DOI https://doi.org/10.15407/csc.2023.01.005
UDC 519.7

B.Ye. RYTSAR, Doctor of Technical Sciences, Professor, L'viv Polytechnic National University,
12 Bandera Str., L'viv, 79013, Ukraine, ORCID: https://orcid.org/0000-0002-2929-2954,
bohdanrytsar@gmail.com

A SIMPLE STUCK-AT-FAULTS DETECTION METHOD IN DIGITAL COMBINATIONAL CIRCUITS

This paper considers the new method of detection (diagnostic) stuck-at-faults (0/1) in digital combinational circuits based on a numerical set-theoretical approach. Compared to known methods and algorithms, the proposed approach differs in simpler implementation of searching for vectors of test codes at arbitrary points of the studied logic circuit. A few simple set-theoretical operations and procedures are sufficient to determine the location and the type of a stuck-at-fault (0/1). This is evidenced by the presented examples of application of the proposed method, that are borrowed from the publications of well-known authors.

Keywords: *digital combinational circuits, stuck-at-faults detection, vectors of test codes, numerical set-theoretical approach, operations and procedures.*

Introduction

The technical diagnostics is an important section of the logical design of digital devices, within the framework of which, the methods of checking the technical condition of devices are developed to ensure the reliability of their functioning. Any violation of the normal functioning of a digital device is called a fault of its operation. A fault in the logic circuit of a digital device can be detected by a sequence of control tests on its inputs and observation of the results obtained on its output or outputs. A test is a combination of input data (test codes) that determines the expected error-free response that the circuit should produce. If the value of the function at the output of the circuit differs from the expected one, this indicates a fault in the tested circuit. The task of diagnostics for any device is to find the minimal number (from all possible 2^n input combinations) of test codes. Typical models of the fault in the design of digital devices are single (at some point of the circuit) fixed log. 0 constants (stuck-at-0 – abb. s-a-0) or log. 1 constants (stuck at-1 – abb. s-a-1) whether multiple stuck-at-fault. In this article we will consider single damages of the stuck-at-fault (0/1) type, that mainly occur in digital combinational devices. To detect such faults, various methods and algorithms [1–16] are known, for example: Truth Table and Fault Vatrix method based on the Boolean derivatives (Boolean difference), the Path Sensitization method and the Searching D-Algorithm based on the test patterns (PODEM), Brach-and-Bount method, etc. However, the vast majority of them are based on an analytical approach and are characterized by their complexity and cumbersomeness of practical implementation, that grows with the increase with amount of variables.

The article proposes a new stuck-at-faults (0\1) detection (diagnoctic) method in combinational digital circuits (schemas), which is based on a numerical set-theoretic approach and differs from the known ones by its simplicity of practical implementation [17, 18].

Theoretical Part

The initial requirement for the implementation of the proposed method is the specification of a logical function $f(x_1, x_2, ..., x_n)$ that describes the operation of the circuit under study, in the set-theoretical form (STF) as a perfect STF $Y^1 = \{m_1, m_2, ..., m_k\}^1$, where $m_1, m_2, ..., m_k$ are the binary minterms of the given function *f* [17, 18].

The peculiarity of the method is that some set (vector) of the test codes (a set of input variables) is used to determine the place in the circuit and the type of stuck-at-fault (s-a-0 or s-a-1). This set (vector) can be easily obtained by a few simple ST operations over the binary minterms of the given function f. Let's consider this in more detail.

Let there be fault of type s-a-0 ("sticking" in the state of log. 0) or s-a-1 ("sticking" in the state of log. 1) at an arbitrary point α of the circuit under study, which led to an undesirable change in the value of $1 \to 0$ or $0 \to 1$ at the output *f* of the circuit. In order to find out exactly at which point of the circuit this change occurred, it is first necessary to determine on which set of variables the value of the function *f* has changed, since this will lead to a change in the perfect STF Y^1. The new formed set will precisely differ from the perfect STF Y^1 by those elements, which are the sought-after test codes, allowing to easy determine the location and the type of stuck-at-fault (0/1). We will call this newly formed set as the pseudoperfect STF of the function *f* and denote it $Y^1_{\alpha/0}$ if fault of the type s-a-0 occurred at point of the circuit, or as $Y^1_{\alpha/1}$ in the case of fault of the type s-a-1 at point α.

The pseudoperfect STFs $Y^1_{\alpha/0}$ and $Y^1_{\alpha/1}$ can be easily obtained after the union of sets: a perfect STF $Y^1 = \{m_1, m_2, ..., m_k\}^1$ with some set $Y^1_{0_j \to 1_j}$ (for $Y^1_{\alpha/0}$) and set $Y^1_{1_j \to 0_j}$ (for $Y^1_{\alpha/1}$). The sets $Y^1_{0_j \to 1_j}$ and $Y^1_{1_j \to 0_j}$ are formed from binary minterms $m_1, m_2, ..., m_k$, where $m_i = (\sigma_1 \cdots \sigma_j \cdots \sigma_n)$, $\sigma_j \in \{0,1\}$, in which the value is replaced by the one corresponding to the conditional stuck-at-fault (0/1): if fault of the s-a-0 type is considered, then the *j*-th position 0_j is replaced by 1_j, i.e. $0_j \to 1_j$, and if fault of type s-a-1 is considered, then the *j*-th position 1_j is replaced by 0_j, i.e. $1_j \to 0_j$. At the same time, the proposed procedures are performed in a polynomial format [18], where pairs of identical elements are eliminated from the consideration:

$$Y^1_{\alpha/0} \Rightarrow \{Y^1 \cup Y^1_{0_j \to 1_j}\}^{\oplus}, \tag{1}$$

$$Y^1_{\alpha/1} \Rightarrow \{Y^1 \cup Y^1_{1_j \to 0_j}\}^{\oplus}. \tag{2}$$

On the basis of pseudoperfect STFs $Y^1_{\alpha/0}$ (1) and $Y^1_{\alpha/1}$ (2), it is possible to construct a truth table of the given and (conditionally) corrupted function *f* in order to compare the difference between them for the further determination of the test codes, i.e. some sets of minterms of a given function *f*.

To determine the variables for which the fault points will be considered, it is convenient to draw demarcation lines in the studied circuit, at the intersection of which these points are located with the lines of communication between the elements of the circuit. At the same time, at the 0-level of delimitation, such points will be the input variables $x_1, x_2, ..., x_n$ of the given function *f*, and at the 1-, 2-, ..., -levels of delimitation, there will be variables formed by the rest of the circuit without elements of the lower levels of the circuit. Thus, newly formed functions are considered for variables of a non-zero level of delimitation, on the minterms of which similar operations are then performed to detect possible fault of the stuck-at-fault (0/1) type there.

Test codes for detecting the location and type of fault at an arbitrary level of the circuit under study can be determined on the basis of $Y^1_{\alpha/0}$ (1) and $Y^1_{\alpha/1}$ (2) using simple ST operations:

$$Y^1 \setminus Y^1_{\alpha/0} = Y^1 \cap Y^0_{\alpha/0}, \tag{3}$$

$$Y^1_{\alpha/0} \setminus Y^1 = Y^1_{\alpha/0} \cap Y^0, \tag{4}$$

$$Y^1 \setminus Y^1_{\alpha/1} = Y^1 \cap Y^0_{\alpha/1}, \tag{5}$$

$$Y^1_{\alpha/1} \setminus Y^1 = Y^1_{\alpha/1} \cap Y^0, \tag{6}$$

where $Y^1_{\alpha/0}$, $Y^1_{\alpha/1}$ and Y^0 the set minterms of STF for the value of the function $f = 0$.

We will illustrate now the proposed method with concrete examples.

Practical Part

Let's consider few examples of using the proposed stuck-at-fault (0/1) type detection method in digital combinational circuits.

Example 1. In the logic circuit (Fig. 1), that implements the function $f = x_1 x_2 \vee \bar{x}_2 x_3$ (*borrowed*

Table 1

«10»	$x_1\,x_2\,x_3$	f	f_{x_1}		f_{x_2}		f_{x_3}	
			$f_{x_1/0}$	$f_{x_1/1}$	$f_{x_2/0}$	$f_{x_2/1}$	$f_{x_3/0}$	$f_{x_3/1}$
0	0 0 0	0	0	0	0	0	0	**1**
1	0 0 1	1	1	1	1	**0**	**0**	1
2	0 1 0	0	0	**1**	0	0	0	0
3	0 1 1	0	0	**1**	**1**	0	0	0
4	1 0 0	0	0	0	0	**1**	0	**1**
5	1 0 1	1	1	1	1	1	**0**	1
6	1 1 0	1	**0**	1	**0**	1	1	1
7	1 1 1	1	**0**	1	1	1	1	1

from [9, p. 399]), determine the test codes at all points of possible of the stuck-at-fault (0/1) type.

Solution. The given function $f = x_1x_2 \vee \overline{x}_2x_3 \Rightarrow$ $\Rightarrow \{(11-),(-01)\}^1$ has a perfect STF $Y^1 = \{1,5,6,7\}^1$. To determine possible points of faults, vertical dashed lines (0-, 1-, 2-level) are drawn in Fig. 1 for their demarcation.

We apply procedures (1) and (2) to the given function *f*. If we fix log. 0 at the input x_1, which corresponds to "grounding", and observe the value of the function *f* when the values at the inputs x_2 and x_3, we get a pseudoperfect STF $Y^1_{x_1/0} = \{1,5\}^1$, then according to (1), we have:

$$Y^1_{x_1/0} \Rightarrow \begin{Bmatrix} (001),(101),\cancel{(110)},\cancel{(111)} \\ \cancel{(101)},\cancel{(101)},\cancel{(110)},\cancel{(111)} \end{Bmatrix}^{\oplus} \Rightarrow \{(001),(101)\}^1 .$$

Similarly, if we fix log. 1 at the input x_1, which corresponds to a high voltage level, then we will get a pseudoperfect STF $Y^1_{x_1/1} = \{1,2,3,5,6,7\}^1$, then according to (2), we have:

$$Y^1_{x_1/1} \Rightarrow \begin{Bmatrix} \cancel{(001)},(101),(110),(111) \\ \cancel{(001)},(001),(010),(011) \end{Bmatrix}^{\oplus} \Rightarrow \{(001),(010),$$

$$(011),(101),(110),(111)\}^1 .$$

From this we can conclude that the sought-test codes due to fault to s-a-0 at the input x_1 are binary minterms $\begin{pmatrix} 110 \\ 111 \end{pmatrix}^0$ that "moved" from the set Y^1 to the set $Y^0_{x_1/0}$, and when fault to s-a-1 at the input x_1 is minterms $\begin{pmatrix} 010 \\ 011 \end{pmatrix}^1$ that "moved" from Y^0 to $Y^1_{x_1/1}$.

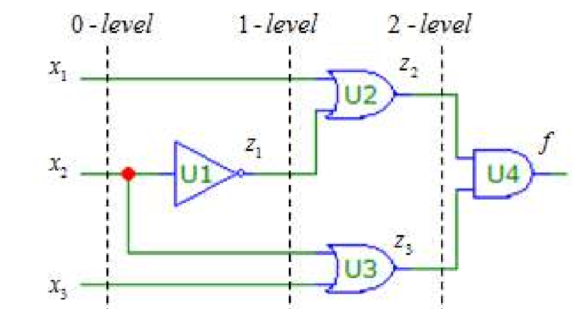

Fig. 1

Applying procedures (1) and (2) to the remaining variables, we obtain the following sets:

$$Y^1_{x_2/0} \Rightarrow \begin{Bmatrix} (001),(101),\cancel{(110)},\cancel{(111)} \\ (011),(111),\cancel{(110)},\cancel{(111)} \end{Bmatrix}^{\oplus} \Rightarrow \{(001),(011),$$

$$(101),(111)\}^1,$$

$$Y^1_{x_2/1} \Rightarrow \begin{Bmatrix} \cancel{(001)},\cancel{(101)},(110),(111) \\ \cancel{(001)},\cancel{(101)},(100),(101) \end{Bmatrix}^{\oplus} \Rightarrow \{(100),(101),$$

$$(110),(111)\}^1$$

$$Y^1_{x_3/0} \Rightarrow \begin{Bmatrix} \cancel{(001)},\cancel{(101)},(110),\cancel{(111)} \\ \cancel{(001)},\cancel{(101)},(111),\cancel{(111)} \end{Bmatrix}^{\oplus} \Rightarrow \{(110),(111)\}^1,$$

Table 2

Stuck-at-fault	x_1	x_2	x_3
s-a-0	$\begin{pmatrix}110\\111\end{pmatrix}^0$	$(110)^0$ $(011)^1$	$\begin{pmatrix}001\\101\end{pmatrix}^0$
s-a-1	$\begin{pmatrix}010\\011\end{pmatrix}^1$	$(001)^0$ $(100)^1$	$\begin{pmatrix}000\\100\end{pmatrix}^1$

$$Y^1_{x_3/1} \Rightarrow \begin{Bmatrix}(001),(101),(110),(111)\\(000),(100),(110),(110)\end{Bmatrix}^{\oplus} \Rightarrow \{(000),(001),(100),(101),(110),(111)\}^1$$

Table 1 contains the truth table of the given function $f(x_1,x_2,x_3)$, as well as the value $f_{x_i/0}$ in the case of fault s-a-0 and the value $f_{x_i/1}$ in the case of fault s-a-1 at the inputs x_i, $i=1,2,3$; values f_{x_i} different from the values of the function *f* are highlighted in bold.

So, at the 0-level for the function *f*, we will obtain the desired test codes after performing ST operations of differences (3)–(6), which reflect non-empty sets:

$Y^1 \setminus Y^1_{x_1/0} = \{1,5,6,7\}^1 \setminus \{1,5\}^1 =$
$= \{1,5,6,7\}^1 \cap \{0,2,3,4,6,7\}^0 = \{6,7\}^0$;

$Y^1_{x_1/0} \setminus Y^1 = \{1,5\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{1,5\}^1 \cap \{0,2,3,4\}^0 = \varnothing$;

$Y^1 \setminus Y^1_{x_1/1} = \{1,5,6,7\}^1 \setminus \{1,2,3,5,6,7\}^1 =$
$= \{1,5,6,7\}^1 \cap \{0,4\}^0 = \varnothing$;

$Y^1_{x_1/1} \setminus Y^1 = \{1,2,3,5,6,7\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{1,2,3,5,6,7\}^1 \cap \{0,2,3,4\}^0 = \{2,3\}^1$;

$Y^1 \setminus Y^1_{x_2/0} = \{1,5,6,7\}^1 \setminus \{1,3,5,7\}^1 =$
$= \{1,5,6,7\}^1 \cap \{0,2,4,6\}^0 = \{6\}^0$;

$Y^1_{x_2/0} \setminus Y^1 = \{1,3,5,7\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{1,3,5,7\}^1 \cap \{0,2,3,4\}^0 = \{3\}^1$;

$Y^1 \setminus Y^1_{x_2/1} = \{1,5,6,7\}^1 \setminus \{4,5,6,7\}^1 =$
$= \{1,5,6,7\}^1 \cap \{0,1,2,3\}^0 = \{1\}^0$;

$Y^1_{x_2/1} \setminus Y^1 = \{4,5,6,7\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{4,5,6,7\}^1 \cap \{0,2,3,4\}^0 = \{4\}^1$;

$Y^1 \setminus Y^1_{x_3/0} = \{1,5,6,7\}^1 \setminus \{6,7\}^1 =$
$= \{1,5,6,7\}^1 \cap \{0,1,2,3,4,5\}^0 = \{1,5\}^0$;

$Y^1_{x_3/0} \setminus Y^1 = \{6,7\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{6,7\}^1 \cap \{0,2,3,4\}^0 = \varnothing$;

$Y^1 \setminus Y^1_{x_3/1} = \{1,5,6,7\}^1 \setminus \{0,1,4,5,6,7\}^1 =$
$= \{1,5,6,7\}^1 \cap \{2,3\}^0 = \varnothing$;

$Y^1_{x_3/1} \setminus Y^1 = \{0,1,4,5,6,7\}^1 \setminus \{1,5,6,7\}^1 =$
$= \{0,1,4,5,6,7\}^1 \cap \{0,2,3,4\}^0 = \{0,4\}^1$.

Table 2 contains the vectors of test codes at the 0-level of the investigated circuit described by the function $f(x_1,x_2,x_3) = x_1x_2 \vee \bar{x}_2x_3$. The superscript (0 or 1) in these vectors symbolizes the value $f=0$ or $f=1$. Note, that some codes may be repeated, such as the code $(110)^0$ (see Tables 1 and 2). In this case, it is necessary to identify the place and the type of fault to the circuit only when $f=0$ taking into account the second code, namely: the fault to s-a-0 at the input x_1 will occur if there is also at code $(111)^0$, and the fault to s-a-0 at the input x_2 will occur if $f=0$ is at code $(110)^0$, and $f=1$ is at code $(111)^0$.

Now we will demonstarte the formation of test codes on the 1-level of the investigated circuit. To do this, we consider a function $f(x_1,z_1,x_3) = x_1\bar{z}_1 \vee z_1x_3 \Rightarrow \{(10-),(-11)\}^1$ that has a perfect STF $Y^1 = \{(011),(100),(101),(111)\}^1$.

As a result of applying procedures (1) and (2) on the 1-level of the circuit, we have[1]:

$$Y^1_{x_1/0} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(111),(100),(101),(111)\end{Bmatrix}^{\oplus} \Rightarrow \{(011),(111)\}^1;$$

$$Y^1_{x_1/1} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(011),(000),(001),(011)\end{Bmatrix}^{\oplus} \Rightarrow \{(000),(001),(011),(100),(101),(111)\}^1;$$

$$Y^1_{z_1/0} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(011),(110),(111),(111)\end{Bmatrix}^{\oplus} \Rightarrow \{(100),(101),(110),(111)\}^1;$$

$$Y^1_{z_1/1} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(001),(100),(101),(101)\end{Bmatrix}^{\oplus} \Rightarrow \{(001),(011),(101),(111)\}^1;$$

1 Here and in the future we will omit the crossing out of identical pairs of minterms.

Table 3

«10»	$x_1\ z_1\ x_3$	f	f_{x_1}		f_{z_1}		f_{x_3}	
			$f_{x_1/0}$	$f_{x_1/1}$	$f_{z_1/0}$	$f_{z_1/1}$	$f_{x_3/0}$	$f_{x_3/1}$
0	0 0 0	0	0	**1**	0	0	0	0
1	0 0 1	0	0	**1**	0	**1**	0	0
2	0 1 0	0	0	0	0	0	0	**1**
3	0 1 1	1	1	**1**	**0**	1	**0**	1
4	1 0 0	1	0	1	1	**0**	1	1
5	1 0 1	1	**0**	1	1	1	1	1
6	1 1 0	0	**0**	0	**1**	0	0	**1**
7	1 1 1	1	1	1	**1**	1	**0**	1

$$Y^1_{x_3/0} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(011),(101),(101),(111)\end{Bmatrix}^{\oplus} \Rightarrow \{(100),(101)\}^1,$$

$$Y^1_{x_3/1} \Rightarrow \begin{Bmatrix}(011),(100),(101),(111)\\(010),(100),(100),(110)\end{Bmatrix}^{\oplus} \Rightarrow \{(010),(011),$$
$$(100),(101),(110),(111)\}^1.$$

Let's construct a Table 3 containing the truth table of the function $f(x_1, z_1, x_3)$ and the values of the functions $f_{x_1/0}$ and $f_{x_1/1}$, $f_{z_1/0}$ and $f_{z_1/1}$, $f_{x_3/0}$ and $f_{x_3/1}$, obtained on the basis of their pseudoperfect STFs $Y^1_{x_1/0}$ and $Y^1_{x_1/1}$, $Y^1_{z_1/0}$ and $Y^1_{z_1/1}$, $Y^1_{x_3/0}$ and $Y^1_{x_3/1}$.

Applying the operations (3)–(6), we obtain the following non-empty sets of test codes on the 1 level of the circuit:

$Y^1 \setminus Y^1_{x_1/0} = \{3,4,5,7\}^1 \setminus \{3,7\}^1 =$
$= \{3,4,5,7\}^1 \cap \{0,1,2,4,5,6\}^0 = \{4,5\}^0,$

$Y^1_{x_1/0} \setminus Y^1 = \{3,7\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{3,7\}^1 \cap \{0,1,2,6\}^0 = \varnothing,$

$Y^1 \setminus Y^1_{x_1/1} = \{3,4,5,7\}^1 \setminus \{0,1,3,4,5,7\}^1 =$
$= \{3,4,5,7\}^1 \cap \{2,6\}^0 = \varnothing,$

$Y^1_{x_1/1} \setminus Y^1 = \{0,1,3,4,5,7\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{0,1,3,4,5,7\}^1 \cap \{0,1,2,6\}^0 = \{0,1\}^1,$

$Y^1 \setminus Y^1_{z_1/0} = \{3,4,5,7\}^1 \setminus \{4,5,6,7\}^1 =$
$= \{3,4,5,7\}^1 \cap \{0,1,2,3\}^0 = \{3\}^0,$

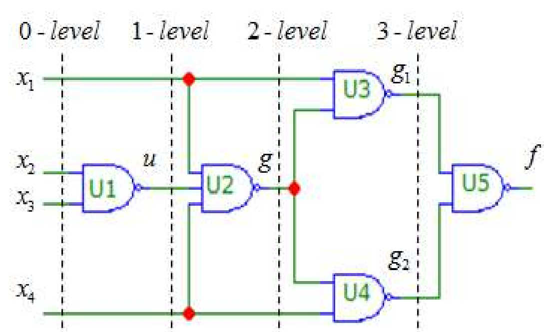

Fig. 2

$Y^1_{z_1/0} \setminus Y^1 = \{4,5,6,7\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{4,5,6,7\}^1 \cap \{0,1,2,6\}^0 = \{6\}^1,$

$Y^1 \setminus Y^1_{z_1/1} = \{3,4,5,7\}^1 \setminus \{1,3,5,7\}^1 =$
$= \{3,4,5,7\}^1 \cap \{0,2,4,6\}^0 = \{4\}^0,$

$Y^1_{z_1/1} \setminus Y^1 = \{1,3,5,7\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{1,3,5,7\}^1 \cap \{0,1,2,6\}^0 = \{1\}^1,$

$Y^1 \setminus Y^1_{x_3/0} = \{3,4,5,7\}^1 \setminus \{4,5\}^1 =$
$= \{3,4,5,7\}^1 \cap \{0,1,2,3,6,7\}^0 = \{3,7\}^0,$

$Y^1_{x_3/0} \setminus Y^1 = \{4,5\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{4,5\}^1 \cap \{0,1,2,6\}^0 = \varnothing,$

$Y^1 \setminus Y^1_{x_3/1} = \{3,4,5,7\}^1 \setminus \{2,3,4,5,6,7\}^1 =$
$= \{3,4,5,7\}^1 \cap \{0,1\}^0 = \varnothing,$

$Y^1_{x_3/1} \setminus Y^1 = \{2,3,4,56,7\}^1 \setminus \{3,4,5,7\}^1 =$
$= \{2,3,4,5,6,7\}^1 \cap \{0,1,2,6\}^0 = \{2,6\}^1.$

Table 4

Stuck-at-fault	x_1	x_2	x_3
s-a-0	$\begin{pmatrix}100\\101\end{pmatrix}^0$	$(011)^0$ $(110)^1$	$\begin{pmatrix}011\\111\end{pmatrix}^0$
s-a-1	$\begin{pmatrix}000\\001\end{pmatrix}^1$	$(001)^0$ $(100)^1$	$\begin{pmatrix}010\\110\end{pmatrix}^1$

Table 4 contains the vectors of test codes for the function $f(x_1, z_1, x_3)$ on the 1-level of the circuit.

On the 2-level of the circuit (see Fig. 1) we have a function $f(z_2, z_3) = z_2 z_3 \Rightarrow \{11\}^1$ for which the Table 5 is valid and taking into account the procedures (1) and (2) in the case of stuck-at-fault (0/1):

$$Y^1_{z_2/0} \Rightarrow \begin{Bmatrix}(11)\\(11)\end{Bmatrix}^{\oplus} \Rightarrow \varnothing,\ Y^1_{z_2/1} \Rightarrow \begin{Bmatrix}(11)\\(01)\end{Bmatrix}^{\oplus} \Rightarrow \{(01),(11)\}^1;$$

$$Y^1_{z_3/0} \Rightarrow \begin{Bmatrix}(11)\\(11)\end{Bmatrix}^{\oplus} \Rightarrow \varnothing,\ Y^1_{z_3/1} \Rightarrow \begin{Bmatrix}(11)\\(10)\end{Bmatrix}^{\oplus} \Rightarrow \{(10),(11)\}^1.$$

Table 6 of test codes on the 2-level of the circuit will be obtained after applying the operation to certain sets (3–6):

$$Y^1 \setminus Y^1_{z_2/0} = \{3\}^1 \setminus \varnothing = \{3\}^0,$$
$$Y^1_{z_2/0} \setminus Y^1 = \varnothing \setminus \{3\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{z_2/1} = \{3\}^1 \setminus \{1,3\}^1 = \varnothing,$$
$$Y^1_{z_2/1} \setminus Y^1 = \{1,3\}^1 \setminus \{3\}^1 = \{1\}^1,$$
$$Y^1 \setminus Y^1_{z_3/0} = \{3\}^1 \setminus \varnothing = \{3\}^0,$$
$$Y^1_{z_3/0} \setminus Y^1 = \varnothing \setminus \{3\}^1 = \varnothing,$$

Table 5

«10»	$x_1\ x_2\ x_3$	f	f_{x_1}		f_{x_2}	
			$f_{x_1/0}$	$f_{x_1/1}$	$f_{x_2/0}$	$f_{x_2/1}$
0	0 0	0	0	0	0	0
1	0 1	0	0	**1**	0	0
2	1 0	0	0	0	0	**1**
3	1 1	1	**0**	1	**0**	1

$$Y^1 \setminus Y^1_{z_3/1} = \{3\}^1 \setminus \{2,3\}^1 = \varnothing,$$
$$Y^1_{z_3/1} \setminus Y^1 = \{2,3\}^1 \setminus \{3\}^1 = \{2\}^1.$$

On the basis of Tables 2, 4 and 6, containing the vectors of test codes, it is possible to build a map of possible faults of the stuck-at-fault (0/1) type at an arbitrary point in the circuit as shown in Fig. 1.

Example 2. Determine the test code vectors for detection the stuck-at-fault (0/1) in the logic circuit shown in Fig. 2. Logical elements of the circuit are described by the following equations (borrowed from [7]):

U5: $f = \overline{g_1 g_2}$; U4: $g_2 = \overline{x_4 g}$; U3: $g_1 = \overline{x_1 g}$;
U2: $g = = \overline{x_1 u x_4}$; U1: $u = \overline{x_2 x_3}$.

Solution. We obtain the function from the given equations

$$f(x_1, x_2, x_3, x_4) = \overline{g_1 g_2} = x_1 g \vee x_4 g =$$
$$= (x_1 \vee x_4)(\overline{x}_1 \vee \overline{x}_4 \vee \overline{u}) = x_1 \overline{x}_4 \vee \overline{x}_1 x_4 \vee x_1 x_2 x_3 \vee$$
$$\vee x_2, x_3, x_4 \Rightarrow \{(1{-}{-}0),(0{-}{-}1),(111{-}),({-}111)\}^1.$$

Therefore, the function *f* has a perfect STF $Y^1 = \{(0001),(0011),(0101),(0111),(1000),(1010),(1100),(1110),(1111)\}^1$.

As a result of the application of procedures (1) and (2), we will get pseudoperfect STFs:

$$Y^1_{x_1/0} \Rightarrow \begin{Bmatrix}(0001),(0011),(0101),(0111),(1000),(1010),(1100),(1110),(1111)\\(1001),(1011),(1101),(1111),(1000),(1010),(1100),(1110),(1111)\end{Bmatrix}^{\oplus} \Rightarrow$$
$$\Rightarrow \{(0001),(0011),(0101),(0111),(1001),(1011),(1101),(1111)\}^1$$

$$Y^1_{x_1/1} \Rightarrow \begin{Bmatrix}(0001),(0011),(0101),(0111),(1000),(1010),(1100),(1110),(1111)\\(0001),(0011),(0101),(0111),(0000),(0010),(0100),(0110),(0111)\end{Bmatrix}^{\oplus} \Rightarrow$$
$$\Rightarrow \{(0000),(0010),(0100),(0110),(0111),(1000),(1010),(1100),(1110),(1111)\}^1$$

Table 6

Stuck-at-fault	x_1	x_2
s-a-0	$(11)^0$	$(11)^0$
s-a-1	$(01)^1$	$(10)^1$

Table 7

Stuck-at-fault	x_1	x_2	x_3	x_4
s-a-0	$\begin{pmatrix}1000\\1010\\1100\\1110\end{pmatrix}^0 \begin{pmatrix}1001\\1011\\1101\end{pmatrix}^1$	$(1111)^0$	$(1111)^0$	$\begin{pmatrix}0001\\0011\\0101\\0111\end{pmatrix}^0 \begin{pmatrix}1001\\1011\\1101\end{pmatrix}^1$
s-a-1	$\begin{pmatrix}0001\\0011\\0101\end{pmatrix}^0 \begin{pmatrix}0000\\0010\\0100\\0110\end{pmatrix}^1$	$(1011)^1$	$(1101)^1$	$\begin{pmatrix}1000\\1010\\1100\end{pmatrix}^0 \begin{pmatrix}0000\\0010\\0100\\0110\end{pmatrix}^1$

Table 8

Stuck-at-fault	x_1	u	x_4
s-a-0	$\begin{pmatrix}100\\110\end{pmatrix}^0 (111)^1$	$(111)^1$	$\begin{pmatrix}001\\011\end{pmatrix}^0 (111)^1$
s-a-1	$\begin{pmatrix}000\\010\end{pmatrix}^1 (011)^0$	$(101)^0$	$\begin{pmatrix}000\\010\end{pmatrix}^1 (110)^0$

Table 9

Stuck-at-fault	x_1	g	x_4
s-a-0	$(110)^0$	$\begin{pmatrix}011\\110\\111\end{pmatrix}^0$	$(011)^0$
s-a-1	$(010)^1$	$\begin{pmatrix}001\\100\\101\end{pmatrix}^1$	$(010)^1$

Continuing to carry out the procedures (1) and (2) by analogy, we will obtain the corresponding sets:

$Y^1_{x_2/0} = \{(0001),(0011),(0101),(0111),(1000),(1100),(1110)\}^1$,

$Y^1_{x_2/1} = \{(0001),(0011),(0101),(0111),(1000),(1011),(1100),(1110),(1111)\}^1$,

$Y^1_{x_3/0} = \{(0001),(0011),(0101),(0111),(1000),(1100),(1110)\}^1$,

$Y^1_{x_3/1} = \{(0001),(0011),(0101),(0111),(1000),(1100),(1101),(1110),(1111)\}^1$,

$Y^1_{x_4/0} = \{(1000),(1001),(1010),(1011),(1100),(1110),(1111)\}^1$,

$Y^1_{x_4/1} = \{(0000),(0001),(0010),(0011),(0100),(0110),(0111),(1110),(1111)\}^1$.

Applying the pseudoperfect STFs $Y^1_{x_1/0}$ and $Y^1_{x_1/1}$, $Y^1_{x_2/0}$ and $Y^1_{x_2/1}$, $Y^1_{x_3/0}$ and $Y^1_{x_3/1}$, $Y^1_{x_4/0}$ and $Y^1_{x_4/1}$ to the operations (3)–(6), we obtain the desired sets of test codes on the 0-level of the circuit:

$Y^1 \setminus Y^1_{x_1/0} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \setminus \{1,3,5,7,9,11,13,15\}^1 = \{8,10,12,14\}^0$,

Table 10

Stuck-at-fault	g_1	g_2
s-a-0	$(11)^1$	$(11)^1$
s-a-1	$(01)^0$	$(10)^0$

$$Y^1_{x_1/0} \setminus Y^1 = \{1,3,5,7,9,11,13,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{9,11,13\}^1,$$
$$Y^1 \setminus Y^1_{x_1/1} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{0,2,4,6,7,8,10,12,14,15\}^1 = \{1,3,5\}^0,$$
$$Y^1_{x_1/1} \setminus Y^1 = \{0,2,4,6,7,8,10,12,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{0,2,4,6\}^1,$$
$$Y^1 \setminus Y^1_{x_2/0} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14\}^1 = \{15\}^0,$$
$$Y^1_{x_2/0} \setminus Y^1 = \{1,3,5,7,8,10,12,14\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{x_2/1} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{1,3,5,7,8,10,11,12,14,15\}^1 = \varnothing,$$
$$Y^1_{x_2/1} \setminus Y^1 = \{1,3,5,7,8,10,11,12,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{11\}^1,$$
$$Y^1 \setminus Y^1_{x_3/0} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14\}^1 = \{15\}^0,$$
$$Y^1_{x_3/0} \setminus Y^1 = \{1,3,5,7,8,10,12,14\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{x_3/1} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{1,3,5,7,8,10,12,13,14,15\}^1 = \varnothing,$$
$$Y^1_{x_3/1} \setminus Y^1 = \{1,3,5,7,8,10,12,13,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{13\}^1,$$
$$Y^1 \setminus Y^1_{x_4/0} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{8,9,10,11,12,13,14,15\}^1 = \{1,3,5,7\}^0,$$
$$Y^1_{x_4/0} \setminus Y^1 = \{8,9,10,11,12,13,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{9,11,13\}^1,$$
$$Y^1 \setminus Y^1_{x_4/1} = \{1,3,5,7,8,10,12,14,15\}^1 \setminus \{0,1,2,3,4,5,6,7,14,15\}^1 = \{8,10,12\}^0,$$
$$Y^1_{x_4/1} \setminus Y^1 = \{0,1,2,3,4,5,6,7,14,15\}^1 \setminus \{1,3,5,7,8,10,12,14,15\}^1 = \{0,2,4,6\}^1.$$

Table 7 contains the vectors of test codes on the 0-level of the circuit at inputs x_1, x_2, x_3 and x_4.

Consider the given function f on the 1-level of the circuit as a function of three variables:

$$f(x_1,u,x_4) = (x_1 \vee x_4)(\overline{x}_1 \vee \overline{x}_4 \vee \overline{u}) = x_1\overline{x}_4 \vee x_1\overline{u} \vee \overline{x}_1 x_4 \vee \overline{u}x_4 \Rightarrow \{(1-0),(10-),(0-1),(-01)\}^1.$$

Hence, we have a perfect STF $Y^1 = \{(001),(011),(100),(101),(110)\}^1$.

After applying the procedures (1) and (2), we will get the following sets:

$$Y^1_{x_1/0} = \{(001),(011),(101),(111)\}^1,$$
$$Y^1_{x_1/1} = \{(000),(001),(010),(100),(101),(110)\}^1,$$
$$Y^1_{u/0} = \{(001),(011),(100),(101),(110),(111)\}^1,$$
$$Y^1_{u/1} = \{(001),(011),(100),(110)\}^1,$$
$$Y^1_{x_4/0} = \{(100),(101),(110),(111)\}^1,$$
$$Y^1_{x_4/1} = \{(000),(001),(010),(011),(100),(101)\}^1.$$

After applying the (3–6), we obtain a set of test codes on the 1-level of the circuit:

$$Y^1 \setminus Y^1_{x_1/0} = \{1,3,4,5,6\}^1 \setminus \{1,3,5,7\}^1 = \{1,3,4,5,6\}^1 \cap \{0,2,4,6\}^1 = \{4,6\}^0,$$
$$Y^1_{x_1/0} \setminus Y^1 = \{1,3,5,7\}^1 \setminus \{1,3,4,5,6\}^1 = \{1,3,5,7\}^1 \cap \{0,2,7\}^1 = \{7\}^1,$$
$$Y^1 \setminus Y^1_{x_1/1} = \{1,3,4,5,6\}^1 \setminus \{0,1,2,4,5,6\}^1 = \{1,3,4,5,6\}^1 \cap \{3,7\}^1 = \{3\}^0,$$
$$Y^1_{x_1/1} \setminus Y^1 = \{0,1,2,4,5,6\}^1 \setminus \{1,3,4,5,6\}^1 = \{0,1,2,4,5,6\}^1 \cap \{0,2,7\}^1 = \{0,2\}^1,$$
$$Y^1 \setminus Y^1_{u/0} = \{1,3,4,5,6\}^1 \setminus \{1,3,4,5,6,7\}^1 = \{1,3,4,5,6\}^1 \cap \{0,2\}^1 = \varnothing,$$
$$Y^1_{u/0} \setminus Y^1 = \{1,3,4,5,6,7\}^1 \setminus \{1,3,4,5,6\}^1 = \{1,3,4,5,6,7\}^1 \cap \{0,2,7\}^1 = \{7\}^1,$$
$$Y^1 \setminus Y^1_{u/1} = \{1,3,4,5,6\}^1 \setminus \{1,3,4,6\}^1 = \{1,3,4,5,6\}^1 \cap \{0,2,5,7\}^1 = \{5\}^0,$$
$$Y^1_{u/1} \setminus Y^1 = \{1,3,4,6\}^1 \setminus \{1,3,4,5,6\}^1 = \{1,3,4,6\}^1 \cap \{0,2,7\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{x_4/0} = \{1,3,4,5,6\}^1 \setminus \{4,5,6,7\}^1 = \{1,3,4,5,6\}^1 \cap \{0,1,2,3\}^1 = \{1,3\}^0,$$
$$Y^1_{x_4/0} \setminus Y^1 = \{4,5,6,7\}^1 \setminus \{1,3,4,5,6\}^1 = \{4,5,6,7\}^1 \cap \{0,2,7\}^1 = \{7\}^1,$$
$$Y^1 \setminus Y^1_{x_4/1} = \{1,3,4,5,6\}^1 \setminus \{0,1,2,3,4,5\}^1 = \{1,3,4,5,6\}^1 \cap \{6,7\}^1 = \{6\}^0,$$
$$Y^1_{x_4/1} \setminus Y^1 = \{0,1,2,3,4,5\}^1 \setminus \{1,3,4,5,6\}^1 = \{0,1,2,3,4,5\}^1 \cap \{0,2,7\}^1 = \{0,2\}^1.$$

Table 11

Stuck-at-fault	*x*	*y*	*z*	*w*
s-a-0	$\begin{pmatrix}1101\\1110\\1111\end{pmatrix}^1$	$\begin{pmatrix}1101\\1110\\1111\end{pmatrix}^1$	$\begin{pmatrix}0010\\0110\\1010\end{pmatrix}^0$	$\begin{pmatrix}0001\\0101\\1001\end{pmatrix}^0$
s-a-1	$\begin{pmatrix}0101\\0110\\0111\end{pmatrix}^0$	$\begin{pmatrix}1001\\1010\\1011\end{pmatrix}^0$	$\begin{pmatrix}0000\\0100\\1000\end{pmatrix}^1$	$\begin{pmatrix}0000\\0100\\1000\end{pmatrix}^1$

Table 12

Stuck-at-fault	*u*	*z*	*w*
s-a-0	$\begin{pmatrix}101\\110\\111\end{pmatrix}^0$	$(110)^0$	$(101)^0$
s-a-1	$\begin{pmatrix}001\\010\\011\end{pmatrix}^1$	$(100)^1$	$(100)^1$

Table 8 contains the test code vectors on the 1-level of the circuit for the function $f(x_1, u, x_4)$.

Test codes on the 2-level of the circuit are obtained for the function $f(x_1, g, x_4) = x_1 g \vee g x_4 \Rightarrow \Rightarrow \{(11-),(-11)\}^1$ that has a perfect STF $Y^1 = = \{(011),(110),(111)\}^1$. Therefore, based on procedures (1) and (2), we have:

$$Y^1_{x_1/0} = \{(011),(111)\}^1,$$
$$Y^1_{x_1/1} = \{(010),(011),(110),(111)\}^1,$$
$$Y^1_{g/0} = \varnothing,$$
$$Y^1_{g/1} = \{(001),(011),(100),(101),(110),(111)\}^1,$$
$$Y^1_{x_4/0} = \{(110),(111)\}^1,$$
$$Y^1_{x_4/1} = \{(010),(011),(110),(111)\}^1.$$

Applying the (3) – 6), we obtain the following sets of test codes on the 2-level of the circuit:

Table 13

Stuck-at-fault	g_1	g_2
s-a-0	$(10)^0$	$(01)^0$
s-a-1	$(00)^1$	$(00)^1$

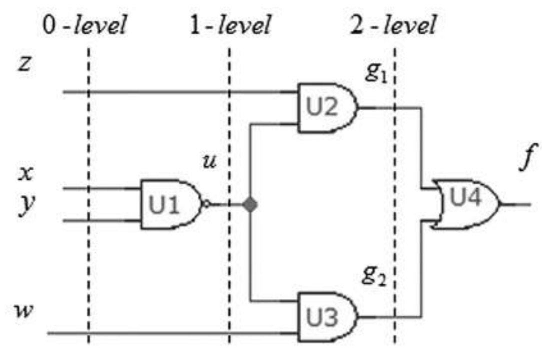

Fig. 3

$$Y^1 \setminus Y^1_{x_1/0} = \{3,6,7\}^1 \setminus \{3,7\}^1 = \{6\}^1,$$
$$Y^1_{x_1/0} \setminus Y^1 = \{3,7\}^1 \setminus \{3,6,7\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{x_1/1} = \{3,6,7\}^1 \setminus \{2,3,6,7\}^1 = \varnothing,$$
$$Y^1_{x_1/1} \setminus Y^1 = \{2,3,6,7\}^1 \setminus \{3,6,7\}^1 = \{2\}^1,$$
$$Y^1 \setminus Y^1_{g/0} = \{3,6,7\}^1 \setminus \varnothing = \{3,6,7\}^0,$$
$$Y^1_{g_1/0} \setminus Y^1 = \varnothing \setminus \{3,6,7\}^1 = \varnothing,$$
$$Y^1 \setminus Y^1_{g/1} = \{3,6,7\}^1 \setminus \{1,3,4,5,6,7\}^1 = \varnothing,$$
$$Y^1_{g/1} \setminus Y^1 = \{1,3,4,5,6,7\}^1 \setminus \{3,6,7\}^1 = \{1,4,5\}^1,$$
$$Y^1 \setminus Y^1_{x_4/0} = \{3,6,7\}^1 \setminus \{6,7\}^1 = \{3\}^0,$$
$$Y^1_{x_4/0} \setminus Y^1 = \{6,7\}^1 \setminus \{3,6,7\}^1 = \varnothing,$$

$Y^1 \setminus Y^1_{x_4/1} = \{3,6,7\}^1 \setminus \{2,3,6,7\}^1 = \varnothing$,

$Y^1_{x_4/1} \setminus Y^1 = \{2,3,6,7\}^1 \setminus \{3,6,7\}^1 = \{2\}^1$.

Found vectors of test codes on the 2-level of the circuit are presented in Table 9 for the function $f(x_1, g, x_4)$.

On the 3-level of the circuit, we have the function $f(g_1, g_2) = \overline{g_1 g_2} = \overline{g}_1 \vee \overline{g}_2 \Rightarrow \{(0-),(-0)\}^1$ and its perfect STF $Y^1 = \{(00),(01),(10)\}^1$. After applying of the procedures (1) and (2), we have:

$$Y^1_{g_1/0} \Rightarrow \begin{Bmatrix}(00),(01),(10)\\(10),(11),(10)\end{Bmatrix}^{\oplus} \Rightarrow \{(00),(01),(10),(11)\}^1,$$

$$Y^1_{g_1/1} \Rightarrow \begin{Bmatrix}(00),(01),(10)\\(00),(01),(00)\end{Bmatrix}^{\oplus} \Rightarrow \{(00),(10)\}^1,$$

$$Y^1_{g_2/0} \Rightarrow \begin{Bmatrix}(00),(01),(10)\\(01),(01),(11)\end{Bmatrix}^{\oplus} \Rightarrow \{(00),(01),(10),(11)\},$$

$$Y^1_{g_2/1} \Rightarrow \begin{Bmatrix}(00),(01),(10)\\(00),(00),(10)\end{Bmatrix}^{\oplus} \Rightarrow \{(00),(01)\}^1.$$

We determine the sets of test codes on the basis of the (3)–(6) and construct the Table 10 accordingly:

$Y^1 \setminus Y^1_{g_1/0} = \{0,1,2\}^1 \setminus \{0,1,2,3\}^1 = \varnothing$,

$Y^1_{g_1/0} \setminus Y^1 = \{0,1,2,3\}^1 \setminus \{0,1,2\}^1 = \{3\}^1$,

$Y^1 \setminus Y^1_{g_1/1} = \{0,1,2\}^1 \setminus \{0,2\}^1 = \{1\}^0$,

$Y^1_{g_1/1} \setminus Y^1 = \{0,2\}^1 \setminus \{0,1,2\}^1 = \varnothing$;

$Y^1 \setminus Y^1_{g_2/0} = \{0,1,2\}^1 \setminus \{0,1,2,3\}^1 = \varnothing$,

$Y^1_{g_2/0} \setminus Y^1 = \{0,1,2,3\}^1 \setminus \{0,1,2\}^1 = \{3\}^1$,

$Y^1 \setminus Y^1_{g_2/1} = \{0,1,2\}^1 \setminus \{0,1\}^1 = \{2\}^0$,

$Y^1_{g_2/1} \setminus Y^1 = \{0,1\}^1 \setminus \{0,1,2\}^1 = \varnothing$.

There is possible to determine the location and the type of possible stuck-at-fault (0/1) in the given logic circuit shown in Fig. 2 by using the test code vectors shown in Tables 7 –10.

Example 3. Determine the vectors of test codes for detection stuck-at-fault (0/1) in the logic circuit shown in Fig. 3, described by a function $f(x,y,z,w) = \overline{(xy)}(z \vee w)$ (*borrowed from* [4]).

Solution. Let's turn the given function f into the set-theoretical form: $f(x,y,z,w) = \overline{xy}(z \vee w) = (\overline{x} \vee \overline{y})(z \vee w) = \overline{x}z \vee \overline{x}w \vee \overline{y}z \vee \overline{y}w \Rightarrow \{(0-1-),(0--1),(-01-),(-0-1)\}^1$.

Hence the perfect STF $Y^1 = \{(0001),(0010),(0011),(0101),(0110),(0111),(1001),(1010),(1011)\}^1$.

As a result of the application of the procedures (1) and (2), we will obtain the pseudoperfect STFs on the 0-level of the circuit:

$Y^1_{x/0} = \{(0001),(0010),(0011),(0101),(0110),(0111),(1001),(1010),(1011),(1101),(1110),(1111)\}^1$,

$Y^1_{x/1} = \{(0001),(0010),(0011),(1001),(1010),(1011)\}^1$,

$Y^1_{y/0} = \{(0001),(0010),(0011),(0101),(0110),(0111),(1001),(1010),(1011),(1101),(1110),(1111)\}^1$,

$Y^1_{y/1} = \{(0001),(0010),(0011),(0101),(0110),(0111)\}^1$;

$Y^1_{z/0} = \{(0001),(0011),(0101),(0111),(1001),(1011)\}^1$,

$Y^1_{z/1} = \{(0000),(0001),(0010),(0011),(0100),(0110),(0111),(1000),(1001),(1010),(1011)\}^1$;

$Y^1_{w/0} = \{(0010),(0011),(0110),(0111),(1010),(1011)\}^1$,

$Y^1_{w/1} = \{(0000),(0001),(0010),(0011),(0100),(0101),(0110),(0111),(1000),(1001),(1010),(1011)\}^1$.

Applying the pseudoperfect STFs $Y^1_{x/0}$ and $Y^1_{x/1}$, $Y^1_{y/0}$ and $Y^1_{y/1}$, $Y^1_{z/0}$ and $Y^1_{z/1}$, $Y^1_{w/0}$ and $Y^1_{w/1}$ to the operations (3)– (6), we obtain sets of test codes on the 0-level of the circuit:

$Y^1 \setminus Y^1_{x/0} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus \{1,2,3,5,6,7,9,10,11,13,14,15\}^1 = \varnothing$,

$Y^1_{x/0} \setminus Y^1 = \{1,2,3,5,6,7,9,10,11,13,14,15\}^1 \setminus \{1,2,3,5,6,7,9,10,11\}^1 = \{13,14,15\}^1$,

$Y^1 \setminus Y^1_{x/1} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus \{1,2,3,9,10,11\}^1 = \{5,6,7\}^0$,

$Y^1_{x/1} \setminus Y^1 = \{1,2,3,9,10,11\}^1 \setminus \{1,2,3,5,6,7,9,10,11\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{y/0} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus \{1,2,3,5,6,7,9,10,11,13,14,15\}^1 = \varnothing$,

$Y^1_{y/0} \setminus Y^1 = \{1,2,3,5,6,7,9,10,11,13,14,15\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \{13,14,15\}^1$,

$Y^1 \setminus Y^1_{y/1} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus$
$\setminus \{1,2,3,5,6,7\}^1 = \{9,10,11\}^0$,

$Y^1_{y/1} \setminus Y^1 = \{1,2,3,5,6,7\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \varnothing$;

$Y^1 \setminus Y^1_{z/0} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus$
$\setminus \{1,3,5,7,9,11\}^1 = \{2,6,10\}^0$,

$Y^1_{z/0} \setminus Y^1 = \{1,3,5,7,9,11\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{z/1} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus$
$\setminus \{0,1,2,3,4,5,6,7,8,9,10,11\}^1 = \varnothing$,

$Y^1_{z/1} \setminus Y^1 = \{0,1,2,3,4,5,6,7,8,9,10,11\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \{0,4,8\}^1$;

$Y^1 \setminus Y^1_{w/0} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus$
$\setminus \{2,3,6,7,10,11\}^1 = \{1,5,9\}^0$,

$Y^1_{w/0} \setminus Y^1 = \{2,3,6,7,10,11\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{w/1} = \{1,2,3,5,6,7,9,10,11\}^1 \setminus$
$\setminus \{0,1,2,3,4,5,6,7,8,9,10,11\}^1 = \varnothing$,

$Y^1_{w/1} \setminus Y^1 = \{0,1,2,3,4,5,6,7,8,9,10,11\}^1 \setminus$
$\setminus \{1,2,3,5,6,7,9,10,11\}^1 = \{0,4,8\}^1$.

Table 11 contains the vectors of test codes on the 0-level of the circuit.

On the 1-level of the circuit we have the function $f(u,z,w) = u(z \vee w)$. We perform the procedures (1) and (2) over the minterms of its perfect STF $Y^1 = \{(101),(110),(111)\}^1$:

$Y^1_{u/0} = \varnothing$,

$Y^1_{u/1} = \{(001),(010),(011,(101),(110),(111)\}$,

$Y^1_{z/0} = \{(101),(111)\}^1$,

$Y^1_{z/1} = \{(100),(101),(110),(111)\}$,

$Y^1_{w/0} = \{(110),(111)\}^1$,

$Y^1_{w/1} = \{(100),(101),(110),(111)\}^1$.

To determine sets of test codes on the 1-level of the circuit, we apply the operations (3)–(6):

$Y^1 \setminus Y^1_{u/0} = \{5,6,7\}^1 \setminus \{\varnothing\}^1 = \{5,6,7\}^0$,

$Y^1_{u/0} \setminus Y^1 = \varnothing$,

$Y^1 \setminus Y^1_{u/1} = \{5,6,7\}^1 \setminus \{1,2,3,5,6,7\}^1 = \varnothing$,

$Y^1_{u/1} \setminus Y^1 = \{1,2,3,5,6,7\}^1 \setminus \{5,6,7\}^1 = \{1,2,3\}^1$;

$Y^1 \setminus Y^1_{z/0} = \{5,6,7\}^1 \setminus \{5,7\}^1 = \{6\}^0$,

$Y^1_{z/0} \setminus Y^1 = \{5,7\}^1 \setminus \{5,6,7\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{z/1} = \{5,6,7\}^1 \setminus \{4,5,6,7\}^1 = \varnothing$,

$Y^1_{z/1} \setminus Y^1 = \{4,5,6,7\}^1 \setminus \{5,6,7\}^1 = \{4\}^1$;

$Y^1 \setminus Y^1_{w/0} = \{5,6,7\}^1 \setminus \{6,7\}^1 = \{5\}^0$,

$Y^1_{w/0} \setminus Y^1 = \{6,7\}^1 \setminus \{5,6,7\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{w/1} = \{5,6,7\}^1 \setminus \{4,5,6,7\}^1 = \varnothing$,

$Y^1_{w/1} \setminus Y^1 = \{4,5,6,7\}^1 \setminus \{5,6,7\}^1 = \{4\}^1$.

Table 12 contains stuck-at-fault (0/1) test codes on the 1-level of the circuit.

The investigated circuit on the 2-level is described by the function $f(g_1, g_2) = g_1 \vee g_2 \Rightarrow \{(1-),(-1)\}^1$ that corresponds to the perfect STF $Y^1 = \{(01),(10),(11)\}^1$. Applying the (1) and (2), we obtain:

$Y^1_{g_1/0} = \{(01),(11)\}^1$,

$Y^1_{g_1/1} = \{(00),(01),(10),(11)\}^1$,

$Y^1_{g_2/0} = \{(10),(11)\}^1$,

$Y^1_{g_2/1} = \{(00),(01),(10),(11)\}^1$.

We apply the operations (3)–(6) to certain sets:

$Y^1 \setminus Y^1_{g_1/0} = \{1,2,3\}^1 \setminus \{1,3\}^1 = \{2\}^0$,

$Y^1_{g_1/0} \setminus Y^1 = \{1,3\}^1 \setminus \{1,2,3\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{g_1/1} = \{1,2,3\}^1 \setminus \{0,1,2,3\}^1 = \varnothing$,

$Y^1_{g_1/1} \setminus Y^1 = \{0,1,2,3\}^1 \setminus \{1,2,3\}^1 = \{0\}^1$,

$Y^1 \setminus Y^1_{g_2/0} = \{1,2,3\}^1 \setminus \{2,3\}^1 = \{1\}^0$,

$Y^1_{g_2/0} \setminus Y^1 = \{2,3\}^1 \setminus \{1,2,3\}^1 = \varnothing$,

$Y^1 \setminus Y^1_{g_2/1} = \{1,2,3\}^1 \setminus \{0,1,2,3\}^1 = \varnothing$,

$Y^1_{g_2/1} \setminus Y^1 = \{0,1,2,3\}^1 \setminus \{1,2,3\}^1 = \{0\}^1$.

Thus, Table 13 for test code vectors of function $f(g_1, g_2)$ on the 2-level of circuit will be presented as follows:

Therefore, with usage of the test code vectors shown in Tables 11–13, it is possible to determine the location and the type of possible stuck-at-fault (0/1) on all three levels of the circuit shown in Fig. 3.

Conclusion

A new method for detection stuck-at-faults (0/1) in digital combinational circuits is proposed, which, compared to known methods and algorithms, is easier to implement due to the use of a numerical set-theoretical approach. To search for vectors of test codes, which can be used to determine the location and the type of stuck-at-faults (0/1) at all levels of the circuit under study, it is enough to perform only a few simple ST operations and procedures. The advantages of the proposed method are illustrated by several examples borrowed from the publications of well-known authors.

REFERENCES

1. *Parag, K. Lala.*, 2009. An Introduction to Logic Circuit Testing. Morgan & Claypool. 111 p.
2. *Kohavi, Z., Jha, N.*, 2010. "Switching and Finite Automata Theory". Cambridge University Press, pp. 206-250.
3. *Fujiwara, H.*, 1986. Logic testing and design for testability. In Comp. Syst. Series. Cambridge, MA: Mass. Inst. Tech., 304 p. DOI: https://doi.org/10.7551/mitpress/4317.001.0001.
4. *Sasao, T.,* 1997. Easily testable realizations for generalized Reed-Muller expressions. IEEE Trans. On Computers, 46 (6), pp. 709-716.
5. *Sasao, T.*, 1999. Switching Theory for Logic Synthesys. Kluwer Academic Publishers, pp. 303–310.
6. *Faraj, K.*, 2011. "Design Error Detection and Correction System based on Reed-Muller Matrix for Memory Protection". International Journal of Computer Applications (0975-8887), 34 (8), pp. 42–48.
7. *Reed, I.S.*, 1973. "Boolean Difference Calculus and Fault Finding". SIAM Journal on Applied Mathematics, 24 (1), pp. 134-141.
8. *Samangadkar, S.S., Dudam, S.S., & Sinha, A.K.*, 2015. "Fault Diagnosis in Combinational Logic Circuits: A Survey". IJSRD, 3 (2), pp. 2051–2054.
9. *Zakrevskiy, A.D., Pottosin, Yu.V., Cheremisinova, L.D.*, 2007. Logicheskiye osnovy proyektirovaniya diskretnykh ustroystv. M.: Fizmatlit. 592 p. [Закревский А.Д., Поттосин Ю.В., Черемисинова Л.Д. Логические основы проектирования дискретных устройств. Физматлит. М.: 2007. 592 с.] (In Russian).
10. *Karkouri, Y, Aboulhamid, M.* Multiple Stuck-at Fault in Logic Circuits. [online]. Available at: <http://www.iro.umontreal.ca/~aboulham/pdfs_sources/KCCVLSI.pdf> [Accessed 1 June 2022].
11. *Y Liu, Q Xu.* On modeling faults in FinFET logic circuits. 2012 IEEE Inter. Test Conf. Paper 11.3. shILAotDjLlJRNNc.pdf.
12. *Liu, Y., Xu, Q.*, 2012. "On modeling faults in FinFET logic circuits". In 2012 IEEE International Test Conference, pp. 1-9.
13. *Sidik Nurcahyo, S.* "General Algorithm for Testing the Combinational Logic Gates inside Digital Integrated Circuits". International Journal of Engineering Research and Applications, 7 (7), pp. 1-5. DOI: 10.9790/9622-0707050105.
14. *Jameil, A. K.*, 2015. "A new single stuck fault detection algorithm for digital circuits". Int. J. Eng. Res. Gen. Sci, 3(1), 1050-1056.
15. *Malihi, L., Malihi, R.*, 2020. "Single stuck-at-faults detection using test generation vector and deep stacked-sparse-autoencoder". SN Applied Sciences, 2, 1715, pp. 1-10. DOI: https://doi.org/10.1007/s42452-020-03460-0.
16. *Zhang, Z., Zhao, J.*, 2017. "A deep belief network based fault diagnosis model for complex chemical processes". Comput Chem Eng., 107, pp. 395-407.
17. *Rytsar, B.Ye.* Teoretyko-mnozhynni optymizatsiyni metody lohikovoho syntezu kombinatsiynykh merezh: dys. doktora tekhn. Lviv, 2004. 348 p. [Рицар Б.Є. Теоретико-множинні оптимізаційні методи логікового синтезу комбінаційних мереж: дис. доктора техн. Львів, 2004. 348 с.] (In Ukrainian).
18. *Rytsar, B.Ye.*, 2015. "A New minimization method of logical functions in polynomial set-theoretical format. 1. Generalized rules of conjuncterms simplification". Upravlyayushchie Sistemy i Mashiny, 2, pp. 39-57.

Received 15.12.2022

Б.Є. Рицар, доктор технічних наук, професор, кафедра радіоелектронних пристроїв та систем,
Національний університет України "Львівська політехніка",
79013, м. Львів, вул. С. Бандери, 12, Україна, ORCID: https://orcid.org/0000-0002-2929-2954,
bohdanrytsar@gmail.com

ПРОСТИЙ МЕТОД ВИЯВЛЕННЯ НЕСПРАВНОСТЕЙ ТИПУ *STUCK-AT FAULT* У ЦИФРОВИХ КОМБІНАЦІЙНИХ СХЕМАХ

Вступ. Важливим розділом логікового проектування цифрових пристроїв є технічна діагностика, у рамках якої розробляються методи перевірки технічного стану пристроїв для забезпечення надійності їх роботи. Будь яке порушення нормального функціювання цифрового пристрою називають несправністю його роботи. Виявити несправність у логіковій схемі цифрового пристрою можна послідовністю контрольних тестів на її входах та спостереження отриманих результатів на її виході чи виходах. Завдання діагностики будь якого пристрою полягає в пошуку мінімальної кількості тестових кодів. Типовими моделями пошкоджень при проектуванні цифрових пристроїв є поодинчі або багаторазові пошкодження. У статті розглянуто поодинчі пошкодження типу *stuck-at-fault* (0/1), які переважно мають місце в цифрових комбінаційних пристроях. Для виявлення таких несправностей відомі різні методи і алгоритми пошуку тестових кодів, серед яких переважна більшість ґрунтується на аналітичному підході та характеризується складністю і громіздкістю практичної реалізації, що зростає зі збільшенням кількості змінних.

Метою статті є запропонувати новий метод виявлення (діагностування) несправностей типу *stuck-at-fault* (0\1) у комбінаційних цифрових схемах, в основі якого лежить числовий теоретико-множинний підхід, який буде відрізнятися від відомих простотою практичної реалізації.

Методи. Запропоновано новий метод виявлення несправностей типу stuck-at-fault (0/1) у комбінаційних цифрових схемах.

Результати. Описано новий метод виявлення (діагностування) несправностей типу *stuck-at-fault* (0/1) у цифрових комбінаційних схемах на основі числового теоретико-множинного підходу. Порівняно з відомими методами й алгоритмами пропонований підхід відрізняється простішою реалізацією пошуку векторів тестових кодів у довільних точках досліджуваної логікової схеми. Для визначення місця і типу несправності *stuck-at-fault* (0/1) достатньо реалізувати кілька простих теоретико-множинних операцій і процедур. Це засвідчують наведені в роботі приклади застосування пропонованого методу, які запозичено з публікацій відомих авторів.

Висновки. Запропонований та описаний у статті метод виявлення несправностей дає змогу виявляти несправності типу *stuck-at-fault* (0/1) у комбінаційних цифрових схемах. Цей метод, порівняно з відомими методами та алгоритмами, відрізняється простішою реалізацією завдяки застосуванню числового теоретико-множинного підходу. Для пошуку векторів тестових кодів, за допомогою яких можна визначити місце і тип несправностей типу *stuck-at-fault* (0/1) на всіх рівнях досліджуваної схеми, достатньо виконати лише кількох простих ТМ-операцій і процедур. Переваги пропонованого методу ілюструють приклади, запозичені з публікацій відомих авторів.

Ключові слова: *цифрові комбінаційні схеми, виявлення несправностей, вектори тестових кодів, числовий теоретико-множинний підхід, операції та процедури.*