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UDC 539

DECOHERENCE IN A $\mathcal{PT}$-SYMMETRIC QUBIT

We investigate the decoherence in a $\mathcal{PT}$-symmetric qubit coupled with a bosonic bath. Using canonical transformations, we map the non-Hermitian Hamiltonian representing the $\mathcal{PT}$-symmetric qubit to a spin boson model. Identifying the parameter α that demarcates the hermiticity and non-hermiticity in the model, we show that the qubit does not decohere at the transition from the real eigen spectrum to a complex eigen spectrum. Using a general class of spectral densities, the strong suppression of the decoherence is observed due to both vacuum and thermal fluctuations of the bath, and the initial correlations hold, as we approach the transition point.

Keywords: $\mathcal{PT}$-symmetry, decoherence, system-bath correlations.

1. Introduction

The fundamentals of quantum mechanics were thought of just as an academic interest, but ever since more and more non-hermitian systems became experimentally accessible [1–4], the notion changed. In fact, recent experiments have shown that the hermiticity postulate of quantum mechanics may not as fundamental as thought [5, 6]. It was just mere convenience to say that every quantum system should be represented by Hermitian operators, as they have real spectrum, but the converse is not necessarily true, one could have real eigen values with non-Hermitian operators as well. One of the examples are $\mathcal{PT}$-symmetric Hamiltonians which have been realized in many different setups, such as optical [7, 8], optomechanical [9] or microcavity-based experiments [10]. In a nutshell, one could define $\mathcal{PT}$-symmetric systems to be those which are invariant under the joint time reversal $\mathcal{T}$ and parity $\mathcal{P}$ operations. It has been shown that $\mathcal{PT}$-symmetric Hamiltonians not only admit a real spectrum, but can also be mapped into Hermitian Hamiltonians with suitable transformations [11].

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When a quantum system of interest interacts with an environment, its evolution becomes non-unitary and displays the decoherence [12]. Decoherence is the fundamental mechanism by which fragile superpositions are destroyed thereby producing a quantum to classical transition [13, 14]. In fact, the decoherence is one of the main obstacles for the preparation, observation, and implementation of multiqubit entangled states. The intensive work on quantum information and computing in recent years has tremendously increased the interest in exploring and controlling the decoherence effects [15–27]. A natural question would pertain to the decoherence in $\mathcal{PT}$-symmetric systems and how the decoherence varies with a change in the "amount of hermiticity" of the Hamiltonian.

It has been observed that the non-hermiticity leads to a slowing of the decoherence [10, 11] in the long time limit of the dynamics. In this work, for the first time we address the question pertaining the decoherence in a $\mathcal{PT}$-symmetric qubit without any approximation on the dynamics. We consider both the situations, where the qubit and bath are initially uncorrelated, as well as correlated; we will show that the decoherence imparted by the initial correlations (as well as in uncorrelated case) is significantly suppressed, as we change the hermiticity in the model.

This work is organized as follows. We introduce the $\mathcal{PT}$-symmetric qubit model in Section 2. This Hamiltonian depends on the parameter α which separates the real and complex eigen spectra of the system. We map the non-Hermitian Hamiltonian to spin boson model with suitable canonical transformations. Assuming the system and bath at the thermal equilibrium at times before $t = 0$, we make a projective measurement on the system only, which results in a bath state that depends on the state vector of the system. In Section 3, we study the decoherence due to this state-dependent bath, as well as due to uncorrelated initial states, and show that the decoherence due to these initial correlations is strongly modified by a change in the parameter α controlling the nature of the model Hamiltonian. We make finally conclusions in Section 4.

2. $\mathcal{PT}$-Symmetric Model Hamiltonian Coupled with a Bosonic Bath

The system under consideration is a $\mathcal{PT}$-symmetric qubit coupled to a bosonic bath described as

$$H = H_S \otimes I_B + I_S \otimes H_B + H_I, \quad (1)$$

where $H_s = i\alpha\sigma^z + \sigma^x$ is a $\mathcal{PT}$-symmetric qubit Hamiltonian [11, 28]. We see that H_s has two eigenvalues $E_\pm = \pm\sqrt{1-\alpha^2}$. Thus, for $|\alpha| \le 1$, we will have real eigenvalues. $\alpha = 1$ would, therefore, correspond to the transition point separating the real and complex eigen spectra. For future references, α will be called the hermiticity or hermiticity parameter and, hence, defines the hermiticity in the Hamiltonian. $H_B = \sum_k \omega_k b_k^\dagger b_k$ represents the bosonic bath with b_k as an annihiliation operator of kth bosonic mode with energy ω_k. The interaction between the qubit and bath is given by $H_I = \sum_k (i\alpha\sigma^z + \sigma^x)(g_k b_k + g_k^* b_k^\dagger)$.

This Jamiltonian can be mapped to a Hermitian Hamiltonian H via an operator T which preserves quantum canonical relations. Identifying

$$T = \Delta^\dagger \begin{pmatrix} s_+ & 0 \\ 0 & s_- \end{pmatrix} \Delta, \quad (2)$$

with $s_\pm = \sqrt{2(1\pm\alpha)}$ and $\Delta = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}$, we can write

$$\tilde{H} = TH_ST^{-1} \otimes I_B + I_S \otimes TH_BT^{-1} + TH_IT^{-1} =$$
$$= E\sigma_x + \sum_k \omega_k b_k^\dagger b_k + \sum_k E\sigma_x (g_k b_k + g_k^* b_k^\dagger), \quad (3)$$

with $E = \sqrt{1-\alpha^2}$. It is clear that the transformed Hamiltonian is Hermitian with $g_k E$ as the effective coupling. Making the transformation $\sigma_x \to \sigma_z$, we get

$$\tilde{H} = E\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sum_k E\sigma_z (g_k b_k + g_k^* b_k^\dagger), \quad (4)$$

which is the well-known spin-boson model. The distinctive feature of this dephasing model is that the average populations of the qubit states do not depend on time.

3. Decoherence

3.1. Uncorrelated state

Suppose that, at time $t = 0$, the state of the composite system is described by the initial density matrix $\rho(0)$. Then, at the time t, the density matrix in the interaction picture is given by

$$\rho(t) = U(t)\,\rho(0)\,U(t)^\dagger, \quad (5)$$

where $U(t) = T\ e^{-i\int_0^t dt' H_I(t')}$ is the time evolution operator, $H_I(t)$ is the interaction Hamiltonian in the interaction picture, and T is the chronological time ordering operator. Our main interest is to calculate the reduced density matrix of the system by tracing over the degrees of freedom of the bath:

$$\rho_s(t) = \mathrm{Tr}_B[U(t)\,\rho(0)\,U(t)^\dagger]. \quad (6)$$

We assume the initial density matrix of the total system as a direct product state:

$$\varrho(0) = \varrho_S(0) \otimes \varrho_B, \quad \varrho_B = \mathrm{e}^{-\beta H_B}/Z_B, \quad (7)$$

where $\beta = 1/k_\mathrm{B}T$, and Z_B is the bath partition function. Note that $\varrho_S(0)$ may be a pure state, as well as a mixed state of the qubit.

Then we write

$$U(t) = Te^{-i\int_0^t d\tau H_I(\tau)} = e^{i\phi(t)}e^{\sigma^z\hat{\Lambda}(t)},$$

where $\phi(t)$ is a function of the time only, and $\hat{\Lambda}(t) = \sum_k[\alpha_k(t)b_k - \alpha_k^*(t)b_k^\dagger]$ with $\alpha_k(t) = Eg_k\frac{e^{-i\omega_k t}-1}{\omega_k}$. Therefore, we can write, for the qubit state $|\psi\rangle = a|0\rangle + b|1\rangle$:

$$\rho_s(t) = \mathrm{Tr}_B[U(t)\,\rho(0)\,U(t)^\dagger] =$$
$$= \mathrm{Tr}_B[e^{\sigma^z\hat{\Lambda}(t)}|\psi\rangle\langle\psi| \otimes \varrho_B e^{-\sigma^z\hat{\Lambda}(t)}] =$$
$$= \begin{pmatrix} |a|^2 & ab^*e^{-\gamma_1(t)} \\ ba^*e^{-\gamma_1(t)} & |b|^2 \end{pmatrix}, \quad (8)$$

with $\gamma_1(t)$ defined as

$$\gamma_1(t) = -\sum_k \ln \left\langle \exp\left[\alpha_k(t)b_k^\dagger - \alpha_k^*(t)b_k\right]\right\rangle_B, \qquad (9)$$

where the symbol $\langle ...\rangle_B$ denotes averages taken with the bath distribution ϱ_B. After a straightforward algebra, we find

$$\gamma_1(t) = (1-\alpha^2)\int\limits_0^\infty d\omega\, J(\omega)\, \coth(\beta\omega/2)\, \frac{1-\cos\omega t}{\omega^2}, \qquad (10)$$

where the continuum limit of the bath modes is performed, and the spectral density $J(\omega)$ is introduced by the rule [12]

$$\sum_k 4|g_k|^2 f(\omega_k) = \int\limits_0^\infty d\omega\, J(\omega) f(\omega).$$

Expression (10) is the exact result for the decoherence function in model (4) under the uncorrelated initial condition (7). We observe that the decoherence function is scaled by the factor $E^2 = 1-\alpha^2$. Thus change of α from zero to 1 results in a suppression of the decoherence. At the transition point from the Hermitian Hamiltonian to a non-Hermitian one, $\alpha = \pm 1$, no decoherence results in making the qubit state maximally robust. In the next ssubection, we will see the same effect in correlated initial states.

3.2. Correlated initial States

We assume the total system plus bath are in the thermal equilibrium state at times $t < 0$, and a measurement is made on such state at the time $t = 0$, when we have [29–31]

$$\rho(0) = \frac{1}{Z}\sum_m \Omega_m e^{-\beta H}\Omega_m^\dagger, \qquad (11)$$

where Ω_m are the projection operators on a desired state of the system and/or bath; Z is the normalization constant called a partition function. Now, we make a particular case of the selective measurement, a projection by taking [30]

$$\Omega_m = |\psi\rangle\langle\psi| \otimes I_B, \qquad (12)$$

where I_B is the identity operation on the state of the bath, and $|\psi\rangle$ is a pure state of the qubit. Therefore, we write

$$\rho(0) = |\psi\rangle\langle\psi| \otimes \rho_B(\psi), \qquad (13)$$

where $\rho_B(\psi) = \frac{1}{Z}\langle\psi|e^{-\beta H}|\psi\rangle$ represents the density matrix of the bath and clearly depends on the state of the qubit $|\psi\rangle$

$$\rho_s(t) = \mathrm{Tr}_B[U(t)\,\rho(0)\,U(t)^\dagger], \qquad (14)$$

$$= \mathrm{Tr}_B[e^{\sigma^z\hat{\Lambda}(t)}|\psi\rangle\langle\psi| \otimes \rho_B(\psi)e^{-\sigma^z\hat{\Lambda}(t)}]. \qquad (15)$$

Now, to evaluate the above expression, we take the general state of the qubit as $|\psi\rangle = a|0\rangle + b|1\rangle$, while we relegate the derivation to appendix 5. We have

$$\rho_s(t) = \begin{pmatrix} |a|^2 & ab^*F(t) \\ ba^*F^*(t) & |b|^2 \end{pmatrix}, \qquad (16)$$

where

$$F(t) = \left[\frac{\begin{bmatrix} |a|^2 e^{-\beta\omega_0/2} e^{i(1-\alpha^2)\Phi(t)} + \\ + |b|^2 e^{\beta\omega_0/2} e^{-i(1-\alpha^2)\Phi(t)} \end{bmatrix}}{|a|^2 e^{-\beta\omega_0/2} + |b|^2 e^{\beta\omega_0/2}}\right] e^{-\gamma_1(t)}, \qquad (17)$$

with $\Phi(t) = \sum_k \frac{4|g_k|^2}{\omega_k^2}\sin\omega_k t$. Using the relations $|a|^2 + |b|^2 = 1$ and $\langle\sigma_z\rangle = |a|^2 - |b|^2$, we can write

$$\frac{|a|^2 e^{-\beta\omega_0/2} e^{i(1-\alpha^2)\Phi(t)} + |b|^2 e^{\beta\omega_0/2} e^{-i(1-\alpha^2)\Phi(t)}}{|a|^2 e^{-\beta\omega_0/2} + |b|^2 e^{\beta\omega_0/2}} =$$

$$= \cos(1-\alpha^2)\Phi(t) - i\frac{\sinh\frac{\beta\omega_0}{2} - \langle\sigma_z\rangle\cosh\frac{\beta\omega_0}{2}}{\cosh\frac{\beta\omega_0}{2} - \langle\sigma_z\rangle\sinh\frac{\beta\omega_0}{2}} \times$$

$$\times \sin(1-\alpha^2)\Phi(t). \qquad (18)$$

In order to get the dephasing or time-dependent frequency shift explicity, we define

$$\tan[\chi(t)] = \frac{\sinh(\beta\omega_0/2) - \langle\sigma_z\rangle\cosh(\beta\omega_0/2)}{\cosh(\beta\omega_0/2) - \langle\sigma_z\rangle\sinh(\beta\omega_0/2)} \times$$

$$\times \tan[(1-\alpha^2)\Phi(t)] \qquad (19)$$

so that $F(t)$ simplifies to $F(t) = e^{i\chi(t)}e^{-\gamma_1(t)-\gamma_c(t)} = e^{i\chi(t)-\gamma(t)}$ with $\gamma(t) = \gamma_1(t) + \gamma_c(t)$ and

$$\gamma_c(t) =$$

$$= -\frac{1}{2}\ln\left[1 - \frac{(1-\langle\sigma_z\rangle^2)\sin^2[(1-\alpha^2)\Phi(t)]}{(\cosh(\beta\omega_0/2) - \langle\sigma_z\rangle\sinh(\beta\omega_0/2))^2}\right]. \qquad (20)$$

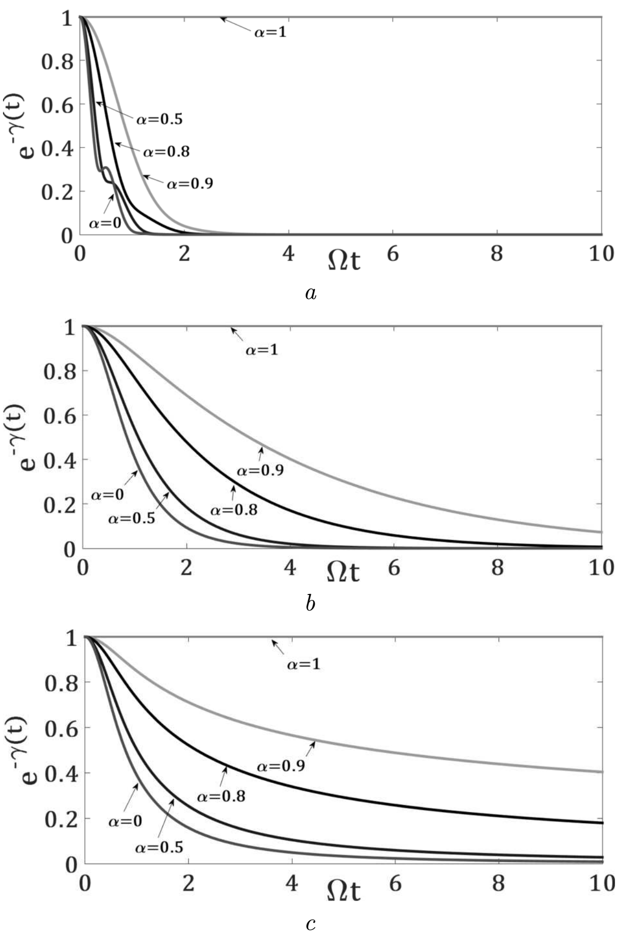

Fig. 1. Time dependence of the exponential of the total decoherence function $e^{-\gamma(t)}$ for the initially correlated state for different values of α with $\beta\omega_0 = 1$ and the initial condition $\langle\sigma_z\rangle = 0, \Omega\beta = 1$ in all the subohmic $s = 0.2$ (*a*) ohmic $s = 1$ (*b*) and superohmic $s = 2$ cases (*c*)

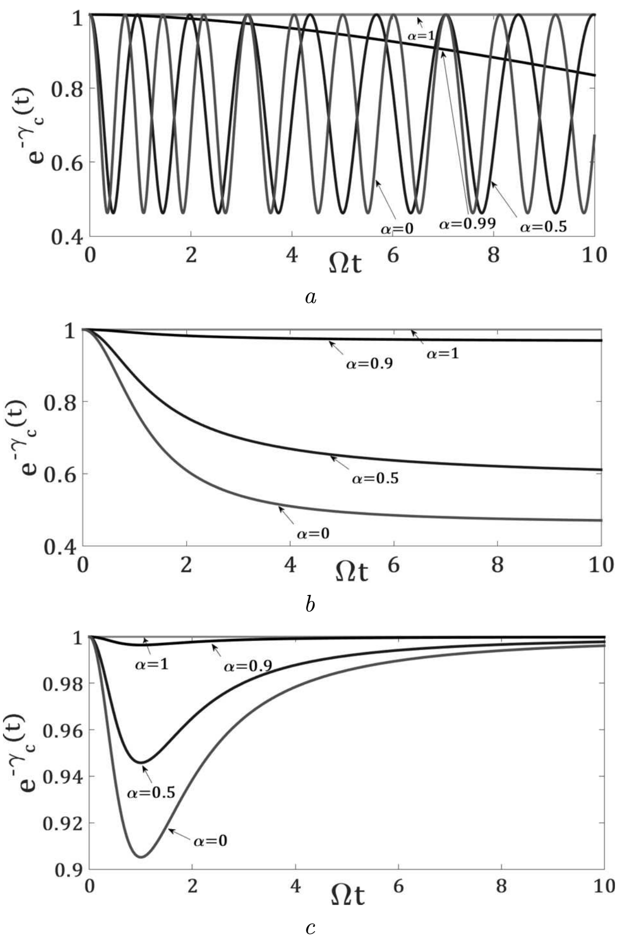

Fig. 2. Time dependence of the exponential of the decoherence due to initial correlations $e^{-\gamma_c(t)}$ for different values of α with $\beta\omega_0 = 1$ and the initial condition $\langle\sigma_z\rangle = 0, \Omega\beta = 1$ in all the subohmic $s = 0.2$ (*a*) ohmic $s = 1$ (*b*) and superohmic $s = 2$ cases (*c*)

The term $\gamma_1(t)$ represents the decoherence due to vaccum and thermal fluctuations of the bath, while $\gamma_c(t)$ represents the decoherence due to initial correlations of the composite system. Thus, we see that the decoherence function $\gamma_1(t)$ gets scaled by a factor of $1 - \alpha^2$, while as a different functional dependence on α is found for $\gamma_c(t)$. The reduced dynamics of a $\mathcal{PT}$-symmetric qubit can be calculated as $T^{-1}\rho_s(t)T$.

In order to understand the effect of the parameter α on the decoherence dynamics, we define a spectral density function for the bath $J(\omega) = \sum_k 4|g_k|^2\delta(\omega - \omega_k)$. It is convenient to describe $J(\omega)$ phenomenologically by assuming the power law form with a certain frequency cut-off. Therefore, we write $J(\omega) = \lambda_s(\omega/\Omega)^s\Omega e^{-\omega/\Omega}$ where λ_s is the dimensionless coupling constant, and Ω is the cutt-off frequency. The values s determine the nature of the bath. If $s = 1$, we call it the ohmic bath. While, if $s < 1$ or $s > 1$, it is called the subohmic or superohmic bath, respectively.

Figure 1 shows the variation of the total decoherence $e^{-\gamma(t)}$ with respect to Ωt for different values of α in the subohmic $s < 1$, ohmic $s = 1$, and superohmic $s = 2$ regimes, where $\gamma(t) = \gamma_1(t) + \gamma_c(t)$. We see from Figure 1, *a* that, for $\alpha = 0$, we observe strong oscillations of $e^{-\gamma(t)}$. However, as α increases from 0 to 1, the oscillations freeze out. The oscillations observed in $e^{-\gamma(t)}$ are due to the initial correlations in the subohmic regime, as can be seen from Figure 2, *a*. The period of oscillations increases from a finite value to the infinite one, and the freeze out occurs at the value of $\alpha = 1$. These features are due to the fact that it takes a longer time for the system to complete one Hilbert space oscillation resulting in the extremely slow dynamics near the boundary $\alpha = 1$ on which the dynamics completely freezes.

Now, we turn to the ohmic case $s = 1$. In this case, using the explicit form of $J(\omega)$, we can write the explicit form of decoherence functions in closed form as [31]

$$\gamma_1(t) = (1-\alpha^2)\left[\frac{\lambda_1}{2}\ln(1+\Omega^2t^2) + \right.$$

$$\left. + 2\lambda_1[\ln\Gamma(1+1/\Omega\beta) - \frac{1}{2}\ln|\Gamma(1+1/\Omega\beta + it/\beta)|^2\right],$$

$$\Phi(t) = \lambda_1 \tan^{-1}(\Omega t). \quad (21)$$

Figures 1, *b* and 2, *b* show the variation of $e^{-\gamma(t)}$ and $e^{-\gamma_c(t)}$ with respect to Ωt for different values of α. It can be seen from the plot that the hermiticity parameter increases from 0 to 1, and the slowing down of the decoherence is observed. We mention the sudden transition at $\alpha = 1$ with no decoherence at all. This feature can be attributed to the frozen dynamics of the system Hamiltonian at $\alpha = 1$.

In the superohmic case, no oscillatory behavior is found unlike the subohmic case (see Figs. 1, *c* and 2, *c*). This is in complete contrast with the subohmic case where the oscillation time period becomes infinite. Although, in both superohmic and subohmic cases, the dynamics kicks off at $t = 0$, the bath has a rapid correlation-dependent effect on the qubit explaining the initial minima in the graphs. But, in the long time limit, the qubit settles in a steady state completely independent of the initial dynamics for the superohmic case, whereas no such steady state is formed in the subohmic case. Nevertheless, both the dynamics have the same physical consequences; namely, the intial correlations disappear. In other words, it would be no matter whether the system was initially prepared independently, or a projective measurement was made on the thermalized system and bath, both will result in approximately the same dynamics near the boundary of separation of the physical and unphysical Hamiltonians.

4. Conclusions

In this work, we have studied a $\mathcal{PT}$-symmetric qubit coupled to a bosonic bath. Using a canonical transformation, we mapped the $\mathcal{PT}$-symmetric model to the well-known spin-boson model which is a purely dephasing model. Using a projective measurement on the system only in a thermalized state of the system plus bath, we arrive at a correlated initial state with the bath state depending on the degrees of freedom of the system. We have shown that the decoherence due to these initial correlations is strongly modified in the subohmic regime. Moreover, it is found that the total decoherence is slowed down with an increase in the hermiticity parameter α. At the transition point that seperates the Hermitian and non-Hermitian regimes, the dynamics of the qubit freezes out making the qubit more robust against external perturbations. A similar dynamics is also observed in the Kibble–Zurek mechanism applied to the one-dimensional Ising model [32, 33]. We see that the decoherence due to initial correlations in all the subohmic, ohmic, and superohmic cases is suppressed in the physically relevant regime for α near to 1. This results in approximately the same dynamics of the initially correlated and uncorrelated states.

5. Author Contribution

JMB and MQL concieved the project and performed the analytical calculations. SD and AF did the numerical calculations. JMB, MQL and GND wrote the paper.

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APPENDIX A

In this appendix, we derive the time evolution of the reduced density matrix $\rho_s(t)$ given in Eq. (16). Since the Hamiltonian $\tilde{H}$ given in Eq. (4) is a purely dephasing model, which results in no dynamics of the diagonal terms of the density matrix $\rho_s(t)$. Observing that

$$e^{-\beta\tilde{H}}|1\rangle = e^{-\beta\mu}e^{-\beta H^+}|1\rangle,$$
$$e^{-\beta\tilde{H}}|0\rangle = e^{\beta\mu}e^{-\beta H^-}|0\rangle,$$

with $H^{\pm} = \sum_k \omega_k b_k^\dagger b_k \pm \sum_k E(g_k b_k + g_k^* b_k^\dagger)$ and $\mu = \frac{\omega_o}{2}$, we can write the $\langle 1|\rho_s(t)|0\rangle = \rho_s^{10}(t)$ as

$$\rho_s^{10}(t) = \frac{ab^*}{Z}(|a|^2 e^{-\beta\mu} \mathrm{Tr}_B[e^{2\hat{\Lambda}(t)} e^{-\beta H^+}] + $$
$$+ |b|^2 e^{\beta\mu} \mathrm{Tr}_B[e^{2\hat{\Lambda}(t)} e^{-\beta H^-}]), \tag{A1}$$

where the partition function Z is given by

$$Z = |a|^2 e^{-\beta\mu} \mathrm{Tr}_B[e^{-\beta H^+}] + |b|^2 e^{\beta\mu} \mathrm{Tr}_B[e^{-\beta H^-}].$$

Now, we define a unitary transformation $O_\pm = \exp[\pm \sum_k \frac{E}{\omega_k} \times \times (g_k b_k - g_k^* b_k^\dagger)]$ such that $H^\pm = O_\pm^{-1}(H_B - \xi)O_\pm$ with $\xi = = \sum_k E^2 \frac{|g_k^2|}{\omega_k}$ and $2\hat{\Lambda}(t) = O_\pm^{-1}[2\hat{\Lambda}(t) \pm iE^2\Phi(t)]O_\pm$. Therefore, we have

$$\mathrm{Tr}_B[e^{2\hat{\Lambda}(t)} e^{-\beta H^\pm}] =$$
$$= \mathrm{Tr}_B[O_\pm^{-1} e^{2\hat{\Lambda}(t) \pm iE^2\Phi(t)} O_\pm O_\pm^{-1} e^{-\beta H_B + \beta\xi} O_\pm] =$$
$$= \mathrm{Tr}_B[O_\pm^{-1} e^{2\hat{\Lambda}(t) \pm iE^2\Phi(t)} e^{-\beta H_B + \beta\xi} O_\pm] =$$
$$= e^{\pm i(1-\alpha^2)\Phi(t)} Z_B e^{-\gamma_1(t)} e^{\beta\xi},$$

where $Z_B = \mathrm{Tr}_B[e^{-\beta H_B}]$. This makes the partition function Z to be

$$Z = Z_B e^{\beta\xi}(|a|^2 e^{-\beta\mu} + |b|^2 e^{\beta\mu}).$$

Substituting all the above results in the expression for the off-diagonal element $\rho_s^{10}(t)$, we have

$$\rho_s^{10}(t) = ab^* \times$$
$$\times \frac{\left[|a|^2 e^{-\beta\frac{\omega_0}{2}} e^{i(1-\alpha^2)\Phi(t)} + |b|^2 e^{\beta\frac{\omega_0}{2}} e^{-i(1-\alpha^2)\Phi(t)}\right]}{|a|^2 e^{-\beta\frac{\omega_0}{2}} + |b|^2 e^{\beta\frac{\omega_0}{2}}} e^{-\gamma_1(t)}.$$

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ДЕКОГЕРЕНЦІЯ
В $\mathcal{PT}$-СИМЕТРИЧНОМУ КУБІТІ

Ми вивчаємо втрату когерентності в $\mathcal{PT}$-симетричному кубіті, який взаємодіє з бозонним оточенням. Використовуючи канонічні перетворення, ми знайшли неермітів гамільтоніан, що описує $\mathcal{PT}$-симетричний кубіт, в моделі бозонів зі спіном. Ми визначили параметр α, який розмежовує області ермітовості і неермітовості в моделі, і знайшли, що кубіт не втрачає когерентності при переході від дійсного власного спектра до комплексного. Використовуючи загальний клас спектральних густин, ми показуємо, що має місце значне зменшення втрати когерентності завдяки вакуумним та тепловим флуктуаціям середовища, і що вихідні кореляції зберігаються при наближенні до точки переходу.

Ключові слова: $\mathcal{PT}$-симетрія, декогеренція, кореляції між системою та середовищем.