based on static measurements [13–15].

It was shown in [11] that the structure of the Bloch point is mainly determined by the exchange energy. Later in [12], the specific energy was calculated, and it was shown that its value is topologically invariant. A family of magnetization textures with local rotation angle γ (in the azimuthal direction) was considered in [12], and it was found that minimizing the magnetostatic energy selects a specific angle $\gamma \approx 112°$ [12]. In order to study the region near the singular point, the Landau magnetic energy [16] was included in [17], and the neglect of a magnetostatic energy was justified. As a result, it was shown that the magnitude of the magnetization vector increases linearly with the radial distance from the center. The magnetiza-

tion field of the Bloch point was calculated with regard for the exchange energy, Landau magnetic energy, and magnetostatic energy in [18]. The Bloch point in the domain wall of a ferromagnet is characterized by a topological (skyrmionic) charge $q = \pm 1$ [19]. There are the infinite number of Bloch point configurations. However, there are three main possible configurations of the Bloch point, namely, a hedgehog configuration in which the magnetization distribution around the Bloch point is spherically symmetric, and the magnetization vector is directed away from the Bloch point (diverging Bloch point $q = +1$) or to the Bloch point (converging Bloch point $q = -1$), vortex or antivortex ($q = +1$ or -1) and spiral ($q = +1$ or -1) configurations, which are obtained by the 90° and 180° rotations of the magnetization of a similar configuration, respectively [20–22]. Direct observation of the stabilized structures of Bloch points with a skyrmion charge $q = +1$, namely, hedgehog-like, vortex, and spiral configurations, is reported in [19]. The in-plane and out-of-plane magnetization components were observed using the magnetic transmission soft X-ray microscopy MTXM [19], and the corresponding structures were determined based on a numerical micromagnetic simulation [19].

"Vortex" or "topological" domain walls with integer topological charges, as well as integer or fractional winding numbers of volume vortices and edge defects, are observed, for example, in ferromagnetic nanowires and nanorings [1]. The dynamics of a domain wall in a ferromagnet depends on the topological charge of the vortices in its structure. The movement of the domain wall leads to the creation, propagation, and annihilation of such defects.

The interest in the dynamics of magnetic vortices and Bloch points is also associated with the discovery of the fast magnetization reversal of the core of a magnetic vortex by alternating external influences (magnetic field [23] or spin current [24]). The numerical micromagnetic simulation of the magnetization reversal of a vortex core [25] showed that the mechanism of annihilation of vortex-antivortex pairs [26] requires the mediation of a magnetization singularity: the "magnetic monopole" or, in other words, the Bloch point [21].

At the same time, the vast majority of theoretical studies of the internal structure of domain walls in ferro- and antiferromagnets are based on a numerical micromagnetic modeling. However, the results of

analysis of exact analytic solutions of the Landau–Lifshitz equations in ferro- and antiferromagnets have shown [27] that there can exist the infinite number of magnetic textures under the same boundary conditions for the magnetization vector (as well as the antiferromagnetism vector), which obviously represents the problem for a numerical micromagnetic simulation.

In a number of works [28, 29], the Landau–Lifshitz–Gilbert–Slonczewski equation was successfully used to describe the dynamics of a domain wall in a free ferromagnetic layer during its long-scale translational motion along this layer in the case of the spin-polarized current flowing perpendicularly to the layers of the layered ferromagnet/nonmagnetic metal/ferromagnet structure under the assumption that the "ballistic conditions" are fulfilled [30].

In this work, we will obtain a 3D exact dynamic solution of the Landau–Lifshitz–Gilbert–Slonczewski equation in a ferromagnet with uniaxial magnetic anisotropy, which describes the motion of a domain wall with a skyrmion-like building block in the internal structure under the influence of an external magnetic field and a spin current. The solution is obtained for an infinite ferromagnet. However, this result is naturally applicable to the free layer of a layered system ferromagnet/nonmagnetic metal/ferromagnet under the condition that the characteristic scale of a skyrmion-like building block, as a component of the internal structure of the domain wall, is much less than the thickness of the free layer. The last condition is well satisfied for skyrmions with sizes of the order of several nm, which are especially popular as promising carriers of information for spintronics.

2. Theory and Calculation

Let us consider a ferromagnet with uniaxial magnetic anisotropy and magnetization $\mathbf{M}$, $|\mathbf{M}| = M_0$, where the absolute value of the magnetization is equal to $M_0 = \text{const}$. The expression for the magnetic energy of a ferromagnet and the equation of magnetization dynamics can be written in terms of the angular variables that are introduced in the standard way:

$$
\begin{aligned}
M_x &= M_0 \sin\theta \cos\varphi, \\
M_y &= M_0 \sin\theta \sin\varphi, \; M_z = M_0 \cos\theta,
\end{aligned}
\tag{1}
$$

where θ and φ are the polar and azimuth angles for the magnetization, and M_x, M_y, and M_z are the Cartesian components of the magnetization vector.

The magnetic energy of a ferromagnet has the form

$$W = M_0^2 \int d\mathbf{r} \left\{ \frac{\alpha_{\text{ex}}}{2} \left\{ \left[\frac{\partial \theta}{\partial x_i} \right]^2 + \right.\right.$$
$$\left.\left. + \sin^2 \theta \left[\frac{\partial \varphi}{\partial x_i} \right]^2 \right\} + \frac{\beta}{2} \sin^2 \theta - \frac{H_0}{M_0} \cos \theta \right\}, \tag{2}$$

where α_{ex} is the nonuniform exchange constant ($\alpha_{\text{ex}} > 0$), β is the uniaxial magnetic anisotropy constant, $\mathbf{H}_0$ is an external magnetic field strength directed along the OZ axis, and the integration in (2) is taken over the volume of a ferromagnet.

The Landau–Lifshitz–Gilbert–Slonczewski equation for a ferromagnet has the form

$$\frac{\partial \mathbf{M}}{\partial t} = -|g| \left[\mathbf{M} \times \mathbf{H}^{\text{eff}} \right] +$$
$$+ \frac{\alpha_G}{M_0} \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \right] + |g| \, \mathbf{T}, \tag{3}$$

where $\mathbf{T} = \mathbf{T}_{\parallel} + \mathbf{T}_{\perp}$ is the spin-transfer torque, i.e., the torque induced upon the magnetization by a spin-polarized current flowing through the ferromagnet,

$$\mathbf{T}_{\parallel} = -\frac{a_J}{M_0} \left[\mathbf{M} \times [\mathbf{M} \times \mathbf{m}_p] \right],$$

$$\mathbf{T}_{\perp} = b_J \left[\mathbf{M} \times \mathbf{m}_p \right],$$

$|g|$ is the gyromagnetic ratio, α_G is the damping factor, $\mathbf{H}^{\text{eff}}$ is the effective field, $\mathbf{H}^{\text{eff}} = -\frac{\delta W}{\delta \mathbf{M}}$, a_J and b_J are the driving torques, and $\mathbf{m}^p$ is the unit vector along the polarization of the current.

The Landau–Lifshitz–Gilbert–Slonczewski equation for a ferromagnet can be written in terms of the angular variables [28, 29]

$$\begin{cases} \sin \theta \dfrac{\partial \varphi}{\partial t} = \dfrac{|g|}{M_0} \dfrac{\delta W}{\delta \theta} + |g| \, b_J m_z^p \left\{ -\sin \theta m_z^p + \right. \\ \left. \quad + \sin \varphi \cos \theta m_y^p + \cos \varphi \cos \theta m_x^p \right\} - \\ - |g| \, a_J \left\{ \sin \varphi m_x^p - \cos \varphi m_y^p \right\} + \alpha_G \dfrac{\partial \theta}{\partial t}, \\ -\sin \theta \dfrac{\partial \theta}{\partial t} = \dfrac{|g|}{M_0} \dfrac{\delta W}{\delta \varphi} + \\ + |g| \, b_J \sin \theta \left\{ \cos \varphi m_y^p - \sin \varphi m_x^p \right\} - \\ - |g| \, a_J \left\{ \sin \theta \cos \theta \cos \varphi m_x^p + \right. \\ \left. \quad + \sin \theta \cos \theta \sin \varphi m_y^p - \sin^2 \theta m_z^p \right\} + \\ + \alpha_G \sin^2 \theta \dfrac{\partial \varphi}{\partial t}. \end{cases} \tag{4}$$

The Landau–Lifshitz–Gilbert–Slonczewski equation can be simplified considering the spin-torque polarization along the OZ axis $m_x^p = m_y^p = 0, m_z^p = \pm 1$:

$$\begin{cases} \sin \theta \dfrac{\partial \varphi}{\partial t} = \dfrac{|g|}{M_0} \dfrac{\delta W}{\delta \theta} - |g| \, b_J \sin \theta m_z^p + \alpha_G \dfrac{\partial \theta}{\partial t}, \\ \sin \theta \dfrac{\partial \theta}{\partial t} = -\dfrac{|g|}{M_0} \dfrac{\delta W}{\delta \varphi} - |g| \, a_J \sin^2 \theta m_z^p - \\ - \alpha_G \sin^2 \theta \dfrac{\partial \varphi}{\partial t}. \end{cases} \tag{5}$$

It is possible to obtain the following equations for the magnetization dynamics substituting the energy of a ferromagnet with uniaxial magnetic anisotropy into the system of Landau–Lifshitz–Gilbert–Slonczewski equations

$$\begin{cases} \sin \theta \dfrac{\partial \varphi}{\partial t} = M_0 |g| \left(\alpha_{\text{ex}} \left\{ -\Delta \theta + \right.\right. \\ \left. + \sin \theta \cos \theta \, (\nabla \varphi)^2 \right\} + \beta \sin \theta \cos \theta + \\ \left. + \dfrac{H_0}{M_0} \sin \theta \right) - |g| \, b_J \sin \theta m_z^p + \alpha_G \dfrac{\partial \theta}{\partial t}, \\ \dfrac{\partial \theta}{\partial t} = M_0 |g| \, \alpha_{\text{ex}} \left\{ \cos \theta \nabla \theta \nabla \varphi + \right. \\ \left. + \sin \theta \Delta \varphi \right\} - |g| \, a_J \sin \theta m_z^p - \\ - \alpha_G \sin \theta \dfrac{\partial \varphi}{\partial t}. \end{cases} \tag{6}$$

3. Results and Discussion

The equations for the magnetization dynamics have the following exact dynamic solution:

$$\begin{cases} \varphi = n\alpha + v_\varphi t + \alpha_0, \\ \text{tg} \dfrac{\theta}{2} = \exp \left(\dfrac{z - vt}{\delta} + n \ln \dfrac{r}{r_0} \right), \end{cases} \tag{7}$$

where

$$\begin{cases} v_\varphi = \dfrac{|g| \, H_0}{1 + \alpha_G^2} - \dfrac{|g| \, m_z^p}{1 + \alpha_G^2} \left(\alpha_G a_J + b_J \right), \\ v = \delta \left\{ \dfrac{\alpha_G}{(1 + \alpha_G^2)} |g| \, H_0 + \right. \\ \left. + \dfrac{|g| \, m_z^p}{(1 + \alpha_G^2)} \left(a_J - \alpha_G b_J \right) \right\}, \\ \delta = \sqrt{\dfrac{\alpha_{\text{ex}}}{\beta}}, \end{cases} \tag{8}$$

n is an arbitrary integer number, α_0 is an arbitrary initial phase, $0 \leq \alpha_0 \leq 2\pi$, δ is the constant with

the dimension of length which characterizes a domain wall thickness, r is a radius vector in the plane XOY, r_0 is the characteristic size of a skyrmion-like building block, and α is the azimuth angle of the radius vector.

In particular at $n = 0$, solution (7) describes the motion of a domain wall of the new type with an oscillating azimuth angle ϕ. This peculiarity distinguishes solution (7) from solutions of the Walker type with constant azimuth angle ϕ. The motion of a domain wall of type (7) can be carried out both due to the external magnetic field and the spin-polarized current in a plane geometry of the type used in [28, 29]. In this case, solution (7) at $n = 0$, which was obtained formally for an infinite medium, exactly satisfies the boundary conditions for the magnetization vector at the interface for any thickness of the plane layer, which satisfies the conditions of applicability of the Landau–Lifshitz–Gilbert–Slonczewski equation.

When $n \neq 0$, solution (7) describes the stationary motion of a complex domain wall. The skyrmion as a domain wall building block can be of any topological charge n (n is equal to both the skyrmion charge and the skyrmion winding number), any size δ, and any initial helicity $\chi = \frac{\alpha_0}{n}$. The analytic model describes the temporary oscillations of the skyrmion helicity from zero to $\chi = \frac{2\pi}{n}$. This means that, during the period of such oscillations $T = \frac{2\pi}{v_\varphi}$, the skyrmion type is transforming from the Néel type at $\alpha_0 = 0$, $\alpha_0 = \pi$, $\alpha_0 = 2\pi$ to the intermediate type at arbitrary $\alpha_0 \neq$ $\neq 0, \alpha_0 \neq \pi, \alpha_0 \neq \pm\frac{\pi}{2}, \alpha_0 \neq 2\pi$, then to the Bloch type $\alpha_0 = \pm\frac{\pi}{2}$, and then again to the Néel type.

Solution (7) can be realized also in another geometry, for example, at the passing of a spin-polarized current directly through a cylindrical ferromagnetic sample or through a conducting system on the surface of a ferromagnetic cylinder (wire). Solution (7) can also be imagined as a domain wall with the reversal of the magnetization in it along the OZ axis, but not of the cylindrical shape, but of a more complex funnel-shaped structure. There is the reversal of the magnetization in the wall but the skyrmion-like internal structure changes with the coordinate r in each transverse section. The experimental implementation of such a configuration is, in principle, possible without switching on the spin-polarized current, for example, if an external magnetic field of the corresponding direction is applied to the ends of a cylindrical ferromagnet of finite height. Then the magnetization

distribution with a structure similar to solution (7) is possible in the sample.

4. Conclusions

The obtained exact dynamic solution (7), (8) of the Landau–Lifshitz–Gilbert–Slonczewski equation in a ferromagnet with uniaxial magnetic anisotropy describes the spin-polarized current-driven ferromagnetic domain wall motion with a skyrmion-like building block. There is the linear dependence of its motion velocity v on the driving torques and the external magnetic field strength according to expression (7). The new exact analytical solution (7), (8) differs from the numerical or analytic solutions presented in the literature by the oscillating time dependence of the azimuthal angle of the magnetization vector. This is a reflection of the fact that there is no uniqueness theorem for solutions of the Landau–Lifshitz equation. For example, specific examples of the existence of an infinite set of exact analytic solutions of the Landau–Lifshitz equation in a specific sample were presented under the same boundary conditions in paper [31]. The temporary oscillations of the skyrmion-like building block type from the Néel to Bloch one during the domain wall motion are predicted according to formula (7). The period of oscillations is $T = \frac{2\pi}{v_\varphi}$.

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РУХ ФЕРОМАГНІТНОЇ ДОМЕННОЇ
СТІНКИ ІЗ ВБУДОВАНИМ СКІРМІОНОМ
ПІД ДІЄЮ СПІН-ПОЛЯРИЗОВАНОГО СТРУМУ

Р е з ю м е

Мета дослідження – побудова аналітичної моделі для опису руху феромагнітної доменної стінки зі скірміон-подібним блоком у її внутрішній структурі під впливом спін-поляризованого струму. Визначено швидкість руху феромагнітної доменної стінки зі скірміон-подібним блоком як функцію рушійних сил та напруженості зовнішнього магнітного поля.