# GENERAL PROPERTIES OF HIGHER-SPIN FERMION INTERACTION CURRENTS AND THEIR TEST IN $\pi N$-SCATTERING

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The currents of higher-spin fermion interactions with zero- and half-spin particles are derived. They can be used for the $N^*(J) \rightleftarrows N\pi$ -transitions ($N^*(J)$ is the nucleon resonance with the $J$ spin). In accordance with the theorem on currents and fields, the spin-tensors of these currents are traceless, and their products with the $\gamma$-matrices and the higher-spin fermion momentum vanish, similarly to the field spin-tensors. Such currents are derived explicitly for $J = \frac{3}{2}$ and $\frac{5}{2}$. It is shown that, in the present approach, the scale dimension of a higher spin fermion propagator equals to $-1$ for any $J \geq \frac{1}{2}$. The calculations indicate that the off-mass-shell $N^*$ contributions to the $s$-channel amplitudes correspond to $J = J_{\pi N}$ only ($J_{\pi N}$ is the total angular momentum of the $\pi N$-system). As contrast, in the usually exploited approaches, such non-zero amplitudes correspond to $\frac{1}{2} \leq J_{\pi N} \leq J$. In particular, the usually exploited approaches give non-zero off-mass-shell contributions of the $\triangle(1232)$-resonance to the amplitudes $S_{31}, P_{31}(J_{\pi N} = \frac{1}{2})$ and $P_{33}, D_{33}(J_{\pi N} = \frac{3}{2})$, but our approach – to $P_{33}$ and $D_{33}$ only. The comparison of these results with the data of the partial wave analysis on the $S_{31}$-amplitude in the $\triangle(1232)$-region shows the better agreement for the present approach.

## 1. Introduction

At present, a lot of higher spin particles (the spin $J \geq 1$) is known. In particular, the masses and the widths of higher spin fermions (HSF) are known quite well, as they are the resonances in the $s$-channel of meson-nucleon scattering. Among the nucleon resonances, $\triangle(1232)$ ($P_{33}(1232)$) occupies a peculiar place. This resonance has the quantum numbers $J = \frac{3}{2}$ (spin) and $I = \frac{3}{2}$ (isospin). It has the simplest theoretical interpretation in the quark model and is studied experimentally better in comparison with another resonances. In quantum chromodynamics, $P_{33}(1232)$ and the nucleon are the different states of the same three coloured quark system. They have different quark spin and isospin orientations only.

It is known that the higher spin particles, as well as the nucleons, pions, and nuclei are not elementary particles. But, in the reactions, the approximation of elementary particles for them is rather good at low and intermediate energies. Therefore, we can assume that the higher spin particles can be considered approximately as elementary particles, similarly to the nucleons and the pions. The non-elementarity of the particles can be taken into consideration by means of the interaction vertex functions. The Feynman rules, projection operators, and vertex functions must be known in the course of calculations of the higher spin particle contributions. All these quontities depend on the formalism used for the description of higher spin particles. The Rarita–Schwinger, Kemmer–Duffin, and Bargmann–Wigner formalisms can be used. As a rule, the Rarita–Schwinger formalism is exploited in the calculations of amplitudes [1–7]. In this formalism, the Feynman rules are known, and the vertex functions describing the interaction of higher-spin particles have fairly simple forms [8–10]. The $\frac{3}{2}$-spin states (fields) are studied theoretically better in comparison with the states of $J > \frac{3}{2}$ (see, e.g., [6, 7, 11, 12]). The investigations of another formalisms are in progress [13–15].

We use the Rarita–Schwinger formalism here. The amplitudes of the transitions like to $N^* \to N\pi$ can be written as

$$V = U(p)_{\mu_1 \ldots \mu_l} \eta(p)_{\mu_1 \ldots \mu_l} \equiv U(p)^l_\mu \eta(p)^l_\mu, \qquad (1)$$

where $U(p)_{\mu_1 \ldots \mu_l} \equiv U(p)^l_\mu$ is the symmetric field spin-tensor for HSF of the $p$ momentum and the $J = l + \frac{1}{2}$ spin, $\eta(p)_{\mu_1 \ldots \mu_l} \equiv \eta(p)^l_\mu$ is the current spin-tensor. In the Rarita–Schwinger formalism, the field spin-tensor obeys the auxiliary conditions

$$p_{\mu_i} U(p)_{\mu_1 \ldots \mu_l} = 0, \qquad (2)$$

$$g_{\mu_i \mu_k} U(p)_{\mu_1 \ldots \mu_l} = 0, \qquad (3)$$

$$\gamma_{\mu_i} U(p)_{\mu_1 \ldots \mu_l} = 0, \tag{4}$$

where $i, k = 1, 2, ..., l$.

As usual, we assume that the HSF interactions are described by the system of inhomogeneous Dirac equations

$$(i\widehat{\partial} - M)U(x)^l_\mu = \eta(x)^l_\mu, \tag{5}$$

where $M$ is the HSF mass. As a rule, the current spin-tensors $\eta(x)^l_\mu$ obey the symmetry condition only. We name the approaches with such current spin-tensors as usual (or simple) approaches. Unfortunately, the usual approaches have some shortcomings discussed in what follows.

### *1.1. Inconsistencies of systems of equations*

Indeed, due to the symmetry and conditions (2)–(4), the field spin-tensors $U(x)^l_\mu$, as well as their Fourier transforms $U(p)^l_\mu$, have $2J + 1 = 2l + 2$ independent components. But, in the usual approaches, the current spin-tensors $\eta(x)^l_\mu$ (and $\eta(p)^l_\mu$) obey the symmetry conditions only. Therefore, they have $N_l = 4 \cdot 4 \cdot 5 \cdot \ldots \cdot (l+3)/l! = 2(l+1)(l+2)(l+3)/3$ independent components. We see that $N_l > 2l + 2$. Let us consider the system of partial differential equations (5) in the momentum representation (i.e., for the Fourier components). Then system (5) becomes a system of $N_l$ linear algebraic equations for $2J + 1$ components of the field spin-tensors $U(p)_{\mu_1 \ldots \mu_l}$. This system of equations is inconsistent. For example, this can be seen, using the Cramer method for its solution. Indeed, for each independent component of the field spin-tensor, we have zero denominator (system's determinant of the order of $N_l$ has rank $2l+2$) whereas the numerator is non-zero (i.e., the solution of this system does not exist). This inconsistency can be considered as the consequence of the Kronecker–Capelli theorem. Indeed, the rank of the matrix of our system is less than that of the matrix of the extended system in the usual approaches ($2l + 2 < N_l$).

### *1.2. Power divergences*

The substitution of the propagators and the vertex functions for higher spin particles into the reaction amplitudes instead of those for 0- and $\frac{1}{2}$-spin particles leads to the power divergences for the amplitudes corresponding to the loop diagrams and the energy increasing for the amplitudes corresponding to the tree diagrams. The usual approaches have two sources of the power divergences.

#### *1.2.1 Propagators*

As is known (see, e.g., [6, 7, 11]), the propagator of a spin - $\frac{3}{2}$ particle includes the term $\widehat{p}p_\mu p_\nu/M^2(p^2 - M^2)$, where $p$ is the HSF momentum. For $J = l + 1/2$, the propagator includes the term

$$\widehat{p}p_{\mu_1} \ldots p_{\mu_l} p_{\nu_1} \ldots p_{\nu_l}/M^{2l}(p^2 - M^2). \tag{6}$$

Thus, the scale dimension of the HSF propagator equals $2l - 1$, i.e., it increases with $p$ and $J$. The HSF momentum can be the integration momentum for loop-diagram amplitudes, and this give the power divergences. For the tree-diagram amplitudes, the HSF momentum is expressed through the external particle momenta, and this leads to the energy increasing at high energy.

#### *1.2.2 Currents*

For the HSF interaction $J(p) \to \frac{1}{2}(k_1) + O(k_2)$, the currents in the usual approaches can be written in three forms

$$\eta_{\mu_1 \cdots \mu_l} = \begin{cases} g k_{1\mu_1} k_{1\mu_2} \ldots k_{1\mu_l} \overline{u}(k_1)\varphi^*(k_2); & (7) \\ g k_{2\mu_1} k_{2\mu_2} \ldots k_{2\mu_l} \overline{u}(k_1)\varphi^*(k_2); & (8) \\ g(k_1 - k_2)_{\mu_1}(k_1 - k_2)_{\mu_2} \ldots (k_1 - k_2)_{\mu_l} \overline{u}(k_1)\varphi^*(k_2), & (9) \end{cases}$$

where $g$ is the coupling constant. We see that the current spin-tensors (7)–(9) include the products of particle momenta, whereas the interaction currents of 0- and $\frac{1}{2}$-spin particles have no any momentum as factors. The momenta in Eqs. (7)–(9) can be expressed in terms of the integration momenta or the external particle momenta. This leads to the power divergences or the power energy increasing. In the simplest case, the vertex functions do not include any form-factors, which are the scalar functions of the scalar products of particle momenta. In this case, the powers of the divergences for loop-diagram amplitudes are proportional to $J$. Moreover, the number of the diverging quantities in field theory increases with $J$. Therefore, as is well known, the theories of higher spin particle interactions are not renormalizable.

In some models, the vertex functions including the form-factors have been considered. As a rule, these form-factors have been chosen to achieve the agreement with experimental data in some reactions in some energy regions. But the power divergences (or energy increasing) remain at such form-factors in other reactions.

### *1.3. Ambiguities of vertex functions*

The use of the current spin-tensors (7)–(9) in vertex functions (1) give different expressions for the amplitudes and even the different powers of the divergences for the loop-diagram amplitudes, as well as the different powers of energy growth for the HSF off-mass-shell contributions to the pole amplitudes [16].

### *1.4. Contradictions with experimental data*

The use of the vertex functions (7)–(9) without form-factors leads to the power increasing of the pole reaction amplitudes with energy and spin $J$. Therefore, such versions of the higher-spin particle interactions give contradictions with experimental data on the cross-sections at high energies. As is known from experiment, the cross-sections cannot increase by any power law at higher energies.

The vertex functions including the form-factors (as a rule, the monopole or dipole form) for the higher-spin resonance contributions to the $s$-channel amplitudes of the $\pi N \to \pi N$, $\gamma N \to \pi N$, $\pi N \to \pi N^*$ reactions allow one to achieve some agreement with experimental data in the resonance region. But the extrapolation of such a model to the high-energy region (e.g., to the region of the application of the Regge pole model) leads to the contradictions with experimental data on the cross-sections. It is due to that the resonance models give no necessary energy decrease at high energies.

Therefore, the resonance contributions are usually thrown away by hands at high energies. However, it is clear that, in the proper models, the resonance contributions must exist at high energies, but they must be very little.

Thus, we conclude that the usual approaches to the description of higher spin particle interactions must be modified. As the shortcomings of the usual approaches exist for all higher-spin particles, we may expect that the general properties for the currents of the of higher spin particles must exist in addition to the properties of the currents for the interaction of zero- and half-spin particles.

In Refs. [17–19], it is shown that, for the currents of the higher-spin boson and fermion interactions, the theorem on currents and fields and the theorem on current asymptotics must be valid. Perhaps, the shortcomings characteristic of the usual approaches must not appear in the approaches satisfying these theorems. The model satisfying these theorems is proposed in Refs. [20, 21] for the vertices of the higher-spin boson interactions with two spinless particles. It is shown [21] that these currents decrease, indeed, with the boson spin $J$ at the higher spin boson momentum $|p_\nu| \to \infty$. In Ref. [22], this model was used for the calculation of the higher-spin boson contributions to the self-energy operator of a spinless particle. These calculations showed that the self-energy operator is finite in the one-loop approximation for the higher spin boson of any spin and mass. This finite value must be compared with the logarithmic divergence for the contribution of two spinless particles to the self-energy operator. Note also that the usual approaches for the higher spin boson contributions to the self-energy operator give different power divergences.

In the present paper, we propose a model for the vertex of a higher-spin fermion with the interaction of 0- and $\frac{1}{2}$-spin particles (e.g., $\pi N \rightleftarrows N^*$). For higher-spin fermion interaction currents, such a vertex is the simplest, as they are determined by one partial amplitude. We will study also the application of this model to the elastic $\pi N$-scattering. We show that the theorem on currents and fields may be tested by means of the partial wave analyses for the elastic $\pi N$-scattering.

## 2. Theorem on Currents and Fields

In accordance with the theorem on currents and fields [19], the system of algebraic linear equations for the Fourier components is consistent only in the case where the current spin-tensors have the same properties as the field spin-tensors. Thus, the conditions similar to (2)–(4) must be valid for the current spin-tensors:

$$j(p)_{\mu_1\cdots\mu_l} p_{\mu_i} = 0, \qquad \partial_{\mu_i} j(x)_{\mu_1\cdots\mu_l} = 0, \tag{10}$$

$$j(p)_{\mu_1\cdots\mu_l} g_{\mu_i\mu_k} = 0, \qquad j(x)_{\mu_1\cdots\mu_l} g_{\mu_i\mu_k} = 0, \tag{11}$$

$$j(p)_{\mu_1\cdots\mu_l} \gamma_{\mu_i} = 0, \qquad \gamma_{\mu_i} j(x)_{\mu_1\cdots\mu_l} = 0, \tag{12}$$

| momentum representation | coordinate representation. |
|---|---|

Note that conditions (10) and (11) [17, 18, 22] must be valid for the current tensors of higher-spin boson interactions. The current spin-tensors $j(p)_{\mu_1\ldots\mu_l} \equiv j(p)^l_\mu$ and $j(x)_{\mu_1\ldots\mu_l} \equiv j(x)^l_\mu$ have $2J+1$ independent components as a consequence of conditions (10)–(12). We name these current spin-tensors as the physical currents. We name the spin-tensors $\eta(p)^l_\mu$ and $\eta(x)^l_\mu$ as the usual currents. It is easy to see that relations (10)–(12) are valid. Indeed, the left-hand sides in the Klein–Gordon and Dirac equations are the scalar operators with respect to the Lorentz transformation. The properties of the field and the current spin-tensor must be the same since the product of a scalar and the representation of some dimension is the representation of the same dimension.

To construct the physical current spin-tensors, we use the modification [16, 19] of the projection operator $\Pi(p)_{\mu_1...\mu_l,\nu_1...\nu_l} \equiv \Pi(p)^l_{\mu,\nu}$ [9, 10]

$$j(p)^l_\mu = (p^2)^l \Pi(p)^l{}_{\mu,\nu}\eta(p)^l_\nu. \quad (13)$$

This projection operator has a rather complicated form. Therefore, we use the contracted projection operator

$$\Pi(p,a,b) = a_{\mu_1}...a_{\mu_l}\Pi(p)_{\mu_1...\mu_l,\quad \nu_1...\nu_l}b_{\nu_1}...b_{\nu_l}, \quad (14)$$

as it has a simple form:

$$\Pi(p,\, a,\, b) = \frac{l\,!}{(2l+1)\,!\,!}(-\widetilde{a}^2)^{\frac{l}{2}}(-\widetilde{b}^2)^{\frac{l}{2}}\times$$

$$\times\left[P'_{l+1}(z) - \frac{\widehat{\widetilde{a}}\,\widehat{\widetilde{b}}}{\sqrt{\widetilde{a}^2\widetilde{b}^2}}P'_l(z)\right]. \quad (15)$$

Here, $P_l(z)$ are the Legendre polynomials, and

$$z = -\frac{\left(\tilde{a}\,\tilde{b}\right)}{\sqrt{\tilde{a}^2\,\tilde{b}^2}},\ \tilde{a}_\mu = a_\mu - p_\mu\frac{(ap)}{p^2},$$

$$\hat{\tilde{a}}_\mu = \tilde{\gamma}_\mu a_\mu = \tilde{\gamma}_\mu\tilde{a}_\mu,$$

$$\tilde{\gamma}_\mu = \gamma_5\left(\gamma_\mu - p_\mu\hat{p}/p^2\right), \tilde{\gamma}_\mu p_\mu = 0, \tilde{\gamma}_\mu\tilde{\gamma}_\mu = -3,$$

$$\{\tilde{\gamma}_\mu\tilde{\gamma}_\nu\} = 2d_{\mu\nu}, d_{\mu\nu} = -g_{\mu\nu} + p_\mu p_\nu/p^2. \quad (16)$$

In the rest-frame of HSF, we have

$$\widetilde{a}_\mu = (0,\mathbf{a}), \qquad \widetilde{\gamma}_0 = 0, \qquad \widetilde{\gamma}_i = \sigma_i\gamma_0. \quad (17)$$

The projection operator $\Pi(p)^l_{\mu,\nu}$ can be derived from the contracted projection operator (15) by means of the differentiations:

$$\Pi(p)_{\mu_1...\mu_l,\nu_1...\nu_l} =$$

$$= \frac{1}{(l!)^2}\cdot\frac{\partial}{\partial a_{\mu_1}}\cdots\frac{\partial}{\partial a_{\mu_l}}\cdot\cdot\frac{\partial}{\partial b_{\nu_1}}\cdots\frac{\partial}{\partial b_{\nu_l}}\cdot\Pi(p,a,b). \quad (18)$$

This projection operator is dimensionless. As a consequence of Eqs. (2)–(4), Eqs. (15) and (18) at any HSF momentum, mass, and $J$ yield

$$\Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,p_{\mu_i} = \Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,p_{\nu_k} = 0, \quad (19)$$

$$\Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,g_{\mu_i\mu_k} = \Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,g_{\nu_i\nu_k} = 0, \quad (20)$$

$$\gamma_{\mu_i}\Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l} = \Pi\,(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,\gamma_{\nu_k} = 0. \quad (21)$$

Now, we consider the propagator for HSF of any spin similarly to the contracted projection operator

$$P\,(p,\,a,\,b) = \frac{\hat{p}+M}{p^2-M^2}\cdot\Pi\,(p,\,a,\,b) =$$

$$= \Pi(p,\,a,\,b)\frac{\hat{p}+M}{p^2-M^2} \quad (22)$$

In particular, from Eq. (22) for $J = \dfrac{3}{2}$, we derive

$$P\,(p)_{\mu\nu} = \frac{\hat{p}+M}{p^2-M^2}\left[d_{\mu\nu} - \frac{1}{3}\tilde{\gamma}_\mu\tilde{\gamma}_\nu\right] =$$

$$= \left[d_{\mu\nu} - \frac{1}{3}\tilde{\gamma}_\mu\tilde{\gamma}_\nu\right]\frac{\hat{p}+M}{p^2-M^2}. \quad (23)$$

We compare our propagator (23) with the usually used propagator for $J = \dfrac{3}{2}$ [6, 7, 11]

$$P(p)^{\text{common}}_{\mu\nu} = \frac{\hat{p}+M}{p^2-M^2}\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \right.$$

$$\left. +2p_\mu p_\nu/3M^2 + \left(\gamma_\mu p_\nu - p_\mu\gamma_\nu\right)/3M\right] =$$

$$= \left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + 2p_\mu p_\nu/3M^2 - \right.$$

$$\left. -\left(\gamma_\mu p_\nu - p_\mu\gamma_\nu\right)/3M\right]\frac{\hat{p}+M}{p^2-M^2}. \quad (24)$$

We can see several distinctions of the HSF propagators in the present method and in the usual approaches: 1) In the present method, the propagators obey equalities similar to Eqs. (19)–(21) for the projection operators $\Pi(p)^l_{\mu\nu}$



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at any HSF momentum, mass, and $J$. In the common approaches, such equalities are valid only at $\hat{p} = M$ (i.e., on the mass shell); 2) In the present method, the operators $\hat{p}+M$ and $\Pi(p)^l_{\mu\nu}$ commute. Therefore, the calculations of virtual HSF contributions to the amplitudes are simplified. In the usual approaches, the operators $\hat{p}+M$ and $\Pi(p)^l_{\mu\nu}$ for virtual HSF do not commute, as we can see, for example, from Eq. (24) for $J = 3/2$; 3) As a consequence of the current conservation (10) and conditions (12), the contributions of the HSF momentum $p_{\mu_i}$ or $p_{\nu_k}$ and of the $\widetilde{\gamma}_{\mu_i}$- or $\widetilde{\gamma}_{\nu_k}$-matrices vanish in the products of the interaction currents $j(p)^l_\mu$ and HSF propagators. Therefore, the non-zero contributions to such products are given by the propagator terms including the metric tensors only [16, 17, 18, 22, 23]. Thus, in our approach, the power divergences due to the HSF propagators vanish as a consequence of the current conservation (10); 4) The scale dimension of our HSF propagators does not depend on the HSF spin value $J$ and equals – 1 (the same as for the $\frac{1}{2}$-spin fermions); whereas, in the usual approaches, the propagator scale dimensions are equal to $2J-2$. In such a way, we see that, in our approach, the HSF propagator must not generate the power divergences in addition to the divergence for $\frac{1}{2}$-spin fermions. In our consideration, we have studied the dependence of the HSF interaction currents on the HSF momentum $p$ only. But the interaction currents depend in reality on the momenta of other particles too. For example, this can be seen from Eqs. (7)–(9). We denote the momenta of other particles in the HSF interaction currents, if required.

## 3. Model for HSF Interactions with 0- and 1/2-Spin Particles

The consistent model for the interaction of a higher spin boson with two spinless particles has been proposed in Refs. [20, 21]. As in the $J(p) \rightleftarrows O(k_1) + O(k_2)$-transition, the orbital momentum has one value, these transitions are the simplest among the interactions of higher spin bosons. The orbital momentum has one value also in the transitions of a higher-spin fermion to the 0- and 1/2-spin particles at any set of the particle parities. Therefore, such interactions are the simplest ones for the higher-spin fermions. These transitions can be studied in the $\pi N$-scattering as the excitation and the decay of a higher spin nucleon resonance $N^*(J)\,(\pi N \to N^*(J) \to N\pi)$ at the intermediate energies. We consider these transitions in details. Using (13), the physical current of the $J\,(p) \to O\,(q_2) + \frac{1}{2}\,(p_2)$ transition can be written down as

$$j(p,q)_{\mu_1\ldots\mu_l} = g_l F_l(p,q')(p^2)^l \varphi^+(q_2)\times$$

$$\times \bar{u}\,(p_2) \left\{ \begin{array}{l} 1 \\ i\gamma_5 \end{array} \right\} \Pi\,(p)_{\mu_1\ldots\mu_l,\nu_1\ldots\nu_l}\, q'_{\nu_1}\ldots q'_{\nu_l}, \tag{25}$$

where $q' = q_2 - p_2, g$ is the coupling constant, $F_l\,(p,q')$ is the form-factor providing for the necessary decreasing of the current at $|p_\nu| \to \infty$ in accordance with the theorem on current asymptotics [19]. The example of such a form-factor is derived in Refs. [20, 21] for the interaction of a higher spin boson with two spinless particles. In Eq. (25), we choose the common current (9). As a consequence of Eq. (19), three common currents (7)–(9) give the physical currents of the same momentum dependence, as they are proportional. The differences between the physical currents derived from three common currents can be taken into account by means of the redefinition of the coupling constants. Thus, we have no ambiguities in the physical currents in the contrast with the usual approaches. Similarly, the physical current of the $O\,(q_1) + \frac{1}{2}\,(p_1) \to J\,(p)$-transition is given by

$$j^+(p,q)_{\mu_1\ldots\mu_l} = g^* F_l^*(p,q)(p^2)^l \Pi(p)_{\mu_1\ldots\mu_l,\nu_1\ldots\nu_l}\times$$

$$\times \left\{ \begin{array}{l} 1 \\ i\gamma_5 \end{array} \right\} u\,(p_1)\,\varphi\,(q_1)\, q_{\nu_1},\ldots,q_{\nu_l}, \quad q = q_1 - p_1. \tag{26}$$

In particular, we derive,
for $J = \frac{3}{2}$,

$$j^+\,(p,q)_\mu = g_1^* F_1^*\,(p,q) \left[ -Q_\mu + \frac{1}{3}\gamma_5 \tilde{\gamma}_\mu \hat{Q} \right] \times$$

$$\times \left\{ \begin{array}{l} 1 \\ i\gamma_5 \end{array} \right\} u\,(p_1)\,\varphi\,(q_1) \tag{27}$$

and, for $J = \frac{5}{2}$,

$$j^+\,(p,q)_{\mu_1\mu_2} = \frac{2}{3} g_2^* F_2^*(p,q) \Big[ 5Q_{\mu_1}Q_{\mu_2} + d_{\mu_1\mu_2}Q^2 -$$

$$-\gamma_5(\widetilde{\gamma}_{\mu_1}Q_{\mu_2} + \widetilde{\gamma}_{\mu_2}Q_{\mu_1})\widehat{Q} \Big] \left\{ \begin{array}{l} 1 \\ i\gamma_5 \end{array} \right\} u(p_1)\varphi(q_1), \tag{28}$$

where $Q_\mu = -d_{\mu\nu} p^2 q_\nu = p^2 q_\mu - p_\mu (p\,q), \ \ (p\,Q) = 0.$

The identity and $i\gamma_5$ matrices in the braces in Eqs. (25)–(28) correspond to the different sets of particle parities. Let us compare current (27) for the $\Delta\,(1232) \to N\pi$ transition and the current in the usual approach. We derive the latter from the Lagrangian [7]

$$L_{\Delta N\pi} = \frac{G}{m}\bar{u}\,(x)\,U\,(x)_\mu\,\partial_\mu\varphi_\pi^+\,(x) + \text{h.c.}, \tag{29}$$

where $G$ is the coupling constant, and $m$ is the nucleon mass. Then we have

$$\eta\,(p_2, q_2)_\mu = i\frac{G}{m}\,q_{2\mu}\,\bar{u}\,(p_2)\,\varphi^+\,(q_2)\,. \tag{30}$$

Current (30) does not obey conditions (10) and (12) for the $\Delta$-resonance off-mass-shell. Currents (27) (with the identity matrix) and (30) coincide with each other only on the mass shell of a $J = \frac{3}{2}$ particle.

## 4. Contributions of HSF Resonances to $\pi N$-Scattering Amplitudes in Covariant Approaches

Let us compare the contributions of $\Delta\,(1232)$ to the $s$-channel $\pi N$-scattering amplitudes calculated in the usual and our approaches. As is known, each resonance contributes to definite multipole (partial) amplitudes. We denote the pion orbital momentum in $\pi N \to \pi N$ as $l_\pi$. Then the multipole amplitudes $f_{l_\pi \pm 1}$ correspond to $J = l_\pi \pm \frac{1}{2}$. The parity of a state with definite $J$ equals $(-1)^{l_\pi + 1}$. In particular, $\Delta\,(1232)$ on the mass shell contributes to the $f_{1+} \equiv P_{33}$ amplitude only [7]. Using current (30) and propagator (24) (with the change in the propagator denominator $p^2 - M^2 \to p^2 - M^2 + iM\Gamma$, where $\Gamma$ is the total width of HSF), we derive the contributions of $\Delta\,(1232)$ to the multipole amplitudes in the usual approach:

$$f_{1+}^\Delta = P_{33}^\Delta = \frac{1}{3}\beta_\Delta(W + M_\Delta)\,(E^2 - m^2),$$

$$f_{0+}^\Delta = S_{31}^\Delta = \frac{2}{3}\beta_\Delta\frac{W^2 - M_\Delta^2}{M_\Delta^2}q_0\times$$

$$\times\left[q_0\,(W + M_\Delta) - M_\Delta\,(E - m)\right],$$

$$f_{1-}^\Delta = P_{31}^\Delta = \frac{2}{3}\beta_\Delta q_0^2\frac{W + M_\Delta}{M_\Delta^2}\times$$

$$\times\,(W - M_\Delta)^2\,\frac{E - m}{E + m},$$

$$f_{2-}^\Delta = D_{33}^\Delta = \beta_\Delta\frac{W - M_\Delta}{3}\,(E - m)^2\,,$$

$$\beta_\Delta = \frac{G^2}{m^2}\frac{E + m}{W^2 - M_\Delta^2 + iM_\Delta\Gamma},\quad W = E + q_0, \tag{31}$$

where $W$, $E$, and $q_0$ are the total energy of the $\pi N$-system, nucleon energy, and pion energy, respectively.

From Eq. (31), we see that the $P_{33}^\Delta$ amplitude is proportional to $W + M_\Delta$, i.e., the $\Delta(1232)$ contribution exists at any $W$. But the amplitudes $S_{31}^\Delta, P_{31}^\Delta, D_{33}^\Delta$ are proportional to $W - M_\Delta$. In other words, the contribution of $\Delta(1232)$ to the states with $J_{\pi N}^p = \frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^-$ vanishes on the mass shell of $\Delta(1232)$ ($J_{\pi N}$ is the total angular momentum of the $\pi N$-system). It is a particular case of the known phenomenon in the usual approaches that the HSF with spin $J$ off the mass shell can change parity and correspond to the states with the angular momenta from $\frac{1}{2}$ to $J$.

The imaginary parts of the amplitudes in Eq. (31) are proportional to $M_\Delta\Gamma/[(W^2 - M_\Delta^2) + M_\Delta^2\Gamma^2]$. The real part of $P_{33}$ and the imaginary parts of the $S_{31}$, $P_{31}$, and $D_{33}$ amplitudes are proportional to $W - M_\Delta$. Therefore, for the test of the usual approach, we propose to study the relations

$$\frac{\text{Im}S_{31}^\Delta}{\text{Re}P_{33}^\Delta} = -2\frac{\Gamma}{M_\Delta}q_0\frac{q_0\,(W + M_\Delta) - M_\Delta\,(E - m)}{(W + M_\Delta)\,(E^2 - m^2)},$$

$$\frac{\text{Im}P_{31}^\Delta}{\text{Re}P_{33}^\Delta} = -2\frac{\Gamma}{M_\Delta}\left(\frac{q_0}{E + m}\right)^2\frac{W - M_\Delta}{W + M_\Delta},$$

$$\frac{\text{Im}D_{33}^\Delta}{\text{Re}P_{33}^\Delta} = -2\frac{M_\Delta\Gamma}{(W + M_\Delta)^2}\frac{E - m}{E + m}. \tag{32}$$

The contribution of any virtual particle to the amplitude is determined by the product of two interaction currents and the virtual particle propagator. For example, such a product for the HSF contributions to the $s$-channel amplitudes of the $\pi N$-scattering corresponds to the diagram in Fig. 1.

Since the propagators for HSF with $J \geq \frac{5}{2}$ are quite complicated, we do not consider the contributions of HSF with any spin to the $\pi N$-scattering amplitudes in

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the usual covariant approaches. We derive the HSF contribution to such a product in our approach. Using the property of the projection operator $\Pi(p)^l_{\mu,\nu}\Pi(p)^l_{\nu,\rho} = (-1)^l\Pi^l_{\mu\rho}$, we have

$$j(p,\, q')^l_\mu \frac{\hat{p}+M}{p^2-M^2}\Pi(p)^l_{\mu,\,\nu} j^+(p,\, q)^l_\nu =$$

$$= (p^2)^{2l}\eta(p,\, q')^l_\rho \Pi(p)^l_{\rho,\mu}\frac{\hat{p}+M}{p^2-M^2}$$

$$\Pi(p)^l_{\mu,\,\nu}\Pi(p)^l_{\nu,\,\sigma}\eta^+(p,\, q')^l_\sigma =$$

$$= (p^2)^{2l}\eta(p,\, q')^l_\mu \Pi(p)^l_{\mu,\,\nu}\frac{\hat{p}+M}{p^2-M^2}\eta^+(p,\, q')^l_\nu. \quad (33)$$

From Eq. (33), we see that such a product (i.e., corresponding to Fig. 1) can be calculated by using the common currents, but the HSF propagator must be taken in accordance with Eq. (22).

Let us consider the contribution of a higher spin nucleon resonance to the $s$-channel amplitude of the $\pi N$-scattering. We exploit the usual form for the $\pi N$-scattering amplitude in c.m.s.:

$$f(\pi N \to \pi N) = f_1(W,\, \cos\theta) +$$

$$+ f_2(W,\, \cos\theta)\boldsymbol{\sigma}\mathbf{q}_2\boldsymbol{\sigma}\mathbf{q}_1/\,|\mathbf{q}|^2. \quad (34)$$

Here, $W^2 = s = (p_1+q_1)^2 = (p_2+q_2)^2$, $\theta$ is the scattering angle, and $\mathbf{q}_1$ and $\mathbf{q}_2$ are the momenta of the initial and final pions, respectively. At the given $J$, the parity of the excited $N^*(J)$ depends on $l_\pi$. The resonances $N^*(J)$ on the mass shell, with the spin-parity $J^p = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+, ...$ contribute to the multipole amplitudes $f_{l_\pi+}$ (i.e., $J = l_\pi + \frac{1}{2}$). They correspond to $l_\pi = 0,\, 1,\, 2,\, 3,\, ...$, respectively. Their excitation is described by currents (25) and (26) with the identity matrix. Similarly, the resonances $N^*(J)$ on the mass shell with $J^p = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, ...$ contribute to the multipole amplitudes $f_{l_\pi-}$ – (i.e., $J = l_\pi - \frac{1}{2}$). They correspond to $l_\pi = 1, 2, 3, 4....$ Their excitation is described by currents (25) and (26) with the $\gamma_5$-matrix.

Using Eqs. (15), (16), (25), (26), and (33), the contribution of $N^*(J)$ to the amplitude can be written as

$$T(\pi N \to \pi N) = |g_l|^2\, F^*_l(p,q)F_l(p,\, q') \times$$

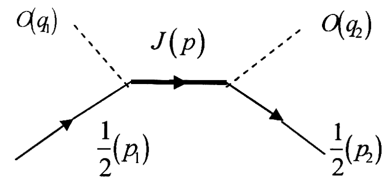


Fig. 1. Feynman diagram for the contributions of higher-spin fermion resonances $N^*(J)$ to the $\pi N$-scattering amplitudes

$$\times \varphi^*(q_2) q'_{\mu_1}...q'_{\mu_l}\bar{u}(p_2)\begin{Bmatrix} 1 \\ i\gamma_5 \end{Bmatrix}\frac{\hat{p}+M}{p^2-M^2+iM\Gamma},$$

$$\Pi(p)_{\mu_1,...\mu_l,\nu_1,\ \nu_l}\begin{Bmatrix} 1 \\ i\gamma_5 \end{Bmatrix} u(p_1)\varphi(q_1)q_{\nu_1}...q_{\nu_l} =$$

$$= |g_l|^2\, F^*_l(p,q)F_l(p,\, q')\varphi^*(q_2)\bar{u}(p_2)\begin{Bmatrix} 1 \\ i\gamma_5 \end{Bmatrix}\times$$

$$\times\frac{\hat{p}+M}{p^2-M^2+iM\Gamma}\Pi(p,\, q', q)\begin{Bmatrix} 1 \\ i\gamma_5 \end{Bmatrix}\times$$

$$\times \bar{u}(p_1\varphi(q_1) = \beta_l\varphi^*(q_2)\chi^*_2\times$$

$$\times\left(1,\ \frac{\boldsymbol{\sigma}\mathbf{q}_2}{E+m}\right)\begin{pmatrix} W\pm M & 0 \\ 0 & -W\pm M\end{pmatrix}\times$$

$$\times\left(P'_{l+1}(\cos\theta) - \frac{\boldsymbol{\sigma}\mathbf{q}_2\boldsymbol{\sigma}\mathbf{q}_1}{\mathbf{q}^2}P'_l(\cos\theta)\right)\times$$

$$\times\begin{pmatrix} 1 \\ -\dfrac{\boldsymbol{\sigma}\mathbf{q}_1}{E+m}\end{pmatrix}\chi_1\varphi(q_1),$$

$$\beta_l = 4^l\,|g_l|^2\, F_l(p,\, q')F^*_l(p,\, q)\frac{l\,!}{(2l+1)\,!!}\times$$

$$\times\frac{W^{4l}\,|\mathbf{q}|^{2l}\,(E+m)}{W^2-M^2+iM\Gamma}, \quad (35)$$

where $\chi^*_2(\chi_1)$ is the spinor of the final (initial) nucleon. From Eq. (35), we see that the contribution of $N^*(J)$ to the amplitude is expressed immediately in terms of the contracted propagator of $N^*(J)$ (22). We derive from Eq. (35) for the contributions to the amplitudes $f_1$ and $f_2$ in Eq. (34)( $J = l + \frac{1}{2}$):

$$f_1 = \beta_l\left[(W\pm M)P'_{l+1}(\cos\theta) -\right.$$

$$-(W \mp M)\frac{E-m}{E+m}P_l'(\cos\theta)\Big],$$

$$f_2 = \beta_l\left[-(W \pm M)P_l'(\cos\theta)+\right.$$

$$\left.+(W \mp M)\frac{E-m}{E+m}P_{l+1}'(\cos\theta)\right]. \qquad (36)$$

Then the partial (multipole) amplitudes corresponding to the contribution of $N^*(J)$ with definite $J^p$ can be written as

a) For the currents with the identity matrix in Eqs. (25) and (26),

$$\left(J^p = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+, ...;\ l = l_\pi,\ \ l_\pi = 0,\ 1,\ 2,\ 3, ...\right)$$

$$f_{l\pi+} = \beta_l(W+M),$$

$$f_{(l\pi+1)-} = \beta_l(W-M)\frac{E-m}{E+m}. \qquad (37)$$

b) For the currents with the $\gamma_5$-matrix in Eqs. (25) and (26)

$$\left(J^p = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, ...;\ l = l_\pi - 1,\ \ l_\pi = 1,\ 2, 3, ...\right)$$

$$f_{l\pi-} = \beta_l(W+M)\frac{E-m}{E+m},$$

$$f_{(l\pi-1)+} = \beta_l(W-M). \qquad (38)$$

From Eqs. (37) and (38), we can see that, in our approach, each $N^*(J)$ off the mass shell contributes to the partial amplitudes corresponding to $J_{\pi N} = J = l + \frac{1}{2} =$ only, but at different parities. Thus, we have the following difference between our and usual approaches: in the usual approach $\frac{1}{2} \leq J_{\pi N} \leq J$, whereas our approach gives $J_{\pi N} = J$.

For $\Delta$ (1232), Eq. (37) yields the following amplitudes:

$$f_{1+}^\Delta = P_{33}^\Delta = \beta_1(W + M_\Delta),$$

$$f_{2-}^\Delta = D_{33}^\Delta = \beta_1(W - M_\Delta)\frac{E-m}{E+m},$$

$$f_{0+}^\Delta = S_{31}^\Delta = 0,\ \ f_{1-}^\Delta = P_{31}^\Delta = 0. \qquad (39)$$

From Eqs. (31) and (39), we see that the usual and our approaches give different results for the $S_{31}$ and $P_{31}$ amplitudes and the same result for the $\mathrm{Im}D_{33}^\Delta/\mathrm{Re}P_{33}^\Delta$ ratio. The difference between the $\Delta$ (1232) contribution to the $S_{31}$- and $P_{31}$-amplitudes, derived in the usual and our approaches can be used to find the proper approach by means of the comparison with experimental data.

## 5. Possibility of Experimental Tests

The theorem on currents and fields for $J = 3/2$ can be tested by the comparison of the predictions derived in our and the usual approaches with the results of partial wave analysis in the $\Delta$ (1232)-region. Indeed, we see from Eq. (31) that, in the usual approach (interaction current (30)) corresponding to the $\Delta$(1232)), the contributions induced by $\Delta$ (1232) ($J^p = \frac{3}{2}^+$, $I = \frac{3}{2}$) to the amplitudes $S_{31}$ ($J^p = \frac{1}{2}^-$, $I = \frac{3}{2}$) and $P_{31}$ ($J^p = \frac{1}{2}^+$, $I = \frac{3}{2}$) must exist for $W \neq M_\Delta$. In contrast with this (corresponding to the $\Delta$ (1232) interaction current (27) which obeys Eqs. (10) and (12)), the contributions induced by $\Delta$ (1232) to $S_{31}$ and $P_{31}$ do not appear in the present method.

The amplitudes $S_{31}$, $P_{31}$, $P_{33}$, $D_{33}$ are known as results of the partial wave analyses, which are derived from the experimental data on differential cross-sections, polarizations, and asymmetries. Therefore, these partial amplitudes may be considered as the experimental results. We consider the results of recent partial wave analyses [24–28]. These amplitudes are the sums of the variety of different terms. In particular, the $S_{31}$ amplitude in the $\Delta$ (1232)-region can include the contributions of the $S_{31}$ (1620) and $S_{31}$ (1900)-resonances. Similarily, the $P_{31}$ - amplitude can include the contribution of $P_{31}$ (1910) resonance. As the masses of these resonances are quite large, their contributions in the $\Delta$ (1232)-region are relatively small. In the $\Delta$ (1232)-region, the particle 3-momenta are small in comparison with the masses of $N$ and $\Delta$(1232). Therefore, the energy dependences of the $S_{31}$-, $P_{31}$-, $D_{31}$-amplitudes can be approximated as linear ones. This is a particular case of the known $s$- and $p$-wave expansion of the amplitudes in the $\Delta$ (1232)-region.

Thus, in the $\Delta$ (1232)-region, we have two predictions for the $S_{31}$- and $P_{31}$-amplitudes related to our and usual approaches. In the present method, these amplitudes are some sums of unknown terms, but without the $\Delta$(1232)-contribution (i.e., we expect the linear energy dependence). In the usual approaches, the $\Delta(1232)$-resonance contributions to $S_{31}$ and $P_{31}$ must be added to these sums (i.e., near $W = M_\Delta$, the $S_{31}$- and $P_{31}$-amplitudes can have nonlinear strong energy dependences). Therefore, it is of great importance to evaluate the $\Delta(1232)$ contribution to the $S_{31}$- and $P_{31}$-amplitudes.

For the evaluation of the $\Delta(1232)$-resonance contributions to these amplitudes, we calculate firstly the $P_{33}^{\Delta}$-amplitude in the usual isobar approach. To calculate the amplitudes $P_{33}$, $S_{31}$, $P_{31}$, and $D_{33}$, we use Eq. (31) with $M_\Delta = 1232$ MeV and the total width, which depends on energy:

$$\Gamma(W) = \Gamma_R \left(\frac{|\mathbf{q}|}{|\mathbf{q}_R|}\right)^3 \left[\frac{1 + (|\mathbf{q}_R| r)^2}{1 + (|\mathbf{q}| r)^2}\right]. \quad (40)$$

Here, $\Gamma_R = 112$ MeV, $|\mathbf{q}|$ is the modulus of the pion 3-momentum, $|\mathbf{q}_R|$ is this modulus calculated at $W = M_\Delta$, $r = 1.11$ fm [24]. Using Eq. (31) and the value of $\mathrm{Im}P_{33} = 0.984$ at $T_{\mathrm{lab}} = 200$ MeV [25] ($T_{\mathrm{lab}}$ is the pion kinetic energy in the laboratory frame), we calculate $\mathrm{Re}P_{33}$, $\mathrm{Im}P_{33}$, $\mathrm{Re}S_{31}$, $\mathrm{Im}S_{31}$, $\mathrm{Re}P_{31}$, $\mathrm{Im}P_{31}$, $\mathrm{Re}D_{33}$, $\mathrm{Im}D_{33}$ in the energy region $M_\Delta - \Gamma_R \le W \le M_\Delta + \Gamma_R$. Note that $\mathrm{Im}P_{33}$ at $W = M_\Delta$ has the maximal value among all amplitudes for any energy. The calculations of the $P_{33}$-amplitude in the usual isobar model by means of Eqs. (31) and (40) are compared with the results of the partial wave analyses [24, 25] given in Fig. 2.

We see that the $\Delta(1232)$ contribution calculated in the usual isobar model gives a good description at $T < 240$ MeV, where $T$ is the pion kinetic energy in the laboratory frame. The calculations of the $\Delta(1232)$ contributions to the amplitudes $S_{31}$, $P_{31}$, $D_{33}$ by means of Eqs. (31) and (40) in the $\Delta(1232)$-region show that the $S_{31}$ amplitude has the largest sensitivity to the $\Delta(1232)$ contribution, and the $D_{33}$-amplitude has the smallest one. This is a consequence of Eq. (31), since $\mathrm{Re}S_{31}^{\Delta} \sim (W - M_\Delta)^2$, $\mathrm{Im}S_{31}^{\Delta} \sim (W - M_\Delta)$, $\mathrm{Im}P_{31}^{\Delta} \sim (W - M_\Delta)^2$, $\mathrm{Re}P_{31}^{\Delta} \sim (W - M_\Delta)^3$, and $D_{33}$ has the small factor $(E - m)^2$. The results of these calculations for $S_{31}^{\Delta}$ are compared with those of the partial wave analysis [24, 25] given in Figs. 3,*a*, 3,*b*.

From Fig. 3, we see that the $\Delta(1232)$-contributions to $\mathrm{Re}S_{31}$ and $\mathrm{Im}S_{31}$, derived in the usual isobar approach with the use of Eqs. (31) and (40), have strong energy dependences for $S_{31}$ $M_\Delta - \Gamma_R \le W \le M_\Delta + \Gamma_R$. As in the usual approach, the $S_{31}$-amplitude is a sum of some linear function and the $\Delta(1232)$ contribution, $\mathrm{Im}S_{31}(W)$ must have the minimum at $T \approx 145$ MeV and $\mathrm{Re}S_{31}(W)$ must have the maximum at $W = M_\Delta$

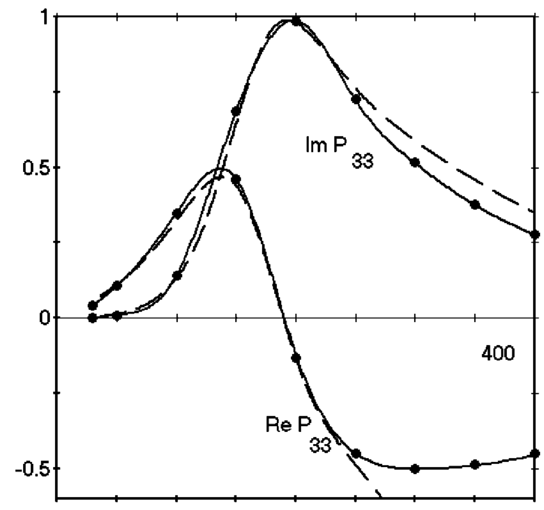


Fig. 2. Comparison of the $\Delta(1232)$-contribution to the $P_{33}$-amplitude derived in the usual isobar model with the results of the partial wave analysis. The full lines correspond to Ref. [24], the dots – to Ref. [25], and the dashed lines – to the calculations based on Eqs. (31)

($T \approx 190$ MeV). It is clear that the total $S_{31}$ amplitude in the $\Delta(1232)$-region is the sum of a linear function of the energy and $S_{31}^{\Delta}$ ((31) or (39)). Thus, we have two predictions: 1) In the usual approach, $S_{31}$ must have a rather strong energy dependence; in particular, the total $\mathrm{Re}S_{31}$ must have a maximum at $T \approx 190$ MeV, and the total $\mathrm{Im}S_{31}$ must have a minimum at $T \approx 145$ MeV; 2) In the present method, the total $\mathrm{Re}S_{31}$ and $\mathrm{Im}S_{31}$ must be approximately some linear functions of the energy, which follows from Eqs. (39).

According to the partial wave analysis (see, e.g., [24–28]) in the $\Delta(1232)$-region, all amplitudes (with the exception of $P_{33}$ and $P_{31}$) are approximately linear functions of the energy. Thus, we may conclude that the $S_{31}$ amplitude has no $\Delta(1232)$-contribution in reality (i.e., $S_{31}^{\Delta} = 0$ in agreement with Eq. (39)). Therefore, we may conclude that the theorem on currents and fields for the $\Delta(1232) \rightleftarrows N\pi$ transitions is valid.

Now, we consider the form-factors $F_l(p, q)$ and $F_l(p, q')$ in the interaction currents (25) and (26). For the interaction of higher-spin bosons with two spinless particles ($J(p) \rightleftarrows O(k_1) + O(k_2)$), the form-factor

$$f(p, q) = (q^2)^{2n_2} \left[(pq)^{2n_1} + a^{4n_1}\right]^{-1} \times$$

$$\times \left[\left(2(pq)^2 - p^2 q^2\right)^{2n_2} + (b^2 q^2)^{2n_2}\right]^{-1},$$

$$p = k_1 + k_2, \ \ q = k_1 - k_2 \quad (41)$$

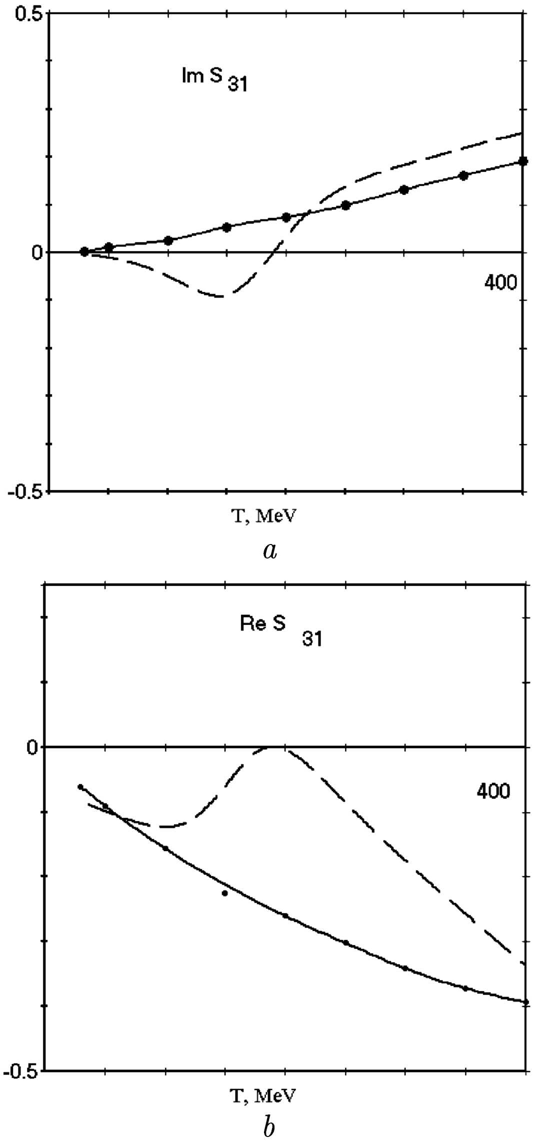


Fig. 3. Comparison of the $\Delta(1232)$ contribution to the Im$S_{31}$ ($a$) and Re$S_3 1$ ($b$) derived by means of the Eqs. (31) (the dashed lines) with the results of the partial wave analysis: Ref. [24] – the full lines, Ref. [25] – the dots

has been derived in Refs. [20, 21]. Here, $a$ and $b$ are positive constants, $n_1$ and $n_2$ are positive integers. The theorem on the current asymptotics for the $J(p) \rightleftarrows O(k_1) + O(k_2)$ transitions can be satisfied for $n_1 \geq 2J + 3$, $n_2 \geq 2 + J/2$. We can try to apply function (41) in currents (25) and (26) and consider the contributions of $N^*(J)$ with these currents to the $s$-channel amplitudes of the elastic $\pi N$-scattering. In the $\pi N$-scattering, $q = q_1 - p_1$, $q' = q_2 - p_2$, $q^2 = (q')^2 = 2(m^2 + \mu^2) - s$ ($\mu$ is the pion mass), $s = p^2 = W^2$, $(pq) = (pq') = \mu^2 - m^2$. Therefore, we have, for the $\pi N$-scattering,

$$f(p,\ q) \sim \left[2(m^2 + \mu^2) - s\right]^{2n_2}. \tag{42}$$

Thus, function (41) vanishes at $s_0 = W_0^2 = 2(m^2 + \mu^2)$, $W_0 = 1341$ MeV, $T = 339$ MeV. We see that the contributions of all resonances $N^*(J)$ for currents (25) and (26) with function (41) to the $s$-channel amplitudes of the $\pi N$-scattering vanish at $W = W_0$. This contradicts the experimental data; e.g., for $\Delta(1232)$, this can be seen from Fig. 2. Thus, function (41) cannot be used for the form-factors $F(p, q)$ and $F(p, q')$ in currents (25) and (26). Therefore, it is of great importance to find the form-factor for the currents of the $N^*(J) \rightleftarrows N\pi$ transitions.

## 6. Conclusion

The considerations of the HSF interactions in Ref. [19] and the present paper have been carried out to achieve a mathematical consistency. We can see that this mathematical consistency leads to the elimination of the problems for HSF interactions additional to the problems of the 0- and 1/2-spin particle interactions. Indeed, from the consistency of the system of equations for the HSF field spin-tensors in the momentum representation, we infer that the HSF interaction currents must obey the same conditions as the HSF fields. It is the content of the theorem on currents and fields.

The HSF propagators coincide in our and usual approaches only on the HSF mass shell. In our approach, the HSF propagator for any spin has the scale dimension – 1, the same as for the 1/2-spin particle. This allows us to eliminate the power divergences appearing from the HSF propagators of the usual approaches.

The convolutions of our HSF propagator (which can be derived from the contracted projection operator by means of (18)) with the HSF momentum $p$ and $\gamma$-matrices vanish at any $p$; whereas, in the common approaches, these convolutions vanish on the HSF mass shell only. As a consequence of the vanishing of these convolutions, the physical currents have the same momentum dependence for any common currents. This allows us to eliminate the ambiguities of the current in the usual approaches. In addition, as a consequence of Eqs. (10)–(12), the partial solution of the inhomogeneous equations may be written as

$$U(x)^{\rm non-hom}_{\mu_1\ldots\mu_l} = \int d^4y \frac{1}{(-\Box_x)^l} S(x-y) j(y)_{\mu_1\ldots\mu_l},$$

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where $S(x-y)$ is the Green function of the 1/2-spin particle.

In our approach, the virtual HSF (e.g., $N^*(J)$ in the $\pi N$ -scattering) can change the parity, but they do not contribute to states with spin less than $J$; whereas, in the usual approaches, such contributions exist. We have tested the predictions of our and usual approaches for the virtual $\Delta(1232)$ in the elastic $\pi N$-scattering. The calculations performed in the usual isobar model show sharp energy dependences of the contributions to the $S_{31}$-and $P_{31}$-amplitudes at $W \approx M_\Delta$. It turns out that the $S_{31}$ amplitude is the most sensitive (the $\Delta(1232)$ contributions to the $D_{33}$ amplitude are very small in our and usual approaches). According to the partial wave analysis, the energy dependences of the real and imaginary parts of the amplitudes are approximately linear, i.e., they differ from the predictions of the usual isobar model. This means that the predictions of our approach are valid. Thus, we have examined the validity of conditions (10) and (12). To examine the validity of conditions (10)–(12), we must consider HSF with spin $J \geq 5/2$. For example, it is of interest to study the contributions of $F_{15}(1680)$ $\left(J^p = \frac{5^+}{2}\right)$ [29] to the $S_{11}$, $P_{11}$, $P_{13}$, $D_{13}$-amplitudes; $F_{35}(1905)$ $\left(J^p = \frac{5^+}{2}\right)$ to the $S_{31}$, $P_{31}$, $P_{33}$, $D_{33}$; $F_{37}(1950)$ $\left(J^p = \frac{7^+}{2}\right)$ to the $S_{31}$, $P_{31}$, $P_{33}$, $D_{33}$, $D_{35}$, $F_{35}$-amplitudes.

For the physical currents of the $\pi N \rightleftarrows \Delta(1232)$ transitions (25) and (26), we need the form-factor $F(p,q)$. Since the form-factor derived in Refs. [20, 21] for the higher spin boson interactions cannot be used in the $\Delta(1232) \rightleftarrows \pi N$-transitions, it is of great importance to modify the form-factor [20, 21] or to derive a new form-factor.

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ЗАГАЛЬНІ ВЛАСТИВОСТІ СТРУМІВ ВЗАЄМОДІЇ ВИСОКОСПІНОВИХ ФЕРМІОНІВ І ЇХ ПЕРЕВІРКА В $\pi N$-РОЗСІЮВАННІ

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Отримано струми взаємодій високоспінових ферміонів з частинками із спіном 0 і 1/2, які можуть бути використані для переходів $N^*(J) \rightleftarrows N\pi$ ($N^*(J)$-нуклонний резонанс із спіном $J$). Згідно з теоремою про поля і струми спін-тензори цих струмів є безслідовими, а їхні добутки на $\gamma$ матриці та імпульс високоспінового ферміона дорівнюють нулю, подібно до польових спін-тензорів. Для спіна $J = \frac{3}{2}$ та $\frac{5}{2}$ такі струми отримані явно. Показано, що в нашому підході масштабна вимірність пропагатора високоспінового ферміона дорівнює $-1$ для будь-якого $J \geq \frac{1}{2}$. Обчислення внесків $N^*(J)$ поза масовою поверхнею в $s$ -канальні амплітуди $\pi N$-розсіювання для будь-якого $J$ в нашому підході показує, що ненульові внески відповідають тільки $J = J_{\pi N}$ ($J_{\pi N}$ – повний кутовий момент $\pi N$-системи). В протилежність цьому в звичайно використовуваних підходах такі ненульові амплітуди відповідають $\frac{1}{2} \leq J_{\pi N} \leq J$. Зокрема, зазвичай, у використовуваних підходах $\Delta(1232)$ поза масовою поверхнею дає ненульові внески у амплітуди $S_{31}$, $P_{31}$ $\left(J_{\pi N} = \frac{1}{2}\right)$ і $P_{33}$, $D_{23}$ $\left(J_{\pi N} = \frac{3}{2}\right)$, а наш підхід – лише у $P_{33}$ і $D_{33}$. Порівняння цих результатів з даними парціально-хвильових аналізів для амплітуди $S_{31}$ в області $\Delta(1232)$ показує краще узгодження з нашим підходом.