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DETERMINATION OF VIBRATION OBJECT' COORDINATE ON SURFACE OF EARTH

Problem of determination of vibrating object's coordinates on Earth's surface is considered for the case in which generated seismic waves are measured with several probes. Analytical dependencies of vibrating object's coordinates as functions of the probes' coordinates, relative delays of the wave front for different probes, and velocity of its propagation are obtained. Configurations involving three and four probes are being investigated, calculations' algorithms and codes are described, and accuracy of the coordinate's determination is studied.

Keywords: *source of vibrations, system of coordinates, seismic wave, analytical dependencies.*

Introduction

There exist a number of practical situations in which an investigated object vibrates thus generating waves propagating inside an elastic medium; these waves can be measured by probes in certain coordinates. A task is stated of distinguishing between the waves from different objects and determination of their locations and properties. As a medium of propagation of the vibrations, there may be air, water, earth's crust, building-construction materials, etc. Water and air conducts longitudinal waves, whereas solid medium — longitudinal and transversal ones. Various machinery (including movable), movements of a human or animal, breaks in the earth's crust or building constructions, processes inside technical and living systems, and so on — all of these may be considered as vibration objects to be investigated.

As to us, we came out of a practical task to determine the coordinates of a vibrating object (VO) situated on the Earth's surface, which generates seismic waves propagating around it, and attain the probes. At that, it is envisaged a possibility of placing several probes on the same surface and standing apart from the each other. These probes must measure the waves, ge-nerated by the VO, and determine the delays of the same wave front arriving at the different probes.

Let us consider alternatives to above described technique of measurement of seismic waves and determination of the VO coordinates.

As known, there exist two types of seismic waves traveling on Earth's surface from a VO: *SH*-waves — horizontally polarized (Love's waves), and *SV*-waves — vertically polarized (Rayleigh's waves) [1–3]. *SV*-waves possess the following useful properties:

– acquires the biggest amount of energy from a VO;

– propagates inside all types of soil;

– can be measured with simple probes, which perceive only vertical soil vibrations and receive uniformly the waves coming from any direction.

Measurement of *SH*-waves opens additional possibilities:

– determination the direction towards VO, since the probes possess different sensitivity to *SH*-waves coming from different directions;

– determination the distance to VO, since travel velocity of *SH*-waves differs from that of *SV*-waves: measurement of delays of the same front for different types of waves may be used for calculation of distance to VO.

But the following problems associated with the uses of these possibilities:

– *SH*-waves propagate not in all types of soil;

– application of lowered sensitivity of probes for certain directions reduces whole sensitivity of the system;

– possible difference in shapes of *SH*- and *SV*-waves may cause difficulties in determination the delays of the same front which measured by different probes;

– probes, possessing the above possibilities, may be of complicated design, since they measures the seismic waves, at least, by two sensors situated under a certain angle relative to one another inside a single case.

The mentioned problems in employing *SH*-waves induce us to use *SV*-waves only, and to measure the relative delays of fronts at locations of remote probes.

Here we consider the aspect of determination of coordinates in supposition that other tasks are fulfilled: the signals from probes is measured and converted into digital form; synchronous arrival of the signals to computer from different probes is provided; the same front arrived from the VO to different probes is distinguished, and their retardations relative to one another are determined.

Formulation of the problem

It is necessary to arrange probes in certain coordinates of a plane, so that coordinates of any point VO can be determined from difference between distances of that point and probe for several pairs of probes. This task can be clarified by follows:

– let us restrict ourselves by a case in which VO and probes are situated in the same plane;

– difference of distances between the VO and the pair of probes can be easily determined from: *i*) relative delays of the same wave front attaining these probes ii) velocity of its propagation;

– VO can be placed in any point of the coordinate plane (in practice, within the distance of sensitivity of the probes). That is, we do not impose conditions for arrangement of VO relative to the probes;

– it is necessary to create an algorithm for determination of the VO coordinates.

We will estimate the configurations of the probes' system in accordance with the following criteria:

– number of probes (the lesser the better, since it makes the realization less expensive);

– the distance between probes: the smaller they are, the smaller the distortions in the transmission of the analog signal to the center where they are processed;

— rate of the processing algorithms — to provide real-time determination of movable objects' coordinates;

— minimal sensitivity to errors of measurements of delays.

As an initial variant of the configuration to be obtained, let us try to determine coordinates of VO situated in arbitrary coordinates $g\ (x,y)$ by using three probes located in known coordinates $d1\ (x_1,y_1)$; $d2\ (x_2,y_2)$; $d3\ (x_3,y_3)$ (Fig. 1).

Measured data from the probes are used for determination of relative delays in arrival of the same wave front, for example, τ_{12}, τ_{13} — to 2nd and 3rd probes relative to 1st one, τ_{23} — to 3rd probe relative to 2nd one. Distances to VO with *x, y* coordinates, from each of the probes can be determined by formulae known from the course of analytical geometry, for example [4]:

$$X_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2};$$
$$X_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}; \qquad (1)$$
$$X_3 = \sqrt{(x - x_3)^2 + (y - y_3)^2}.$$

Let us denote V being velocity of propagation of the waves; then the following dependences will be true for chosen VO:

$$X_2 - X_1 = \tau_{12} \cdot V,$$
$$X_3 - X_1 = \tau_{13} \cdot V, \qquad (2)$$
$$X_3 - X_2 = \tau_{23} \cdot V.$$

After substituting into relationships (2) values for distances $X1$, $X2$, and $X3$ expressed through the known coordinates of probes and unknown coordinates of VO, it becomes possible to find the unknown coordinates. Each of the equations (2) is a hyperbola equation, i.e. geometric locus of points, difference of distances from which to two given points (coordinates of the pair of probes) is constant (as known from the analytical geometry course). Intersection of the two hyperbolas drawn for different pairs of probes gives us possible VO coordinates.

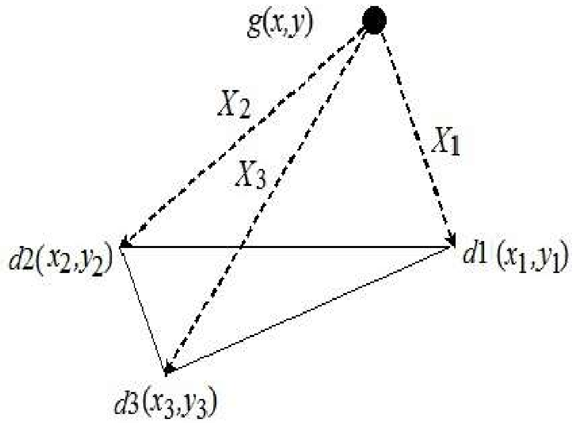

Fig. 1. On-plane arrangement of probes $d1$, $d2$, $d3$, measuring the same wave front from $g\ (x,y)$. X_1, X_2, and X_3 – distances from VO to the probes

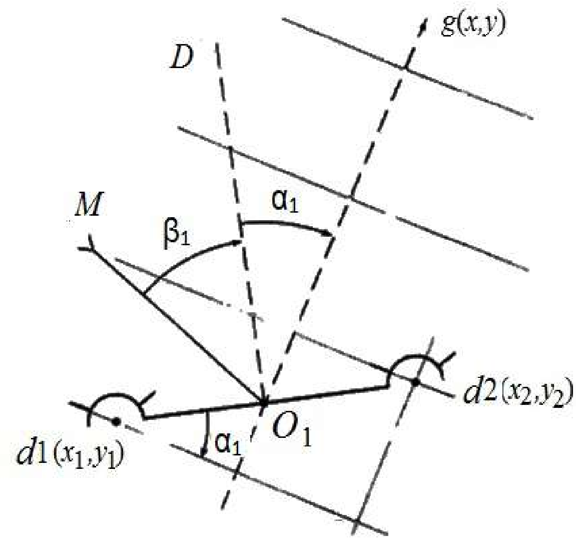

Fig. 2. Determination of direction towards VO: O_1 – middle of segment connecting probes $d1(x_1,y_1)$ and $d2(x_2,y_2)$. Angle α_1 determines direction towards VO relative to perpendicular D (directrix) drawn from point O_1. Angle β_1 determines direction of directrix D relative to meridian M

Existing approaches

Determination of direction towards source of vibrations using two probes

In works [5, 6] a supposition of flat front is made as to wave propagating from a VO towards a pair of probes situated remotely. Such a supposition is the more admissible (calculation error is lesser), the lesser the angle of the pair of probes seen from

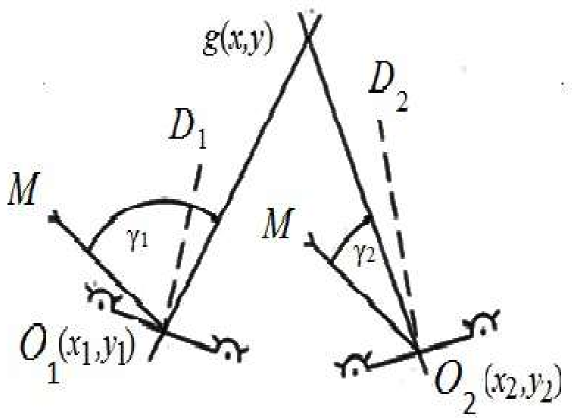

Fig. 3. Determination coordinates $g(x,y)$ as the points of intersection of the two lines, which are determined in manner according Fig. 2.

VO. The error also reduces if direction towards VO approaches to directrix *D*. The corresponding geometrical arrangements are depicted in Fig. 2 and 3.

From Fig. 2, we obtain the following ratio:

$$\alpha_1 = \arcsin\left(\frac{\tau_{12} \cdot V}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}\right),$$

with τ_{12} — being relative delay of the wave front at the probes, V — velocity of the wave, i.e., the numerator equals length of the triangle's cathet opposing to angle α_1 (triangle's hypotenuse is the distance between probes). As to such a determination of the direction, we should make the following notations:

– when calculating, the sign of τ_{12} is important, since the direction symmetric relative to directrix *D* yields the same angle value but of reversed sign;

– VO situated symmetrically relative to the line between probes, yields equal delays; thus, it is impossible to distinguish the side of this line the wave comes from. Only preliminary knowledge of the half-plane, where the VO can be located, will eliminate ambiguity;

– more accuracy in determination of the direction towards VO can be achieved if we draw the hyperbola according to corresponding equation from (2). In this case, one of the asymptotes of this hyperbola coincides with the direction towards VO, defined by a manner depicted in Fig. 2. We tested this statement by modeling on a computer, although it is probably possible to confirm their identity by equivalent transformations of the relevant analytical expressions.

Equation of line directed towards VO, if we issue from: *i*) coordinates of point $O_1(x_1,y_1)$; *ii*) direction $\gamma_1 = \alpha_1 + \beta_1$ relative to the meridian, $\gamma = -tg\,\gamma_1 \times x + (y_1 + x_1 \times tg\gamma_1)$. Similar pair of probes in different coordinates gives us another line directed towards the same VO (Fig. 3): $y = -tg\gamma_2 \cdot x + (y_2 + x_2 \times tg\gamma_2)$. Intersection of these two lines determines the VO coordinates:

$$x = \frac{y_2 + x_2\mathrm{tg}\gamma_2 - y_1 - x_1\mathrm{tg}\gamma_1}{\mathrm{tg}\gamma_2 - \mathrm{tg}\gamma_1};$$

$$y = -\frac{y_2 + x_2\mathrm{tg}\gamma_2 - y_1 - x_1\mathrm{tg}\gamma_1}{\mathrm{tg}\gamma_2 - \mathrm{tg}\gamma_1} \cdot \mathrm{tg}\gamma_1 + y_1 + x_1\mathrm{tg}\gamma_1.$$

Drawbacks of the above considered approach:

– inaccuracies in determining coordinates due to the roughness of the model (supposition of flat front of wave);

– ambiguity in the determination of VO coordinates — necessity of preliminary define of an area containing VO;

– long distance between locations of pairs of the probes — a processing center acquires information from remote probes; it means that measurements can be distorted. From the other point of view, the long distance between the probes' pairs possess a positive side as to remote VO — the longer is distance the bigger is angle, at which the direction lines intersect one another; this means that sensitivity to inaccuracy of measurements is lesser.

Usually, the above-described scheme is used for determination of coordinates of a shot whose sound is received by pairs of probes, and directions toward it relative to the meridian are transferred to a center in which the coordinates of VO should be determined.

Optimization procedure for solving the system of equations

In work [7], it is suggested to give a general solution to the task of determination of VO coordinates provided that *i*) number of probes is arbitrary; *ii*) the coordinates to be determined are situated not only 2D but also 3D.

Dependences (1) and (2) from previous section are generalized as follows:

$$R_i = \sqrt{(x_i - x) + (y_i - y)^2 + (z_i - z)^2}; \qquad (1^*)$$

$$R_t - R_j = \tau_{ij} \cdot V, \qquad (2^*)$$

where R_i is distance from VO to i-th probe; x_i, y_i, z_i are 3D coordinates of i-th probe; x, y, z — coordinates of VO; τ_{ij} — delays between comings to probes i and j of a certain wave front; V — velocity of the wave.

In [7] it is proposed *i*) to take the number of probes to be more than minimally needed, *ii*) to determine unknown VO coordinates by selecting their values in a manner that minimizes the functional:

$$\Phi = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \left((R_i - R_j - V \cdot \tau_{ij}\right)^2,$$

with m being number of probes.

The described approach possesses follows drawbacks:

– it supposes excessive number of probes thus increasing the expenses on hardware;

– the optimizing involves a selection algorithm which is time-consuming for real time processing, especially in determination of movable objects' coordinates.

We shall try to resolve equation systems similar to (1) and (2) by analytical manner with the use of minimal number of probes.

Three-probe configuration

It is obvious that two probes are insufficient for determination of delays between the fronts, since only one equation similar to (2) can be formulated; this means that the location can only be confined with a corresponding hyperbola.

Intersection of two hyperbolas takes place, in general case, in 4 points. To select one point from the four, 3rd equation is needed: each of the obtained solutions of the selected two equations must be verified — if it meets the 3rd equation. But a question rises: if the 3rd equation allows isolation of only one point.

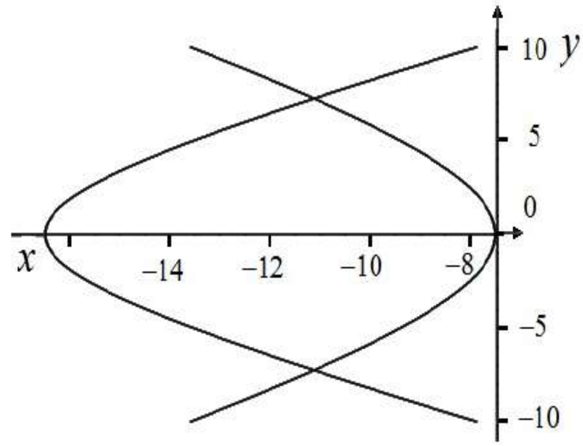

Fig. 4. Hyperbolas drawn for two different pairs of probes situated along *x* axis – the curves are symmetric relative to this axis

A case, in which all 3 probes are arranged along a single line, is more obvious (Fig. 4).

In this case, all hyperbolas, related to a certain pair of probes, do intersect in points symmetric relative to the line connecting the probes. This mean that it is impossible to distinguish in which of the two points the VO is localized. Furthermore, any number of probes placed on a single line not allows us to determine real location of VO. Therefore, three probes must not be placed along the same line.

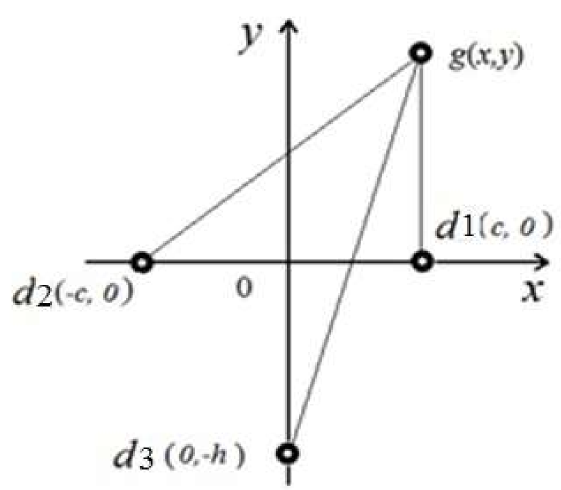

Fig. 5. Arrangement of probes in apexes of equilateral triangle with apexes' coordinates *d*1 (*c*,0), *d*2 (-*c*,0), *d*3 (0, -*h*) for determination coordinates of any point *g* (*x*,*y*) on the plane

In supposition that three probes located out of a single line are sufficient for unambiguous determination of the VO coordinates, we have formed an algorithm for the following VO configuration. We arranged, on the coordinate plane, a system of 3 probes placed in triangle's tops (Fig. 5). To simplify the calculations, we accepted the following conditions. The probes are placed on tops of equilateral triangle ($d1$, $d2$, $d3$) with side 2·c. The coordinate axes are oriented as follows: *i*) *OX* axis coincides with one of the sides of triangle; *ii*) the coordinate center located in central point of that side. In the chosen coordinate system, probes are located with coordinates $d1$ (c,0); $d2$ (-c,0); $d3$ (0, -h), where $h = c \cdot 3^{1/2}$.

In our notations, distances from an arbitrary point g (x,y) to the probes, on a plane are expressed by the following equations:

$$g1 = \sqrt{(x-c)^2 + y^2}; \quad (3)$$

$$g2 = \sqrt{(x+c)^2 + y^2}; \quad (4)$$

$$g3 = \sqrt{x^2 + (y+h)^2}. \quad (5)$$

Differences between each pair of the distances are determined, in real conditions, by measuring delays of the same wave front received by different probes. The delays and the difference of distances can be related by the following dependences:

$$g2 - g1 = \tau_{12} \cdot V; \quad (6)$$

$$g3 - g1 = \tau_{13} \cdot V; \quad (7)$$

$$g3 - g2 = \tau_{23} \cdot V, \quad (8)$$

where $\tau_{12}, \tau_{13}, \tau_{23}$ — time delays in receiving the wave front by probes $1 \to 2, 1 \to 3, 2 \to 3$ respectively; V — wave velocity. In so doing, negative values of the delays are indicatives of reversed direction from which the wave front arrives as compared with that pointed by the arrow. Let us denote: $\tau_{12} \cdot V = 2 \cdot a$; $\tau_{13} \cdot V = 2 \cdot d$; $\tau_{23} \cdot V = 2 \cdot b$. Then equations $(6 \div 8)$ can be written in the form:

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2 \cdot a; \quad (9)$$

$$\sqrt{x^2 + (y+h)^2} - \sqrt{(x-c)^2 + y^2} = 2 \cdot d; \quad (10)$$

$$\sqrt{x^2 + (y+h)^2} - \sqrt{(x-c)^2 + y^2} = 2 \cdot b. \quad (11)$$

To model the functioning of system of coordinates determination, we shall preset coordinates (x,y) and determine the values of a, b, d from equations (9) to (11). Our algorithm must determine the coordinates (x,y) on the basis of preset values of a, b, d, that is, it must resolve a problem reversed to that mentioned in the previous sentence. As the algorithm's results, there must be coordinates corresponding to those preset for calculation of parameters a, b, d. If we succeed in achieving such a correspondence for any point of the coordinate plane, then our algorithm can be employed for determination of VO coordinates basing on measurements of delays.

The algorithm for determination of VO coordinates is composed from next steps:

a) coming out from values of a, b, d, the pair of equations — (9, 10), (9, 11), or (10, 11) — should be chosen for being resolved relative to x, y. The pair must be selected in such a manner that the corresponding hyperbolas intersect under the angle the most close to the right one. In this case, inaccuracies of measuring parameters a, b, d, exert a minimal impact to the result;

b) resolving the system of selected pair of equations and obtaining, in a general case, 4 pairs of their solutions. Equivalent transformations of equations $(9 \div 11)$ lead to the following radicals-free equation:

$$x^2 = a^2 + y^2 \cdot \lambda, \text{ where } \lambda = \frac{a^2}{c^2 - a^2}, \quad (9^*)$$

$$m \cdot x^2 + n \cdot y^2 + p \cdot x \cdot y + s \cdot x + q \cdot y + r = 0, \quad (10^*)$$

where $m = c^2 - 4d^2$; $n = 3c^2 - 4d^2$; $p = 2hc$; $s = 2c(4d^2 + F_d)$; $q = 2hF_d$; $r = F_d^2 - 4c^2d^2$; $F_d = c^2 - 2d^2$.

$$Mx^2 + Ny^2 - Pxy + Sx + Qy + R = 0, \quad (11^*)$$

where $M = c^2 - 4b^2$; $N = 3c^2 - 4b^2$; $P = 2hc$; $S = 2c \cdot (4b^2 + F_b)$; $Q = 2hF_b$; $R = F_b^2 - 4c^2b^2$; $F_b = c^2 - 2b^2$

To determine the unknown coordinates x, y, we resolve the system of the two selected equations by the substitution technique. After that, we obtain equation of 4th degree relative to one of the unknown variables, for example y:

$$A \cdot y^4 + B \cdot y^3 + C \cdot y^2 + D \cdot y + E = 0. \quad (12)$$

Coefficients of this equation depend of the selected pair: (9^*, 10^*); (9^*, 11^*); or (10^*, 11^*). In a general case, equation of 4th degree gives 4 roots. Two of them may be complex numbers but two others must be real ones if initial parameters *a, b, d* are calculated on the basis of real VO coordinates, since there exists a real point of intersection of the corresponding hyperbolas. The second unknown variable — *x* should be determined for each of the determined real solutions y (existence of real value x is substantiated likewise to value of *y* — by real existence of intersection point of the hyperbolas). So, we obtain 4 variants of the VO coordinates according to the pair of chosen equations. Solutions of equation of 4th degree can be expressed analytically [8] — this is the highest degree of a polynomial of general form that can be resolved in radicals [9] — equation of a higher degree can be resolved only by numerical methods (by successive approximations), which are more time-consuming [10].

c) elimination the solutions which do not meet 3rd equation that was not used ((11), (10), or (9), see item *a*)). Substitution of roots obtained in item *b*) into the previously unused equation must transform it into an identity if the pair of coordinates is "right". However, because the calculations possess an error, it is worthwhile to calculate a difference between RHS and LHS of 3rd (for verifying) equation, and solution with the minimal difference should be considered as a real one.

As a result of conducted experiments, areas on the coordinate plane were allocated for which the program, formed in accordance with the above-described algorithm, showed inability to point out the preset coordinates. Fig. 6 shows hyperbolas drawn for various pairs of probes situated on apexes of triangle in coordinates corresponding to Fig. 5 with values c = 10, $h = c \cdot \sqrt{3}$ =17,321. Parameters of equations $(9 \div 11)$ are determined for VO located in coordinates x = 25, y = 20: a = 15,3113, d = =19,9202, b = 4,6089. As seen from Fig. 6, the three hyperbolas intersect not only in the point with chosen coordinates, but also in point with coordinates x = 13,7, y = 9,6 (program, formed according to our algorithm was not able to distinguish between these solutions).

When investigating other points of coordinate plane, we come to the following conclusion. Ambiguity and incorrectness of the algorithm show themselves in wide strips of the coordinate values

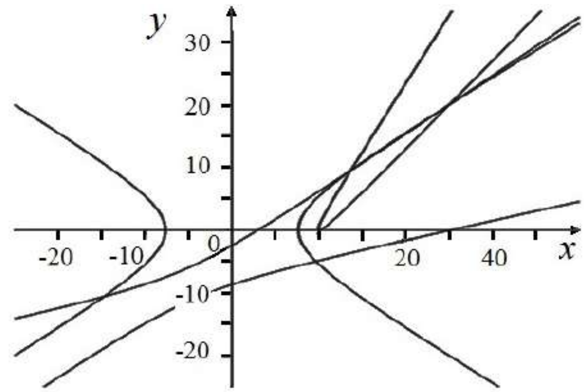

Fig. 6. Three hyperbolas $(9 \div 11)$ were built in such a manner that to have intersection in one point. But, as it turned out, all three lines are intersected also in another point for some predefined points

adjacent to the semi-lines with starting points in apexes of the triangle (where the probes are placed) and which are elongations of its sides. Determination of coordinates of VO, located in other areas of the coordinate plane is accomplished correctly. The ambiguity emerges in connection with shortening of the distance between symmetric branches of the hyperbolas, when VO approaches to the semi-line, which is the elongating the triangle's side (Fig. 6). Two other hyperbolas, which pass through a point preset, are also passing through the other point, adjacent to the preset one, but residing on another, symmetric, branch of the narrow hyperbola.

Additional information on order in which the wave front arrives to the probes, in most cases, does not allow us to eliminate the incorrect solution. This is seen, in particular, from Fig. 6: the order of arriving the wave's front is the same for 2 points of intersection of all the three hyperbolas.

It may be supposed that any 3-probe configuration contain the areas of ambiguity. We do not have exact theoretical and experimental substantiations of this assertion. But it is quite believable, since the factor of ambiguity — too sharp angles between the sides of triangle, with the probes in his apexes, is impossible to increase simultaneously for all the angles of triangle.

So, the 3-probe configuration may be employed for monitoring certain areas, but not the whole surrounding area. It is probable that it is possible to control the ambiguity areas by selecting the mutual

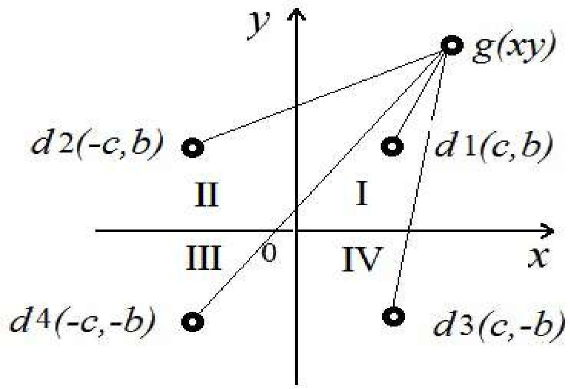

Fig. 7. The probes are placed in rectangle's apexes with coordinates: $d1$ (c,b), $d2$ $(-c,b)$, $d3$ $(c,-b)$, $d4$ $(-c,-b)$ for determination of coordinates of any point g (x,y) on the plane

arrangement of the probes as well by their general placement in a coordinate plane.

Four-probe configuration

With a goal to use a minimal number of the probes, let us consider the configuration involving 4 probes situated in one plane; we will determine the VO coordinates in the same plane.

Variant with 4 probes, two pairs of which are far from each other, and which is described earlier, does not satisfy us, since:

a) it does not meet the condition of compactness;

b) it is supposed that it is known the half-planes with VO;

c) it employs approximate models of wave's propagation from VO, which are admissible at large distances but give essential errors near to probes.

Let us conduct our study in accordance with the same scheme as for the 3 probes: we select a convenient coordinate system and the probes' arrangement to obtain simple dependences similar to (9÷11). The corresponding configuration of probes is shown in Fig. 7.

Let us write dependences, for distances between g (x,y) and the probes, which is similar to (9÷11) ones. Here we restrict ourselves by pairs of probes situated in sides of the rectangle; we exclude two still possible equations for the pairs of probes situated in the rectangle's diagonals (we leave it for further investigations).

$$g2-g1=\sqrt{(x+c)^2+(y-b)^2}-\sqrt{(x-c)^2+(y-b)^2}=a_0\,; \tag{13}$$

$$g3-g1=\sqrt{(x-c)^2+(y+b)^2}-\sqrt{(x-c)^2+(y-b)^2}=a_1; \tag{14}$$

$$g4-g3=\sqrt{(x+c)^2+(y+b)^2}-\sqrt{(x-c)^2+(y+b)^2}=a_2. \tag{15}$$

To determine the VO coordinates, we use the algorithm described earlier for 3 probes with taking into account features of the given configuration. We shall select the pair of equations dependently of quadrant number (I ÷ IV) in which VO is situated. The general approach is to choose 2 equations from (13÷16) — to provide minimal results' dependence of inaccurate measurements. This aim can be reached, if the corresponding hyperbolas intersect under an angle closest to the 90°. Since it was difficult to ensure such a condition, we adopted a simplified method: a couple of equations will be chosen depending on the quadrant (I÷IV in Fig. 7), where VO is located. The quadrant is determined from a signs of distance differences from VO to a pair of probes. It is sufficient to analyze one of the pairs of values a_0, a_1 or a_2, a_3, as is directly seen from Fig.7: Iq — $a_0>0$, $a_1>0$; IIq — $a_0<0$, $a_1>0$; IIIq — $a_0<0$, $a_1<0$; IVq — $a_0>0$, $a_1<0$. So, for the VO placed in quadrant I, we have to choice equations (13) and (14) for determination the four possible coordinates. One of the unused equations, for example (15) is used to select one of the 4th solutions obtained at the previous stage. According to this example, equations were selected for each of the quadrants: Iq — (13)&(14)→(15); IIq — (13)&(16)→(15); IIIq — (15)&(16)→(13); IVq — (16)&(14)→(13).

Just as in the previous paragraph for a configuration of 3 sensors, we will get rid of radicals in equations of (13–16) by their equivalent transformations. As a result, we obtain two pairs of equations for horizontal and vertical sides of the rectangle are of a same structure but with different values of coefficient:

• for equations (14), (16) (the vertical sides): $M \cdot y^2 - N \cdot x^2 \pm Q \cdot x + R = 0$, where $M = 16b^2 - 4a_i^2$; $N = 4a_i^2$; $Q = 8ca_i^2$; $R = a_i^4 - 4a_i^2(b^2 + c^2)$;

$+Q$, $i = 1$ for equation (14), and

$-Q$, $i = 3$ for equation (16);

• for equations (13), (15) (the horizontal sides): $mx^2 - ny^2 \pm qy + r = 0$, where $m = 16c^2 - 4a_j^2$; $n = 4a_j^2$; $q = 8ba_j^2$; $r = a_j^4 - 4a_j^2(c^2 + b^2)$;

$+q$, $j = 0$ for equation (13), and

$-q$, $j = 2$ for equation (15).

To find the unknown coordinates *x, y*, we solve a system of two equations chosen depending on the quadrant in which the VO is situated (4 variants of conjunction of these equations are possible). It would be convenient to express *x* from equation (13)/(15) with further substitution into equation (14) /(16), or vice versa – to express y from (14) /(16) with successive substitution into (13)/(15); we have chosen the first variant of substitution. In so doing, we obtained the equation (12) of 4th degree relative to *y*: $A{\cdot}y^4 + B{\cdot}y^3 + C{\cdot}y^2 + D{\cdot}y + E = 0$.

Values of coefficients of this equation depend on the selected pair of entry equations. For each quadrant, we get own variants of the values of the coefficients of the equation (12), but the structure of expressions for coefficients *A, B, C, D, E* remains unchanged.

$$A = F^2;\ B = 2F \cdot H;\quad C = 2F \cdot G + H^2 - Q^2 \cdot n / m;$$

$$D = 2G \cdot H + Q^2 q / m;\quad E = G^2 + Q^2 r / m;$$

$$F = M - N \cdot n / m;\ H = N \cdot q / m;\ G = R + r \cdot N / m.$$

The simple expression for coefficients of equation of 4th degree is achieved due to symmetric, relative to the coordinate axes, arrangement of probes on apexes of the rectangle. Other variants of the probes' arrangement lead to more complicated expressions.

After that, we apply the general scheme of resolving the 4th degree equation, and solutions being obtained (four, in general case) should be verified with one of the unemployed equations according to the algorithmic scheme described in previous item

As a result of experimental calculations being conducted, the following specific features of the developed program are elucidated:

- 4 horizontal half-straight $y = b$ and $y = -b$ out of the rectangle's sides (where $x > c$; $x < -c$) and 4 vertical half-straight $x = c$ and $x = -c$ out of the rectangle's sides (where $y > b$; $y < -b$), represent a multitude of VO coordinates that can not be processed by the general algorithm. This peculiarity is connected with equalizing to zero the *m*-coefficient in equations (13) and (15), or *M*-coefficient — in (14) and (16). To take into account this peculiarity, special branch is inserted into the algorithm, in which values *x* or *y* are calculated in a simplified manner with adopting a known value of y (*c* or *-c*) or *x* (*b* or *-b*).
- within the area of x = -0,05 to 0,05 and y = =-0,05 to 0,05, the algorithm does not determine the correct values of coordinates (4th-degree equation has no solutions), that is why it was taken the zero coordinates for the VO between these borders. Cause of this phenomenon, possibly, is associated with erroneous calculations, and needs to be investigated additionally (since, in theory, any equation of 4th degree possesses 4 roots, possibly, complex ones). If the area of uncertainty has to be shrinked, the calculations accuracy should be improved.

There is no exist other points of coordinate plane, whose coordinates are determined incorrectly as compared with the preset ones — for which the inter-probe distance is used as a parameter of the algorithm. Therefore, the configuration, in which the probes are placed on apexes of the rectangle, can be used for unambiguous determination of coordinates of VO, situated in any points of the plane.

A deviation from the rectangular form, in our estimate, will lead to complication of the algorithm and add more sensitivity to measurement errors. Square is the best variant as to relationships between its sides — errors of measurements from the short sides restrict the general accuracy in determination of coordinates. The rectangular form can be used in the cases in which complications emerge when placing the probes along sides of a square. Symmetric trapezoid widens, to some extent, flexibility in the probes arrangement and, by our estimations, introduces a minor complication into the calculations (this needs to be investigated additionally).

Transition from the chosen local coordinate system to any other (global) one is possible with a help of formulae of coordinates transformation known from the course of analytical geometry, for example [4]:

$$X = x\cos\alpha - y\sin\alpha + q_x;$$
$$Y = x\sin\alpha + y\cos\alpha + q_y, \quad (17)$$

where X, Y are VO coordinates in the global coordinate system; x, y — coordinates of VO in the local coordinate system; q_x, q_y — coordinates of the beginning of the axes of the local coordinate system relative to the global one; α — angle of turning of axes of the local system relative to the global one.

Investigation of the algorithm's sensitivity to errors in measurements of delays

For any point, residing in a plane, and two given points in a certain coordinates, a hyperbola can be found, i.e. line that includes all the points for which the difference of distances is the same as for the given point. This means that the system of two equations, chosen amongst (13 ÷ 16) with parameters ai, $i = 0...3$, which correspond to a real coordinate on the plane in which (coordinate) the corresponding hyperbolas intersect, must obligatory have a real solution (that is, two real ones, since only even number of complex roots must exist). With arbitrary values of parameters a_i, $i = 0$ to 3, the real roots, theoretically, may be absent. That is why we discard such roots (we leave the technique of employing the complex roots for further investigations).

The case, in which errors of measurements (and the digitalization as well) lead to minor deviations of the parameters corresponding to a certain VO coordinate, is the most probable. We have simulated such deviations: certain VO coordinates were preset, differences of distances a_0, a_1 were determined (we restricted ourselves by experiments with rectangle $c=b$), and we introduced errors in distance differences before the calculation is being accomplished.

For the VO distances within $4{\cdot}c$ from the origin (rectangle's center), error of $0{,}01{\cdot}c$ lead to coordinate value mistake of $0{,}06{\cdot}c$, i.e., it increases by 6 times. If the distance equals to $8{\cdot}c$, the same error of $0{,}01{\cdot}c$ gives the mistake of $0{,}5{\cdot}c$, that is, it increases by 50 times. Therefore, the bigger the distance from probes, the lesser the accuracy of determination of coordinates — the remote VO becomes more sensitive to inaccuracy of measurements. This fact may be explained if we take into account that, at larger distances from the probes, angles of the hyperbolas' intersection become sharper, hence, minor changes in location of each hyperbola leads to considerable deviations of the intersection point. The measurements were conducted in units c — the half-length of side of the square with probes placed in its apexes. This means that elongation of c will lead to elongation the real distance of VO, for the set accuracy.

Based on the fact that the values a_i, $i = 0...3$, can not physically exceed $2{\cdot}c$ ($2{\cdot}b$) — the maximum difference in distances between probes, we believe that there has been an error in measurements and reduce such a measurement to $2{\cdot}c$ ($2{\cdot}b$).

Conclusions

To determine the coordinates of the source (VO), which create waves propagated in an elastic environment along the plane, at least 4 probes which are placed in known coordinates are required. Delays in the receipt of the same front of the wave on these probes, as well as the known speed of propagation of elastic waves allow determining the coordinates of the VO. The algorithm is described and computer program has been created that calculates unknown VO coordinates for any point on the plane. The fastest and most accurate method of calculation is implemented: *i*) analytical solutions of the system of nonlinear equations, which accurately describe the geometry of the plane and the propagation of waves, are used (errors in calculations arise only due to the limited bitness of the presentation of data); *ii*) placing probes on the tops of the rectangle, symmetrical relative to coordinate axes, is used to simplify equations. The computer program can also be used to study the accuracy of determining the coordinates of the VO depending on the errors of measurements. It turned out that measurement errors have different effects on coordinate determination errors; it is depending on the distance to the source. This influence can be reduced by increasing the distance between the probes — the accuracy of the calculation results is influenced by the ratio of distances between the probes and distance to VO. The theoretical explanation of the measurement results is quite simple: the intersection of lines (hyperboles), which determine the coordinates of the VO at long distances, occurs at a more acute angle.

As a result of experiments with three probes, there were areas of ambiguous determination of the coordinates of the VO. Such configurations can also be used, taking into account the limitations of areas in which the coordinates of VO are determined unambiguously. Configurations with 4 sensors with a more general placement of probes (for example, at the tops of the correct trapezoid) require additional research and the creation of appropriate programs. Ideolo-gically, such programs are similar to those considered; require only neat symbolic transformations, which are convenient to perform using the corresponding mathematical packages MAPLE, MATLAB or similar.

Configurations for determining the spatial coordinates of the VO require additional probes and the solution of a system of 3 equations with 3 unknowns (the spatial coordinate z is added to the considered coordinates x, y, which determine the point on the plane). An analytical solution of the equation of the 6th degree obtained by the method of substitution relative to one of the coordinates x, y or z is impossible; it is proved [9] that the 4th degree of the equation is the highest for such a possibility. Therefore, the algorithm for determining coordinates in space will have to be determined in one of the numerical ways.

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ВИЗНАЧЕННЯ КООРДИНАТ ОБ’ЄКТА, ЩО ВІБРУЄ НА ПОВЕРХНІ ЗЕМЛІ

Вступ. Розглядається задача визначення координат об'єкта, що вібрує на поверхні землі (ВО), на основі затримок фронту породженої ним хвилі на датчиках, що її реєструють. Задача розв’язується для випадку, коли датчики та об’єкт розміщено на площині, а також відомі координати датчиків та швидкості поширення хвиль.

Мета статті. Аналіз відомих підходів та обґрунтування алгоритму для визначення координат ВО на основі інформації про хвилі, які він породжує. Критеріями обрано простоту системи реєстрації, швидкість і точність розрахунків. Результати розрахунків передбачається використовувати в системі для ідентифікації та визначення координат рухомих об’єктів на поверхні Землі.

Методи. У роботі було використано математичні методи розв'язання систем нелінійних рівнянь та комп'ютерне моделювання.

Результат. Отримано точні аналітичні вирази для координат ВО в залежності від координат датчиків, відносних затримок фронту хвилі на різних датчиках і швидкості поширення хвилі. На цій основі створено комп'ютерну програму. За її допомогою досліджено точність визначення координат залежно від похибок при вимірах затримок.

Висновки. Для визначення координат ВО, які поширюються в площині, потрібно щонайменше чотири датчики, які реєструють відносні затримки фронту хвилі на різних датчиках. Три датчики не забезпечують однозначності визначення координат ВО, що розташовані в певних областях площини відносно датчиків. Для симетрично розміщених датчиків отримано прості аналітичні залежності; при довільному розміщенні датчиків аналітичні вирази координат виявляються суттєво складнішими.

Розрахунки координат через аналітичні залежності передбачають обчислення елементарних функцій (через проміжні змінні в декілька етапів) та оцінку варіантів розв'язків. Кількість розв'язків визначається ступенем алгебраїчного рівняння відносно однієї змінної після підстановки виразів для інших невідомих. Для розглянутої моделі отримано рівняння 4-го ступеня — це найвищий ступінь рівняння, для якого розв'язки можливо отримати в аналітичному вигляді. Для складніших випадків, наприклад, коли потрібно визначати координати ВО в просторі, остаточне рівняння відносно однієї змінної має 6-й ступінь, а отже не має аналітичного розв'язку. Його потрібно буде розв'язувати чисельно, перебираючи варіанти. Чисельний алгоритм потребує більше часу і не забезпечує «абсолютної» точності — його необхідно зупиняти при досягненні потрібної точності результату. При використанні аналітичних залежностей, отриманих на основі точних геометричних співвідношень, похибки в розрахунках виникають лише через обмежену розрядність представлення даних.

На точність результатів розрахунків (координат ВО) впливає точність вхідних даних. У роботі досліджено залежність точності визначення різниці відстаней від датчиків до ВО на точність визначення координат. Вплив похибок вимірів затримок на похибки у визначенні координат ВО залежать від відстані датчиків до джерела: чим вона більша, тим більший цей вплив.

Ключові слова: *джерело вібрацій, система координат, реєстрація фронту сейсмічні хвилі, аналітичні залежності.*