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ABOUT TRANSVERSE MOMENTUM DISTRIBUTIONS OF NEGATIVE PIONS IN p^{12}C AND π^{-12}C COLLISIONS AT HIGH ENERGIES

Collision centrality dependences of the transverse momentum distributions of negative pions produced in $p^{12}C$ *and* $\pi^{-12}C$ *collisions at 9.9 and 40 GeV/c, respectively, are investigated. The shapes and widths of the normalized transverse momentum distributions, as well as the average values of the transverse momentum of the negative pions, do not depend within the uncertainties on the collision centrality in* $p^{12}C$ *and* $\pi^{-12}C$ *collisions at 9.9 and 40 GeV/c in experiment, which is confirmed by the modified FRITIOF model calculations and minimum* χ^2 *fits of the experimental spectra with the two- and three-temperature Hagedorn model functions. Modified FRITIOF model calculations underestimate the average values of transverse momenta of the negative pions in experiment and do not reproduce the tail of the experimental transverse momentum distributions of the negative pions in both collision types. It is obtained that the transverse momentum distributions of the negative pions exhibit two temperature (two-slope) shapes in peripheral and central* $p^{12}C$ *collisions at 9.9 GeV/c in agreement with the early works on nucleus-nucleus collisions at incident energies of the order of a few GeV/c. However, the transverse momentum distributions of the negative pions in peripheral and central* $\pi^{-12}C$ *collisions at 40 GeV/c can be described well assuming the three temperature (three-slope) shapes of these spectra.*

Keywords: pion production, collision centrality, relativistic nuclear collisions.

1. Introduction

It is well known that the largest fraction of the collision energy in relativistic nuclear collisions goes to the production of pions, which have the lowest production threshold energy among all hadrons. In Refs. [1–3], we investigated the mean multiplicities and various kinematic characteristics of the charged pions, produced in π^{-12}C interactions at 40 GeV/c, ^{12}C^{12}C, and ^{12}C^{181}Ta collisions at 4.2 A GeV/c at various collision centralities. It was established that the mean numbers of negative pions per one participant proton and the average values of their full and transverse momenta, emission angles, and partial inelasticity coefficients in $^{12}C^{12}C$ collisions at 4.2 A GeV/c [3] do not depend within statistical uncertainties on the degree of collision centrality. It was also deduced that the projectile nucleons interact with target nucleons independently from each other. In $^{12}C^{181}Ta$ collisions at 4.2 A GeV/c, an increase of the degree of absorption of negative pions by a target nucleus with the collision centrality was observed [1]. The present analysis is a continuation of the previous works [1–3] and devoted to the comparative analysis of the characteristics of negative pions in $p^{12}C$ collisions at 9.9 GeV/c and $\pi^{-12}C$ interactions at 40 GeV/c at various collision centralities. The experimental data are compared with the calculations within a modified FRITIOF model [4–7]. For the separation of the collision events on peripheral, semicentral, and central collision events, we used, as was done in previous works [1–3], the mean multiplicities per event of the participant protons, $\langle \nu_p \rangle$, from a target nucleus, calculated for the whole ensemble of inelastic interactions, and the number of participant protons, ν_p, in each individual collision event. The protons having momentum $0.22 \leqslant p \leqslant 0.75$ GeV/c in the laboratory frame in $p^{12}C$ collisions at 9.9 GeV/c and $\pi^{-12}C$ interactions at 40 GeV/c were considered as participant protons. All positive singly charged particles with momentum $p \geqslant 0.75$ GeV/c in the laboratory frame in $\pi^{-12}C$ interactions at 40 GeV/c were taken as π^+ mesons. The admixture of protons with momentum $p > 750$ MeV/c among π^+ mesons was estimated to be ≈12% in $\pi^{-12}C$ interactions at 40 GeV/c [8]. Therefore, in $\pi^{-12}C$ interactions at 40 GeV/c, we included the fast protons among π^+ mesons with momentum $p > 750$ MeV/c, applying the 12% weightage, to the number of participant protons.

The events with $\nu_p \leqslant \langle \nu_p \rangle$, $\langle \nu_p \rangle < \nu_p \leqslant 2\langle \nu_p \rangle$, and with $\nu_p > 2\langle \nu_p \rangle$ were selected to be peripheral, semicentral, and central collision events, respectively [1–3]. The same criteria were applied to the collision events generated with the use of a modified FRITIOF model [4–7].

The experimental data were collected using a 2-meter propane (C_3H_8) bubble chamber of the Laboratory of High Energies (LHE) of the Joint Institute for Nuclear Research (JINR, Dubna, Russia) exposed to π^- mesons accelerated to a momentum of 40 GeV/c at HEPI (High Energy Physics Institute, Russia) accelerator and exposed to protons accelerated to a momentum of 9.9 GeV/c at the Dubna synchrophasotron. The experimental data consist of the measured 18,941 $p^{12}C$ and 15,841 $\pi^{-12}C$ collision events with practically all the secondary charged particles of reactions measured in the full (4π) solid angle. All singly charged negative particles were taken as the negative pions. An admixture of protons with momentum $p > 750$ MeV/c among π^+ mesons was estimated to be about 12% of the total multiplicity of positive pions [8]. The relative uncertainty ($\Delta p/p$) of momentum measurements of the charged pions for the majority of the measurements did not exceed ∼14–15% with the average value $\langle \Delta p/\ p \rangle \approx 11\%$ for the total ensemble of momentum measurements. The relative uncertainty of the momentum measurements of the charged pions depends on the length of their tracks in a propane chamber exposed to the magnetic field reaching the minimal value $(\Delta p/\ p) \approx 5\%$ for the track lengths exceeding 40 cm. The average threshold for the registration and measurement of momenta of the charged pions was about 50 MeV/c, corresponding to a track length of around 5 cm in a propane (C_3H_8) chamber. The so-called leading particles were excluded from the further analysis. The particle was considered to be a leading one in $\pi^{-12}C$ interactions at 40 GeV/c, if it had, in a given collision event, the maximum momentum, whose value exceeds one-third (1/3) of the initial (incident) momentum, i.e., $p > 13.33$ GeV/c [2]. It should be noted that, in our experiments, we registered and measured all the directly produced charged pions and all those pions produced via the strong decay of excited baryon resonances, vector mesons (such as ρ^0, w^0, and f^0 mesons), *etc.* Pions produced via the secondary weak decays such as those coming from the weak decay $K^0 \to \pi^+\pi^-$ were not registered.

Practically all the spectator nucleons of a target carbon nucleus have momenta $p_n < p^{\rm F}_{\rm max}$, where $p^{\rm F}_{\rm max}$ is the maximum Fermi momentum in the nucleus rest frame. The maximum Fermi momentum is approximately 0.22 GeV/c for carbon nuclei. Therefore, the participant protons are the ones, which remain after excluding the spectator protons. The other methodological procedures of the experiment are presented in Refs. [8–10].

To estimate the possible systematic (methodological) uncertainties, the momenta of the protons that stopped in a chamber and have track length longer than 40 cm were measured three times, and their momenta were calculated based both on their path length and trajectory curvature in the magnetic field in the chamber. The results obtained using these two methods coincided within the uncertainties of less than 5%, which showed the practical absence of such systematic (methodological) uncertainties [2]. Besides it, the presence or absence of systematic (methodological) uncertainties in the experiment was examined by the reconstruction of π^0 mass based on the measurements of 2 gamma quanta as the decay products. To estimate such systematic uncertainties, the momenta of the electrons and positrons produced from the chain reaction $\pi^0 \to 2\gamma \to 2*(\gamma \to e^+e^-)$ in the propane bubble chamber were measured [2]. The measurements were made for 10 such reactions of π^0 decay in the chamber, and the mass of a π^0 meson was reconstructed from the momentum measurements of the electrons and positrons with the sufficiently long tracks in the chamber. In all such measurements, the reconstructed masses of π^0 practically coincided with the known mass, 134.98 MeV, of π^0 within the precision of less than 1% [2].

For comparison with the experimental data, we simulated 30,000 collision events for p^{12}C and π^{-12}C collisions using the modified FRITIOF model [4–7]. The model parameters used in the simulation were the same as given in Ref. [5]. In experiment, all the collision events regardless of the impact parameter of a collision were measured. That is why the collision events in the modified FRITIOF model were simulated for minimum bias events without restrictions on the impact parameter of a collision.

The average numbers per one collision event of the participant protons in experiment and those calculated using the modified FRITIOF model for p^{12}C and π^{-12}C collisions are shown in Table 1. As observed from Table 1, the average numbers of the participant protons practically coincide in the experiment and model calculations for both collision types, being practically equal to (1). Therefore, according to the above-mentioned criteria, the collision events with the number of participant protons (ν_p) in a collision event $0 \leqslant \nu_p \leqslant 1$, $\nu_p = 2$, and $\nu_p \geqslant 3$ were classified as the peripheral, semicentral, and central collision events, respectively. It is natural that the average number of participant nucleons per collision event should be approximately twice as large as the corresponding $\langle \nu_p \rangle$.

Because the transverse momenta of the secondary particles are determined not only by their full momentum, but also their emission angles, we will consider the average values of all these three quantities in three selected collision centrality groups. The average values of the full momentum (P), transverse momentum (P_t), and emission angle (Θ) of the negative pions in p^{12}C and π^{-12}C collisions for the analyzed three collision centrality groups are presented in Table 2. The data on π^{-12}C collisions presented in Table 2 were obtained in our previous work [2]. The average values in Table 2 are calculated for the negative pions with $P_{\text{lab}} \geqslant 50$ MeV/c. In π^{-12}C collisions at 40 GeV/c, the leading particles, π^- mesons, were excluded from the further analysis. The negative pion was considered to be a leading one, if, in a given collision event, it had maximum momentum, whose value exceeds one-third (1/3) of the initial (incident) momentum, i.e., $p > 13.33$ GeV/c [2].

As seen from Table 2, the average values of the full momentum in experiment, as well as in the model, decrease with an increase in the collision centrality for both collision types. As seen from Table 2, in p^{12}C collisions at 9.9 GeV/c, the model average values of the full momentum practically coincide with those of the experiment for peripheral and semicentral collisions. For central collisions, as seen from Table 2, the model average values of the full momentum are larger than those of the experiment. Table 2 shows that the average values of the transverse momentum coincide within uncertainties for peripheral, semicentral, and central collisions for both π^{-12}C interactions at 40 GeV/c and p^{12}C collisions at 9.9 GeV/c, i.e., they do not depend on the collision centrality in the experiment and in the model. However, the modified FRITIOF model underestimates noticeably

Table 1. **The average numbers per one collision event of the participant protons in experiment and calculated with the use of the modified FRITIOF model**

Quantity	Collision type			
	p^{12}C		π^{-12}C	
	Experiment	Mod. FRITIOF	Experiment	Mod. FRITIOF
$\langle \nu_p \rangle$	0.99 ± 0.01	1.01 ± 0.010	0.98 ± 0.01	0.99 ± 0.01

Table 2. **The average values of the total momentum (P), transverse momentum (P_t), and width of P_t spectrum, DP_t and the emission angles (θ) of negative pions in the laboratory frame for the selected three groups of collision centralities of π^{-12}C collisions at 40 GeV/c in experiment and calculated with the use of the modified FRITIOF model. Statistical errors are shown. The average values are calculated for the negative pions with $P_{lab} \geqslant 50$ MeV/c**

Quantity	Colision type	Colision centrality					
		Peripheral		Semicentral		Central	
		Exp.	Model	Exp.	Model	Exp.	Model
$\langle P\rangle$, Gev/c	π^{-12}C	3.17 ± 0.01	3.13 ± 0.02	2.93 ± 0.03	2.72 ± 0.02	2.39 ± 0.04	2.60 ± 0.0
	p^{12}C	0.98 ± 0.01	1.00 ± 0.01	0.88 ± 0.01	0.90 ± 0.01	0.79 ± 0.01	0.84 ± 0.0
$\langle P_t\rangle$, Mev/c	π^{-12}C	356 ± 2	315 ± 1	353 ± 3	314 ± 2	349 ± 4	315 ± 2
	p^{12}C	296 ± 2	275 ± 1	291 ± 3	277 ± 3	292 ± 3	271 ± 3
$\langle\Theta\rangle$, degrees	π^{-12}C	19.5 ± 0.2	17.1 ± 0.1	22.4 ± 0.3	20.0 ± 0.2	25.9 ± 0.4	21.0 ± 0
	p^{12}C	34.0 ± 0.3	32.2 ± 0.3	38.4 ± 0.5	34.9 ± 0.4	42.3 ± 0.5	35.4 ± 0
DP_t, Mev/c	π^{-12}C	310 ± 4	206 ± 3	306 ± 5	208 ± 4	313 ± 6	166 ± 5
	p^{12}C	208 ± 3	168 ± 3	208 ± 4	172 ± 4	204 ± 5	166 ± 5

the average values of the transverse momenta of negative pions in experiment. The independence of the average values of the transverse momenta of negative pions of the collision centrality is probably due to the compensation effects arising when the data on transverse momenta are averaged over the whole phase space [11]. In our case, the peripheral collisions can be considered as pion-nucleon or proton-nucleon collisions. In Ref. [11], it was shown that the average values of the transverse momenta of charged pions in π^-N and π^{-12}C collisions at 40 GeV/c do not depend on the collision type (π^-N or π^{-12}C). For finding the nature of such a behavior of the average values of the transverse momenta of pions, the dependence of the average transverse momentum on the longitudinal momentum in the center-of-mass system of a "$\pi^- p$" collision at 40 GeV/c was investigated [11]. It was discovered [11] that, in the forward hemisphere, the average transverse momentum is systematically larger in π^{-12}C collisions compared to π^-N collisions, and, in the backward hemisphere, it is vice versa. Therefore, when we average the data on transverse momenta over the whole phase space, the difference in the average values of the transverse momenta of pions gets cancelled and disappears [11].

As seen from Table 2, the average values of the emission angles of negative pions increase with the collision centrality both in the experiment and the model, and the model underestimates the experimental values of $\langle\Theta\rangle$. At the same time, the values of $\langle\Theta\rangle$ grow at a higher rate in experiment as compared to the model, which is especially pronounced for the central collision events for both collision types.

We also examined the transverse momentum distributions of the negative pions for both collision types in the experiment and the model, because the coincidence of the average values of the transverse momentum distributions does not necessarily mean the coincidence of the shapes and widths of the spectra. Figure 1 presents the experimental transverse momentum distributions of the negative pions normalized per one negative pion in peripheral and central p^{12}C and π^{-12}C collisions at 9.9 and 40 GeV/c, respectively. The corresponding transverse momentum distributions calculated using the modified FRITIOF model [4–7] are also given as histograms in Fig. 1.

As seen from Fig. 1 and last two rows of Table 2, the shapes and widths of transverse momentum spectra of the negative pions in experiment, as well as in the model, do not depend on the collision centrality for both collision types. Figure 1 shows that the model describes satisfactorily the experimental transverse momentum spectra of the negative pions in the region $p_t < 0.7$ GeV/c and underestimates the experimental data in the region $p_t > 0.7$ GeV/c. This is especially pronounced in π^{-12}C collisions at 40 GeV/c: the calculated model spectrum breaks at $p_t \sim 2.1$ GeV/c, whereas the experimental spectrum extends up to $p_t \sim 3$ GeV/c values. This observation is probably due to the influence of the nuclear medium and nuclear effects on the transverse momentum spectra, which is not fully accounted in the model. This observation requires a further investigation.

In Ref. [12], it was obtained that the experimental transverse momentum spectra of π^- mesons in d^{12}C, ^{4}He^{12}C, and ^{12}C^{12}C collisions at 4.2 A GeV/c were fitted nicely and significantly better by using the Hagedorn model function with two temperatures, T_1 and T_2, as compared to the one-temperature fits. The dominant contribution ($R_1 \sim 90\%$) to the total multiplicity of π^- mesons came from the spectral temperature $T_1 \sim 78$–84 MeV, while the relative yield of the high-temperature, $T_2 \sim 146$–155 MeV, component was much lower ($R_2 \sim 10\%$).

Within the framework of the Hagedorn Thermodynamic Model [13, 14], the normalized transverse momentum (p_t) distribution of hadrons can be described by the expression

$$\frac{dN}{Np_t dp_t} = A(m_t T)^{1/2} \exp\left(-\frac{m_t}{T}\right), \tag{1}$$

where N depending on the choice of normalization is the total number of inelastic events or the total number of the respective particles, $m_t = \sqrt{m^2 + p_t^2}$ is the transverse mass, T is the spectral temperature (inverse slope parameter), and A is the fitting constant. This relation (1) is referred as a one-temperature Hagedorn (model) function [12]. Correspondingly, in the case of two temperatures, T_1 and T_2, the above formula is modified as

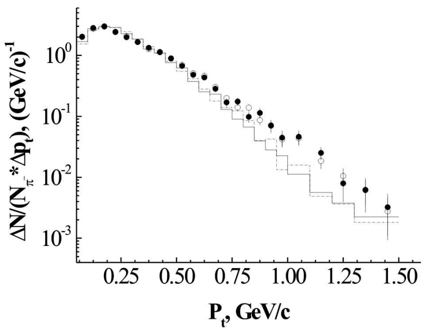
$$\frac{dN}{Np_t dp_t} = A_1(m_t T_1)^{1/2} \exp\left(-\frac{m_t}{T_1}\right) + \\ + A_2(m_t T_2)^{1/2} \exp\left(-\frac{m_t}{T_2}\right) \tag{2}$$

referred to as the two-temperature Hagedorn (model) function [12] in this work. The above expressions (1) and (2) were derived assuming that $m_t \gg T$ [15, 16].

To describe the experimental transverse momentum spectra of the negative pions in Fig. 1, we can rewrite relation (2) as

$$\frac{dN}{Ndp_t} = A_1 p_t (m_t T_1)^{1/2} \exp\left(-\frac{m_t}{T_1}\right) + \\ + A_2 p_t (m_t T_2)^{1/2} \exp\left(-\frac{m_t}{T_2}\right). \tag{3}$$

The results of the minimum χ^2 fits by the two-temperature Hagedorn function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions in peripheral and central p^{12}C and π^{-12}C collisions at 9.9 and 40 GeV/c, respectively, are shown in Fig. 2, a and b, respectively. The obtained corresponding values of the parameters of minimum χ^2 fittings by the two-temperature Hagedorn function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions in peripheral and central p^{12}C and π^{-12}C collisions at 9.9 and 40 GeV/c are presented in Table 3. It should be noted that all the fits of the experimental transverse momentum spectra with the theoretical (model) functions in the present paper are conducted using the minimum χ^2 method including the statistical er-



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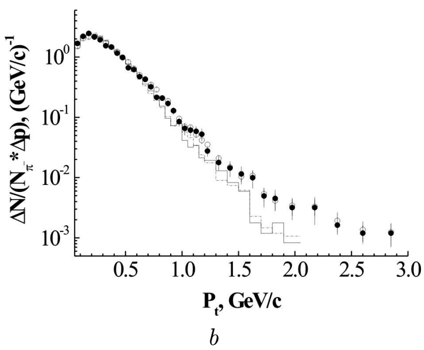
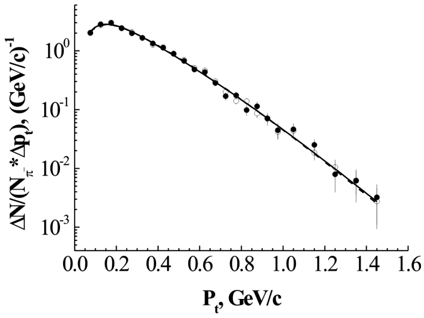


b

Fig. 1. The experimental transverse momentum distributions of the negative pions normalized per one negative pion in peripheral (○) and central (•) p^{12}C (a) and π^{-12}C (b) collisions at 9.9 and 40 GeV/c, respectively. The histograms are the results of corresponding modified FRITIOF model calculations for the peripheral (dashed histogram) and central (solid histogram). The distributions are constructed for the negative pions with $P_t \geq 50$ MeV/c. Statistical errors are shown

Table 3. **The obtained values of the parameters of minimum χ^2 fittings by the two-temperature Hagedorn function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions in region $P_t \geqslant 50$ MeV/c in peripheral and central p^{12}C and π^{-12}C collisions at 9.9 and 40 GeV/c, respectively; *n.d.f.* denotes the number of degrees of freedom**

Collision type	Collision centrality	A_1, (GeV/c)$^{-3}$	T_1, MeV	A_2, (GeV/c)$^{-3}$	T_2, MeV	$\chi^2/n.d.f.$
p^{12}C, 9.9 GeV/c	Peripher.	1708 ± 346	66 ± 6	286 ± 58	130 ± 4	1.47
	Central	1737 ± 380	68 ± 7	250 ± 69	133 ± 5	1.12
π^{-12}C, 40 GeV/c	Peripher.	555 ± 90	105 ± 6	45 ± 11	186 ± 6	2.36
	Central	837 ± 195	90 ± 9	64 ± 16	177 ± 7	2.10



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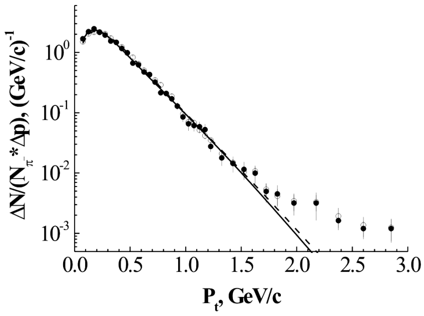

b

Fig. 2. The experimental transverse momentum distributions of the negative pions normalized per one negative pion in peripheral (○) and central (•) p^{12}C (*a*) and π^{-12}C (*b*) collisions at 9.9 and 40 GeV/c, respectively. The curves are the results of minimum χ^2 fits by the two-temperature Hagedorn function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions in peripheral (dashed curve) and central (solid curve) collisions. The distributions are constructed for the negative pions with $P_t \geqslant 50$ MeV/c. Statistical errors are shown

rors as the weighting factors $(1/(\text{error})^2)$ for the data points during the fitting procedures.

As seen from Fig. 2, *a* and $\chi^2/n.d.f.$ values in Table 3, the two-temperature Hagedorn function (Eq. 3) describes satisfactorily the experimental transverse momentum distributions of the negative pions in peripheral and central p^{12}C collisions at 9.9 GeV/c. Table 3 shows that the obtained values of the parameters of minimum χ^2 fittings of the experimental transverse momentum distributions of the negative pions by the two-temperature Hagedorn function (Eq. (3)) coincide with each other within fitting errors in peripheral and central p^{12}C collisions at 9.9 GeV/c. This confirms our above observation that the shapes and widths of transverse momentum distributions of the negative pions in p^{12}C collisions at 9.9 GeV/c do not depend within the uncertainties on the degree of collision centrality. As seen from the comparison of the parameters A_1 and A_2 in Table 3, the dominant contribution to the transverse momentum distribution of π^- mesons in peripheral and central p^{12}C collisions at 9.9 GeV/c comes from the lower spectral temperature (inverse slope) $T_1 \sim 70$ MeV, while the contribution of the high-temperature (inverse slope), $T_2 \sim 130$ MeV, component is much smaller. This is in agreement with the similar result obtained for transverse momentum distributions of the negative pions in d^{12}C, ^{4}He^{12}C, and ^{12}C^{12}C collisions at 4.2 A GeV/c in Ref. [12]. It was obtained in Ref. [12] that the spectral temperatures, T_1 and T_2, and their relative contributions do not depend, within fitting errors, on the degree of collision centrality in ^{12}C^{12}C collisions at 4.2 A GeV/c. This result is in agreement with the independence within the uncertainties of the shapes and widths of transverse momentum distributions of the negative pions in p^{12}C collisions at 9.9 GeV/c observed in the present work of the degree of collision centrality.

The two-temperature shapes of the energy and transverse momentum distributions of pions were also observed in nucleus-nucleus collisions at incident energies of a few GeV/nucleon in early works [9, 16–19]. In Ref. [17], the two-temperature shape of the center-of-mass system (c.m.s.) kinetic energy spectra of negative pions in Ar + KCl collisions at 1.8 GeV/nucleon was observed. The existence of two temperatures, T_1 and T_2, in this work was explained by two channels of pion production: pions coming from the Δ resonance decay (T_1) and pions produced directly in nucleon-nucleon collisions (T_2). In Ref. [18], the two-temperature shape of the kinetic energy spectrum of pions emitted at 90° in c.m.s. of central La + La collisions at 1.35 GeV/nucleon was interpreted by the different contributions of deltas originating in the early and later stages of heavy ion collisions. The two-temperature behavior was also seen in the c.m.s. energy, as well as p_t, spectra of π^- mesons in Mg + Mg collisions [16] at an incident momentum of 4.2–4.3 A GeV/c. In Ref. [9], the two-temperature shape of experimental c.m.s. energy spectra of π^- in $^{12}C^{12}C$ collisions at 4.2 A GeV/c was expained by the superposition of partial contributions of different sources (decays of resonances, direct reactions, *etc.*) from the analysis of the spectra of π^- originating from different sources in the framework of the Quark-Gluon-String-Model (QGSM), modified to intermediate energies. In contrast to the above explanations, it was stated in Ref. [19] that it would be naive to believe that the origin of pions in a minimum bias $^{12}C^{12}C$ collisions at 4.2 A GeV/c can be explained only by two thermal sources [19]. The collective flow has become the well-established phenomenon in relativistic nuclear collisions. The inverse slope parameter, T, or an apparent temperature of the emitting source, of transverse momentum distributions of hadrons was assumed to consist of two components: a thermal part, T_{thermal}, and a second component resembling the collective expansion with an average transverse velocity $\langle\beta_t\rangle$ [19, 20]. Therefore, the observed two-temperature shape of the energy and transverse momentum spectra of negative pions in $^{12}C^{12}C$ collisions at 4.2 A GeV/c [19] could also be explained by the collective flow effects.

As seen from Fig. 2, *b*, the two-temperature Hagedorn function (Eq. (3)) describes satisfactorily the experimental transverse momentum distributions of the negative pions in peripheral and central $\pi^{-12}C$ collisions at 40 GeV/c up to transverse momentum values $p_t \sim 1.7$ GeV/c. Table 3 shows that the obtained values of the parameters of minimum χ^2 fittings by the two-temperature Hagedorn function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions also overlap with each other within fitting errors in peripheral and central $\pi^{-12}C$ collisions at 40 GeV/c. This supports our above observation that the shapes and widths of transverse momentum distributions of the negative pions in $\pi^{-12}C$ collisions at 40 GeV/c do not depend within the uncertainties on the degree of collision centrality. However, as seen from Fig. 2, *b*, the two-temperature Hagedorn function (Eq. (3)) fails to reproduce the high p_t tail of the transverse momentum distributions of the negative pions in peripheral and central $\pi^{-12}C$ collisions at 40 GeV/c in region $p_t > 1.7$ GeV/c.

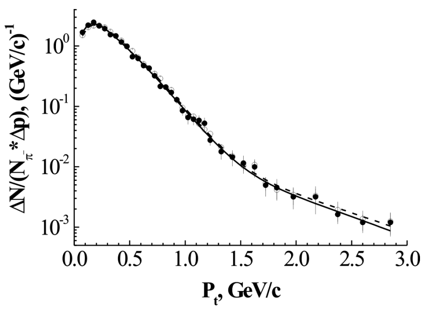

Fig. 3. The experimental transverse momentum distributions of the negative pions normalized per one negative pion in peripheral (○) and central (•) $\pi^{-12}C$ collisions at 40 GeV/c, respectively. The curves are the results of minimum χ^2 fits by the three-temperature Hagedorn function (Eq. (4)) of the experimental transverse momentum distributions of the negative pions in peripheral (dashed curve) and central (solid curve) collisions. The distributions are constructed for the negative pions with $P_t \geqslant 50$ MeV/c. Statistical errors are shown

In order to describe satisfactorily the experimental transverse momentum distributions of the negative pions in peripheral and central $\pi^{-12}C$ collisions at 40 GeV/c in the whole measured range $P_t \geqslant 50$ MeV/c, we fitted the experimental transverse momentum spectra by the three-temperature Hagedorn function given by

$$\frac{dN}{Ndp_t} = A_1 p_t (m_t T_1)^{1/2} \exp\left(-\frac{m_t}{T_1}\right) + A_2 p_t (m_t T_2)^{1/2} \exp\left(-\frac{m_t}{T_2}\right) + A_3 p_t (m_t T_3)^{1/2} \exp\left(-\frac{m_t}{T_3}\right). \quad (4)$$

Table 4. **The obtained values of the parameters of minimum χ^2 fittings by the three-temperature Hagedorn function (Eq. (4)) of the experimental transverse momentum distributions of the negative pions in the region $P_t \geqslant 50$ MeV/c in peripheral and central π^{-12}C collisions at 40 GeV/c. *n.d.f.* denotes the number of degrees of freedom**

Collision centrality	A_1, (GeV/c)$^{-3}$	T_1, MeV	A_2, (GeV/c)$^{-3}$	T_1, MeV	A_3, (GeV/c)$^{-3}$	T_1, MeV	$\chi^2/n.d.f.$
Peripher.	611 ± 262	82 ± 7	178 ± 38	150 ± 5	0.09 ± 0.07	504 ± 84	0.84
Central	1133 ± 336	71 ± 7	178 ± 44	149 ± 7	0.10 ± 0.08	479 ± 94	0.88

As observed from Fig. 3 and $\chi^2/n.d.f.$ values in Table 4, the three-temperature Hagedorn function (Eq. 4) describes very well the experimental transverse momentum distributions of the negative pions in peripheral and central π^{-12}C collisions at 40 GeV/c. Table 4 shows that the obtained values of the parameters of minimum χ^2 fittings by the three-temperature Hagedorn model function (Eq. (3)) of the experimental transverse momentum distributions of the negative pions overlap with each other within fitting errors in peripheral and central π^{-12}C collisions at 40 GeV/c. This confirms again our above observation that the shapes and widths of transverse momentum distributions of the negative pions in π^{-12}C collisions at 40 GeV/c do not depend within the uncertainties on the degree of collision centrality. As observed from the value of the parameter A_3 in Table 4, the contribution of the high-temperature (inverse slope) T_3 component to the transverse momentum distribution of the negative pions in π^{-12}C collisions at 40 GeV/c is very low and much smaller than the contributions of the T_1 and T_2 components.

In summary, we can conclude that the average values, shapes, and widths of transverse momentum distributions of negative pions in p^{12}C and π^{-12}C collisions at 9.9 and 40 GeV/c, respectively, do not depend within the uncertainties on collision centrality both in the experiment and the modified FRITIOF model calculations. The latter underestimate the average values of transverse momenta of the negative pions in the experiment and do not reproduce the tail of the experimental transverse momentum distributions of the negative pions in both collision types. The Hagedorn model function with the two-temperature parameters (two inverse slope parameters) describes well the transverse momentum distributions of the negative pions in peripheral and central p^{12}C collisions at 9.9 GeV/c. However, the two-temperature Hagedorn model function describes satisfactorily the experimental transverse momentum distributions of the negative pions in peripheral and central π^{-12}C collisions at 40 GeV/c only up to transverse momentum values $p_t \sim 1.7$ GeV/c, failing to reproduce the high p_t tail of the transverse momentum spectra of the negative pions in π^{-12}C collisions in the region $p_t > 1.7$ GeV/c. The Hagedorn model function with the three-temperature parameters (three inverse slope parameters) describes very well the experimental transverse momentum distributions of the negative pions in peripheral and central π^{-12}C collisions at 40 GeV/c in the whole measured range $P_t \geqslant 50$ MeV/c.

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ПРО РОЗПОДІЛ ПОПЕРЕЧНОГО ІМПУЛЬСУ НЕГАТИВНИХ ПІ-МЕЗОНІВ У p^{12}C І π^{-12}C ЗІТКНЕННЯХ ПРИ ВИСОКИХ ЕНЕРГІЯХ

Р е з ю м е

Вивчено залежності розподілів поперечного імпульсу негативних пі-мезонів, народжених у p^{12}C і π^{-12}C зіткненнях при 9,9 і 40 Гев/c від центральності зіткнень. Форми і ширини нормалізованих розподілів і середні величини поперечного імпульсу в межах помилок не залежать від центральності, що підтверджується розрахунками в модифікованій FRITIOF моделі і мінімальною по χ^2 підгонкою експериментальних спектрів модельними функціями Хагедорна з двома і трьома температурами. Розрахунки по модифікованій FRITIOF моделі дають менші величини середніх поперечних імпульсів і не відтворюють хвости розподілів. Знайдено, що розподіли мають два нахили (дві температури) для периферичних і центральних p^{12}C зіткнень при 9,9 Гев/c, що узгоджується з ранніми роботами для енергій порядку декількох Гев/c. Розподіли поперечних імпульсів негативних пі-мезонів для периферичних і центральних π^{-12}C зіткнень при 40 Гев/c мають форму спектрів з трьома нахилами (три температури).