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EFFECTS OF RESERVOIR INPUT FIELDS ON THE NON-CLASSICAL FEATURES OF QUANTUM BEAT CASCADE LASER

The quantum features and quantum statistical properties of a cavity-mode radiation emitted from the coherently prepared degenerate three-level laser have been investigated, by using the standard quantum electrodynamics approach and accounting for the light-matter interaction. We considered the vacuum reservoir, squeezed vacuum reservoir, and thermal reservoir to see the effect of reservoir input fields on the statistical and squeezing nature on the cavity radiation. It is found that the squeezed vacuum reservoir has enhancement effect on the squeezing property, as well as the brightness of the cavity radiation compared to those of the vacuum and thermal reservoirs. It is also observed that the radiation emitted from the cavity is in the squeezed state with super-Poissonian photon statistics regardless of the reservoir nature.

Keywords: super-Poissonian, squeezed state, quantum features, light-matter interaction.

1. Introduction

The light-matter interaction [1–6] is the heart of quantum optics. When an atom interacts with a light mode in a certain confinement such as in the cavity, there would be emission of a photon through the atomic transition either spontaneously, or stimulated. Spontaneous transmissions are assumed to be the cause for noise in the cavity radiation. On contrast, stimulated emission produces light modes which are assumed to be coherent, in-phase, and focused. Following M.O. Scully [7], researchers did a series of papers on the development of atomic systems in the field of quantum optics. For instance, the interaction of a three-level atomic system with the cavity field has been studied in [8–25]. It is confirmed that the three-level atomic system generates a light mode with non-classical natures by different mechanisms. These systems are a source of non-classical light, when the atom is in coherent superposed states and coupled to the enhanced reservoir modes [23, 25, 26].

The non-classical behaviors such as the squeezing, entanglement, and photon statistics in the cavity mode are assumed to be a consequence of the coupling and atomic coherence. The generation of squeezed and entangled light from various schemes of coherent superposed three-level atomic system passing through the cavity has been studied in [23, 25, 26]. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, two photons are emitted. If the two photons have the same frequency, the three-level atom is called degenerate; otherwise it is known to be non-degenerate.

Several authors were researching the quantum optical systems coupled to a reservoir modes, for example, cavity mode radiation fluctuations coupled to a squeezed vacuum reservoir [16, 25, 26], vacuum reservoir [27], and thermal reservoir [28] using standard approaches. In a three-level laser, we could have different configurations,such as cascade [8, 9, 20, 23, 25–28], V-shaped [12, 21, 29], and lambda-types [14–19, 22]. One of the fundamental processes in a three-

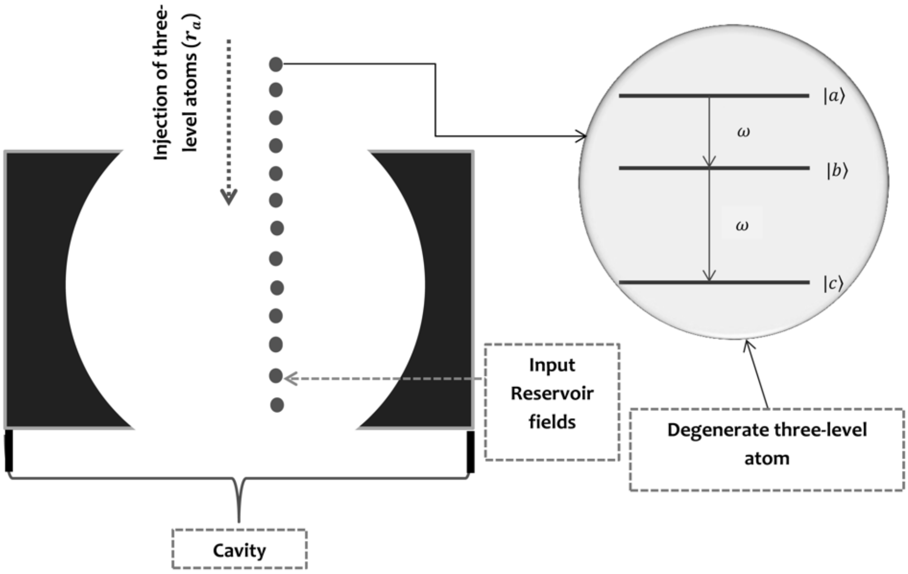

Fig. 1. Schematic representations of moving three-level atoms passing through a cavity with quantized electromagnetic fields. The atoms stay for a time τ in the cavity such that they have sufficient decay time

level system is the establishment of population inversion in which three-level atoms are pumped from the lower electronic state to the upper one to produce a coherent light [23,26]. Moreover, in the establishment of a superposed initial atomic state, there would be the atomic coherence [25,30]. In one way, the quantifiers of the light beams generated from the atomic system in an optical cavity are explained by the atomic probability difference [23, 25, 26]. Even though it is possible to quantify the properties of a light beam in relation to the atomic probability difference, it is more convenient to see the role of the atomic coherence in the quantifier dynamics.

On the other hand, it is well known that the system features are affected, when it immersed into a reservoir. Thus, light beams produced from an optical cavity which contains three-level atoms are highly disturbed by the fields of the reservoir besides the cavity parameters. Therefore; the quantifiers like the quadrature squeezing, entanglement, quantum discord, and intensity of the cavity radiation produced through the system are changed in a certain manner.

Taking this as motivation, in this work, we formulate the atomic coherence for a typical three-level atom passing through a cavity. To achieve this goal, we have employed the existing atomic probability difference expression used in [26]. In this context, the influences of the atomic coherence and reservoir field are analyzed using analytically approaches. This helps us to figure out the relationship between the atomic coherence and the quantifiers of light beams without omitting the reservoir effect. Therefore, such study can facilitate the establishment and comprehensive understanding of the inherent degree of the quantifiers due to the atomic coherence and reservoir fields.

2. Model and Quantum Electrodynamics

In Fig. 1, we show the schematic representation of coherently superposed three-level atoms passing through the cavity. The cavity is composed of a single ported mirror which allows the reservoir field to enter the cavity via one side and a perfectly reflected mirror on the other side. In this paper, we consider the degenerate cascade three-level atoms initially prepared in a coherent superposition of the top and bottom levels that are injected at a constant rate r_a and removed from the laser cavity after some time τ. We denote the top, intermediate, and bottom levels of a three-level atom by $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively. We assume the cavity mode to be at resonance with the two transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$, and with direct transition between levels $|a\rangle$ and $|c\rangle$ to be dipole forbidden. To this end, we write the initial state of a single atom as

$$|\psi_A^{(0)}\rangle = c_a^{(0)}|a\rangle + c_c^{(0)}|c\rangle, \tag{1}$$

where $c_a^{(0)}$ and $c_c^{(0)}$ are the real probability amplitudes of an atom in the states $|a\rangle$ and $|c\rangle$, respectively. The corresponding density of states for the atom is represented by

$$\hat{\rho}_A^{(0)} = \rho_{aa}^{(0)}|a\rangle\langle a| + \rho_{cc}^{(0)}|c\rangle\langle c| + \rho_{ac}^{(0)}|a\rangle\langle c| + \rho_{ca}^{(0)}|c\rangle\langle a|, \tag{2}$$

where $\rho_{aa}^{(0)}$ and $\rho_{cc}^{(0)}$ are the probabilities for the atom to be on the upper level and lower one, respectively. Furthermore, $\rho_{ac}^{(0)}$ and $\rho_{ca}^{(0)}$ are cross-correlations which can be named as the atomic coherence. These cross-correlations signifies the probability of an atom to be found in one electronic energy state at the same time. The superposed state of an atom at the initial time is responsible for the induction of the atomic coherence. This can be explained using quantum mechanics purely. Hence, we may not found the classical analogy for such correlations.

In the rotating approximation scheme and assuming $\hbar = 1$, the quantum Hamiltonian which describes the interaction of a single atom with the cavity mode can be written as

$$\hat{H}_I = ig\left[(\hat{\sigma}_{ab} + \hat{\sigma}_{bc})\,\hat{a} - \hat{a}^\dagger(\hat{\sigma}_{ab}^\dagger + \hat{\sigma}_{bc}^\dagger)\right], \tag{3}$$

where $\hat{a}$ is the annihilation operator for the cavity mode, and g is the atom-field coupling constant which must be positive ($g > 0$). The operators $\hat{\sigma}_{ab} = |a\rangle\langle b|$ and $\hat{\sigma}_{bc} = |b\rangle\langle c|$ are atomic lowering operators.

In this section, however, we found it useful to include a highlight for the derivation of the time evolution of the system's density operator describing the interaction of the cavity mode generated by a three-level laser coupled to a squeezed vacuum reservoir in order to make the paper more self-contained.

We denote the density operator of squeezed vacuum reservoir modes by $\hat{\chi}(t)$. Then the density operator for the system alone is given by

$$\hat{\rho}(t) = \mathrm{Tr}_R[\hat{\chi}(t)], \tag{4}$$

where Tr_R indicates the trace over the reservoir variable only. The density operator $\hat{\chi}(t)$ evolves in time according to

$$\frac{d}{dt}\hat{\chi}(t) = \frac{1}{i\hbar}\left[\hat{H}(t), \hat{\chi}(t)\right], \tag{5}$$

where $\hat{H}(t)$ is the Hamiltonian which governs the interaction between the system and the reservoir. By considering initially the system and the reservoirs to be uncorrelated, it is possible to write, for the density operator of the system and the reservoirs at the initial time ($t = 0$), that $\hat{\chi}(0) = \hat{\rho}(0) \otimes \hat{R}$ [31], where $\hat{\rho}(0)$ and $\hat{R}$ are the density operators of the system and the reservoir at the initial time, respectively. In view of these relations, Eq. (5) could be written as

$$\frac{d}{dt}\hat{\chi}(t) = \frac{1}{i\hbar}\left[\hat{H}(t),\, \hat{\rho}(0)\otimes\hat{R}\right] - $$
$$-\frac{1}{\hbar^2}\int_0^t \left[\hat{H}(t'),\left[\hat{H}(t'),\, \hat{\chi}(t')\right]\right] dt'. \tag{6}$$

Applying the weak coupling approximation which implies that $\hat{\chi}(t') = \hat{\rho}(t') \otimes \hat{R}$, it follows that

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{i\hbar}\mathrm{Tr}_R\left\{\left[\hat{H}(t),\, \hat{\rho}(0)\otimes\hat{R}\right]\right\} - $$
$$-\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{\left[\hat{H}(t),\left[\hat{H}(t'),\, \hat{\rho}(t')\otimes\hat{R}\right]\right]\right\} dt', \tag{7}$$

Let the system with a single-mode light with frequency ω be coupled to a single-mode continuum squeezed vacuum reservoir. Therefore, the interaction between the system and the squeezed vacuum reservoir is described by the Hamiltonian

$$\hat{H}(t) = i\hbar\sum_i \mu_i\left((\hat{a}^\dagger\hat{A}_i e^{i(\omega-\omega_i)} - \hat{a}\hat{A}_i^\dagger e^{-i(\omega-\omega_i)}\right). \tag{8}$$

Here, $\hat{A}_i$ is the annihilation operator for the reservoir mode with frequency of ω_i. The coefficient μ_i is the coupling constant describing the interaction between the intracavity mode and the reservoir mode. Applying the cyclic property of the trace and the relation $\mathrm{Tr}_R\big(\hat{R}\otimes\hat{H}(t)\big) = \langle\hat{H}(t)\rangle_R$ and with the fact that, for squeezed vacuum reservoirs

$$\langle\hat{A}_i\rangle_R = \langle\hat{A}_i^\dagger\rangle_R = 0, \tag{9}$$

one can get

$$\mathrm{Tr}_R\left\{\left[\hat{H}(t),\, \hat{\rho}(0)\otimes\hat{R}\right]\right\} = 0. \tag{10}$$

Therefore; Eq. (7) becomes

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{\left[\hat{H}(t),\left[\hat{H}(t'),\, \hat{\rho}(t')\otimes\hat{R}\right]\right]\right\} dt'. \tag{11}$$

After the lengthy, but straightforward integration, Eq. (11) takes the form

$$\frac{d}{dt}\hat{\rho} = \frac{\kappa N}{2}\left[2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^{\dagger} - \hat{a}\hat{a}^{\dagger}\hat{\rho}\right] + \\ + \frac{\kappa(N+1)}{2}\left[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\right] + \\ + \frac{\kappa M}{2}\left[2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{\rho}\right] + \\ + \frac{\kappa M^*}{2}\left[2\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho}\right], \quad (12)$$

where $\kappa = 2\pi g(\omega)\mu^2(\omega)$ is the cavity damping constant. The squeezed vacuum effects are incorporated through the mean photon number of the reservoir mode $N = \sinh^2(r)$, and the constant $M = \sqrt{N(N+1)}$ represents the phase property of the reservoir.

Applying the adiabatic approximation scheme [31] for a three-level laser, the master equation for the system coupled to a squeezed vacuum reservoir could be written as

$$\frac{d}{dt}\hat{\rho} = \frac{\xi}{2}\left[2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^{\dagger} - \hat{a}\hat{a}^{\dagger}\hat{\rho}\right] + \\ + \frac{\eta}{2}\left[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\right] + \\ + \frac{\Omega}{2}\left[2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{\rho}\right] + \\ + \frac{\Omega^*}{2}\left[2\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho}\right]. \quad (13)$$

Here, $\xi = A\rho_{aa}^{(0)} + \kappa N$, $\eta = A\rho_{cc}^{(0)} + \kappa(N+1)$, $\Omega = \kappa M + A\rho_{ac}^{(0)}$, $\Omega^* = \kappa M + A\rho_{ca}^{(0)}$. The constant $A = \frac{2r_a g^2}{\gamma^2}$ is the linear gain coefficient with γ to be the spontaneous decay rate.

3. Atomic Probabilities

It is worth mentioning that the quantum properties of the light generated by a three-level system are determined using the master equation as a tool for the derivation of stochastic differential equations. At the initial time $t = 0$, the probability for the atom to be on the level $|a\rangle$ is $\rho_{aa}^{(0)}$ and the probability of finding the atom on the level $|c\rangle$ is $\rho_{cc}^{(0)}$ which satisfies the relation $\rho_{aa}^{(0)} + \rho_{cc}^{(0)} = 1$. Using the fact that the amplitudes are real, we can write, in Eq. (1), $c_a^{(0)} = \sin\phi$ and $c_c^{(0)} = \cos\phi$. Given that $\rho_{cc}^{(0)} - \rho_{aa}^{(0)} = \cos 2\phi$, we now introduce the atomic coherence $\rho_{ac}^{(0)} = \rho_{ca}^{(0)} = \beta$ such that

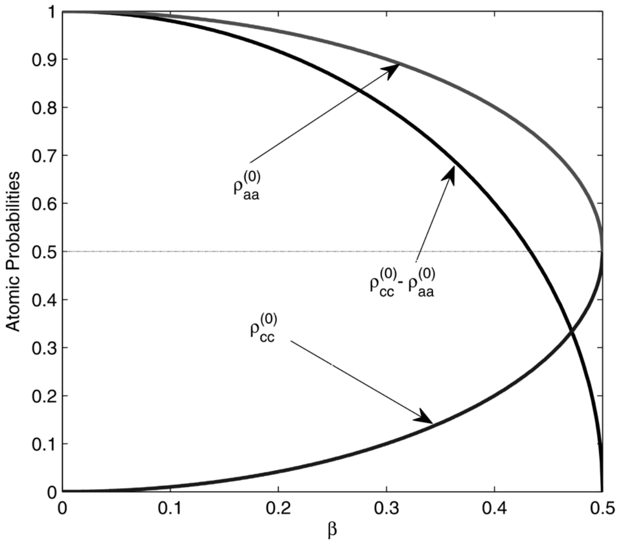

Fig. 2. Plots of the atomic probabilities ($\rho_{cc}^{(0)}$ and $\rho_{aa}^{(0)}$) and the probability difference ($\rho_{cc}^{(0)} - \rho_{aa}^{(0)}$) against the atomic coherence in a single three-level system

$$\beta = c_a^{(0)} c_c^{(0)} = \cos\phi\sin\phi = \frac{\sin 2\phi}{2}. \quad (14)$$

Hence, we can write the probability difference in terms of the atomic coherence as

$$\rho_{cc}^{(0)} - \rho_{aa}^{(0)} = \sqrt{1 - 4\beta^2}. \quad (15)$$

In view of this, we can write the probability of an atom to be on the upper energy level as

$$\rho_{aa}^{(0)} = \frac{1 - \sqrt{1 - 4\beta^2}}{2}. \quad (16)$$

Using straightforward algebra, the probability of an atom to be on the lower level can be formulated in terms of the atomic coherence as

$$\rho_{cc}^{(0)} = \frac{1 + \sqrt{1 - 4\beta^2}}{2}. \quad (17)$$

In Fig. 2, the red curve represents the probability of an atom to be found on the upper energy level, while the blue curve represents the probability of an atom to be found on the lower energy level. However, the black curve reads as the atomic probability difference. As we infer from this figure, the atomic coherence is bounded in between 0 and 0.5 ($0 \leq \beta \leq 0.5$). At the extreme point $\beta = 0.5$ which physically tells us an atom has equal probabilities to be found on both the upper and lower energy levels. At $\beta = 0$, the atom occupies the lower state with a probability of 100% for which $\rho_{cc}^{(0)} = 0$. Furthermore, the probability difference is decreasing, as the atomic coherence is increasing, and becomes zero at $\beta = 0.5$.

4. Time Evolution of the Cavity Mode

Applying the cyclic property of a trace, $\frac{d}{dt}\langle\hat{G}(t)\rangle = \mathrm{Tr}\left(\frac{d}{dt}\hat{\rho}(t)\hat{G}\right)$ and the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$, the cavity mode evolves in time according to

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = -\frac{\varepsilon}{2}\langle\hat{a}(t)\rangle. \tag{18}$$

It can be also verified that

$$\frac{d}{dt}\langle\hat{a}^2(t)\rangle = -\varepsilon\langle\hat{a}^2(t)\rangle + \Omega, \tag{19}$$

and

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = -\varepsilon\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle + \xi. \tag{20}$$

In which $\varepsilon = \kappa + A\sqrt{1-4\beta^2}$.

5. Squeezing of Cavity-Mode Radiation

One of the measures of the non-classical nature of a light beam generated from a certain quantum optical system is achieved by calculating the quadrature variance of the fields in the system analytically. Quadrature squeezing is useful to detect the non-classical properties of a cavity radiation. We can study the quadrature squeezing of a three-level laser by considering the cavity field fluctuations associated with the noise incident into the cavity. Here, we study the quadrature variance of a cavity field which leads us to formulate the radiation squeezing. First, we denote a positive quadrature operator by $\hat{Q}_+$ and define it in terms of the cavity mode operator by

$$\hat{Q}_+ = \hat{a}^\dagger + \hat{a} \tag{21}$$

and its conjugate

$$\hat{Q}_- = i\left(\hat{a}^\dagger - \hat{a}\right). \tag{22}$$

Quadrature squeezing is occurred in one of the above quadratures, either in a positive quadrature or in the negative one. If the cavity field has the squeezing property in the positive quadrature, its conjugate will be noisy. To show this, we seek to calculate the quadrature variances. The variance of the quadrature given in Eqs. (21) and (22) can be expressed as

$$\Delta Q_\pm^2 = 1 + \langle:\hat{Q}_\pm, \hat{Q}_\pm:\rangle. \tag{23}$$

In this notation, we use :: to represent the normal ordering of the cavity mode operator and the expression $\langle\hat{Q}_\pm, \hat{Q}_\pm\rangle = \langle\hat{Q}_\pm^2\rangle - \langle\hat{Q}_\pm\rangle\langle\hat{Q}_\pm\rangle$. With the help of this relation, one can show that

$$\Delta Q_\pm^2 = 1 + 2\langle\hat{a}^\dagger\hat{a}\rangle \pm \left[\langle\hat{a}^2\rangle + \langle\hat{a}^{\dagger 2}\rangle\right]. \tag{24}$$

Using the large time approximation, which leads to steady state solutions of Eqs. (18), (19) and (20), one can write Eq. (24) as

$$\Delta Q_\pm^2 = 1 + \frac{2}{\varepsilon}\left(\xi \pm \Omega\right). \tag{25}$$

We now recall the Heisenberg uncertainty principle to detect on which quadrature of the field is squeezed, inequality of variances product should satisfy

$$\Delta Q_+ \Delta Q_- \geq 1 \tag{26}$$

together with either $\Delta Q_+^2 < 1$ and $\Delta Q_-^2 > 1$ or $\Delta Q_-^2 < 1$ and $\Delta Q_+^2 > 1$. In the following sections, we consider specific conditions of a quantum beat cascade laser such as in the squeezed vacuum, thermal, and ordinary vacuum reservoirs. Now, we test the maximum possibility to have a quantum beat cascade laser (QBCL).

5.1. QBCL in squeezed vacuum reservoir

Using the standard approaches and steady state solutions, we realize the positive and negative quadrature variances for the three-level system coupled to a squeezed vacuum reservoir that

$$\Delta Q_\pm^2 = 1 + \frac{A\left[1 - \sqrt{1-4\beta^2} \pm 2\beta\right] + 2\kappa(N \pm M)}{\kappa + A\sqrt{1-4\beta^2}}. \tag{27}$$

With the Heisenberg uncertainty relation between the positive and negative quadrature operators, it is detected that the negative quadrature variance is below the classical limit. Hence, the cavity field is in a squeezed state for the system coupled to a squeezed vacuum reservoir.

Figure 3 shows the plots quadrature variance against the atomic coherence for a three-level quantum beat laser coupled to a squeeze vacuum reservoir. In this figure, it is indicated that the system generates light beams which are in the squeezed state in the negative quadrature for certain values of A and N. We will have a particular value for β at which the curve changes its nature. Furthermore, this special value of β depends on the linear gain coefficient for fixed values of the other parameters. For example, for the curve with $A = 100$, the quadrature variance decreases, as the value of β increases in the interval $0 < \beta < 0.464$, and the quadrature variance increases, as the value of β increases in the interval $0.464 < \beta < 0.5$. The reason to have such dynamics of the quadrature variance is associated with the nature of probability fluctuations for an atom to be on either the upper energy level or lower one. This must be due to that the probability of finding the atom to be on the lower energy level increases, while the probability of an atom to be on the upper energy level decreases. If the atom has equal probabilities to be found on the upper and lower energy levels, the quadrature variance is inflated, and, hence, the light beam emitted in the cavity loses its squeezing nature. Consequently, the light beams are associated with a high noise. However, such increment of the noise can be suppressed using the input fields from the squeezed vacuum reservoir.

5.2. QBCL in a thermal vacuum reservoir

In this section, we let the system to interact with the fields of a thermal reservoir. The fields from such baths are assumed to be in the thermal equilibrium at a certain temperature. Thus, the mean number of photons indecent into the cavity is calculated according to the following relation:

$$N_c = \frac{1}{e^{\hbar\omega/K_{\rm B}T} - 1}. \qquad (28)$$

Here, $\hbar$ is the reduced Planck constant, $K_{\rm B}$ is the Boltzmann constant, and T is the temperature.

At this point, we replacie a squeezed vacuum reservoir by a thermal reservoir, make limit $N \to N_c$, and, hence, set the parameter $M = 0$. Then Eq. (27) can be reduced to

$$\Delta Q_-^2 = 1 + \frac{A\left[1 - \sqrt{1 - 4\beta^2} - 2\beta\right] + 2\kappa N_c}{\kappa + A\sqrt{1 - 4\beta^2}}. \qquad (29)$$

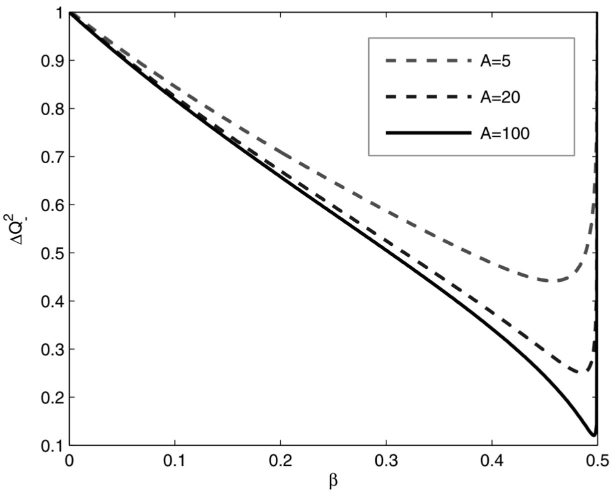

Fig. 3. Plots of the quadrature variance of the cavity mode radiation ($N = 10$) against the atomic coherence for a squeezed vacuum reservoir for the cavity damping constant $\kappa = 0.8$, with different values of the linear gain coefficient $A = 5$ (the red dashed curve), $A = 20$ (the blue dashed curve) and $A = 100$ (the black solid curve)

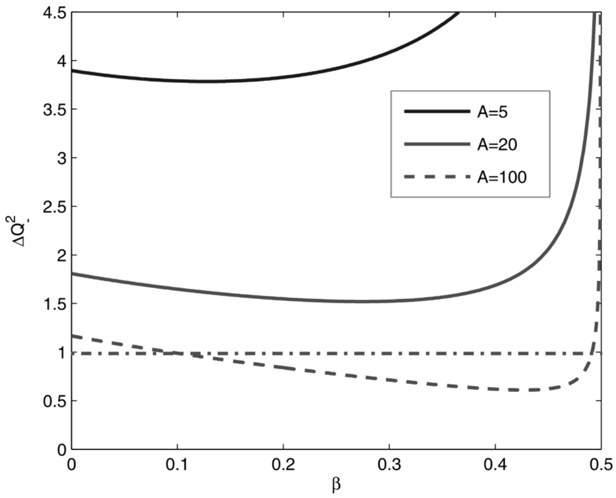

Fig. 4. Plots of the quadrature variance of the cavity mode radiation ($N = N_c = 10$) against the atomic coherence for a thermal reservoir for the cavity damping constant $\kappa = 0.8$, with different values of the linear gain coefficient $A = 5$ (the blue solid curve), $A = 20$ (the red solid curve) and $A = 100$ (the red dashed curve)

In Fig. 4, we plot the quadrature variance against the atomic coherence for a three-level cascade beat laser system coupled to a thermal reservoir. In this case, we infer that the quadrature variance in the region $0 < \beta < 0.5$ spans above the classical standard

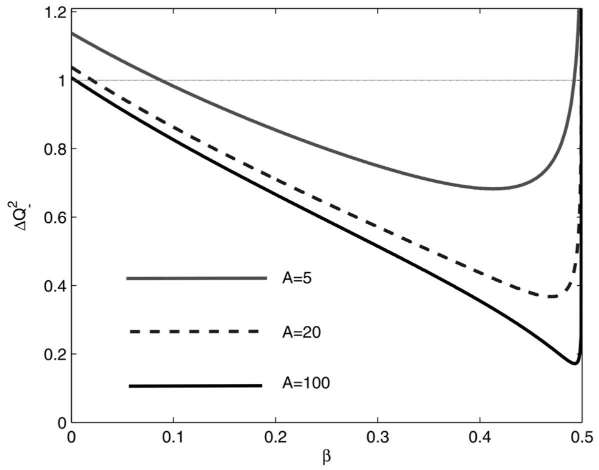

Fig. 5. Plots of the quadrature variance of the cavity mode radiation ($N = 0$) against the atomic coherence for an ordinary vacuum reservoir for the cavity damping constant $\kappa = 0.8$, with different values of the linear gain coefficient $A = 5$ (the red solid curve), $A = 20$ (the blue dashed curve) and $A = 100$ (the black solid curve)

for small values of A. Therefore; we immediately get from the figure that the light beams generated in the cavity are not in the squeezed state. This must be due to the high noise nature of the thermal light incident into the cavity via a coupler. This detects that, in a three-level cascade beat laser, the light beam generated from the cavity has no squeezing nature, when it coupled to the thermal bath.

According to the information presented in Fig. 4, it is well known that there is no quadrature squeezing at all. It occurs for small values of the linear gain coefficients $A = 5$ and $A = 20$. But, for $A = 100$ solely, the squeezed states of the cavity mode radiation would appear in the small region of the atomic coherence $(0.093 < \beta < 0.491)$. This result shows that there would be induced the noise in the cavity which may force the cavity filed to fluctuate beyond the classical fluctuation limit. Thus, the linear gain coefficient would be enough to be large to suppress such fluctuations due to thermal photons. It is a not denied fact that even though the thermal photons cause the noise, they make the cavity mode radiation to become bright. Our findings and results in this paper are in agreement with the works cited in [28].

5.3. QBCL in vacuum reservoir

In this section, we study the quadrature variance and the corresponding quadrature squeezing for a vacuum reservoir. To do this, if we replace a thermal reservoir by an ordinary vacuum one by applying the approximation $N_c = 0$. Then Eq. (29) is reduced to

$$\Delta Q_-^2 = 1 + \frac{A\left[1 - \sqrt{1-4\beta^2} - 2\beta\right]}{\kappa + A\sqrt{1-4\beta^2}}. \tag{30}$$

This condition is more practical [4] to be used and has the intermediate effect on the quantum nature of a cavity field radiation. Here, there is nothing enters the cavity. Then the quadrature squeezing of the system depends only on the atomic coherence which is induced by a coherent superposition. The linear gain coefficient, and the photon decay rate into a vacuum reservoir are described by Eq. (30). As the atomic coherence β changes from 0.0 to 0.5, which is shown in Fig. 5, the variance of a cavity radiation decreases to a minimal value near to the maximum atomic coherence and raised again, while we approach the value $\beta = 0.5$.

In Fig. 3, it is observed that the quadrature variance ΔQ_-^2 is minimum for $A = 100$ and at $\beta = 0.023$. The quadrature variance decreases, as the values of β increases in the region $0 < \beta < 0.023$, and it increases in the region $0.023 < \beta < 0.5$. At the maximum value of β, the cavity light beams totally lose their quantum nature in general and squeezing property in particular. In addition, any effect of the reservoir is not observed for this case. But we are still observing the squeezing for the light beam generated through the system.

This shows that the quadrature squeezing can be increased for the light beams generated through a three-level cascade laser by coupling it with a squeezed vacuum reservoir. It can be taken as a way for the noise reduction. Therefore; we can get light with a higher squeezing which is generated from a three-level cascade laser coupled with a squeezed vacuum reservoir than light beams generated from a three-level cascade laser coupled to a vacuum reservoir.

6. Photon Statistics

The statistical properties of a light beam generated through a certain quantum optical system can be studied in the three regimes such as sub-Poissonian [32–35], super-Poissonian [36], and Poissonian ones. These photon statistical distributions are characterized by comparing the variance of the photon

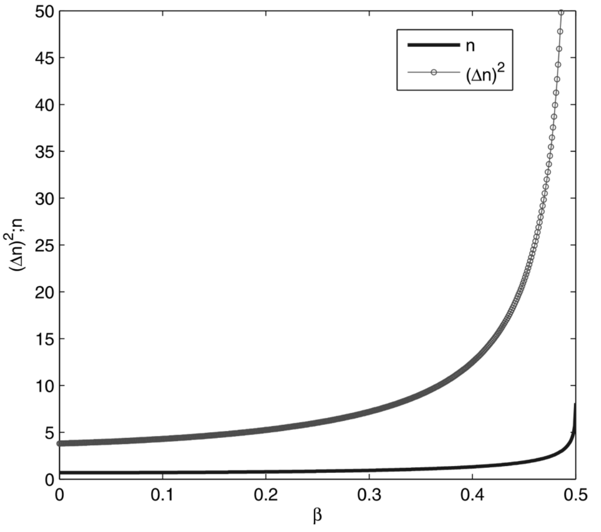

Fig. 6. Plots of the quadrature variance of the cavity mode radiation against the atomic coherence for a squeezed vacuum reservoir for the cavity damping constant $\kappa = 0.8$, linear gain coefficient $A = 5$, and $N = 10$

number and the mean number of photons for the cavity mode light.

The light beams which possess the super-Poissonian photon statistics satisfy the mathematical inequality $\Delta n^2 > n$. Such light beams are termed as chaotic light. On contrast, when the light beams possess the sub-Poissonian distribution, the governing inequality is $\Delta n^2 < n$. This photon statistics indicates that such light beams are squeezed light. In the Poissonian photon statistics, the variance of the photon number us equal to the mean number of photon $\Delta n^2 = n$.

The mean number of photons for a single-mode radiation which denoted by n is defined as

$$n = \langle \hat{a}^\dagger \hat{a} \rangle. \tag{31}$$

The mean number of photons for the cavity radiation containing n photons can be obtained from the solution of Eq. (20) in a steady state:

$$n = \frac{1}{2\kappa + 2A\sqrt{1-4\beta^2}} \Big[A(1-\sqrt{1-4\beta^2}) + \kappa N\Big]. \tag{32}$$

This mean number of photons represents the cavity light generated from a three-level laser coupled to a broad-band squeezed vacuum reservoir. Furthermore,

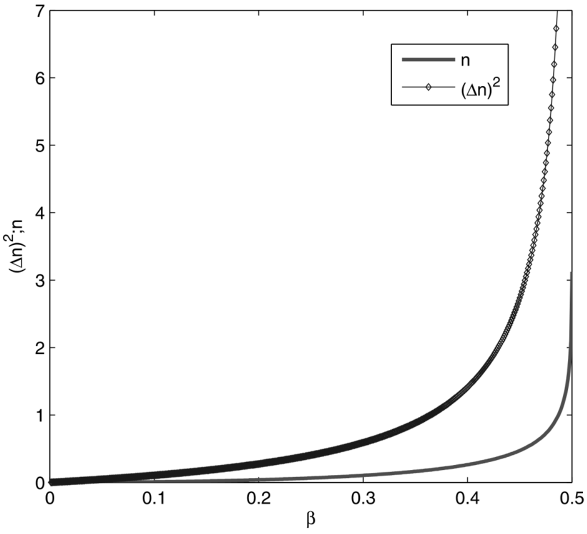

Fig. 7. Plots of the quadrature variance of the cavity mode radiation against the atomic coherence for a vacuum reservoir for the cavity damping constant $\kappa = 0.8$ and the linear gain coefficient $A = 5$

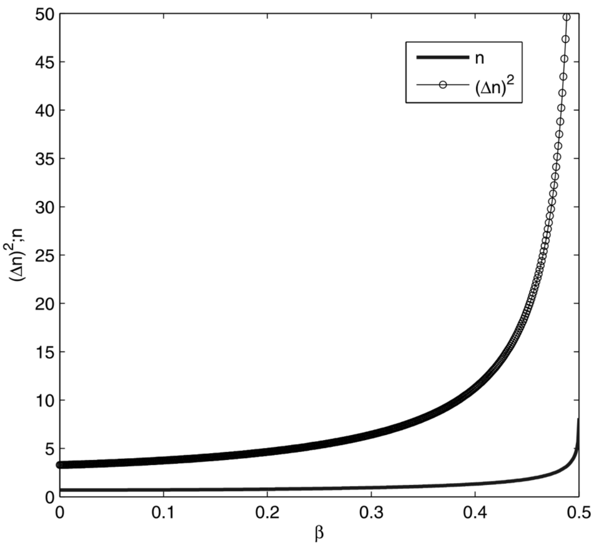

Fig. 8. Plots of the quadrature variance of the cavity mode radiation against the atomic coherence for a thermal reservoir for the cavity damping constant $\kappa = 0.8$, linear gain coefficient $A = 5$, and $N_c = 10$

the photon number variance of a single-mode radiation is defined by

$$\Delta n^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2, \tag{33}$$

where the term $\langle \hat{n}^2 \rangle$ represents the dispersion of cavity light beams. Thus, the variance of the photon number of the cavity radiation can be expressed by

$$\Delta n^2 = \left\langle \hat{a}^2 \hat{a}^{\dagger 2} \right\rangle - n^2 - 3n - 2. \qquad (34)$$

On the basis of the compression between the mean number of photons n and the variance of the photon number, we showed the nature of the cavity mode radiation for the system under consideration. It is the sufficient condition to show system's photon statistics distribution, when it is coupled to different reservoirs. To be specific, as clearly shown in the plots of Fig. 6 (for the case of a quantum beat cascade laser coupled to a squeezed vacuum reservoir), Fig. 8 (for the case of a quantum beat cascade laser coupled to a thermal reservoir) and Fig. 7 (for the case of a quantum beat cascade laser coupled to a vacuum reservoir). As we observe from all these figures, the photon statistics regardless of the nature of a reservoir is super-Poissonian.

7. Conclusions

In this work, the quantum properties of the cavity radiation emitted from a degenerate cascade three-level laser coupled to different reservoirs have been analyzed. The most influencing factors for the quantum features are the atomic coherence and the reservoir nature. It is confirmed that, with the optimum value of the atomic coherence, the quantum properties of a degenerate three-level laser coupled to reservoir modes could be enhanced. Based on the analytic calculations and simulations, we have categorized the system under consideration using the nature of a reservoir coupled to it. The system belongs to bright squeezed radiation generators, when it is coupled to a squeezed vacuum reservoir accompanied with the high pumping rate of three-level atoms into the cavity. The system may be also categorized under dim squeezed radiation generators, when it is coupled to an ordinary vacuum environment. On contrary, the system may also be in the category of bright noisy generators, when it is coupled to a thermal reservoir. It is found that radiation emitted from a degenerate three-level laser has the most squeezing property, when the system is coupled to a squeezed vacuum reservoir. In the case, where the system is coupled to a thermal bath, it loses quantum features due to the entirely classical nature of the thermal bath. It is also observed from our work that, when the three-level laser is coupled to a vacuum reservoir, the radiation emitted from the cavity is highly dependent only on the cavity parameters such as the linear gain of the medium and the atomic coherence. Overall, the reservoir nature is a very influential factor for the cavity radiation. For example, when the system is coupled to a squeezed vacuum reservoir, the squeezing property of the cavity radiation is enhanced, while the mean number of photons is increased. Hence, we obtain the bright squeezed light in this case.

The study of quantum statistical properties of the system was another issue. According to our statistical analysis, the cavity mode radiation emitted from a degenerate three-level laser has the super-Poissonian photon statistics. Different papers stated that the squeezed states of light have the sub-Poissonian photon statistics [see for example, [7, 9, 10]]. But in our analysis, we observe the possibility of a squeezed light with the super-Poissoian photon statistics. Thus, this work has confirmed that a degenerate three-level laser coupled to different reservoir modes possesses the super-Poissonian photon statistics. These results give a new insight for a non-classical light which may possess the super-Poissonian photon statistics.

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С. Ешете

ВПЛИВ ПОЛІВ НА ВХОДІ У ПОРОЖНИНУ НА НЕКЛАСИЧНІ ВЛАСТИВОСТІ ІМПУЛЬСНОГО КАСКАДНОГО ЛАЗЕРА

Досліджуються квантові і статистичні властивості моди випромінювання порожнини когерентного виродженого трирівневого лазера із застосуванням стандартних методів квантової електродинаміки та з урахуванням взаємодії випромінювання із речовиною. Розглянуто вакуумну, стиснуту вакуумну та термальну порожнини для того, щоб визначити вплив полів на вході порожнини на статистичні властивості і природу стиснення випромінювання з неї. Встановлено, що для стиснутої вакуумної порожнини вплив на стискування та на яскравість випромінювання найбільші порівняно з вакуумною та термальною порожнинами. Знайдено також, що випромінювання із порожнини знаходиться в стиснутому стані з суперпуассонівською статистикою фотонів незалежно від типу порожнини.

Ключові слова: суперпуассонівський, стиснутий стан, квантові властивості, взаємодія світла із речовиною.