V. GRYTSAY

Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14b, Metrolohichna Str., Kyiv 03680, Ukraine; e-mail: vgrytsay@bitp.kiev.ua)

UDC 577.3

CHARACTERISTICS OF THE INVARIANT MEASURE OF THE STRANGE ATTRACTOR OF THE BACTERIA MATHEMATICAL MODEL

The bacteria metabolic process of open nonlinear dissipative system far from equilibrium point is modeled using classical methods of synergetics. The invariant measure and its convergence in the phase space of the system was obtained in strange attractor mode. The distribution of point density of trajectory intersection of phase space cells with maximum invariant measure and convergence in time of its average value was obtained. The result concluded is that the value of an invariant measure can be a characteristic of the transitional process of adaptation of cell metabolic process to change outside environment.

Keywords: mathematical model, metabolic process, strange attractor, phase space, invariant measure, convergence.

1. Introduction

This article investigates the mathematical model of bacteria metabolic process constructed in [1–3]. The experimental results of biochemical process of *Arthrobacter globiformis* cells in bioreactor [4, 5] were used as a base for construction of mathematical model. The model shows the main metabolic connections of oxygen-breathing bacteria. The metabolic process in a cell was considered as an open dissipative system. The system has two main self organized subsystems of the dissipative system: substrate transformation and a breath chain.

2. Mathematical Model

The mathematical model was constructed according to the general scheme of cell metabolic process Fig. 1 and was described in the system (1)–(10) [6–10]:

$$\frac{dG}{dt} = \frac{G_0}{N_3 + G + \gamma_2\psi} - l_1V(E_1)V(G) - \alpha_3 G, \quad (1)$$

$$\frac{dP}{dt} = l_1V(E_1)V(G) - l_2V(E_2)V(N)V(P) - \alpha_4 P, \quad (2)$$

$$\frac{dB}{dt} = l_2V(E_2)V(N)V(P) - k_1V(\psi)V(B) - \alpha_5 B, \quad (3)$$

$$\frac{dE_1}{dt} = E_{10}\frac{G^2}{\beta_1 + G^2}\left(1 - \frac{P + mN}{N_1 + P + mN}\right) - l_1V(E_1)V(G) + l_4V(e_1)V(Q) - a_1E_1, \quad (4)$$

$$\frac{de_1}{dt} = -l_4V(e_1)V(Q) + l_1V(E_1)V(G) - \alpha_1 e_1, \quad (5)$$

$$\frac{dQ}{dt} = 6lV(2 - Q)V(O_2)V^{(1)}(\psi) - l_6V(e_1)V(Q) - l_7V(Q)V(N), \quad (6)$$

$$\frac{dO_2}{dt} = \frac{O_{20}}{N_5 + O_2} - lV(2 - Q)V(O_2)V^{(1)}(\psi) - \alpha_7 O_2, \quad (7)$$

$$\frac{dE_2}{dt} = E_{20}\frac{P^2}{\beta_2 + P^2}\frac{N}{\beta + N}\left(1 - \frac{B}{N_2 + B}\right) - l_{10}V(E_2)V(N)V(P) - \alpha_2 E_2, \quad (8)$$

$$\frac{dN}{dt} = -l_2V(E_2)V(N)V(P) - l_7V(Q)V(N) + k_2V(B)\frac{\psi}{K_{10} + \psi} + \frac{N_0}{N_4 + N} - \alpha_6 N, \quad (9)$$

$$\frac{d\psi}{dt} = l_5V(E_1)V(G) + l_8V(N)V(Q) - \alpha\psi. \quad (10)$$

where $V(X) = X/(1 + X)$, $V^{(1)}(\psi) = 1/(1 + \psi^2)$.

The parameters of the model are $l = l_1 = k_1 = 0.2$; $l_2 = l_{10} = 0.27$; $l_5 = 0.6$; $l_4 = l_6 = 0.5$; $l_7 = 1.2$; $l_8 = 2.4$; $k_2 = 1.5$; $E_{10} = 3$; $\beta_1 = 2$; $N_1 = 0.03$; $m = 2.5$;

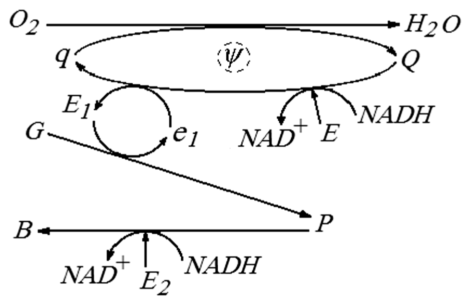

Fig. 1. The main scheme of a cell metabolic process

$\alpha = 0.033$; $a_1 = 0.007$; $\alpha_1 = 0.0068$; $E_{20} = 1.2$; $\beta = 0.01$; $\beta_2 = 1$; $N_2 = 0.03$; $\alpha_2 = 0.02$; $G_0 = 0.019$; $N_3 = 2$; $\gamma_2 = 0.2$; $\alpha_5 = 0.014$; $\alpha_3 = \alpha_4 = \alpha_6 = \alpha_7 = 0.001$; $O_{20} = 0.015$; $N_5 = 0.1$; $N_0 = 0.003$; $N_4 = 1$; $K_{10} = 0.7$.

The nonlinear differential system (1)–(10) was solved using Runge–Kutta–Merson method with accuracy 10^{-12}.

The study of the solutions of the mathematical model (1)–(10) was performed using nonlinear differential equation theory [11–14] and developed methods of mathematical modeling of biochemical systems by the author and other researchers [15–43].

Using this model, all possible modes of the metabolic process as a function of small parameter were investigated. Self organization modes and dynamical chaos were found [28–30]. Liapunov exponents were obtained. A spectral analysis of the system solutions was carried out. The stability of the modes dynamic were studied.

3. Results of Studies

In the work, we investigate experimental modes of the cell metabolic process that may arise in bioreactor. In previous papers, we investigated and described kinetic curves of self organization modes in detail. Strange attractors modes can not be described by kinetic curves. It is because calculation and experimental characteristics are not comparable because of exponential trajectory run off and hypersensitivity of the system to initial data.

The author suggests to describe such types of modes of the system by calculating invariant measure. It defines probability of existing trajectory in different region of the phase space. The work continues investigation of invariant measures for strange attractors of the systems started in work [37].

From Krylov–Bogolyubov theorem, in a case continuous mapping and compact phase space of dynamical system (1)–(10), there exists at least one invariant measure μ_i [11]. Obtained phase portraits and invariant measures confirmed that the system complies with these requirements.

Let us investigate strange attractor mode of the system 13×2^x ($\alpha = 0.03217$).

The strange attractor was created as result a funnel. In this area the its trajectories are mixing.

Let us investigate properties of the invariant measure.

A convergence graph of invariant measure for the strange attractor was constructed as shown in Fig. 2. Calculations show that changing of amount of mapping points does not influence the probability of visiting the trajectory of each cell. Time shifting along trajectory does not influence the probability too. It means an invariance of a measure for the strange attractor. The peak of the invariant measure indicates an attracting set of the strange attractor in the mixing funnel.

Let us obtain attractor convergence 13×2^x ($\alpha = 0.03217$), for each of 10000 iterations.

From the graph of convergence measure it can be seen that the measure tends to converge to its average value. The value of the measure is decreasing as $1/t$.

Let us investigate this situation. Lets obtain a density distribution of points of intersection of strange attractor trajectory for cell of phase space with maximum invariant measure.

Let us obtain and compare invariant measure for $N = 200^{10}$ (Fig. 3, *a, b, c*) and $N = 1000^{10}$ (Fig. 3, *d, e, f*).

The change with time of distribution density of intersection points can be seen in Fig. 3. Where X is the first intersection point of the cell. In both cases, intersection points group in compressed areas.

Let us obtain a convergence of the mean over time in the cell with maximum invariant measure ($N = 1000^{10}$, $\sum n = 3103$, $t = 2 \times 10^8$) (Fig. 4).

It can be concluded from the graph that variation of the mean slowly decreases with time. The system stabilizes in a new auto-oscillating mode.

Obtained invariant measure and its convergence show adaptive capabilities of metabolism in cell in

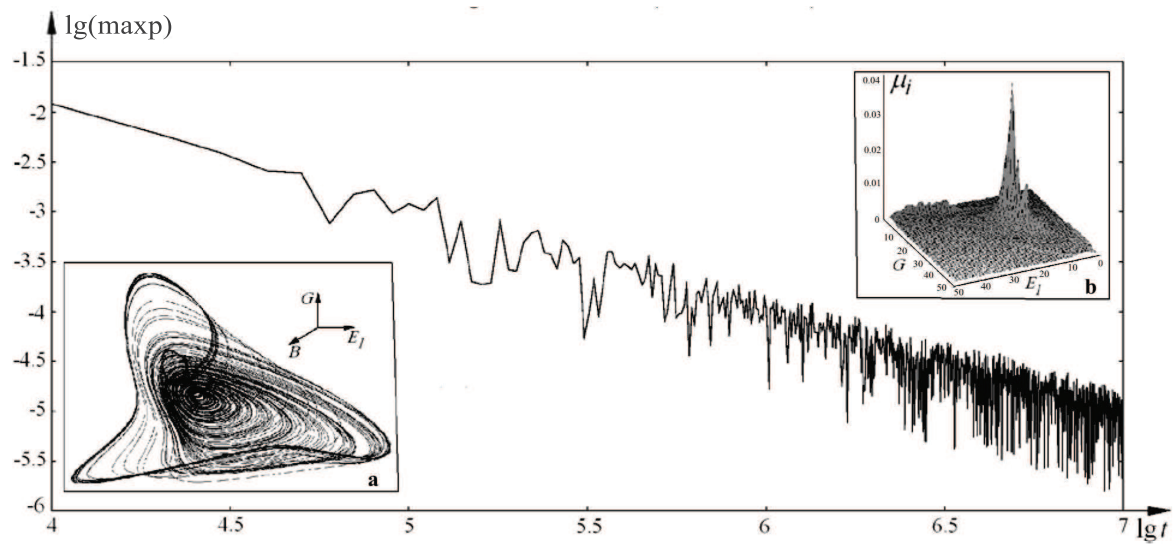

Fig. 2. Graph of convergence of the invariant measure of the strange attractor for the system 13×2^x ($\alpha = 0.03217$), where: **a** – a projection of the phase portrait of the attractor in 3d phase space E_1, G, B; **b** – a histogram of the projection of the invariant measure of the strange attractor onto the plane G, E_1

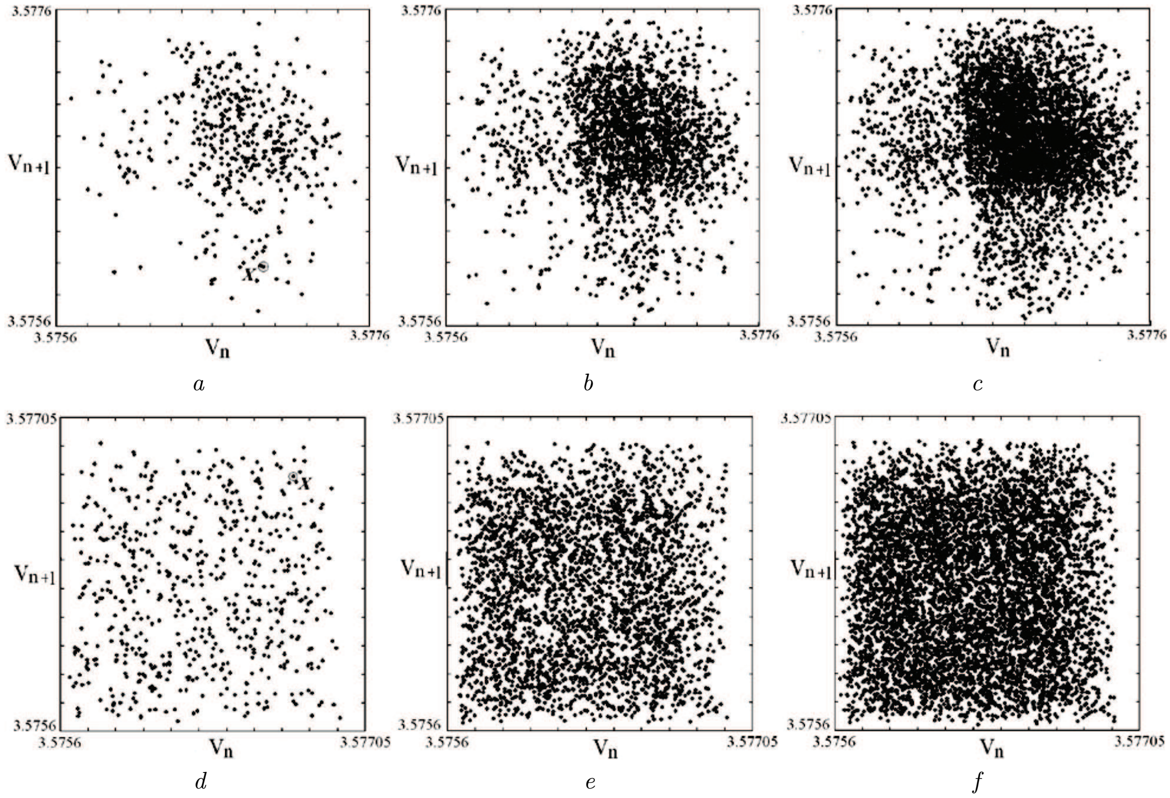

Fig. 3. The evolution for density points distribution of intersection strange attractor trajectory 13×2^x cells of phase space with maximum invariant measure for $N = 200^{10}$ cells: *a* ($\sum n = 465$, $t = 4 \times 10^6$), *b* ($\sum n = 2298$, $t = 2 \times 10^7$), *c* ($\sum n = 4592$, $t = 4 \times 10^7$); for $N = 1000^{10}$ cells: *d* ($\sum n = 598$, $t = 4 \times 10^7$), *e* ($\sum n = 3103$, $t = 2 \times 10^8$), *f* ($\sum n = 6110$, $t = 4 \times 10^8$)

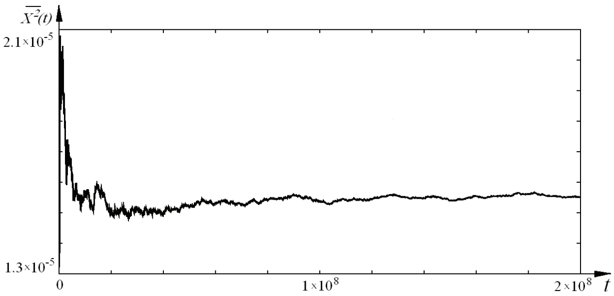

Fig. 4. The graph of convergence of the mean over time for strange attractor of the system 13×2^x in the cell with maximum invariant measure ($N = 1000^{10}$, $\sum n = 3103$, $t = 2\times10^8$)

self organization process to environment of the dissipative system. The metabolic process is maintained by the cell at the vicinity of average level of its metabolites.

4. Conclusions

Strange attractor mode of the cell metabolic process was investigated by the mathematical model. The possibility of application of the calculation of the invariant measure for chaotic modes of the model has been investigated. The distribution density of intersection points of trajectories of the cell in a phase space correspondent to the maximum invariant measure is found. The convergence in time of its average value is demonstrated. It is concluded that the value of invariant measure and its convergence show adaptive capabilities of cell metabolism during self organization as a response to change in environment. Maintenance of cell metabolites around their average values is demonstrated.

The work was supported by project PKNo 0120U101347 the National Academy of Science of Ukraine.

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Received 03.08.22

В.Й. Грицай

ОСОБЛИВОСТІ ІНВАРІАНТНОЇ
МІРИ ДИВНОГО АТРАКТОРА МАТЕМАТИЧНОЇ
МОДЕЛІ БАКТЕРІЇ

Використовуючи класичні методи синергетики, проведено моделювання метаболічного процесу бактерії – відкритої нелінійної дисипативної системи, далекої від рівноваги. В режимі дивного атрактора розраховується інваріантна міра та її збіжність у фазовому просторі системи. Розраховано розподіл густини точок перетину траєкторією комірки фазового простору з максимумом інваріантної міри та збіжність по часу її середнього значення. Зроблено висновок: величина інваріантної міри може бути характеристикою перехідного процесу адаптації метаболізму клітини до змін у навколишньому середовищі.

Ключові слова: математична модель, метаболічний процес, дивний атрактор, фазовий простір, інваріантна міра, збіжність.