https://doi.org/10.15407/ujpe69.4.223

A.K. HASAN, H.H. ABED

Department of Physics, College of Education for Girls, University of Kufa
(Najaf, Iraq; e-mail: alikh.alsinayyid@uokufa.edu.iq, baqrakeel87@gmail.com)

CALCULATION OF ENERGY LEVELS B(E2) AND B(M1) FOR ^{58,59}Cu ISOTOPES BY USING NushellX@MSU CODE

In this study, the NushellX@MSU code was applied to compute energy levels, B(E2) and B(M1), values for ^{58}*Cu and* ^{59}*Cu isotopes, using the jj44pn shell and the jun45pn effective interaction. The model space encompassed all possible nucleon configurations within the* ($f_{5/2}$, $p_{3/2}$, $p_{1/2}$, *and* $g_{9/2}$) *orbits. Overall, the computed probabilities of electromagnetic transitions and energy levels for the* ^{58}*Cu and* ^{59}*Cu isotopes demonstrate a reasonable consistency with available experimental data.*

Keywords: B(E2), B(M1), jj44pn shell, ^{58,59}Cu isotopes, effective interaction.

1. Introduction

The nucleus represents a quantum mechanical arrangement, comprising protons and neutrons. The behavior of these nuclei is influenced by the motion of nucleons located beyond the dense central core, which is composed of valence particles. A precise understanding of how the valence particles interact is crucial in order to adequately explain the characteristics of the nucleus. Numerous nuclear models have elucidated the characteristics of a multitude of nuclei [1]. Among these models, the accomplished nuclear shell model stands out. This model identifies the energy states, their placements, and the transitions occurring between those states, thereby providing a significant portrayal of nuclear attributes [2, 3]. In the realm of shell-model calculations, there are two primary components: the interaction between nucleons (N–N interaction) and the configuration space allotted for valence particles. Essentially, one can undertake shell-model calculations using either a genuine N–N interaction within an extensive configuration space or an adjusted effective interaction within a more constrained configuration space [4]. Due to the short-range correlation and medium effects, the realistic nucleon-nucleon (NN) interactions must be renormalized when applied to shell-model computations [5]. The foundational prerequisites for shell-model calculations encompass combinations of energies associated with individual particles and matrix elements governing the interactions between pairs of particles. These sets are referred to as the "effective interaction" or the "Hamiltonian space model". The fundamental approach to deriving the effective interactions between nucleons (NN) within the nucleus involves initiating from the of (NN) abstraction interaction while treating the interacting nucleons as integral components of the A-nucleon system (albeit intricate). An interaction thus formulated is termed an "active interaction". [6]. In the domain of light nuclei, a number of established "typical" effective interactions are present, including the Cohen–Kurath [7] and USD

Citation: Hasan A.K., Abed H.H. Calculation of energy levels, B(E2) and B(M1) for ^{58,59}Cu isotopes by using NushellX@MSU code. *Ukr. J. Phys.* **69**, No. 4, 223 (2024). https://doi.org/10.15407/ujpe69.4.223.

Цитування: Хасан А.К., Абед Х.Х. Розрахунок енергії рівнів B(E2) та B(M1) для ізотопів ^{58,59}Cu з використанням коду NushellX@MSU. *Укр. фіз. журн.* **69**, №4, 223 (2024).

[3] interactions tailored for the p and sd shells, respectively. Further, in the subsequent significant shell, namely, the jj44pn-shell, there also exist customary interactions; an instance of this is the jj44bpn interaction [8]. In this study, the NuShellX@MSU software was utilized to calculate energy levels, electromagnetic transitions, and charge density distribution pertaining to the ^{58,59}Cu isotopes. This was achieved by employing the jun45pn interaction within the jj44pn-shell. Research of a theoretical nature has previously been undertaken on the studied isotopes by [9, 10].

2. Theory

There are two approaches to describing the model space Hamiltonian. The first approach, referred to as "realistic", is tailored to a specific shell model space using established data about the free nucleon-nucleon force. The second approach, known as the "experimental" method, involves determining the parameters based on the agreement between measured level energies and shell-model eigenvalues [11]. Within classical shell-model computations, the customary approach shifts away from determining the complete energy of the system. Instead, the focus lies in evaluating energy levels pertaining to an individual nucleon positioned outside the doubly magical core concerning a closed shell configuration. In situations involving multiple nucleons beyond the core, the energy is presumed to embody an eigenvalue of the Hamiltonian H_0. This, in turn, leads to the formulation of the comprehensive Hamiltonian in the following manner [12]:

$$H = \sum_{k=1} (H_0)_k + \sum_{k\leq l} V_{kl}. \qquad (1)$$

The residual 2-body interaction, denoted as V_{kl} is present alongside the conventional shell-model potential. This can be expressed in the following manner [12, 13]:

$$\sum_{k<l} V_{kl} = \sum_{JM} \sum_{ja\geq jb} \sum_{jc\geq jd} \langle j_a j_b \mid V_{12} \mid j_c j_d \rangle_J \times$$
$$\times\, a^{+}_{JM}(j_a j_b) a_{JM}(j_c j_d), \qquad (2)$$

$\langle\ \rangle$ represents the matrix element of the residual 2-body interaction. $a_J M^{+}(j_a j_b)$ is the operator responsible for generating a nucleon pair within the single-particle states j_a and j_b, characterized by a collective angular momentum of JM. $a_J M(j_c j_d)$ represents the operator that destroys a pair of particles in states j_c and j_d.

Is the one that destroys a pair of particles in states j_c and j_d; it is the Hermitian adjoin operator to $a^{+}_{JM}(j_a j_b)$.

3. Results and Calculations

The energy levels and reduced electromagnetic transition probabilities for the isotopes ^{58}Cu and ^{59}Cu have been computed using the NushellX@MSU code [14] designed for Windows operating systems. This software is specifically designed for precise calculations of energies, eigenvectors, and spectroscopic overlaps related to low-lying states within shell-model Hamiltonian matrix computations. These computations involve extremely large basis dimensions. The calculations utilize a j-coupled proton-neutron basis and can accommodate J-scheme matrix dimensions, reaching magnitudes of up to 100 million. For this study, the effective interaction jun45pn was employed within the jj44pn model space. Notably, this model space includes orbitals ($f_{5/2}$, $p_{3/2}$, $p_{1/2}$ and $g_{9/2}$) located above the closed core of ^{56}Ni.

3.1. ^{58}Cu Isotope

3.1.1. Energy levels

The foundational state of the ^{58}Cu nucleus demonstrates a configuration to the closed ^{56}Ni nucleus, where two nucleons reside beyond the enclosed shell. These additional nucleons are distributed within the jj44pn model space. Upon implementing the NuShellX@MSU code on the ^{58}Cu nucleus and evaluating its energy levels through the shell model can conclude:

1. The total angular momentum and ground state parity were matched for the 1^{+}_{1} level when compared with the available practical values.

2. The agreement is compatible for the values of energies calculated theoretically (0.406, 0.312, 1.476, 1.712, 1.995, 2.611) MeV corresponding to the angular momentum 3^{+}_{1}, 0^{+}_{1} 1^{+}_{2}, 2^{+}_{1}, 2^{+}_{2}, 4^{+}_{2}, when we compared it with the available experimental data.

3. This study confirmed the total angular momentum and parity for the practically uncertain energies (2.065, 1.549, 1.647, 3.280, 8.228, 4.065) MeV corresponding to angular momentum 5^{+}, 4^{+}, 3^{+}, 4^{+}, 9^{+}, 7^{-}.

Table 1. **A comparison between the theoretical values of the energy levels relative to the ground state of the ^{58}Cu isotope with the experimental data [15] using the jun45pn interaction**

Theoretical values of E (MeV)		Experimental values		Theoretical values of E (MeV)		Experimental values	
J	Jun45pn results	E (MeV)	J^π	J	Jun45pn results	E (MeV)	J^π
1_1^+	0	0	1+	5_1^-	4.327	–	–
0_1^+	0.312	0.202	0^+	3_6^+	4.998	–	–
3_1^+	0.406	0.443	3^+	3_2^-	5.267	–	–
1_2^+	1.476	1.051	1^+	0_3^+	5.561	–	–
2_1^+	1.125	1.427	2^+	3_3^-	5.964	–	–
5_1^+	1.181	2.065	(5^+)	5_2^-	5.971	–	–
4_1^+	1.337	1.549	(4^+)	7_2^-	5.253	5.190	(7^+)
3_2^+	1.56	1.647	(3^+)	5_3^-	6.002	–	–
2_2^+	1.995	1.652	2^+	4_2^-	6.018	–	–
2_4^+	2.682	2.815	–	2_2^-	6.100	–	–
1_4^+	2.932	2.949	$(1)^+$	6_2^-	6.180	–	–
3_3^+	2.946	2.840	–	4_3^-	6.376	–	–
2_1^-	3.179	3.230	–	3_4^-	6.829	–	–
2_3^+	2.22	2.249	–	6_3^-	6.843	–	–
0_2^+	2.528	2.270	–	4_4^-	6.89	–	–
4_2^+	2.611	2.690	4^+	5_4^-	6.998	–	–
1_3^+	2.641	2.780	–	5_5^-	7.076	–	–
1_5^+	3.185	3.310	–	4_5^-	7.229	–	–
2_5^+	3.325	–	–	5_6^-	7.679	–	–
3_4^+	3.504	3.512	–	6_4^-	7.701	–	–
3_5^+	3.531	3.570	–	9_1^+	7.974	8.228	(9^+)
6_1^-	3.647	3.820	–	4_6^-	8.107	–	–
4_3^+	3.239	3.280	$(0^+ : 4^+)$	2_9^+	9.228	8.900	–
2_6^+	3.949	3.890	–	7_1^+	9.202	–	–
4_1^-	4.066	4.010	–	1_8^+	9.272	9.209	$(1)^+$
2_7^+	4.201	4.210	–	0_4^+	9.335	–	–
3_1^-	4.272	–	–	5_2^+	9.545	–	–
1_6^+	4.039	4.720	$(1)^+$	3_7^+	9.582	9.680	–
7_1^-	4.342	4.065	(7^-)	4_4^+	9.914	–	–
2_8^+	4.628	–	–	6_1^+	10.300	–	–
1_7^+	4.686	5.160	$(1)^+$	8_1^+	10.346	10.776	–

Total angular momentum is confirmed for the practically uncertain energy (5.190) MeV corresponding to angular momentum 7 but in negative parity.

4. Total angular momentum is only confirmed for the practically uncertain energies (2.949, 4.720, 5.160, 9.209) MeV corresponding to angular momentum 1^+, 1^+, 1^+, 1^+.

5. From our calculation, we expected that the total angular momentum and the parity of the experimental energies (2.840, 3.230, 3.310, 3.512, 3.570, 3.820, 3.890, 4.010, 4.210, 8.900, 9.680, 10.776, 2.249, 2.270, 2.780, 2.815) MeV is 3_3^+, 2_1^-, 1_5^+, 3_4^+, 3_5^+, 6_1^-, 2_6^+, 4_1^-, 2_7^+, 2_9^+, 3_7^+, 8_1^+, 2_3^+, 0_2^+, 1_3^+, 2_4^+ because to the convergence of practical values with our theoretical values.

6. In our calculations, (28) levels were obtained with total angular momentum and parity that have not been matched by any practical value so far. We noticed that the highest calculated energy value is

theoretically (10.346) MeV while the highest experimental energy value is (24.000) MeV

Table 1 presents a comparison between the theoretical energy level predictions for ^{58}Cu using the jun45pn interaction and the available practical values [15].

3.1.2. Electromagnetic transition probability B(E2), B(M1)

Utilizing the NushellX@MSU code, we computed the electromagnetic transition probability within the ^{58}Cu nucleus, employing the jun45pn interaction. The calculations were conducted using the harmonic oscillator potential (HO, b), where $b > 0$ was applied to each in-band transition. To account for the primary polarization effect in the jun45pn interaction, we opted for effective charges for protons and neutrons ($e_p = 2.1$, $e_n = 1.0$). Furthermore, adjustments were made to the g factor to be Compatible with practical data of the magnetic transitions' ground state ($g_s p = 8.75$, $g_s n = -6.826$) within the jj44bpn interaction. Tables 2 and 3 juxtapose certain theoretical outcomes for electric and magnetic transitions derived from the effective jun45pn interaction with experimental data [15].

Table 2. **Comparison of the B(E2) results by using jun45pn interaction in unit e^2 fm^4 for ^{58}Cu isotope with the experimental data [15]**

$J_i \to J_f$	Theoretical B(E2), e^2 fm^4 Jun45pn results $e_p = 2.1$, $e_n = 1.0$	Experimental B(E2), e^2 fm^4
$3_1^+ \to 1_1^+$	103.2000	102.687
$2_1^+ \to 3_1^+$	40.4400	<36.007
$2_1^+ \to 0_1^+$	0.7888	<4.668
$2_1^+ \to 1_1^+$	60.2700	<133.359
$4_1^+ \to 3_1^+$	15.0000	<400.078
$3_2^+ \to 3_1^+$	2.1040	<14.67
$3_2^+ \to 1_1^+$	11.2300	<37.341
$2_2^+ \to 0_1^+$	105.8000	120.023
$4_1^- \to 2_1^-$	0.6150	–
$7_1^- \to 6_1^-$	94.5400	–

Table 3. **Comparison of the B(M1) results by using jun45pn interaction in unit μ^2 for ^{58}Cu isotope with the experimental data [15]**

$J_i \to J_f$	Theoretical B(M1) μ^2 Jun45pn results $g_s p = 8.75$, $g_s n = -6.826$	Experimental B(M1) μ^2
$1_2^+ \to 0_1^+$	0.7380	0.77
$2_1^+ \to 3_1^+$	0.0200	<0.003
$2_1^+ \to 1_1^+$	0.0002	<0.018
$4_1^+ \to 3_1^+$	0.00003	<0.057
$3_2^+ \to 3_1^+$	0.0026	<0.004
$2_2^+ \to 1_1^+$	0.1291	0.269
$2_2^+ \to 3_1^+$	1.9870	0.555
$3_1^- \to 2_1^-$	33.0400	–
$5_1^- \to 6_1^-$	7.7120	–

Using the jun45pn interaction and noticing the aforementioned tables, we observed a good compatibility between the electric transitions B(E2) $3_1 \to 1_1$, B(E2) $2_1 \to 3_1$, B(E2) $2_2 \to 0_1$ with the available experimental data. Additionally, the magnetic transitions B(M1) compatibility for the transitions B(M1) 12 → 01, B(M1) $2_1 \to 3_1$, B(M1) $2_1 \to 1_1$, B(M1) $4_1 \to 3_1$, B(M1) $3_2 \to 3_1$, B(M1) $2_1 \to 1_1$ was good with available experimental data. At the same time, the compatibility for the remaining transfers was reasonable, and through our calculations, we also discovered new transitions for which there are currently no experimental values.

3.2. ^{59}Cu isotope

3.2.1. Energy levels

In accordance with the nuclear shell model, the enclosed core of the ^{59}Cu nucleus (with neutron number $N_n = 30$ and proton number $N_p = 29$) is equivalently described as a ^{56}Ni nucleus hosting three nucleons outside the closed shell, arranged in orbits ($f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$). To calculate the energy levels of this nucleus, the jun45pn interaction was utilized. Through the down table and comparing the results using the jun45pn interaction with the practical results [16] of the ^{59}Cu isotope, we deduce:

1. The total angular momentum and ground state parity of the $3/2_1^-$ level were matched when compared with the available practical values.

2. The agreement was appropriate for the values of energies calculated theoretically (0.422, 1.655, 1.457, 2.205, 2.328, 2.704, 2.679, 2.707, 2.936, 3.002, 3.104, 3.394, 3.421, 3.513, 3.516, 3.622,3.709, 3.915, 4.109, 4.176, 4.222, 4.30, 4.50, 4.822, 5.009, 5.275, 6.693, 6.783, 7.005, 7.906) MeV corresponding to the angular momentum $1/2_1^-$, $5/2_1^-$, $7/2_1^-$, $9/2_1^-$, 3/2-2, $5/2_2^-$,

Table 4. **A comparison between the theoretical values of the energy levels relative to the ground state of the ^{59}Cu isotope with the experimental data [16] using the jun45pn interaction**

Theoretical values of E (MeV)		Experimental values		Theoretical values of E (MeV)		Experimental values	
J	Jun45pn results	E (MeV)	J^π	J	Jun45pn results	E (MeV)	J^π
$3/2^-_1$	0	0	$3/2^-$	$3/2^+_1$	4.822	4.973	$3/2^+$, $5/2^+$
$1/2^-_1$	0.422	0.491	$1/2^-$	$11/2^-_3$	4.852	–	–
$5/2^-_1$	1.655	0.914	$5/2^-$	$13/2^+_1$	4.988	–	–
$7/2^-_1$	1.457	1.398	$7/2^-$	$9/2^-_5$	4.999	–	–
$9/2^-_1$	2.205	2.390	$9/2^-$	$7/2^+_1$	5.009	4.932	$7/2^+$, $9/2^+$
$3/2^-_2$	2.328	2.324	$3/2^-$	$7/2^-_9$	5.023	5.043	–
$1/2^-_2$	2.652	2.318	1/2(–), 5/2(–)	$9/2^-_6$	5.624	5.220	9/2
$5/2^-_2$	2.704	2.706	$5/2^-$	$11/2^-_4$	5.114	–	–
$11/2^-_1$	2.679	2.587	$11/2^-$	$1/2^-_8$	5.187	5.105	($1/2^-$, 3/2, $5/2^-$)
$7/2^-_2$	2.707	2.715	$7/2^-$	$7/2^-_{10}$	5.205	5.225	–
$13/2^-_1$	2.936	3.447	$13/2^-$	$3/2^+_2$	5.257	–	–
$5/2^-_3$	3.002	3.114	$5/2^-$	$9/2^+_2$	5.257	–	–
$3/2^-_3$	3.104	3.129	$3/2^-$	$1/2^-_9$	5.275	5.230	$1/2^-$
$1/2^-_3$	3.23	3.438	(1/2)	$11/2^+_1$	5.322	–	–
$5/2^-_4$	3.394	3.550	$5/2^-$	$17/2^+_1$	5.334	5.427	(17/2+)
$7/2^-_3$	3.409	3.437	(7/2+, 9/2+)	$1/2^-_{10}$	5.447	5.431	–
$3/2^-_4$	3.421	3.615	$3/2^-$	$5/2^+_2$	5.465	5.482	($5/2^-$)
$5/2^-_5$	3.400	3.574	5/2, 7/2	$1/2^+_2$	5.564	5.584	–
$9/2^+_1$	3.513	3.042	$9/2^+$	$7/2^+_2$	5.676	5.589	–
$3/2^-_5$	3.516	3.741	$3/2^-$	$9/2^-_7$	5.644	–	–
$5/2^-_6$	3.501	3.725	3/2, 5/2	$9/2^-_8$	5.76	5.777	–
$7/2^-_4$	3.622	3.699	$7/2^-$	$7/2^+_3$	5.745	5.620	7/2(–)
$1/2^-_4$	3.709	3.654	$1/2^-$, $3/2^-$	$13/2^+_2$	5.766	–	–
$7/2^-_5$	3.724	3.758	5/2(+), 7/2, 9/2(–)	$19/2^+_1$	5.833	5.801	–
$5/2^-_7$	3.906	4.072	(3/2, 5/2, 7/2)(–)	$11/2^+_2$	5.902	5.957	–
$3/2^-_7$	3.915	3.884	$3/2^-$	$3/2^+_3$	5.993	5.941	3/2, 5/2
$9/2^-_2$	3.959	–	–	$9/2^+_3$	6.039	5.950	$(9/2)^+$
$9/2^-_3$	3.99	4.183	5/2, 9/2(–)	$5/2^+_3$	6.042	6.104	($5/2^+$)
$7/2^-_6$	4.014	–	–	$9/2^-_9$	6.043	–	–
$1/2^-_5$	4.109	4.051	$1/2^-$, $3/2^-$	$9/2^-_{10}$	6.106	–	–
$3/2^-_7$	4.176	3.904	$3/2^-$	$5/2^+_4$	6.113	6.201	3/2, 5/2
$3/2^-_8$	4.222	4.108	$3/2^-$	$15/2^+_1$	6.235	6.174	($15/2^+$)
$7/2^-_7$	4.245	4.207	5/2, 7/2(–)	$11/2^-_5$	6.444	–	–
$9/2^-_4$	4.26	4.258	–	$11/2^+_3$	6.297	–	–
$3/2^-_9$	4.30	4.818	$3/2^-$	$7/2^+_4$	6.33	6.381	–
$7/2^-_8$	4.4	4.465	5/2(+), 7/2, 9/2(–)	$9/2^+_4$	6.336	6.310	$(9/2)^+$
$5/2^-_8$	4.401	4.301	5/2(–)	$5/2^+_5$	6.37	6.336	($5/2^+$)
$1/2^+_1$	4.425	4.411	–	$1/2^+_3$	6.414	6.410	–
$13/2^-_2$	4.47	4.527	($13/2^+$)	$3/2^+_4$	6.461	6.419	3/2(–)
$11/2^-_2$	4.491	–	–	$11/2^+_4$	6.638	6.662	–
$5/2^-_9$	4.514	4.774	$3/2^-$, $5/2^-$	$5/2^+_6$	6.681	6.727	($5/2^+$)
$5/2^+_1$	4.547	4.914	5/2(+), 7/2, 9/2(–)	$7/2^+_5$	6.693	6.669	$7/2^+$, $9/2^+$
$1/2^-_6$	4.56	4.500	$(1/2)^-$	$15/2^+_2$	6.726	–	–
$5/2^-_{10}$	4.594	–	–	$11/25/2^-_6$	6.555	–	–
$1/2^-_7$	4.738	4.710	$(1/2)^-$	$3/2^+_5$	6.756	6.760	($3/2^-$)
$3/2^-_{10}$	4.743	–	–	$9/2^+_5$	6.77	6.836	($9/2^+$)

Continuation of the Tabl. 4

Theoretical values of E (MeV)		Experimental values		Theoretical values of E (MeV)		Experimental values	
J	Jun45pn results	E (MeV)	J^π	J	Jun45pn results	E (MeV)	J^π
$9/2^+_6$	6.783	6.905	$9/2^+$	$23/2^-_1$	9.233	9.333	$(23/2^-)$
$13/2^+_3$	6.823	–	–	$19/2^-_1$	9.342	–	–
$7/2^+_6$	6.80	–	–	$3/2^+_{10}$	9.53	9.280	$+$
$11/2^+_5$	6.928	–	–	$17/2^+_5$	9.752	9.780	$+$
$5/2^+_7$	6.932	6.923	$(5/2^+)$	$15/2^+_2$	9.811	–	–
$13/2^+_4$	6.942	–	–	$15/2^+_{10}$	9.224	–	–
$7/2^+_7$	7.005	7.042	$7/2^+$, $9/2^+$	$13/2^-_4$	9.877	–	–
$11/2^+_6$	7.071	–	–	$17/2^+_6$	10.03	–	–
$9/2^+_7$	7.155	–	–	$11/2^+_8$	10.036	–	–
$11/2^+_7$	7.284	–	–	$17/2^-_1$	10.269	–	–
$7/2^+_8$	7.215	7.209	$(7/2^-)$	$21/2^-_1$	10.241	10.657	$(21/2^-)$
$17/2^+_2$	7.215	7.074	$(17/2^+)$	$11/2^-_9$	10.261	–	–
$9/2^+_8$	7.248	–	–	$17/2^-_2$	10.415	–	–
$15/2^+_3$	7.288	–	–	$13/2^-_5$	10.471	–	–
$9/2^+_9$	7.444	–	–	$19/2^-_2$	10.227	–	–
$3/2^+_6$	7.349	7.332	3/2	$13/2^-_6$	10.698	–	–
$13/2^+_5$	7.372	–	–	$15/2^-_3$	10.756	–	–
$7/2^+_9$	7.374	7.450	$7/2^+$, $9/2^+$	$1/2^+_6$	10.781	10.5	$+$
$11/2^+_8$	7.439	7.473	–	$11/2^-_{10}$	10.892	–	–
$5/2^+_8$	7.476	7.444	$(3/2^+, 5/2^+)$	$15/2^-_1$	11.245	–	–
$7/2^+_{10}$	7.427	–	–	$19/2^-_3$	11.038	–	–
$15/2^+_4$	7.504	7.503	–	$17/2^-_3$	11.137	11.1	–
$9/2^+_{10}$	7.505	7.523	–	$17/2^-_4$	11.315	–	–
$13/2^+_6$	7.573	7.531	–	$21/2^-_2$	11.45	–	–
$13/2^+_7$	7.732	7.765	–	$13/2^-_7$	11.486	–	–
$1/2^+_4$	7.765	7.770	–	$15/2^-_5$	11.506	–	–
$11/2^+_9$	7.802	7.802	–	$15/2^-_6$	11.626	–	–
$17/2^+_3$	7.846	7.827	$(17/2^+)$	$21/2^-_3$	11.65	–	–
$11/2^+_{10}$	7.853	7.857	–	$13/2^-_8$	11.349	–	–
$5/2^+_9$	7.906	7.920	$3/2^+$, $5/2^+$	$19/2^-_4$	11.751	–	–
$15/2^+_5$	8.025	8.013	–	$17/2^-_5$	11.761	–	–
$13/2^+_8$	8.038	8.016	–	$13/2^-_9$	11.915	–	–
$3/2^+_7$	8.103	8.223	3/2(–), 5/2	$13/2^-_{10}$	11.953	–	–
$15/2^+_6$	8.194	8.182	–	$15/2^-_7$	12.222	–	–
$5/2^+_{10}$	8.21	8.193	$(5/2)^+$	$19/2^-_5$	12.277	–	–
$17/2^+_4$	8.279	8.227	–	$17/2^-_6$	12.318	–	–
$3/2^+_8$	8.947	8.315	–	$15/2^-_8$	12.562	–	–
$13/2^+_9$	8.399	8.333	–	$19/2^-_6$	12.999	–	–
$15/2^+_7$	8.483	8.400	–	$17/2^-_7$	12.759	–	–
$13/2^+_{10}$	8.531	8.515	–	$21/2^-_4$	12.907	–	–
$1/2^+_5$	8.666	8.667	–	$15/2^-_9$	13.043	–	–
$3/2^+_9$	8.943	8.831	–	$15/2^-_{10}$	13.103	–	–
$15/2^-_1$	8.986	8.989	–	$17/2^-_8$	13.29	–	–
$11/2^-_7$	9.03	9.001	–	$17/2^-_9$	13.251	–	–
$15/2^+_8$	9.084	9.014	–	$17/2^-_{10}$	13.508	–	–
$15/2^+_9$	9.131	9.112	–	$19/2^-_7$	13.85	–	–
$13/2^-_3$	9.005	9.252	–	$25/2^+_1$	14.186	12.859	$(25/2^-)$

End of the Tabl. 4

Theoretical values of E (MeV)		Experimental values		Theoretical values of E (MeV)		Experimental values	
J	Jun45pn results	E (MeV)	J^{π}	J	Jun45pn results	E (MeV)	J^{π}
$17/2^+_7$	14.666	–	–	$19/2^+_2$	14.895	–	–
$19/2^-_8$	15.143	–	–	$23/2^+_1$	15.106	15.900	–
$1/2^+_7$	15.822	–	–	$19/2^+_3$	16.254	–	–
$17/2^+_8$	16.013	–	–	$17/2^+_9$	17.33	–	–
$21/2^+_1$	14.638	14.700	–	$21/2^+_2$	19.608	–	–

$11/2^-_1$, $7/2^-_2$, $13/2^-_1$, $5/2^-_3$, $3/2^-_3$, $5/2^-_4$, $3/2^-_4$, $9/2^+_1$, $3/2^-_5$, $7/2^-_4$, $1/2^-_4$, $3/2^-_6$, $1/2^-_5$, $3/2^-_7$, $3/2^-_8$, $3/2^-_9$, $5/2^-_9$, $3/2^+_1$, $7/2^+_1$, $1/2^-_9$, $7/2^+_5$, $9/2^+_6$, $7/2^+_7$, $5/2^+_9$.

3. We expected that the parity for the practical energies (3.758, 4.072, 4.465, 5.22 3.574) MeV corresponding to the angular momentum 7/2, 5/2, 7/2, 9/2, 5/2 is negative. Also, we expected the positive parity for the practical energies (5.941, 6.201, 7.332) MeV corresponding to the angular momentum 3/2, 5/2, 3/2.

4. This study confirmed only parity for the practically uncertain energies (4.183, 4.207, 4.301, 4.914, 2.318) MeV corresponding to angular momentum $9/2^-$, $7/2^-$, $5/2^-$, $5/2^+$, $1/2^-$; it expected positive parity for the practical energies (6.419, 8.223,5.620) MeV corresponding to the angular momentum 3/2, 3/2, 7/2.

5. Through our calculations, only the angular momentum was confirmed and the parity as a negative parity of the practical energy was determined (3.438) MeV corresponding to the angular momentum 1/2.

The angular momentum was confirmed and the parity was predicted as a negative parity of the practical energies (4.527, 3.437) MeV corresponding to the angular momentum 7/2, 13/2.

While positive parity was predicted and angular momentum was confirmed only for practical energies (6.760, 7.209, 12.859, 5.482) MeV corresponding to the angular momentum 3/2, 7/2, 25/2, 5/2.

6. This study confirmed the total angular momentum and parity for the practically uncertain energies (5.105, 4.072, 5.427, 6.104, 6.174, 6.336, 6.727, 6.836, 6.923, 7.074, 7.444, 7.827, 9.333, 10.657) MeV corresponding to angular momentum $1/2^-$, $5/2^-$, $17/2^+$, $5/2^+$,$15/2^+$, $5/2^+$, $5/2^+$, $9/2^+$, $5/2^+$, $17/2^+$, $5/2^+$, $17/2^+$, $23/2^-$, $21/2^-$.

7. The total angular momentum is only confirmed for the practically uncertain energies (4.710, 5.950, 6.310, 8.193, 4.5) MeV corresponding to angular momentum $1/2^-$, $9/2^+$, $9/2^+$, $5/2^+$, $1/2^-$.

8. We expected that the total angular momentum and the parity for the experimental energies (5.589, 5.777, 5.801, 5.957, 6.381, 6.410, 6.662, 7.473, 7.503, 7.523, 7.531, 7.765, 7.77, 7.802, 7.857, 8.013, 8.016, 8.182, 8.227, 8.315, 8.333, 8.400, 8.515, 8.667, 8.831, 8.989, 9.001, 9.014, 9.112, 9.252, 11.1, 14.7, 15.9, 4.258, 4.411, 5.093, 5.225, 5.431, 5.584) MeV is $7/2^+_2$, $9/2^-_8$, $19/2^+_1$, $11/2^+_2$, $7/2^+_4$, $1/2^+_3$, $11/2^+_4$, $11/2^+_8$, $15/2^+_4$, $9/2^+_{10}$, $13/2^+_6$, $13/2^+_7$, $1/2^+_4$, $11/2^+_9$, $11/2^+_{10}$, $15/2^+_5$, $13/2^+_8$, $15/2^+_6$, $17/2^+_4$, $3/2^+_8$, $13/2^+_9$, $15/2^+_7$, $13/2^+_{10}$, $1/2^+_5$, $3/2^+_9$, $15/2^-_1$, $11/2^-_7$, $15/2^+_8$, $15/2^+_9$, $13/2^-_3$, $17/2^-_3$, $21/2^+_1$, $23/2^+_1$, $9/2^-_4$, $1/2^+_1$, $7/2^-_9$, $7/2^-_{10}$, $1/2^-_{10}$, $1/2^+_2$ because to the convergence of practical values with our theoretical values.

9. We expected that the angular momentum of practical energies (9.280, 9.780, 10.5) MeV is $3/2^+_{10}$, $17/2^+_5$, $1/2^+_6$.

10. From our calculations, (80) levels were obtained with total angular momentum and parity that have not been matched by any practical value so far. Well we noticed that the highest calculated energy value is theoretically (19.608) MeV while the highest experimental energy value is (28.134) MeV.

Table 4 allows the comparison between the theoretical energy levels for this interaction and the available practical values [16].

3.2.2. Electromagnetic transition probability B(E2), B(M1)

Using the NushellX@MSU code, we have computed the electromagnetic transition probability within the ^{59}Cu nucleus, employing the jun45pn interaction. The calculations were carried out by implement-

Table 5. **Comparison of the B(E2) results by using jun45pn interaction in unit e^2 fm^4 for ^{59}Cu isotope with the experimental data [16]**

$J_i \to J_f$	Theoretical B(E2), e^2 fm^4 Jun45pn results $e_p = e_n = 1.999$	Experimental B(E2), e^2 fm^4
$1/2_1^- \to 3/2_1^-$	4052	<4093.019
$5/2_1^- \to 3/2_1^-$	553.3	<39.566
$7/2_1^- \to 5/2_1^-$	173.2	17.736
$7/2_1^- \to 3/2_1^-$	1770	231.938
$5/2_9^- \to 7/2_2^-$	0.2904	23.194
$5/2_9^- \to 5/2_1^-$	5.871	8.186
$5/2_9^- \to 1/2_1^-$	3.906	20.329
$3/2_2^- \to 5/2_1^-$	75.69	272.868
$5/2_3^- \to 1/2_1^-$	329.3	95.504
$5/2_3^- \to 3/2_1^-$	5.4	49.116
$5/2_1^+ \to 9/2_1^+$	1778	231.938
$5/2_3^+ \to 3/2_1^+$	1.121	0.096
$9/2_3^+ \to 9/2_1^+$	2.712	1.091
$3/2_1^+ \to 1/2_1^+$	167.2	–
$7/2_1^+ \to 9/2_1^+$	25.7800	–

Table 6. **Comparison of the B(M1) results by using jun45pn interaction in unit μ^2 for ^{59}Cu isotope with the experimental data [16]**

$J_i \to J_f$	Theoretical B (M1), μ^2 Jun45pn results $g_s p = 1.90$, $g_s n = -2.826$	Experimental B(M1), μ^2
$1/2_1^- \to 3/2_1^-$	0.3178	> 0.322
$5/2_1^- \to 3/2_1^-$	0.0007	< 0.045
$7/2_1^- \to 5/2_1^-$	0.0220	0.125
$3/2_2^- \to 5/2_1^-$	0.0504	0.02
$3/2_2^- \to 3/2_1^-$	0.0604	0.113
$5/2_3^- \to 3/2_1^-$	0.0011	0.034
$5/2_3^+ \to 3/2_1^+$	0.0059	0.002
$5/2_9^- \to 5/2_1^-$	0.1300	0.029
$9/2_2^+ \to 9/2_1^+$	0.0077	0.134
$9/2_3^+ \to 7/2_1^+$	0.0051	0.125
$9/2_3^+ \to 9/2_1^+$	0.0050	1.235
$3/2_3^- \to 3/2_1^-$	0.0001	–
$1/2_2^- \to 3/2_1^-$	0.0922	–

ing the harmonic oscillator potential (HO, b), where $b > 0$ for each in-band transition. In order to align with observed practical values of ground-state magnetic transitions, adjustments were made to the g factor ($g_s p = 1.9$, $g_s n = -2.826$). Additionally, the primary polarization effect was considered by selecting effective charges for protons and neutrons ($e_p = e_n = 1.999$) within the jun45pn interaction. Theoretical values for specific electric and magnetic transitions, based on the effective jun45pn interaction, are compared against experimental values [16] in Tables 5 and 6.

As noted in the above table, the reasonable agreement between the electric transitions B(E2) $1/2_1^- \to 3/2_1^-$, B(E2) $5/2_9^- \to 5/2_1^-$, B(E2) $3/2_3^+ \to 3/2_1^+$, B(E2) $9/2_3^+ \to 9/2 + 1$ and the available experimental data using the JJ44BPN interaction. Additionally, the magnetic compatibility for the transitions B(M1) $1/2_1^- \to 3/2_1^-$, B(M1) $5/2_1^- \to 3/2_1^-$, B(M1) $7/2_1^- \to 5/2_1^-$, B(M1) $3/2_2^- \to 5/2_1^-$, B(M1) $3/2_2^- \to 3/2_1^-$, B(M1) $5/2_3^- \to 3/2_1^-$, B(M1) $5/2_3^+ \to 3/2_1^+$, B(M1) $5/2_9^- \to 5/2_1^-$, B(M1) $9/2_2^+ \to 9/2_1^+$, and B(M1) $9/2_3^+ \to 7/2_1^+$ was well based on the experimental data that were available, while the compatibility for the transition was B(M1) $9/2_3^+ \to 9/2_1^+$ reasonable. Our calculations also led to the discovery of new transitions for which there aren't any experimental values yet.

4. Conclusions

The present study demonstrates that the interaction files utilized in calculation yield favorable outcomes, aligning well with the computed energy levels and electromagnetic transition probabilities B(E2) and B(M1) when compared to practical measurements. Generally speaking, the calculations for level spectra within the jj44pn framework for both isotopes yielded results that are in a reasonable accordance with practical data. This agreement encompassed numerous confirmed energy levels, including cases where momentum values and parity that do not have practical data so far. Additionally, we also obtained energy levels that have no practical values yet. Similarly, the computations of B(E2) and B(M1) also exhibited a satisfactory consistency with empirical data.

1. J.M. Blatt, V.F. Weisskopf. *Theoretical Nuclear Physics* (Springer, 1979) [ISBN: 978-0-486-66827-7].
2. A. Hasan, F. Obeed, A. Rahim. Positive parity levels of ^{21,23}Na isotopes by using the nuclear shell. *Ukr. J. Phys.* **65**, 3 (2020).
3. B.A. Brown, B. Wildenthal. Status of the nuclear shell model. *Annu. Rev. Nucl. Part. Sci.* **38**, 29 (1988).

4. P.C. Srivastava, I. Mehrotra. Large-scale shell model calculations for even-even ^{62,66}Fe isotopes. *J. Phys. G: Nucl. Part Phys.* **36**, 105106 (2009).
5. A. Hasan. Shell model calculations for 18, 19, 20O isotopes by using USDA and USDB interactions. *Ukr. J. Phys.* **63**, 189 (2018).
6. A.A. Al-Sammarrae, F.I. Sharrad, A. Aziz, N. Yusof, H.A. Kassim. Application of USDA and SDBA Hamiltonians in calculating the excited states of odd-A magnesium isotopes. *Eur. Phys. J. Plus* **129**, 1 (2014).
7. S. Cohen, D. Kurath, Effective interactions for the 1p shell. *Nucl. Phys.* **73**, 1 (1965).
8. A. Lisetskiy, B.A. Brown, M. Horoi, H. Grawe. New $T = 1$ effective interactions for the $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$ model space: Implications for valence-mirror symmetry and seniority isomers. *Phys. Rev. C* **70**, 044314 (2004).
9. F.A. Majeed, H. M. Tawfeek, S. M. Obaid. Large basis shell model calculations of some nuclei around doubly-magic ^{56}Ni. *Int. J. Nucl. Energy Sci. Technol.* **12**, 370 (2018).
10. M. Hossain. Energy levels in ^{59}Cu. *Nuov. Cim. A* **60**, 157 (1980).
11. A. Hasan, H. Abed. Calculation of energy levels and B(E2) for ^{20}Ne isotope by using nuclear shell model. *Int. J. Sci. Res. Publ.* **9**, 8861 (2019).
12. P.J. Brussaard, P.W Glaudemans. Shell-model applications in nuclear spectroscopy. *Phys. Today* **31**, 68 (1978).
13. A. Hasan, A. Subber. Level structure of ^{210}Po by means of surface delta interaction. *Turk. J. Phys.* **37**, 348 (2013).
14. B. Brown, W. Rae. The shell-model code NuShellX@ MSU. *Nucl. Data Sheets* **120**, 115 (2014).
15. C. Nesaraja S. Geraedts, B. Singh. Nuclear data sheets for $A = 58$. *Nucl. Data Sheets* **111**, 897 (2010).
16. S. Basunia. Nuclear data sheets for $A = 59$. *Nucl. Data Sheets* **151**, 1 (2018).

Received 24.10.23

А.К. Хасан, Х.Х. Абед

РОЗРАХУНОК ЕНЕРГIЇ РIВНIВ B(E2) ТА B(M1) ДЛЯ IЗОТОПIВ ^{58,59}Cu З ВИКОРИСТАННЯМ КОДУ NushellX@MSU

З використанням коду NushellX@MSU і ефективної взаємодії оболонок jj44pn та jun45pn розраховано енергії рівнів B(E2) і B(M1) для ізотопів ^{58}Cu і ^{59}Cu. Модель враховує всі можливі конфігурації нуклонів на орбітах ($f_{5/2}$, $p_{3/2}$, $p_{1/2}$ і $g_{9/2}$). Обчислені ймовірності електромагнітних переходів та енергії рівнів добре узгоджуються з експериментом.

Ключовi слова: B(E2), B(M1), оболонка jj44pn, ізотопи ^{58,59}Cu, ефективна взаємодія.