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ANISOTROPIC DARCY–BRINKMAN MAGNETIC FLUID CONVECTION UNDER THE INFLUENCE OF A TIME-DEPENDENT SINUSOIDAL MAGNETIC FIELD

The impact of the sinusoidal mode of a magnetic field involving time fluctuations on the threshold of the ferromagnetic smart liquid convection in a saturated permeable medium is investigated using the regular perturbation technique. The Darcy–Brinkman model with anisotropic permeability is used to describe the flow through porous media. The thermal anisotropy is implemented in the energy equation. The problem might be useful in thermal engineering applications such as dynamic loudspeakers and computer hard discs and in medical applications like the treatment of tumor cells and the cell separation, to name a few. The regular perturbation technique is based on the minimum amplitude of a magnetic field modulation, and the onset criterion is dealt with in terms of a correction in the critical Rayleigh number and wavenumber. The thermal Rayleigh number correction depends on the magnetic field modulation frequency, magnetic force, anisotropies, porosity, and Prandtl number. At moderate values of the magnetic field modulation frequency, the impact of various physical factors is perceived to be noteworthy. The influences of the magnetic mechanism, Prandtl number, porosity parameter, and Brinkman number are shown to augment the destabilizing effect of the magnetic field modulation for moderate values of the frequency of a modulation. However, the destabilizing effect of the magnetic field modulation is diminished due to an increase in the values of the mechanical anisotropy parameter and thermal anisotropy parameter. The study reveals that the effect of the magnetic field modulation could be exploited to control the convective instability in an anisotropic porous medium saturated by a ferromagnetic fluid.

Keywords: magnetic field modulation, stability, ferromagnetic fluid, perturbation method, porous medium.

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1. Introduction

Ferromagnetic fluids, often known as ferrofluids or magnetic fluids, are a special type of smart fluids that are magnetized via magnetic fields. Ferromagnetic fluids are manufactured by dissolving the microscopic magnetic (iron – Fe, cobalt – Co, nickel – Ni, *etc.*) granules in a non-magnetic liquid transporter (ester, petrol, hydrocarbons, *etc.*). These granules are wrapped with a surfactant-like organic solution to avoid the aggregation of granules, when a magnetic field is present. However, many researchers and technologists are fascinated by colloidal magnetite (Fe_3O_4), the most comprehensively studied ferrofluid, in view of its diverse utilization in the fields of thermal engineering, bio-medical and aerospace engineering [1, 2]. The applicability of ferroconvection toward the thermal expansion in a layer enclosing the ferrofluid is comparable to the conventional Bénard convection and has garnered abundant vigilance in the literature owing to its prospective value as a heat exchanger. Finlayson [3] initially outlined how the horizontal surface of a ferrofluid with variable magnetic susceptibility creates a stress in the force of a magnetic field that leads to the thermomagnetic ferroconvection. Later, many authors investigated the effect of various constraints on the commencement of magnetic fluid convection [4–11].

The modulation of an appropriate parameter may have significant impact on the motion of various sectors such as charges in an electrode material and can result in the greater stability. The outcome of fluctuations in the magnetic field in respect of time during the starting point of the advection in a magnetic liquid and the collision between harmonic and subharmonic conditions using the Floquet theory, Chebyshev pseudospectral procedure and the QZ method has been respectively elucidated in detail [12–14]. Further, in the experimental work [15], it was shown that the characteristics of the onset of the thermomagnetic advection of a ferromagnetic smart liquid significantly affect the stationary and periodically modulated magnetic fields. In article [16], it was found that the onset of the magnetic field-modulated ferroconvection in a sparsely arranged permeable structure can be delayed or advanced by controlling the parameters of the study. Of late, on the basis of Stokes microcontinuum theory, the combined effect of couple stresses signifying non-Newtonian characteristics of the ferrofluid and the magnetic field modulation was reported in the theoretical work [17], where it was revealed that the effect of a couple stress delays the starting point of the ferroconvection.

The convective heat transfer through fluid-saturated porous materials has elicited a lot of attention in view of its natural phenomenon in addition to its diverse utilization in science and technology including the geothermal power resource usage, nuclear waste eradication, building thermal shielding, waste removal in aquifers, drying processes, and so forth. The pioneering work on the fluid-saturated permeable structure located between two identically flat surfaces and heated directly beneath in the traditional composition was elucidated by Harton and Rogers [18] and Lapwood [19]. The overall problem is now known as the Horton–Rogers–Lapwood or Darcy–Bénard one. However, several researchers have dealt with the topic in depth, and the expanding body of research in this area is well documented [20, 21]. Recently, it was found that the onset of the Brinkman–Bénard triple-diffusive Marangoni magnetoconvection in a two-layer system can be postponed, by increasing the values of the modified internal Rayleigh number for the fluid layer and the solute Marangoni numbers, the Darcy number, and the viscosity ratio [22]. The majority of scientific and experimental research on the advection of a flow in porous environments has focused on isotropic materials. However, in many real scenarios, the mechanical and thermal assets of porous materials are anisotropic, which can be seen in several industrial and environmental situations as a result of the irregular pattern of a permeable matrix.

The thermohaline advection in a permeable structure was theoretically inspected by Tyvand [23] assuming that the layer is anisotropic and homogenous and has an infinite horizontal extent. According to the literature [24], assuming a symmetry axis making an angle of $(90^\circ - \theta)$ against the perpendicular motion in a porous material with anisotropic thermal diffusivity results in two distinct convection cells. In addition, the anisotropic permeable matrix subjected to the inclined layer, time-periodic temperature/gravity, rotation, and double diffusivity has been reported in [25–29]. The impact of the g-jitter on the advent of the ferroconvection in Darcian permeable materials confirms that subcritical unsteadiness might exist, when the frequency of a g-jitter is minimal [30]. A weakly nonlinear instability in a rotating permeable anisotropic magnetic fluid layer using the Runge–Kutta–Gill numerical technique has been carried out in recent years [31]. The impact of a time-dependent electric field on the commencement of the electroconvection in a densely packed anisotropic porous layer saturated with a Boussinesq dielectric fluid is reported in work [32]. It was revealed that the anisotropic parameters greatly influence the stability criterion for moderate and large values of the frequency of the electric field modulation. Of late, the influence of the throughflow and gravity fluctuation on the thermosolutal convection in an anisotropic porous medium with the Darcy–Brinkman effect is examined numerically [33]. The results show that the mechanical anisotropy parameter and the Lewis number have a destabilizing effect, while the thermal anisotropy parameter, Darcy number, solutal Rayleigh number, throughflow parameter, and gravity parameter have a stabilizing effect on the stationary and oscillatory convection.

The convection control is a phenomenon that is vital and intriguing in a wide range of magnetic fluid technologies. At the same time, it is conceptually challenging. The problem of unamplified Rayleigh-Bénard convection in ferromagnetic liquids has received a lot of attention. However, no significant attention has been devoted to studying the influence of a time-dependent sinusoidal magnetic field on the threshold of the thermal convection in a sparsely packed anisotropic porous layer saturated by a ferromagnetic smart liquid. Such investigations might be extremely useful in dynamic loudspeakers, diagnostic systems, in-line polarized fiber modulator, treatment of tumor cells, geophysics, climatology, and in zero-gravity application situations involving a ferromagnetic smart liquid as the working medium. Motivated by these gaps, we will investigate the problem of the Rayleigh–Bénard ferroconvection in an anisotropic porous medium induced by the magnetic field modulation with a focus on how the stability criterion for the ferroconvection in an anisotropic porous medium changes in the presence of a magnetic field modulation. The analysis presented is based on the assumption that the magnetic field modulation dimension is very minimal, and the convective currents are weak, resulting in the avoidance of nonlinear effects. As a result, depending on the frequency of the magnetic field modulation, the advent of the ferroconvection can be advanced or delayed in the presence of an anisotropic porous medium. The statement of the problem and basic equations are described in Section 2. The quiescent basic state of the fluid is discussed in Section 3. In Section 4, the linear stability analysis is performed. The method of solution is described in Section 5. The results obtained are presented graphically and discussed in Section 6. The conclusions are presented in Section 7.

2. Mathematical Formulation

We consider a Boussineq ferromagnetic smart liquid-saturated anisotropic porous medium placed between two horizontal infinite planes positioned at $z = 0$ and $z = d$ in the presence of a sinusoidally time-varying external magnetic field $\mathbf{H}_0^{\text{ext}}(t) = H_0(1 + \varepsilon \cos \omega t)\hat{k}$, acting vertically upward and the gravity $g = -g\hat{k}$ acting downward with g being the gravitational acceleration, H_0 is the uniform magnetic field, ε the small amplitude, ω the frequency, and t being the time. The top and bottom surfaces are retained at different uniform temperatures with a gradient ΔT. The geometry of the problem and the coordinate system are shown in Fig. 1. The Cartesian coordinate system (x, y, z) is used with the origin at the bottom of the surface and the z-axis vertically upward. The fluid flow through the porous medium is described by the extended Darcy equation commonly known as the Brinkman equation [34]. The Brinkman model involves viscous shearing stresses acting on a volume element of the fluid, whereas, in the Darcy model, only the damping force of the porous mass has been retained. In this work, the Brinkman model is taken into account, and the porous medium is assumed to possess the anisotropic distribution along the vertical plane and isotropic distribution along the horizontal plane. The basic equations governing the flow of the incompressible ferrofluid saturating a layer of the anisotropic porous medium with the magnetic field modulation effect are as follows [3, 18, 23, 29, 34, 35].

The general form of the continuity equation is

$$\varepsilon_p \frac{D\rho}{Dt} + \rho\,(\nabla \cdot \mathbf{q}) = 0. \quad (1)$$

Equation (1) for a fluid in the Boussinesq approximation reduces to

$$\nabla \cdot \mathbf{q} = 0. \quad (2)$$

The density is a linear function of the temperature, and the same is given by

$$\rho = \rho_0 \left[1 - \alpha\,(T - T_a)\right]. \quad (3)$$

The momentum equation for a ferromagnetic fluid in the Boussinesq approximation with the magnetic field modulation and the Brinkman model is

$$\rho_0 \left[\frac{1}{\varepsilon_p}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon_p^2}\,(\mathbf{q}\cdot\nabla)\cdot\mathbf{q}\right] = -\nabla p + \rho \mathbf{g} +$$
$$+ \nabla \cdot (\mathbf{HB}) \; - \mu_f\,\mathbf{K}\cdot\mathbf{q} + \bar{\mu}_f \nabla^2 \mathbf{q}, \quad (4)$$

where ε_p is the porosity, $\mathbf{q}$ is the velocity vector field of the fluid flow, ρ is the fluid density, ρ_0 is the reference density of the fluid, α is the coefficient of thermal expansion, T is the temperature, T_a is the reference temperature, μ_f is the dynamic viscosity, $\bar{\mu}_f$ is the effective viscosity, $\mathbf{H}$ is the magnetic field, $\mathbf{B}$ is the magnetic induction, and the anisotropic permeability $\mathbf{K} = K_x^{-1}(\hat{i}\,\hat{i} + \hat{j}\,\hat{j}) + K_z^{-1}(\hat{k}\,\hat{k})$.

The thermal conductivity in the energy equation is assumed to possess an anisotropic distribution along the vertical plane and the isotropic one along the horizontal plane. Hence, the energy equation in the present work is of the form

$$\varepsilon_p C_1 \frac{DT}{Dt} + \mu_0\, T \left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,\,H} \cdot \frac{D\mathbf{H}}{Dt} + \\ + (1 - \varepsilon_p)\,(\rho_0 C)_s \frac{\partial T}{\partial t} = \nabla \cdot (\mathbf{K}_T\, \nabla T), \quad (5)$$

where $\mathbf{M}$ is the magnetization, μ_0 the magnetic permeability, $C_1 = \rho_0 C_{V,\,H} - \mu_0 \mathbf{H} \cdot (\partial \mathbf{M}/\partial T)_{V,\,H}$, $C_{V,\,H}$ the specific heat at the constant volume and magnetic field, and $\mathbf{K}_T = K_{T_x}(\hat{i}\,\hat{i} + \hat{j}\,\hat{j}) + K_{T_z}(\hat{k}\,\hat{k})$ is the anisotropic thermal conductivity. The ferromagnetic fluid considered in this work consists typically of a suspension of submicron-sized particles of magnetite in a nonmagnetic liquid carrier. In addition, the ferromagnetic fluid obeys the Maxwell equations. In writing the Maxwell equations, one has to keep in mind that the conductivity of a ferromagnetic fluid is very small. Therefore, we assume that the fluid is electrically nonconducting with the current density zero. Hence, the magnetic field equations, neglecting the displacement current, are [3, 6, 35, 36]

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0. \quad (6)$$

The defining relation connecting $\mathbf{B}$ and $\mathbf{M}$ is

$$\mathbf{B} = \mu_0\,(\mathbf{H} + \mathbf{M}). \quad (7)$$

We assume that the magnetization is aligned with the magnetic field, but allows a dependence on the magnetic field strength, as well as the temperature, in the form

$$\mathbf{M} = \frac{\mathbf{H}}{H} M\,(H\,T). \quad (8)$$

The magnetic equation of state is linearized about H_0 and T_a to become

$$M = M_0 + \chi_m\,(H - H_0) - K_m\,(T - T_a), \quad (9)$$

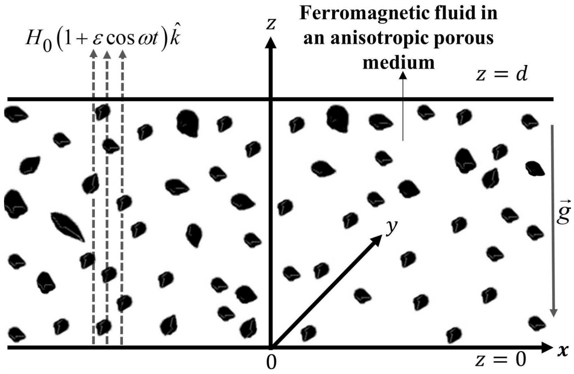

Fig. 1. Schematic of the Rayleigh–Bénard ferroconvection problem

where M_0 is the reference magnetization, χ_m is the differential magnetic susceptibility, and K_m is the pyromagnetic coefficient.

3. Basic State

The quiescent basic state is represented by

$$\begin{aligned} &\mathbf{q} = \mathbf{q}_b = \mathbf{0},\; p = p_b(z), \\ &\rho = \rho_b(z),\; T = T_b(z), \\ &\mathbf{M} = \mathbf{M}_b = (0, 0, M_0(z)), \\ &\mathbf{B} = \mathbf{B}_b = (0, 0, B_0(z)), \\ &\mathbf{H} = \mathbf{H}_b = (0, 0, H_0(z,t)) = H_0^{\text{ext}}(t)\hat{k}. \end{aligned} \quad (10)$$

In the basic state, the pressure, magnetic field, temperature, magnetic induction, and magnetization equations are as follows:

$$\begin{aligned} &-\frac{\partial p_b}{\partial z} - \rho_b g + B_0 \frac{\partial H_b}{\partial z} = 0, \\ &T_b = T_a + \Delta T \left(\frac{1}{2} - \frac{z}{d}\right), \\ &B_b = \mu_0\,(M_0 + H_0), \\ &\rho_b = \rho_0 \left(1 - \alpha\,\Delta T \left(\frac{1}{2} - \frac{z}{d}\right)\right), \\ &H_b = \left[1 + \frac{\gamma_0\,\Delta T}{(1+\chi_0)} \left(\frac{1}{2} - \frac{z}{d}\right)\right] \frac{H_0\,(1 + \varepsilon J)}{(1+\chi_0)}, \\ &M_b = \left[M_0 + \frac{H_0 \gamma_0\,\Delta T}{(1+\chi_0)} \left(\frac{1}{2} - \frac{z}{d}\right)\right] \frac{(1+\varepsilon J)}{(1+\chi_0)}, \end{aligned} \quad (11)$$

where $\chi_0 = \frac{M_0}{H_0}$, $\gamma_0 = \frac{\chi_0}{T_a}$ and $J = \mathrm{Re}\left(e^{-i\omega t}\right) = \cos\omega t$.

4. Linear Stability Investigation

To verify the stability of the basic state, we use the perturbation technique which involves superimposing small perturbations on the basic state of the form

$$\begin{aligned} &\mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad p = p_b + p', \\ &\rho = \rho_b + \rho', \quad T = T_b + T', \\ &\mathbf{H} = \mathbf{H}_b + \mathbf{H}', \quad \mathbf{M} = \mathbf{M}_b + \mathbf{M}', \\ &\mathbf{B} = \mathbf{B}_b + \mathbf{B}', \end{aligned} \tag{12}$$

where primes represent perturbed quantities. Substituting (12) into Eqs. (2)–(9) and using the basic state solution, we obtain the following equations:

$$\nabla \cdot \mathbf{q}' = 0, \tag{13}$$

$$\rho' = -\alpha \rho_0 T', \tag{14}$$

$$\begin{aligned} &\frac{\rho_0}{\varepsilon_p}\left[\frac{\partial \mathbf{q}'}{\partial t}\right] = -\nabla p' - \rho' g \hat{k} - \mu_f \,\mathbf{K} \cdot \mathbf{q}' + \\ &+ \bar{\mu}_f \nabla^2 \mathbf{q}' + \mu_0 \left(M_0 + H_0\right) \frac{\partial \mathbf{H}'}{\partial t} - \\ &- \left(\frac{\mu_0\, \chi_0\, H_0\, (1+\varepsilon J)\, \Delta T}{T_a\, (1+\chi_0)\, d}\right) \frac{\partial \phi'}{\partial z} \hat{k} + \\ &+ \left(\frac{\mu_0\, \chi_0^2 H_0^2 (1+\varepsilon J)^2 \Delta T}{T_a^2 (1+\chi_0)^3 d}\right) T' \,\hat{k}, \end{aligned} \tag{15}$$

$$\begin{aligned} &C_3 \frac{\partial T'}{\partial t} - \varepsilon_p\, C_3 \left(\frac{\Delta T}{d}\right) w' + \\ &+ \frac{\varepsilon_p\, \mu_0 \chi_0 H_0^2}{T_a (1+\chi_0)^2} \left(\frac{\partial T'}{\partial t} - w' \,\frac{\Delta T}{d}\right) (1+\varepsilon J)^2 + \\ &+ \frac{\mu_0 \chi_0^2 H_0^2 \Delta T}{T_a (1+\chi_0)^3 d} (1+\varepsilon J)^2 w' - \\ &- \frac{\mu_0 \chi_0}{(1+\chi_0)} \left(\frac{\partial \phi'}{\partial z}\right) \frac{\partial}{\partial t} H_0\, (1+\varepsilon J) - \\ &- \frac{\mu_0 \chi_0\, H_0}{T_a (1+\chi_0)^2}\, (1+\varepsilon J)\, T' \frac{\partial}{\partial t} H_0\, (1+\varepsilon J) - \\ &- \frac{\mu_0 \chi_0 H_0\, (1+\varepsilon f)}{(1+\chi_0)} \frac{\partial}{\partial t} \left(\frac{\partial \phi'}{\partial z}\right) = \\ &= K_{T_z} \left[\eta\, \nabla_1^2 T' + \frac{\partial^2 T'}{\partial z^2}\right], \end{aligned} \tag{16}$$

$$(1+\chi_0)\, \nabla^2 \phi' - \left(\frac{H_0\, (1+\varepsilon J)\, \chi_0}{T_a\, (1+\chi_0)}\right) \frac{\partial T'}{\partial z} = 0, \tag{17}$$

where $\mathbf{q}' = (u', v', w')$, $C_3 = \varepsilon_p\, C_2 + (1-\varepsilon_p)\, (\rho_0\, C)_s$, $C_2 = \rho_0\, C_{V,\,H}$, $\mathbf{H} = \nabla \phi'$, with ϕ' being the magnetic potential. In Eq. (15), the pressure term can be eliminated by applying the curl twice and then condensing the resulting equations (13)–(17), by adopting the dimensionless scaling over the transformations $(x^*,\, y^*,\, z^*) = \left(\frac{x}{d},\, \frac{y}{d},\, \frac{z}{d}\right)$, $W^* = \frac{w'}{\left(\frac{K_{T_z}}{C_2\, d}\right)}$, $T^* = \frac{T'}{\Delta T}$, $t^* = \frac{t}{\left(\frac{C_2 d^2}{K_{T_z}}\right)}$, $\omega^* = \frac{\omega'}{\left(\frac{K_{T_z}}{C_2 d^2}\right)}$ and $\phi^* = \frac{\phi'}{\left(\frac{K_m \Delta T d}{(1+\chi_0)}\right)}$ [3, 16] to obtain (for simplicity, asterisks (*) are dropped)

$$\begin{aligned} &\left(\frac{1}{\Pr} \frac{\partial}{\partial t} \nabla^2 + D_a^2 \left(\nabla_1^2 + \frac{1}{\xi} \frac{\partial^2}{\partial z^2}\right) - \Lambda \nabla^4\right) W = \\ &= \left[R + RM_1 (1+\varepsilon J)^2\right] \nabla_1^2 T - \\ &- RM_1 (1+\varepsilon J)^2 \frac{\partial}{\partial z} \left(\nabla_1^2 \phi\right), \end{aligned} \tag{18}$$

$$\begin{aligned} &\lambda_p \frac{\partial T}{\partial t} - W + \frac{M_2}{\varepsilon_p} \psi^2 W + \frac{M_2}{\chi_0} \psi (1+\varepsilon J) \Gamma - \\ &- \frac{M_2}{H_0} \frac{\partial H_0}{\partial t} \left(\frac{\partial \phi}{\partial z}\right) \psi - \frac{M_2}{H_0} T \frac{\partial}{\partial t} H_0\, (1+\varepsilon J) - \\ &- \frac{M_2}{H_0} \psi (1+\varepsilon J) \frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial z}\right) = \eta\, \nabla_1^2 T + \frac{\partial^2 T}{\partial z^2}, \end{aligned} \tag{19}$$

$$\nabla^2 \phi = \frac{\partial T}{\partial z}, \tag{20}$$

with $\psi = \frac{(1+\varepsilon f)}{(1+\chi_0)}$ and $\Gamma = \frac{\partial T}{\partial t} - W$.

The dimensionless parameters appearing in Eqs. (18) through (20) are as follows: $\xi = \frac{K_x}{K_z}$ is the mechanical anisotropy parameter, $\eta = \frac{K_{T_x}}{K_{T_z}}$ is the thermal anisotropy parameter, $\Pr = \frac{\varepsilon_p \mu_f C_2}{\rho_0 K_{T_z}}$ the Darcy–Prandtl number, $R = \frac{\alpha g \Delta \rho_0 T d^3 C_2}{\mu_f K_{T_z}}$ the Darcy–Rayleigh number, $M_1 = \frac{\mu_0 \Delta T \chi_0^2 H_0^2}{T_a^2 (1+\chi_0)^3 \alpha \rho_0 g d}$ the buoyancy-magnetization parameter, $M_2 = \frac{\mu_0 \chi_0^2 H_0^2}{C_2 (1+\chi_0) T_a}$ the magnetization parameter, $RM_1 = \frac{\mu_0 \chi_0^2 (\Delta T)^2 H_0^2 C_2}{\mu_f K_{T_z} (1+\chi_0)^3 T_a^2}$ the magnetic Rayleigh number, $D_a^2 = \frac{d^2}{K_z}$ the porosity parameter, $\Lambda = \frac{\bar{\mu}_f}{\mu_f}$ the Brinkman number, and $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$ the Laplacian differential operator with $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Since the typical values of the parameter M_2 are equivalent to the order of 10^{-6} for all kinds of ferromagnetic fluids, the parameter M_2 can be omitted in further calculations [3, 8, 12]. The suitable boundary conditions are [26, 31]:

$$W = \frac{\partial^2 W}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0,\, 1. \tag{21}$$

It is suitable to state the whole problem in terms of the vertical component of the velocity W. By combining Eqs. (18)–(20), we obtain the equation

$$L_1 L_2 \nabla^2 W = R\,\nabla^2\,\nabla_1^2 W + RM_1(1+\varepsilon f)^2 \nabla_1^4 W, \quad (22)$$

with

$$L_1 = \frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2 + D_a^2\left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right) - \Lambda\,\nabla^4$$

and

$$L_2 = \frac{\partial}{\partial t}\nabla^2 - \nabla^4.$$

The boundary conditions for Eq. (21) can also be expressed in terms of W in the form [37–40]

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = \frac{\partial^8 W}{\partial z^8} = 0 \text{ at } z = 0,\,1. \quad (23)$$

Now, the disturbances in the normal modes can be expressed as [40]

$$W = w(z,\,t)e^{i(\alpha_x x + \alpha_y y) + bt}, \quad (24)$$

where $w(z,\,t)$ is a periodic function of the time with the same period as the magnetic field modulation. The quantities α_x, α_y are the wavenumbers of the disturbances in the x and y directions, respectively, and $b = b_r + ib_i$ is the growth rate of the disturbances. Let b_r^* be the eigenvalue with the greatest real part. The basic state, with respect to the infinitesimal disturbances, is unstable, if the real part $b_r^* > 0$, or stable, if $b_r^* < 0$. Here, unstable means that a disturbance experiences the net growth over each modulation cycle or grows during a part of the cycle, but ultimately decays, while stable means that every disturbance decays at every instant. At the neutral stable state, $b_r^* = 0$. If the imaginary part b_i^* is also zero simultaneously, the disturbance is synchronous with the periodic basic state. We consider in the present study only the synchronous mode.

Substituting the normal modes (24) into (22), we obtain

$$L_3 L_4 w = R\delta\alpha^2 w + RM_1(1+\varepsilon f)^2 \alpha^4 w, \quad (25)$$

with

$$L_3 = \frac{1}{\Pr}\frac{\partial}{\partial t}\delta + D_a^2\left(\alpha^2 + \frac{1}{\xi}D^2\right) - \Lambda\delta^2,$$

$$L_4 = \frac{\partial}{\partial t}\delta^2 - \delta^3, \quad \delta = D^2 - \alpha^2,$$

$$D = \frac{\partial}{\partial z} \text{ and } \alpha^2 = \alpha_x^2 + \alpha_y^2.$$

The associated boundary conditions are

$$w = D^2 w = D^4 w = D^6 w = D^8 w = 0 \text{ at } z = 0, 1. \quad (26)$$

5. Solution Procedure

The eigenfunctions w and the eigenvalues R associated with the system of equations (25)–(26) are sought for a modulated magnetic field that is different from the constant magnetic field by a small quantity of order ε. The eigenfunction w and eigenvalue R should be functions of ε, and they should be obtained for a given modulation frequency ω, mechanical anisotropy ξ, thermal anisotropy η, magnetic parameter M_1, Prandtl number Pr, porosity parameter D_a, and Brinkman number Λ. Since $\varepsilon < 1$ for the problem under consideration, we expand these eigenfunctions and eigenvalues in power series in ε. We assume the solution of Eq. (25) in the form [41]

$$\begin{aligned} w &= w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \ldots, \\ R &= R_0 + \varepsilon R_1 + \varepsilon^2 R_2 + \ldots. \end{aligned} \quad (27)$$

Here, w_0 and R_0 are the eigenfunctions and eigenvalues, respectively, of the unmodulated system, w_j and R_j $(j \geq 1)$ are the corrections to w_0 and R_0 in the presence of a magnetic field modulation. Substituting (27) into (25) and equating the corresponding terms upto $O\left(\varepsilon^2\right)$, we obtain the following system of equations:

$$Lw_0 = 0, \quad (28)$$

$$Lw_1 = R_1\nabla^2\nabla_1^2 w_0 + R_1 M_1 \nabla_1^4 w_0 + {} + 2fR_0 M_1 \nabla_1^4 w_0, \quad (29)$$

$$Lw_2 = R_1\nabla^2\nabla_1^2 w_1 + R_2\nabla^2\nabla_1^2 w_0 + R_1 M_1\nabla_1^4 w_1 + {} + R_2 M_1 \nabla_1^4 w_0 + 2fR_0 M_1 \nabla_1^4 w_1 + 2fR_1 M_1 \nabla_1^4 w_0, \quad (30)$$

where

$$L = L_3 L_4 - R_0\left[D^2 + (1+M_1)\,\alpha^2\right]\alpha^2.$$

Each of w_n is required to satisfy the boundary conditions (26). Equation (28) which is obtained at $O\left(\varepsilon^0\right)$ is the one used in the study of the ferroconvection in a fluid-saturated anisotropic permeable medium in the absence of a magnetic field modulation. The marginally stable solutions of that problem are

$$w_0^n = \sin(n\pi z) \tag{31}$$

with corresponding eigenvalues

$$R_0^n = \frac{L_5\left(n^2\pi^2 + \eta\,\alpha^2\right)\left(n^2\pi^2+\alpha^2\right)}{\alpha^2\left[n^2\pi^2 + (1+M_1)\,\alpha^2\right]}, \tag{32}$$

for

$$L_5 = D_a^2\left(\frac{n^2\pi^2}{\xi}+\alpha^2\right) + \Lambda(n^2\pi^2+\alpha^2)^2.$$

For a fixed wavenumber α, the least eigenvalue occurs for $n = 1$ and is given by

$$R_0 = \frac{L_6(\pi^2+\eta\,\alpha^2)(\pi^2+\alpha^2)}{\alpha^2\left[\pi^2+(1+M_1)\alpha^2\right]}, \tag{33}$$

with

$$L_6 = D_a^2\left(\frac{\pi^2}{\xi}+\alpha^2\right)+\Lambda\left(\pi^2+\alpha^2\right)^2$$

corresponding to

$$w_0 = \sin(\pi z), \tag{34}$$

where R_0 is the critical Rayleigh number of the problem in the absence of a modulation. Since changing the sign of ε leads to a shift in the time origin by half a period, and such a shift does not affect the stability of the problem, it follows that all the odd coefficients R_1, R_3, R_5, ... in Eq. (27) must vanish [32, 37]. Following the existing analysis [16, 17], we obtain the expression for R_2 (i.e., R_2 is the first non-zero correction to R_0)

$$R_2 = -\frac{2R_0^2M_1^2\alpha^6}{\left[\pi^2+(1+M_1)\,\alpha^2\right]}\sum_{n=1}^{\infty}\left(\frac{G_1}{G_1^2+G_2^2}\right), \tag{35}$$

where

$$G_1 = -\frac{\omega^2\delta_1^2}{\mathrm{Pr}} + D_a^2\delta_1\delta_2\delta_3 + \Lambda\delta_1^3\delta_2 - - R_0\alpha^2\left[n^2\pi^2+(1+M_1)\,\alpha^2\right],$$

$$G_2 = -\frac{\omega}{\mathrm{Pr}}\delta_1^2\delta_2 - D_a^2\delta_1\delta_3 - \Lambda\delta_1^3$$

with

$$\delta_1 = n^2\pi^2+\alpha^2,\ \delta_2 = n^2\pi^2+\eta\alpha^2,\ \delta_3 = \frac{n^2\,\pi^2}{\xi}+\alpha^2.$$

In the absence of anisotropic effects (that is, when $\xi = \eta = 1$), the expression for R_2 reduces to that of Balaji *et al.* [16].

If R_{2c} is positive, the supercritical instability exists. On the other hand, when R_{2c} is negative, the subcritical instability is possible.

6. Results and Discussion

The effect of a time-fluctuating sinusoidal magnetic field on the commencement of the thermal convection in a ferromagnetic smart liquid-saturated anisotropic sparsely packed permeable structure is inspected. Adopting the normal mode analysis, the analytic solution is obtained by means of the regular perturbation technique [17, 41]. The correction to the critical Rayleigh number R_{2c} is computed as a function of the modulation frequency ω, mechanical anisotropy parameter ξ, thermal anisotropy parameter η, buoyancy magnetization M_1, Prandtl number Pr, porosity parameter D_a, and Brinkman number Λ. The underlying stability process is based on the minimal magnitude of a magnetic field modulation. We note that the starting point of the convection in the electrically nonconducting fluid is affected by the time-varying magnetic field. This is due to the suspended microscopic magnetic granules in the nonmagnetic liquid carrier medium which are wrapped with a surfactant. The microscopic magnetic granules make the fluid magnetically responding in addition to being thermally responding. We notice that the presence of suspended microscopic magnetic granules in the liquid carrier increases the viscosity of the ferromagnetic fluid. In the absence of a magnetic field modulation, the viscosity of the ferromagnetic fluid depends on the concentrations of magnetic granules and a surfactant. Whereas, under the influence of the magnetic field which is varied sinusoidally in the time, the viscosity of the ferromagnetic fluid depends on the modulation frequency. If the modulation frequency is low, then the applied magnetic field modulation increases the viscosity of the ferromagnetic fluid. For moderate and large values of the frequency, it reduces the viscosity of the ferromagnetic fluid. In view of this, we consider values of the Prandtl number for the ferromagnetic fluid higher than those of the carrier liquid without suspended granules. Figures 2 through 7 are used to summarize the results of the current investigation. The stabilizing or destabilizing impact of the magnetic field modulation is determined by the sign of R_{2c}. The supercritical instability occurs provided $R_{2c} > 0$. On the other hand, the subcritical instability is possible, when $R_{2c} < 0$. In this work, one of the most sophisticated scientific application packages, Wolfram Mathematica, is used to extract the numerical values and to plot the graphs.

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Figure 2 illustrates the effect of the buoyancy magnetization parameter M_1 on the stability of the system over the critical correction Rayleigh number R_{2c}, with ξ, η, Pr, D_a, and Λ are being fixed. The parameter M_1 is the ratio of the magnetic force to the gravitational force. It is noticed that, in the interval of the frequency $0 < \omega \leq 100$ (i.e., when ω is small), R_{2c} gradually moves from negative to positive values, as M_1 increases. This is observed, because, for small ω, the viscosity is high in ferromagnetic fluids due to the suspended microscopic magnetic granules in the carrier fluid wrapped with a surfactant. Ultimately, for small ω, the suspended magnetic granules in the ferromagnetic fluid takes time to expand in the fluid and, hence, delays the onset of the ferroconvection. As a result, the influence of M_1 enhances the stabilizing effect of the magnetic field modulation over a small interval of values of the frequency. On the other hand, the viscosity of the ferromagnetic fluid decreases over moderate and larger ω. As a result, R_{2c} decreases with an increase in M_1, indicating that the effect of increasing M_1 is to destabilize the system. It is clear that, when M_1 increases, either the magnetic force increases or gravitational force decreases, which shows that increasing M_1 increases the magnetic force and makes the system more unstable in the interval of the frequency $100 \leq \omega \leq 500$ (i.e., when ω is moderate and large) and stable, when ω is small enough.

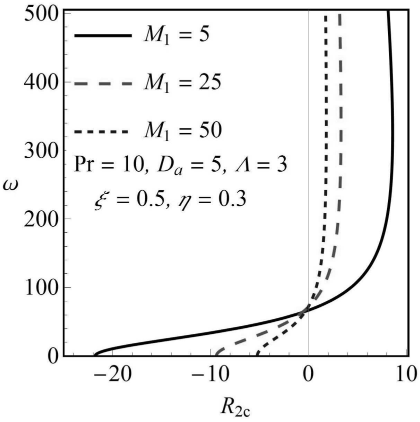

Fig. 2. Variation of the critical correction Rayleigh number R_{2c} with respect to the frequency of a magnetic field modulation ω and the buoyancy magnetization parameter M_1

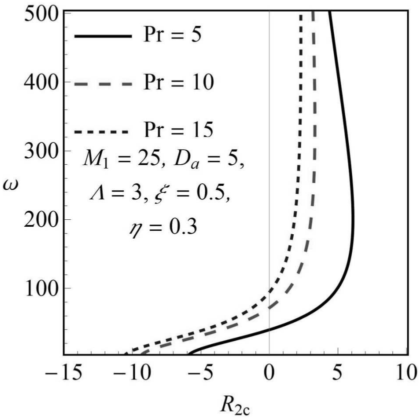

Fig. 3. Variation of the critical correction Rayleigh number R_{2c} with respect to the frequency of a magnetic field modulation ω and Prandtl number Pr

The influence of the Prandtl number Pr on R_{2c} with the frequency ω is shown in Fig. 3. The Prandtl number is the ratio of the speed of momentum propagation (kinematic viscosity) to that of heat transport (thermal diffusivity). In general, the kinematic viscosity varies much more widely than that of heat transport. So, high Prandtl number fluids are very viscous ones. For instance, ferrofluids are of this type. We observe from Fig. 3 that the critical correction Rayleigh number R_{2c} is let down by increasing the value of Pr over a wide interval of frequencies (i.e., $0 < \omega < 500$), which indicates that the Prandtl number Pr enhances the destabilizing effect of a magnetic field modulation on the threshold of the ferroconvection in an anisotropic porous layer. This is observed, because, in the ferroconvection problem, the magnetic equation of state is a function of both the magnetic field and temperature. Hence, the applied temperature gradient causes a spatial variation in the magnetization of suspended magnetic granules. It is apparent that the suspended magnetic granules quickly spread the temperature throughout the fluid medium, and the thermal diffusivity dominates the viscous effect in a ferromagnetic fluid. As a result, incrementing the value of Pr makes the system more unstable irrespective of the frequency. Further, we find that the effect of Pr is less significant, when the frequency is small, and becomes significant for moderate and large frequencies.

Figure 4 depicts the influence of the porosity parameter D_a over the critical correction Rayleigh number R_{2c} with other parameters being fixed. It is worth to note that, for small and moderate frequencies (i.e., $0 < \omega \leq 300$), an increase in the values of D_a, decreases R_{2c} indicating that the effect of D_a reduces the influence of a magnetic field modulation. This

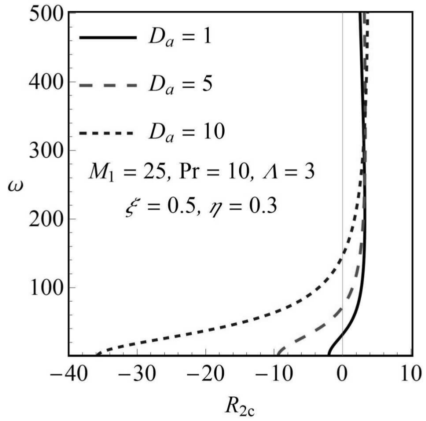

Fig. 4. Variation of the critical correction Rayleigh number R_{2c} with respect to the frequency of a magnetic field modulation ω and the porosity parameter D_a

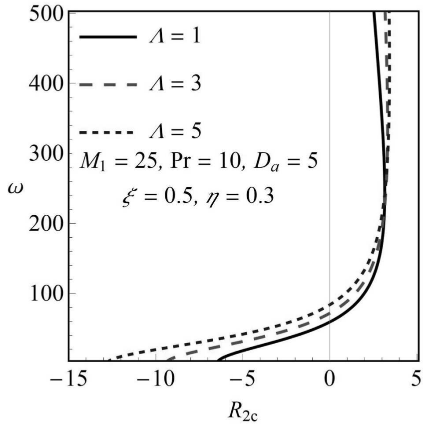

Fig. 5. Variation of the critical correction Rayleigh number R_{2c} with respect to the frequency of a magnetic field modulation ω and the Brinkman number Λ

may be due to the fact that the porous medium considered in this work is sparsely packed. As D_a increases, the porous medium becomes more and more sparse, and it permits the fluid to move freely. Therefore, the onset of the ferroconvection takes place at an early point. Thus, R_{2c} is small in this case. On the other hand, R_{2c} becomes positive and increases with an increase in D_a over the frequency interval $300 < \omega \leq 500$ (i.e., when ω is large), indicating that the impact of D_a diminishes the destabilizing effect of a magnetic field modulation. This is due to a reduction in the permeability of the porous medium following an increase in D_a. We note that, for small and moderate ω, the porosity parameter plays a dominant role, whereas, for very large values of ω, the porosity parameter plays a passive role.

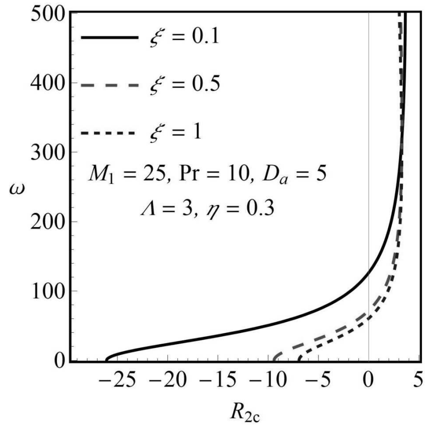

Fig. 6. Variation of the critical correction Rayleigh number R_{2c} with respect to the frequency of a magnetic field modulation ω and the mechanical anisotropy parameter ξ

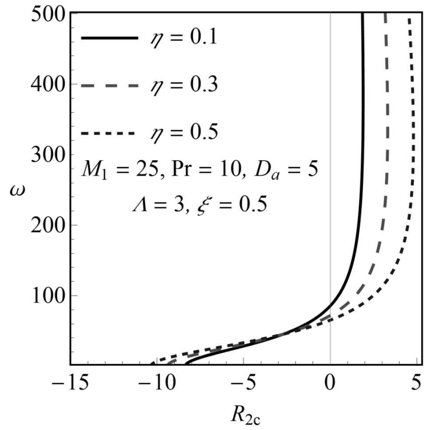

Fig. 7. Variation of the critical correction Rayleigh number R_{2c} the frequency of a magnetic field modulation ω and the thermal anisotropy parameter η

The effect of the Brinkman number Λ on R_{2c} is depicted in Fig. 5 by fixing all other parameters. The parameter Λ is the ratio of the effective viscosity to the dynamic viscosity (fluid viscosity). It is clear that when Λ increases, either the effective viscosity increases or the fluid viscosity decreases. It is also important to note that Λ has a destabilizing effect on the system, when the values of ω are small and moderate. As mentioned earlier, for small ω, the viscosity of the ferromagnetic fluid is high due to the suspended microscopic magnetic granules in the carrier fluid wrapped with a surfactant. But, in this case, increasing Λ decreases the fluid viscosity in the frequency interval 0 to 250. As a result, an increase in Λ accelerates the onset of the ferroconvection. Consequently, R_{2c} is small. But, in the frequency interval from 250 to 500, an increase in Λ increases R_{2c} indicating that the effect of Λ stabilizes the system. In this situation, Λ advances the viscous effect and makes the system more stable.

The variation of R_{2c} with ω for different values of the mechanical anisotropy parameter ξ is shown in Fig. 6 by fixing the parameters M_1, η, Λ, Pr and D_a. We note that, in the interval of modulation frequencies $0 < \omega \leq 300$, R_{2c} increases, as the value of ξ increases, indicating that the effect of increasing ξ is to stabilize the system. This is due to a reduction in the anisotropic permeability distribution along the vertical plane and the isotropic distribution along the horizontal plane, which is responsible for slowing down the ferroconvective instability in this case. On the other hand, the opposite effect occurs in the interval of frequencies $300 \leq \omega < 500$. In this situation, the subcritical motion occurs at the onset of the ferroconvection in a porous medium.

The variability of R_{2c} with ω for various values of the thermal anisotropy parameter η is shown in Fig. 7, and the values of other parameters are fixed. The thermal anisotropy parameter is the ratio of the isotropic thermal conductivity along a horizontal plane (abscissa) to that of the anisotropic thermal conductivity along a vertical plane (ordinate). We observed that the value of R_{2c} reduces, as the thermal anisotropy component η grows, provided frequency is very minimum. However, in the interval $30 < \omega \leq 500$, an increase in η increases the value of R_{2c}. This is due to the fact that increasing the thermal anisotropy parameter either increases the thermal conductivity in a horizontal plane or decreases the thermal conductivity along the vertical direction. In addition, the anisotropic substance possesses a high thermal conductivity in some directions and a low thermal conductivity in other directions. This means that they can simultaneously dissipate heat from a local heating in directions with a high thermal conductivity, while providing the thermal insulation in other directions. Thus, the effect of thermal anisotropy enhances or diminishes the influence of a magnetic field modulation in the intervals $0 < \omega \leq 30$ and $30 < \omega \leq 500$, respectively.

7. Conclusions

The influence of the time-dependent sinusoidal magnetic field on the threshold of the thermal convection in a sparsely packed anisotropic porous layer saturated by a ferromagnetic smart liquid is inspected by means of the regular perturbation technique. The effects of the buoyancy magnetization parameter, Prandtl number, porosity parameter, Brinkman number, mechanical anisotropy parameter, and thermal anisotropy parameter are established. The results obtained are presented graphically, and the following consequences are drawn.

1. The buoyancy magnetization parameter M_1 enhances the stabilizing effect of a magnetic field modulation for small frequencies, while, for moderate and large frequencies, its effect is to augment the destabilizing effect of a magnetic field modulation.

2. Prandtl number Pr augments the amplifying effect of a magnetic field modulation irrespective of the interval of frequencies.

3. Darcy-number D_a and Brinkman-number Λ augment the destabilizing effect of a magnetic field modulation for small and moderate frequency rates, while the opposite occurs for significantly larger frequency rate.

4. The impact of the mechanical anisotropy on the time-fluctuating sinusoidal magnetic field is to stabilize the system at low and intermediate frequencies and destabilize at high frequencies. The consequence of the thermal anisotropy parameter is exactly opposite to that of the mechanical anisotropy.

5. The impact of a time-fluctuating magnetic force, anisotropy parameters, and magnetic parameter dissipate, when the frequency rate is considerably large.

In a nutshell, the influence of a magnetic field modulation in the presence of a sparsely packed anisotropic porous medium can hasten or postpone the threshold of the ferroconvective instability depending on the frequency of a magnetic field modulation. The effect of a magnetic field modulation could be used to control the convection in a sparsely packed anisotropic porous medium saturated by a ferromagnetic fluid. The results obtained here can be useful for dynamic loudspeakers, diagnostic systems, in-line polarized fiber modulators, treatment of tumor cells, geophysics, climatology, and in zero-gravity situations involving a ferromagnetic smart liquid as the working medium. On the other hand, the Floquet theory could be used to attack the current problem alternatively. The Landau–Ginzburg and Lorenz techniques are some of the useful nonlinear approaches to the solution of the problem which offer the reliable information about the stability boundaries.

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АНІЗОТРОПНА КОНВЕКЦІЯ
МАГНІТНОЇ РІДИНИ ДАРСІ–БРІНКМАНА
ПІД ВПЛИВОМ ЗАЛЕЖНОГО ВІД ЧАСУ
СИНУСОЇДАЛЬНОГО МАГНІТНОГО ПОЛЯ

Вплив синусоїдального режиму магнітного поля, що флуктуює у часі, на поріг конвекції феромагнітної смарт-рідини в насиченому проникному середовищі досліджено за допомогою методу регулярних збурень. Для опису течії через пористі середовища використовується модель Дарсі–Брінкмана з анізотропною проникністю. Теплова анізотропія реалізована в рівнянні енергії. Дана задача може бути корисною, зокрема, у таких технічних застосуваннях, як динамічні гучномовці та комп'ютерні жорсткі диски, а також у медицині, зокрема для лікування пухлин. Метод регулярного збурення базується на мінімальній амплітуді модуляції магнітного поля, а критерій початку розглядається в термінах поправки критичного числа Релея та хвильового числа. Поправка до числа Релея залежить від частоти модуляції магнітного поля, амплітуди цього поля, анізотропії, пористості та числа Прандтля. Показано, що вплив магнітного механізму, числа Прандтля, параметра пористості та числа Брінкмана посилює дестабілізуючий ефект модуляції магнітного поля для помірних значень частоти модуляції. Проте цей дестабілізуючий ефект зменшувався за рахунок збільшення значень параметрів механічної та теплової анізотропії та параметра теплової анізотропії. Дослідження показує, що ефект модуляції магнітного поля можна використовувати для утримування під контролем конвективної нестабільності в анізотропному пористому середовищі, насиченому феромагнітною рідиною.

Ключові слова: модуляція магнітного поля, стабільність, феромагнітна рідина, метод збурень, пористе середовище.