UDC 519.718

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A Simple Minimization Method of the Variables Number in the Complete and Incomplete Logic System Functions. Part 2

Предложен новый метод минимизации числа переменных в полных и неполных логических функциях, основанный на процедуре расцепления конъюнктермов. Преимущества предложенного метода показаны на примерах определения несущественных переменных в функциях, заимствованных автором из известных публикаций в порядке сравнения.

Ключевые слова: минимизация числа переменных, логическая функция, несущественная переменная, конъюнктерм, процедура расцепления.

Запропоновано новий метод мінімізації кількості змінних у повних і неповних логікових функціях, що ґрунтується на процедурі розчеплення кон'юнктермів. Переваги пропонованого методу показано на прикладах визначення неістотних змінних у функціях, запозичених автором із відомих публікацій з метою порівняння.

Ключові слова: мінімізації кількості змінних, логікова функція, неістотна змінна, кон'юнктерм, процедура розчеплення.

A new minimization method of the variables number in complete and incomplete logic functions, based on the procedure of conjuncterms splitting is proposed. The advantages of the proposed method are illustrated by examples of determining nonessential variables in the functions, which are borrowed from the well-known publications.

Keywords: minimization of the variables number, logic function, nonessential variable, conjuncterm, splitting procedure.

In the second part we consider the application of the proposed method for the determination of the nonessential variables in the complete and incomplete logic system functions.

In general case the system of the functions $F(X)=\{f_1(X),f_2(X),...,.f_s(X)\}$, $X=\{x_1,x_2,...,x_n\}$, in the set-theoretical format is reflected by the system of perfect STF $\{Y_i^1,Y_i^*\}$, $i=1,2,...,s$ [10]:

$$\begin{cases} Y_1^1=\{m_{11},m_{12},...,m_{1k_1}\}^1, Y_1^*=\{m_{k_1+1},m_{k_1+2},...,m_{2^n-k_1-v_1}\}^* \\ Y_2^1=\{m_{21},m_{22},...,m_{2k_2}\}^1, Y_2^*=\{m_{k_2+1},m_{k_2+2},...,m_{2^n-k_2-v_2}\}^* \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ Y_s^1=\{m_{s1},m_{s2},...,m_{sk_s}\}^1, Y_s^*=\{m_{k_s+1},m_{k_s+2},...,m_{2^n-k_s-v_s}\}^* \end{cases}, \quad v_i<2^n-k_i, \qquad (2)$$

where m_{ij}, $i=1,2,...,s$, $j=1,2,...,(2^n-k_i-v_i)$, are numeric minterms of the i-th function f_i of system (2); the mark * replaces the symbol ~ or 0 depending on the fact which part of n-dimensional boolean space of the system (2) belongs to indefinite values of the functions f_i for all $i=1,2,...,s$, namely: if $Y_i^*\equiv Y_i^\sim$, here $|Y_i^\sim|\le|Y_i^0\cup Y_i^1|$, then (2) is the system of inpredeterminated (incomplete) functions reflected by the system of perfect STF $\{Y_i^1,Y_i^\sim\}$, if $Y_i^*\equiv Y_i^0$, here $|Y_i^\sim|>|Y_i^0\cup Y_i^1|$, then (2) is the system of weakly determinated (incomplete) functions reflected by the system of perfect STF $\{Y_i^1,Y_i^0\}$, if $Y_i^*\equiv\varnothing$, then (2) is the system of complete functions reflected by the system of perfect STF $\{Y_i^1\}$.

Nonessential variables in the complete and/or incomplete system (2) can be common to all functions of the system, we call them *systemic nonessential variables*, and if nonessential variables are only some of the functions of the system (2), we call them *own nonessential variables* of these functions. Obviously, the system of complete functions (2) does not have the systemic nonessential variables, if at least one of its function has an odd number of minterms. Then, search for nonessential variables is executed for the subsystem of the system functions (2) having an even number of minterms.

For the definition of certain nonessential variables by the method of splitting minterms m_{ij} of the system (2) a set Y_S^1 of the systemic minterms $(m)_s$ is formed, where $S \in \{1,2,...,s\}$ is the set of indices s of the system functions (2) [10]. In this case we implement the compatible method of splitting conjuncterms, similarly as in the case of system functions minimization [9,10].

Over the systemic minterms $(m)_s$, as in the case of a single function (see p. 2 and p. 3), the splitting procedure is performed using the matrix M_n^{n-1}. Elements of this matrix are *systemic splitted conjuncterms* of $(n-1)$-rank $(\theta_i^{n-1})_s$ and formed by imposition of the masks of literals of $(n-1)$-rank on the systemic mintems $(m)_s$. Among $(\theta_i^{n-1})_s$ pairs of equal by value and having the same indexes are distinguished by underlining. Conseguently, if the matrix M_n^{n-1} is covered by such elements of i-th row, then according to the Theorem (*see* Section 2), the given system $F(X)$ has a systemic nonessential variable x_p, and if the matrix M_n^{n-1} is covered by two or more rows, then (see corollary Theorem) the given system $F(X)$ has two or more systemic nonessential variables. It is clear, that the certain functions of a given system can have their own nonessential variables. These variables are defined on the next step of splitting procedure in a similar way (see Section 2). After the determining of the systemic nonessential variables and/or own nonessential variables of functions, the given final form of the system $F(X)$ is obtained with procedure of distribution of the functions by their indices [10], and, if necessary, by further minimization procedure.

5. Definition of nonessential variables in the complete system functions. Definition of the nonessential variables in the complete system functions is based on the splitting conjuncterms method for the single function (described in Section 2) and by procedural features of the system functions described above. Let us illustrate it with a following example.

Example 8. Determining the nonessential variables using the splitting conjuncterms method in the complete system functions $F(X)=\{f_1(X),f_2(X),f_3(X)\}$, $X=\{x_1,x_2,x_3,x_4\}$, given in the perfect STF

$$\begin{cases} Y_1^1=\{(0011),(0111),(1000),(1010),(1100),(1110)\}^1 \\ Y_2^1=\{(0001),(0011),(0101),(0111)\}^1 \\ Y_3^1=\{(0001),(0101),(1000),(1100)\}^1 \end{cases}.$$

Solution. Define the STF $Y_{1,2,3}^1$ of the system minterms $(m)_{1,2,3}$ of the given system $F(X)$ and execute the splitting procedure using a matrix M_4^3, where the systemic conjuncterms-copies with the same indices of function (the indices of system functions are shown about the matrix column) are underlined:

$$Y_{1,2,3}^1=\{(0011)_{1,2},(0111)_{1,2},(1000)_{1,3},(1010)_1,(1100)_{1,3},(1110)_1,(0001)_{2,3},(0101)_{2,3}\}^1 \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} lll- \\ ll-l \\ l-ll \\ -lll \end{Bmatrix} = \begin{bmatrix} \overset{1,2}{001-} & \overset{1,2}{011-} & \overset{1,3}{100-} & \overset{1}{101-} & \overset{1,3}{110-} & \overset{1}{111-} & \overset{2,3}{000-} & \overset{2,3}{010-} \\ 00-1 & 01-1 & 10-0 & 10-0 & 11-0 & 11-0 & 00-1 & 01-1 \\ \underline{0-11} & \underline{0-11} & \underline{1-00} & \underline{1-10} & \underline{1-00} & \underline{1-10} & \underline{0-01} & \underline{0-01} \\ -011 & -111 & -000 & -010 & -100 & -110 & -001 & -101 \end{bmatrix} \overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow} \{(0-11)_{1,2},(1-00)_{1,3},(1-10)_1,(0-01)_{2,3}\}^1.$$

Thus, according to the Theorem, the given system $F(X)$ has the nonessential variable system x_2.

After the procedure of the functions distribution by their indices we get the simplified system

$$F(X)=\begin{cases} Y_1^1=\{(0-11),(1-00),(1-10)\}^1=\{(0-11),(1--0)\}^1 \\ Y_2^1=\{(0-11),(0-01)\}^1 \Rightarrow \{(0--1)\}^1 \\ Y_3^1=\{(1-00),(0-01)\}^1 \end{cases},$$

where the fuction f_2 after the minimization procedure has also its own nonessential variable x_3.

Answer. The given system $F(X)$ has the nonessential variable system x_2 and its function f_2 has also its own nonessential variable x_3.

6. Definition of nonessential variables in the incomplete system functions. In the case of the incomplete system functions (2) the detection of the nonessential variables gets much more complicated because of the multivariants of the possible solutions. On the one hand, this is due to the predetermining of the individual functions, on the other hand, this is due to the need to ensure the compatible system solution. The peculiarity of the proposed method of determining nonessential variables in the incomplete system functions is that the splitting of the systemic minterms procedure implements a compatible solution of the system, providing simultaneous predetermination of its functions. If the given system has at least one systemic nonessential variable, then according to the Theorem (Section 2), in the splitting matrix M_n^{n-1} of the systemic minerms $(m)_s$ one row of systemic conjuncterms-copies of $(n-1)$-rank appears and they cover it.

Example 9. The incomplete system functions $F(X)=\{f_i(X)\}$, $X=\{x_1,x_2,x_3,x_4,x_5\}$, $i=1,2,...,6$, given in the Table 2.1, to reduce the system functions of the essential variables (*this system is borrowed from* [6, p. 68]).

Table 2.1

	0	3	4	6	11	12	14	16	17	19	20	22
	0	0	0	0	0	0	0	1	1	1	1	1
	0	0	0	0	1	1	1	0	0	0	0	0
	0	0	1	1	0	1	1	0	0	0	1	1
	0	1	0	1	1	0	1	0	0	1	0	1
	0	1	0	0	1	0	0	0	1	1	0	0
Y_1	1	0	1	0	1	0	1	0	0	0	1	1
Y_2	0	0	1	1	0	1	1	0	1	1	0	0
Y_3	0	0	1	1	0	0	1	0	0	0	1	0
Y_4	1	1	0	0	1	0	0	0	1	0	0	0
Y_5	0	0	0	0	0	1	0	0	0	0	0	1
Y_6	0	0	0	0	0	0	0	1	0	0	0	0

Solution. From the Table 2.1 define the systemic minterms $(m)_s$ that form the perfect STF Y_s^1 and the perfect STF Y_s^0 of the given system $F(X)$, where $s\in\{1,2,3,4,5,6\}$:

$$Y_s^1=\{(00000)_{1,4},(00011)_4,(00100)_{1,2,3},(00110)_{2,3},(01011)_{1,4},(01100)_{2,5},(01110)_{1,2,3},$$
$$(10000)_6,(10001)_{2,4},(10011)_2,(10100)_{1,3},(10110)_{1,5}\}^1$$
$$Y_s^0=\{(00000)_{2,3,5,6},(00011)_{1,2,3,5,6},(00100)_{4,5,6},(00110)_{1,4,5,6},(01011)_{2,3,5,6},(01100)_{1,2,4,6},$$
$$(01110)_{4,5,6},(10000)_{1,2,3,4,5},(10001)_{1,3,5,6},(10011)_{1,3,4,5,6},(10100)_{2,4,5,6},(10110)_{2,3,4,6}\}^1$$

From Table 2.1 we can also define the perfect STF $Y_s^{\sim}=\mathbf{E}_2^5\setminus\{Y_s^1\cup Y_s^0\}$, where $\mathbf{E}_2^5=\{0,1,...,31\}$. In decimal format the perfect STF $Y_s^{\sim}=\{1,2,5,7,8,9,10,13,15,18,21,23,24,25,26,27,28,29,30,31\}^{\sim}$, whose minterms (the numbers are absent in Table 2.1) participate in the predetermination procedure.

Let us execute the splitting procedure by using the matrix M_6^5 for the systemic minterms of the STF Y_s^1 (the numbers above the columns of the matrix are numbers of the given system $F(X)$):

$$\begin{Bmatrix} llll- \\ lll-l \\ ll-ll \\ l-lll \\ -llll \end{Bmatrix} = \begin{bmatrix} \scriptstyle 1,4 & \scriptstyle 4 & \scriptstyle 1,2,3 & \scriptstyle 2,3 & \scriptstyle 1,4 & \scriptstyle 2,5 & \scriptstyle 1,2,3 & \scriptstyle 6 & \scriptstyle 2,4 & \scriptstyle 2 & \scriptstyle 1,3 & \scriptstyle 1,5 \\ 0000- & 0001- & 0010- & 0011- & 0101- & 0110- & 0111- & \mathbf{1000-} & \mathbf{1000-} & 1001- & 1010- & 1011- \\ 000-0 & 000-1 & \underline{001-0}_{2,3} & \underline{001-0}_{2,3} & 010-1 & \underline{011-0}_2 & \underline{011-0}_2 & 100-0 & \underline{100-1}_2 & \underline{100-1}_2 & \underline{101-0}_1 & \underline{101-0}_1 \\ \underline{00-00}_1 & 00-11 & \underline{00-00}_1 & 00-10 & 01-11 & 01-00 & 01-10 & \mathbf{10-00} & 10-01 & 10-11 & \mathbf{10-00} & 10-10 \\ 0-000 & \underline{0-011}_4 & \underline{0-100}_2 & \underline{0-110}_{2,3} & \underline{0-011}_4 & \underline{0-100}_2 & \underline{0-110}_{2,3} & 1-000 & 1-001 & 1-011 & 1-100 & 1-110 \\ \mathbf{-0000} & \mathbf{-0011} & \underline{-0100}_{1,3} & \mathbf{-0110} & -1011 & -1100 & -1110 & \mathbf{-0000} & -0001 & \mathbf{-0011} & \underline{-0100}_{1,3} & \mathbf{-0110} \end{bmatrix}$$

In the matrix M_6^5 the systemic conjuncterms are highlighted in bold font. We don't take them into further consideration because their minterms belong to Y_s^0. For example, $(1000-)_6 = \{(10000)_6,(10001)\}$ contain the minterm $(10001) \in Y_6^0$ or $(-0000)_{1,4} = \{(00000)_{1,4},(10000)\}$, where $(10000) \in Y_{1,4}^0$ and etc.. In the matrix M_6^5 we underline equal elements with the indices of common functions defined by their intersection. For example, the conjuncterm $(011-0)_2$ has the index 2, because it is defined by intersection $(2,5) \cap (1,2,3) = (2)$ and etc.. The remaining elements of the matrix M_6^5 have indices of generating systemic minterms that are not specified here.

As one can see, in the matrix M_6^5 there is not any row that would cover it. Thus, according to the theorem (*see* Section 2) the given system $F(X)$ does not have systemic nonessential variables.

To determine the nonessential variables in the certain system functions we take only those elements from each row of the matrix M_6^5 which have an index of specific function. This set doesn't include elements of the row containing at least one element that was eliminated due to the loss of index of the corresponding function. For example, for the function f_1 such rows are 3-th and 6-th and for the function f_2 are 1-th and 6-th rows.

Consequently, the elements of the matrix M_6^5 with index 1 of the function f_1 are STF

$$Y_1^1 = \left\{ \begin{pmatrix} \mathbf{0000-} \\ 000-0 \\ 0-000 \end{pmatrix}, (\mathbf{0010}-), \begin{pmatrix} \mathbf{0101-} \\ 010-1 \end{pmatrix}, (\mathbf{0111}-), \begin{pmatrix} \mathbf{1010-} \\ 101-0 \\ 1-100 \end{pmatrix}, \begin{pmatrix} \mathbf{1011-} \\ 101-0 \\ 1-110 \end{pmatrix} \right\}^1 .$$

According to the Theorem the function f_1 has the nonessential variable x_5. The minimal STF Y_1^1 is obtained after the splitting procedure of the obtained conjuncterms of the 4-rank using the matrix M_5^4:

$$Y_1^1 = \{(0000-),(0010-),(0101-),(0111-),(1010-),(1011-)\}^1 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} lll-- \\ ll-l- \\ l-ll- \\ -lll- \end{Bmatrix} = \begin{bmatrix} 000-- & 001-- & 010-- & 011-- & \underline{101--} & \underline{101--} \\ \underline{00-0-} & \underline{00-0-} & \underline{01-1-} & \underline{01-1-} & 10-0- & 10-1- \\ 0-00- & 0-10- & 0-01- & 0-11- & 1-10- & 1-11- \\ -000- & \underline{-010-} & -101- & -111- & \underline{-010-} & -011- \end{bmatrix}$$

$$\overset{C}{\Rightarrow} \{(00-0-),(01-1-),(101--)\}^1 .$$

To verify the obtained results let us transform the minimal STF Y_1^1 into the perfect STF $Y_1^1 = \{(0,1,4,5),(10,11,14,15),(20,21,22,23)\}^1 \Rightarrow \{0,\mathbf{1},4,\mathbf{5},\mathbf{10},11,14,\mathbf{15},20,\mathbf{21},22,\mathbf{23}\}^1$, where the predeterminated minterms from the set $Y_s^{\sim}$ are highlighted in bold.

The elements with the index 2 of the matrix M_6^5 form a set with two rows for the function f_2:

$$Y_2^1=\left\{\begin{pmatrix}001-0\\0-100\end{pmatrix},\begin{pmatrix}001-0\\0-110\end{pmatrix},\begin{pmatrix}011-0\\0-100\end{pmatrix},\begin{pmatrix}011-0\\0-110\end{pmatrix},\begin{pmatrix}100-1\\1-001\end{pmatrix},\begin{pmatrix}100-1\\1-001\end{pmatrix}\right\}^1\Rightarrow$$

$$\Rightarrow\begin{cases}1.\ \{(001-0),(011-0),(100-1)\}^1\Rightarrow\{(0-1-0),(100-1)\}^1\\2.\ \{(0-100),(0-110),(1-001),(1-011)\}^1\Rightarrow\{(0-1-0),(1-0-1)\}^1\end{cases}.$$

Hence, we see that the function f_2 for the 2nd solution has two nonessential variables x_2 and x_4, that after minimization is reflected by STF Y^2, this way (for verification) we transform into the perfect STF $Y_2^1=\{(4,6,12,14),(17,19,25,27)\}^1\Rightarrow\{4,6,12,14,17,19,\mathbf{25},\mathbf{27}\}^1$, where the predeterminated minterms from the set $Y_s^{\sim}$ are highlighted in bold.

For the function f_3 we have STF $Y_3^1=\left\{\begin{pmatrix}\mathbf{0010-}\\001-0\end{pmatrix},\begin{pmatrix}\mathbf{0011-}\\001-0\\00-10\\0-110\end{pmatrix},\begin{pmatrix}\mathbf{0111-}\\01-10\\0-110\end{pmatrix},\begin{pmatrix}\mathbf{1010-}\\1-100\end{pmatrix}\right\}$, where the elements with nonessential variable x_5 are highlighted in bold. Minimal STF Y_3^1 of the function f_3 is obtained after the splitting procedure of these elements:

$$Y_3^1=\{(0010-),(0011-),(0111-),(1010-)\}^1\overset{S}{\Rightarrow}\begin{Bmatrix}lll--\\ll-l-\\l-ll-\\-lll-\end{Bmatrix}=\begin{bmatrix}\underline{001--} & \underline{001--} & 011-- & 101--\\00-0- & 00-1- & 01-1- & 10-0-\\0-10- & \underline{0-11-} & \underline{0-11-} & 1-10-\\\underline{-010-} & -011- & -111- & \underline{-010-}\end{bmatrix}\overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow}\{(-010-),(0-11-)\}^1.$$

Thus, the function f_3 has the minimal STF $Y_3^1=\{(4,5,20,21),(6,7,14,15)\}^1\Rightarrow\{4,\mathbf{5},6,\mathbf{7},14,\mathbf{15},20,\mathbf{21}\}^1$, where the predeterminated minterms of the perfect STF Y_3^1 are highlighted in bold.

Executing the similar procedure for the function f_4, we get its nonessential variable x_2:

$$Y_4^1=\left\{\begin{pmatrix}000-0\\\mathbf{0-000}\end{pmatrix},\begin{pmatrix}000-1\\00-11\\\mathbf{0-011}\end{pmatrix},\begin{pmatrix}010-1\\01-11\\\mathbf{0-011}\end{pmatrix},\begin{pmatrix}10-01\\\mathbf{1-001}\end{pmatrix}\right\}\Rightarrow\{(0-000),(0-011),(1-001)\}^1.$$

From here STF $Y_4^1=\{(0,8),(3,11),(17,25)\}^1\Rightarrow$, i.e. the perfect STF $Y_4^1=\{0,3,\mathbf{8},11,17,\mathbf{25}\}^1$.

For the function f_5 we have STF $Y_5^1=\left\{\begin{pmatrix}\mathbf{0110-}\\\mathbf{01-00}\end{pmatrix},\begin{pmatrix}\mathbf{1011-}\\\mathbf{10-10}\\1-110\end{pmatrix}\right\}\Rightarrow\begin{cases}1.\ \{(0110-),(1011-)\}^1\\2.\ \{(01-00),(10-10)\}^1\end{cases}$. From here we obtain two solutions: 1) with the nonessential variable x_5 that corresponds to STF $Y_5^1=\{(12,13),(22,23)\}^1$, i.e. $Y_5^1=\{12,\mathbf{13},22,\mathbf{23}\}^1$, and 2) with the nonessential variable x_3 that corresponds to STF $Y_5^1=\{(8,12),(18,22)\}^1$, i.e. $Y_5^1=\{\mathbf{8},12,22,\mathbf{23}\}^1$.

For f_6 we get $Y_6^1=\left\{\begin{pmatrix}100-0\\1-000\end{pmatrix}\right\}\overset{C}{\Rightarrow}\begin{cases}1.\ \{(100-0)\}^1\\2.\ \{(1-000)\}^1\end{cases}\Rightarrow\{(1-0-0)\}^1$, i.e. $Y_6^1=\{16,\mathbf{18},\mathbf{24},\mathbf{26}\}^1$.

Thus, the given system $F(X)$ after "screening" its nonessential variables becomes the following:

$$F(x_1,x_2,x_3,x_4,x_5) \Rightarrow \begin{cases} f_1(x_1,x_2,x_3,x_4,-) \\ f_2(x_1,-,x_3,-,x_5) \\ f_3(x_1,x_2,x_3,x_4,-) \\ f_4(x_1,-,x_3,x_4,x_5) \\ f_5(x_1,x_2,x_3,x_4,-)\,/\,f_5(x_1,x_2,-,x_4,x_5) \\ f_6(x_1,-,x_3,-,x_5) \end{cases}$$

Comparing with [6], the obtained result is better due to the compatible solution of the given system by conjuncterms splitting method, since the two more nonessential variables are found. In particular, in the functions f_2 and f_6 one more nonessential variable x_4 is found in addition to x_2 and the function f_5 has two solutions from which it is worth keeping $f_5(x_1,x_2,x_3,x_4,-)$, since the functions f_1 and f_3 have the nonessential variable x_5 too.

Example 10. The system of incomplete weakly determinated functions $F(X)=\{f_i(X)\}$, $i=1,2,...,5$, $X=\{x_1,x_2,x_3,x_4,x_5,x_6\}$ given in Table 2.2, must be reduced to the system functions from essential variables (*this system is borrowed from* [1, p. 275]).

Table 2.2

x_1	x_2	x_3	x_4	x_5	x_6	f_1	f_2	f_3	f_4	f_5
1	0	1	1	0	0	0	0	1	0	1
1	1	0	0	0	0	0	1	0	1	1
0	0	1	0	1	1	1	1	1	0	1
1	1	1	0	1	0	1	0	1	0	0
0	1	0	0	1	0	1	0	1	0	0

Solution. From Table 2.2 we get such STF of the given system $F(X)$:

$$Y_s^1=\{(101100)_{3,5},(110000)_{2,4,5},(001011)_{1,2,3,5},(111010)_{1,3},(010010)_{1,3}\}^1$$
$$Y_s^0=\{(101100)_{1,2,4},(110000)_{1,3},(001011)_{4},(111010)_{2,4,5},(010010)_{2,4,5}\}^0$$
$$Y_s^{\sim}=\mathbf{E}_2^6\setminus\{Y_s^1\cup Y_s^0\},\ \text{где}\ \mathbf{E}_2^6=\{0,1,...,63\}$$

We execute the splitting procedure using the matrix M_6^5 with the systemic minterms of the STF Y_s^1:

$$Y_s^1 \overset{s}{\Rightarrow} \begin{Bmatrix} lllll- \\ llll-l \\ lll-ll \\ ll-lll \\ l-llll \\ -lllll \end{Bmatrix} = \begin{bmatrix} \overset{3,5}{10110-} & \overset{2,4,5}{11000-} & \overset{1,2,3,5}{00101-} & \overset{1,3}{11101-} & \overset{1,3}{01001-} \\ 1011-0 & 1100-0 & 0010-1 & 1110-0 & 0100-0 \\ 101-00 & 110-00 & 001-11 & 111-10 & 010-10 \\ 10-100 & 11-000 & 00-011 & 11-010 & 01-010 \\ 1-1100 & 1-0000 & 0-1011 & 1-0010 & 0-0010 \\ -01100 & -10000 & -01011 & -11010 & -10010 \end{bmatrix}.$$

Here none of the systemic conjuncterm of the 5-rank $(\theta_i^5)_{s'}$, predeterminated by minterms of the set $Y_{s'}^{\sim}$, does not intersect with any systemic minterm $(m_i)_{s'}$ of the set $Y_{s'}^0$, i.e. $(\theta_i^5)_{s'}\cap Y_{s'}^0=\varnothing$, where $s'\subseteq s$ is a set of numbers of the functions, which includes $(\theta_i^5)_{s'}$. For example, the intersection $(10110-)_{3,5}\cap Y_{\{3,5\}}^0=\varnothing$, where $(10110-)_{3,5}=\{(101100)_{3,5},(101101)_{3,5}\}$ (here minterm $(101101)_{3,5}$ belongs $Y_s^{\sim}$) and $Y_{\{3,5\}}^0=\{(110000)_3,(111010)_5,(010010)_5\}$, and therefore $\{(101100)_{3,5},(101101)_{3,5}\}\cap$ $\cap\{(110000)_3,(111010)_5,(010010)_5\}=\varnothing$. Since all the elements of the matrix M_6^5 within the minimal predetermination (one of minterm from $Y_s^{\sim}$) do not intersect with the elements of the set $Y_{s'}^0$, and according to the Theorem (*see* Section 2) all the variables of the given incomplete system functions are illusive nonessential variables. However, the number of nonessential variables decreases if to increase the power of predetermination set by reducing the rank of systemic splitted conjuncterms. This can be seen at the following levels of the splitting procedure for the given systemic minterms using the matrices M_6^4, M_6^3 and M_6^2 (the matrix M_6^1 can not be considered because all its rows have predeterminated conjuncterms of the 1-rank, containing the elements of the set $Y_{s'}^0$).

Consider the matrix M_6^2, where the elements containing the set $Y_{s'}^0$ are highlighted in bold (for example, $(11{-}{-}{-}{-})_{2,4,5} \cap Y_{\{2,4,5\}}^0 \neq \varnothing$). Rows with elements are not taken into consideration to simplify the matrix M_6^2 up to four rows:

$$
Y_s^1 \overset{S}{\Rightarrow} \left\{\begin{array}{l}
ll{-}{-}{-}{-} \\ l{-}l{-}{-}{-} \\ l{-}{-}l{-}{-} \\ l{-}{-}{-}l{-} \\ l{-}{-}{-}{-}l \\ {-}ll{-}{-}{-} \\ {-}l{-}l{-}{-} \\ {-}l{-}{-}l{-} \\ {-}l{-}{-}{-}l \\ {-}{-}ll{-}{-} \\ {-}{-}l{-}l{-} \\ {-}{-}l{-}{-}l \\ {-}{-}{-}ll{-} \\ {-}{-}{-}l{-}l \\ {-}{-}{-}{-}ll
\end{array}\right\} =
\begin{array}{ccccc}
{\scriptstyle 3,5} & {\scriptstyle 2,4,5} & {\scriptstyle 1,2,3,5} & {\scriptstyle 1,3} & {\scriptstyle 1,3} \\
\left[\begin{array}{c}
10{-}{-}{-}{-} \\ 1{-}1{-}{-}{-} \\ 1{-}{-}1{-}{-} \\ \mathbf{1}{-}{-}{-}\mathbf{0}{-} \\ \mathbf{1}{-}{-}{-}{-}\mathbf{0} \\ {-}\mathbf{01}{-}{-}{-} \\ {-}0{-}1{-}{-} \\ {-}0{-}{-}0{-} \\ {-}0{-}{-}{-}0 \\ {-}{-}11{-}{-} \\ {-}{-}1{-}0{-} \\ {-}{-}1{-}{-}0 \\ {-}{-}{-}10{-} \\ {-}{-}{-}1{-}0 \\ {-}{-}{-}{-}\mathbf{00}
\end{array}\right. &
\begin{array}{c}
\mathbf{11}{-}{-}{-}{-} \\ 1{-}0{-}{-}{-} \\ \mathbf{1}{-}{-}\mathbf{0}{-}{-} \\ \mathbf{1}{-}{-}{-}\mathbf{0}{-} \\ \mathbf{1}{-}{-}{-}{-}\mathbf{0} \\ {-}\mathbf{10}{-}{-}{-} \\ {-}\mathbf{1}{-}\mathbf{0}{-}{-} \\ {-}1{-}{-}0{-} \\ {-}\mathbf{1}{-}{-}{-}\mathbf{0} \\ {-}{-}\mathbf{00}{-}{-} \\ {-}{-}0{-}0{-} \\ {-}{-}\mathbf{0}{-}{-}\mathbf{0} \\ {-}{-}{-}00{-} \\ {-}{-}{-}\mathbf{0}{-}\mathbf{0} \\ {-}{-}{-}{-}\mathbf{00}
\end{array} &
\begin{array}{c}
00{-}{-}{-}{-} \\ 0{-}1{-}{-}{-} \\ 0{-}{-}0{-}{-} \\ 0{-}{-}{-}1{-} \\ 0{-}{-}{-}{-}1 \\ {-}01{-}{-}{-} \\ {-}0{-}0{-}{-} \\ {-}0{-}{-}1{-} \\ {-}0{-}{-}{-}1 \\ {-}{-}10{-}{-} \\ {-}{-}1{-}1{-} \\ {-}{-}1{-}{-}1 \\ {-}{-}{-}01{-} \\ {-}{-}{-}0{-}1 \\ {-}{-}{-}{-}11
\end{array} &
\begin{array}{c}
\mathbf{11}{-}{-}{-}{-} \\ 1{-}1{-}{-}{-} \\ \mathbf{1}{-}{-}\mathbf{0}{-}{-} \\ 1{-}{-}{-}1{-} \\ \mathbf{1}{-}{-}{-}{-}\mathbf{0} \\ {-}11{-}{-}{-} \\ {-}\mathbf{1}{-}\mathbf{0}{-}{-} \\ {-}1{-}{-}1{-} \\ {-}\mathbf{1}{-}{-}{-}\mathbf{0} \\ {-}{-}10{-}{-} \\ {-}{-}1{-}1{-} \\ {-}{-}1{-}{-}0 \\ {-}{-}{-}01{-} \\ {-}{-}{-}\mathbf{0}{-}\mathbf{0} \\ {-}{-}{-}{-}10
\end{array} &
\left.\begin{array}{c}
01{-}{-}{-}{-} \\ 0{-}0{-}{-}{-} \\ 0{-}{-}0{-}{-} \\ 0{-}{-}{-}1{-} \\ 0{-}{-}{-}{-}0 \\ {-}\mathbf{10}{-}{-}{-} \\ {-}\mathbf{1}{-}\mathbf{0}{-}{-} \\ {-}1{-}{-}1{-} \\ {-}\mathbf{1}{-}{-}{-}\mathbf{0} \\ {-}{-}\mathbf{00}{-}{-} \\ {-}{-}0{-}1{-} \\ {-}{-}\mathbf{0}{-}{-}\mathbf{0} \\ {-}{-}{-}01{-} \\ {-}{-}{-}\mathbf{0}{-}\mathbf{0} \\ {-}{-}{-}{-}10
\end{array}\right]
\end{array} \Rightarrow
$$

$$
\Rightarrow
\begin{array}{c}
\begin{array}{ccccc}
{\scriptstyle 3,5} & {\scriptstyle 2,4,5} & {\scriptstyle 1,2,3,5} & {\scriptstyle 1,3} & {\scriptstyle 1,3}
\end{array} \\
\left[\begin{array}{ccccc}
1{-}1{-}{-}{-} & 1{-}0{-}{-}{-} & 0{-}1{-}{-}{-} & 1{-}1{-}{-}{-} & 0{-}0{-}{-}{-} \\
{-}0{-}{-}0{-} & {-}1{-}{-}0{-} & {-}0{-}{-}1{-} & {-}1{-}{-}1{-} & {-}1{-}{-}1{-} \\
{-}{-}1{-}0{-} & {-}{-}0{-}0{-} & {-}{-}1{-}1{-} & {-}{-}1{-}1{-} & {-}{-}0{-}1{-} \\
{-}{-}{-}10{-} & {-}{-}{-}00{-} & {-}{-}{-}01{-} & {-}{-}{-}01{-} & {-}{-}{-}01{-}
\end{array}\right]
\end{array}
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
$$

Hence, we have four possible solutions of this problem, where the positions of the dashes in the systemic conjuncterms of the 2-rank indicate the nonessential variables of the given incomplete (weakly determinated) system functions $F(X)$. The validity of the obtained result can be easily checked by writing the STF Y_s^1 for each of these solutions to ensure that the resulting conjuncterm $(\theta_i^2)_{s'}$ of a function contains the systemic minterm $(m)_{s'}$, i.e. $(\theta_i^2)_{s'} \supset (m)_{s'}$, and the rest of minterms belong to $Y_s^{\sim}$. We illustrate this with an example of the 4-th solution, where the given system reflects the STF $Y_s^1 = \{({-}{-}{-}10{-})_{3,5}, ({-}{-}{-}00{-})_{2,4,5}, ({-}{-}{-}01{-})_{1,2,3,5}\}^1$:

$$
Y_1^1 = \{({-}{-}{-}01{-})\}^1 \supset \begin{pmatrix} 001011 \\ 111010 \\ 010010 \end{pmatrix}, \quad Y_2^1 = \{({-}{-}{-}0{-}{-})\}^1 \supset \begin{pmatrix} 110000 \\ 001010 \end{pmatrix},
$$

$$Y_3^1=\{(---10-)(---01-),\}^1\supset\begin{pmatrix}101100\\001011\\111010\\010010\end{pmatrix},\ Y_4^1=\{(---00-)\}^1\supset(110000),$$

$$Y_5^1=\{(---10-),(---00-),(---01-)\}^1\supset\begin{pmatrix}101100\\110000\\001011\end{pmatrix}.$$

The validity result confirms the correctness of 4-th solution. Similar conclusion should be made for the remaining solutions for this problem.

The given system $F(X)$ based on the minimization of its functions with the essential variables are presented as follows reflecting four solutions of STF $\{Y_i^1\}$, $i=1,2,...,5$:

$$1.\ F(x_1,-,x_3,-,-,-)=\begin{cases}Y_1^1=\{(0-1---),(1-1---),(0-0---)\}^1\Rightarrow\{(0-----),(--1---)\}^1\\Y_2^1=\{(1-0---),(0-1---)\}^1\\Y_3^1=\{(1-1---),(0-1---),(0-0---)\}^1\Rightarrow\{(--1---),(0-----)\}^1\\Y_4^1=\{(1-0---)\}^1\\Y_5^1=\{(1-1---),(1-0---),(0-1---)\}^1\Rightarrow\{(1-----),(--1---)\}^1\end{cases},$$

where x_2, x_4, x_5, x_6 are the systemic nonessential variables;

$$2.\ F(-,x_2,-,-,x_5,-)=\begin{cases}Y_1^1=\{(-0--1-),(-1--1-)\}^1\Rightarrow\{(----1-)\}^1\\Y_2^1=\{(-1--0-),(-0--1-)\}^1\\Y_3^1=\{(-0--0-),(-0--1-),(-1--1-)\}^1\Rightarrow\{(-0----),(----1-)\}^1\\Y_4^1=\{(-1--0-)\}^1\\Y_5^1=\{(-0--0-),(-1--0-),(-0--1-)\}^1\Rightarrow\{(----0-),(-0----)\}^1\end{cases},$$

where x_1, x_3, x_4, x_6 are the systemic nonessential variables and the function f_1 has also the nonessential variable x_2;

$$3.\ F(-,-,x_3,-,x_5,-)=\begin{cases}Y_1^1=\{(--1-1-),(--0-1-)\}^1\Rightarrow\{(----1-)\}^1\\Y_2^1=\{(--0-0-),(--1-1-)\}^1\\Y_3^1=\{(--1-0-),(--1-1-),(--0-1-)\}^1\Rightarrow\{(--1---),(----1-)\}^1\\Y_4^1=\{(--0-0-)\}^1\\Y_5^1=\{(--1-0-),(--0-0-),(--1-1-)\}^1\Rightarrow\{(----0-),(--1---)\}^1\end{cases},$$

where x_1, x_2, x_4, x_6 are the systemic nonessential variables and the function f_1 has also the nonessential variable x_3;

$$4.\ F(-,-,-,x_4,x_5,-)=\begin{cases}Y_1^1=\{(---01-)^1\\Y_2^1=\{(---00-),(---01-)\}^1\Rightarrow\{(---0--)\}^1\\Y_3^1=\{(---10-),(---01-)\}^1\\Y_4^1=\{(---00-)\}^1\\Y_5^1=\{(---10-),(---00-),(---01-)\}^1\Rightarrow\{(----0-),(---0--)\}^1\end{cases},$$

where x_1, x_3, x_3, x_6 are the systemic nonessential variables and the function f_2 has also the nonessential variable x_5.

Note that in [1, p. 276] there is only the 2-nd solution of definition of the nonessential variables without minimization of the system functions.

Conclusion

An application of conjuncterms splitting method for the reduction of a number of the nonessential variables in complete and incomplete system functions is described. The peculiarities and advantages of the proposed method are analyzed in the first part of the article. They are also inherent to the mentioned system functions that are illustrated by the examples borrowed from the well-known publications.

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Поступила 21.05.2017

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