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## ANOMALIES IN INTERNAL CONVERSION COEFFICIENTS FOR HINDERED ROTATIONAL $\gamma$ -TRANSITIONS

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New data concerning the penetration effect in the  $M1$  component of hindered rotational  $\gamma$ -transitions have been obtained. The effect is caused by different selection rules for  $\gamma$ -emission and intranuclear conversion matrix elements. The magnitude of penetration parameter for the  $M1$ -component of the 129-keV  $\gamma$ -transition in  $^{191}\text{Ir}$  nucleus is determined for the first time. The results obtained for similar transitions in  $^{163}\text{Er}$ ,  $^{165}\text{Er}$ , and  $^{177}\text{Hf}$  nuclei are also reported. By comparing the experimental penetration matrix elements with those calculated from the Nilsson model, the following renormalization coefficients are found for the gyromagnetic ratios for the spin-multipole interaction in  $M1$ -conversion transitions:  $g_s(M1)/g_s^{\text{free}}(p) = 0.574 \pm 0.023$  and  $g_s(M1)/g_s^{\text{free}}(n) = 0.59 \pm 0.07$ .

### 1. Introduction

The internal conversion of  $\gamma$ -rays is a process, in which an atomic nucleus, while transiting from one excited state into another one characterized by a lower energy, transfers the excitation energy immediately to one of the electrons in the atomic shell. This results in that the electron leaves the atom. The ratio between the number of conversion electrons that escaped from the atom,  $N_e$ , and the number of  $\gamma$ -quanta that left the nucleus within the same time interval,  $N_\gamma$ , is called the internal conversion coefficient (ICC) of  $\gamma$ -rays [1],

$$\alpha = N_e/N_\gamma. \quad (1)$$

As early as in the first theoretical works, the conclusion was drawn that the ICC is a parameter of the nuclear transition, which does not depend on nuclear structure details. The method, which used the ICC to determine the spins and the parity of nuclear levels, was considered for a long time as completely independent of any theoretical assumptions concerning the structure of the

nucleus, except the most general requirements dealing with the energy, angular momentum, and parity conservation laws in radiative and conversion nuclear transitions. Nevertheless, as it turned out later [2], this conclusion is valid only in the case where the probability of  $\gamma$ -emission at this transition is not small in comparison with that of a one-particle transition. However, if the transition is strongly forbidden in this sense, the ICC value can substantially depend on the nuclear structure.

The case in point concerns anomalies in the ICCs of  $\gamma$ -rays associated with the penetration effect. By the penetration effect or the intranuclear conversion in the internal conversion theory is meant a correction to the ICC, which arises, when the transition electromagnetic potentials (they emerge at the moment, when the nucleus transits from one nuclear level onto another) calculated for a point-like nucleus are substituted by those calculated for a nucleus with finite dimensions.

According to the results of work [2], the matrix element of the conversion transition calculated with regard for finite dimensions of a nucleus is divided into two components,

$$\langle M_e \rangle = \langle M_e \rangle_{r < R} + \langle M_e \rangle_{r > R}, \quad (2)$$

in the regions  $r < R$  and  $r > R$ , where  $r$  and  $R$  are the coordinates of an electron and a nucleon, respectively, that interact at the transition. As it was in the case of a point-like nucleus, the matrix element  $\langle M_e \rangle_{r > R}$  turns out proportional to the matrix element of  $\gamma$ -transition,  $\langle U_\gamma \rangle$ . Therefore, the ICC equals

$$\alpha = \left| \frac{\langle M_e \rangle}{\langle U_\gamma \rangle} \right|^2 = \sum_{\mathcal{x}} |M_{\mathcal{x}} + i\Delta_{\mathcal{x}}|^2, \quad (3)$$

where  $M_{\varkappa}$  is the a normal conversion matrix element independent of the nuclear variables; the quantity

$$\Delta_{\varkappa} = \frac{\langle M_e \rangle_{r < R}}{\langle U_{\gamma} \rangle} \quad (4)$$

reflects the contribution of the intranuclear conversion; the quantum number  $\varkappa = (l - j)(2j + 1)$  characterizes the electron state after the conversion process; and  $j$  and  $l$  are the total and orbital, respectively, electron angular momenta.

To simplify calculations, it is convenient to expand the Hankel and Bessel functions, as well as the electron wave functions, in the expression for the intranuclear conversion matrix element  $\langle M_e \rangle_{r < R}$  in power series in  $r$ , because  $r < R_0$  inside the nucleus. Then, the integration over the electron variables is carried out explicitly, and  $\Delta_{\varkappa}$  can be represented as a linear combination of the so-called nuclear penetration and electronic parameters.

The penetration effect in the case of magnetic multipole transitions can be described with the use of a single nuclear parameter [3]

$$\lambda = \frac{\langle f || \hat{\mathbf{J}}_n \mathbf{T}_L^{(0)*} \left( \frac{R}{R_0} \right)^{L+2} || i \rangle}{\langle f || \hat{\mathbf{J}}_n \mathbf{T}_L^{(0)*} \left( \frac{R}{R_0} \right)^L || i \rangle}, \quad (5)$$

where  $\hat{\mathbf{J}}_n$  is the operator of nuclear current at the transition,  $\mathbf{T}_L$  are the vector spherical functions,  $R_0 = 1.2A^{1/3}$  fm is the nucleus radius, and  $L$  is the multipolarity order. Formula (5) defines  $\lambda$  as a ratio between dimensionless penetration and emission matrix elements. The intranuclear conversion matrix element differs from the radiation one by both the form of the integrand and the integration limits.

As a rule, the corrections stemming from the penetration effect do not exceed 2% and do not affect the magnitude of ICC considerably. The absolutely different situation takes place in the case of hindered  $\gamma$ -transitions. The hindered character of a  $\gamma$ -transition means that the denominator in formula (5) is substantially reduced. In this case, the contribution from the intranuclear conversion can dominate and determine, alone, the ICC magnitude. Certainly, it is necessary that the selection rules, which are responsible for a reduction of the  $\gamma$ -emission probability, should not affect (or do it appreciably) the probability of the intranuclear conversion. Really, in certain cases such as the so-called “random” forbiddenness of  $M1$ -transitions, the  $l$ -forbiddenness, and the forbiddenness according to the selection rule for asymptotic quantum numbers in deformed nuclei, the selection rules

for matrix elements of  $\gamma$ -emission and intranuclear conversion turn out different [4], and the ICCs are anomalous. It is the former case that is a subject of our researches.

## 2. Selection of Objects for Studying

In the generalized model of nucleus, the reduced probability of transition  $M1$  between the rotational band levels (for  $K \neq 1/2$ ) is determined by the formula [5]

$$B(M1, I + 1 \rightarrow I) = \frac{3}{4\pi} \left( \frac{e\hbar}{2mc} \right)^2 (g_K - g_R)^2 \times \frac{K^2(I + 1 + K)(I + 1 - K)}{(I + 1)(2I + 3)}. \quad (6)$$

The gyromagnetic ratios for internal,  $g_K$ , and collective,  $g_R$ , motions, which are included into formula (6), are connected with the magnetic moment  $\mu_0$  of the ground state of a band by the relation

$$\mu_0 = \frac{I_0}{I_0 + 1} (g_K I_0 + g_R). \quad (7)$$

They can be calculated with the help of experimentally measured values for  $B(M1)$  and  $\mu_0$ . In some cases, the gyromagnetic ratios for the frame,  $g_R$ , and the particle beyond it,  $g_K$ , turn out close by value. This circumstance gives rise to a “random” forbiddenness of  $M1$ -transitions in deformed nuclei.

Experimental values of  $g_R$  for nuclei with odd  $A$ 's are concentrated in a vicinity of about 0.3 in the middle of the deformation range and approach the value  $g_R \cong Z/A$ ; the latter estimate was obtained in the framework of the generalized model and is based on the assumption about a regular motion of the charged nuclear substance [6]. At the same time, the  $g_K$ -factor changes in wider limits, and it is a characteristic quantity, which can be used for the configuration identification [7]. By analyzing the data on magnetic  $g_K$ -factors for odd deformed nuclei reported in work [8], we came to a conclusion that the “random” forbiddenness of  $M1$ -transitions can be observed in rotational bands constructed of both one-particle states of the unpaired proton (3/2[402] and 7/2[404]) and one-particle states of the unpaired neutron (5/2[523] and 7/2[514]). Since the expected contribution to the ICC associated with the penetration effect, according to theoretical estimations, would not exceed several percent [9], the precision measurements for the coefficients of internal conversion are needed for those

**Table 1.** Main characteristics of hindered rotational  $M1$ -transitions in deformed nuclei and experimental values of nuclear penetration parameters

Nucleus	$E_\gamma$ , keV	$K[Nn_Z\Lambda]$	$J_i^\pi, J_f^\pi$	$F_W$	$\lambda^{\text{exp}}$	Source
$^{161}_{66}\text{Dy}$	57.2	3/2[521]	$5/2^-, 3/2^-$	$19 \pm 4$	$-7_{-8}^{+6}$	[12]
$^{163}_{68}\text{Er}$	84.0	5/2[523]	$7/2^-, 5/2^-$	$700 \pm 100$	$-(4.0 \pm 4.2)$	[13]
$^{165}_{67}\text{Ho}$	94.7	7/2[523]	$9/2^-, 7/2^-$	$3.6 \pm 0.2$	$2 \leq \lambda \leq 5$	[14]
$^{165}_{68}\text{Er}$	77.3	5/2[523]	$7/2^-, 5/2^-$	$870 \pm 70$	$4.4_{-3.9}^{+2.1}$	[15]
$^{175}_{71}\text{Lu}$	113.8	7/2[404]	$9/2^+, 7/2^+$	$28 \pm 1$	$-$	[16]
$^{177}_{72}\text{Hf}$	113.0	7/2[514]	$9/2^-, 7/2^-$	$2300 \pm 500$	$8.4_{-2.5}^{+2.2}$	[17]
$^{177}_{72}\text{Hf}$	136.7	7/2[514]	$11/2^-, 9/2^-$	$900 \pm 400$	$-(3 \pm 1)$	[18]
$^{181}_{73}\text{Ta}$	136.3	7/2[404]	$7/2^+, 5/2^+$	$14.7 \pm 0.9$	$(3.3 \pm 4.1)^*$	[18]
$^{191}_{77}\text{Ir}$	129.4	3/2[402]	$5/2^+, 3/2^+$	$39 \pm 1$	$(3.8 \pm 4.1)^*$	[18]
$^{193}_{77}\text{Ir}$	138.9	3/2[402]	$5/2^+, 3/2^+$	$32 \pm 1$	$\approx 2$	[19]
$^{177}_{72}\text{Hf}$	113.0	7/2[514]	$9/2^-, 7/2^-$	$2300 \pm 500$	$-(3 \pm 1)$	[20]
$^{177}_{72}\text{Hf}$	136.7	7/2[514]	$11/2^-, 9/2^-$	$900 \pm 400$	$-(0.6 \pm 2.0)$	[21]
$^{181}_{73}\text{Ta}$	136.3	7/2[404]	$7/2^+, 5/2^+$	$14.7 \pm 0.9$	$5.6 \pm 8.4$	[22]
$^{191}_{77}\text{Ir}$	129.4	3/2[402]	$5/2^+, 3/2^+$	$39 \pm 1$	$-$	
$^{193}_{77}\text{Ir}$	138.9	3/2[402]	$5/2^+, 3/2^+$	$32 \pm 1$	$-$	

\*The  $\delta^2(E2/M1)$ -values were taken from different works

transitions. The measurements can be executed only on a high-resolution magnetic  $\beta$ -spectrometer and, since the aperture ratio of such devices is small, only for those  $\gamma$ -transitions, which are sufficiently intense and have a suitable half-life period of the parent nucleus.

In Table 1, we give the basic characteristics of  $M1$ -transitions, which are the most promising ones for such researches, as well as experimental values of nuclear penetration parameters  $\lambda$  taken from works of other authors. The prohibition factors for  $\gamma$ -emission,  $F_W$ , with respect to Weisskopf's estimates were calculated using the data in work [10]. The authors of review [11] reported the results for two more rotational  $M1$ -transitions (in  $^{161}\text{Dy}$  and  $^{165}\text{Ho}$ ). Those transitions are characterized by large differences  $|g_K - g_R|$  and, as a consequence, small prohibition factors. The corresponding data were also included into Table 1.

Table 1 demonstrates that the largest prohibition factors for  $M1$ -transitions are observed for  $^{163}\text{Er}$ ,  $^{165}\text{Er}$ , and  $^{177}\text{Hf}$  nuclei. The largest ICC anomalies should also be expected for those nuclei. For  $^{163}\text{Er}$  and  $^{165}\text{Er}$  nuclei, the nuclear penetration parameters have not been measured at all; for  $^{177}\text{Hf}$ , the data are inconsistent. The determination error for  $\lambda$  is too large for other  $M1$ -transitions as well. In particular, for  $^{175}\text{Lu}$ , the weighted average value is  $\lambda = -0.7 \pm 1.4$ . Our researches aimed at specifying the ICC data for hindered rotational transitions in

$^{163}\text{Er}$ ,  $^{165}\text{Er}$ ,  $^{177}\text{Hf}$ , and  $^{191}\text{Ir}$  nuclei and obtaining experimental values of penetration parameters, which would be suitable for further analysis.

### 3. Analysis Procedure of ICC Anomalies in Mixed $M1 + E2$ -Transitions

The penetration,  $\lambda$ , and mixture,  $\delta$ , parameters were determined by solving the corresponding system of equations for absolute or relative ICC. For any  $i$ -th subshell of the mixed  $M1 + E2$ -transition with regard for the penetration effect in the  $M1$ -component, the experimental ICCs look like [23]

$$\alpha_{i,\text{exp}} = \frac{\alpha_i(M1)(1 + B_1^i\lambda + B_2^i\lambda^2) + \delta^2\alpha_i(E2)}{1 + \delta^2}, \quad (8)$$

where  $B_1^i$  and  $B_2^i$  are parameters that depend only on the electron wave functions and are tabulated in work [23];  $\alpha_i(M1)$  and  $\alpha_i(E2)$  are theoretical ICC values for the  $i$ -th subshell for the  $M1$ - and  $E2$ -transitions, respectively; and  $\alpha_{i,\text{exp}}$  are experimental ICC values for the  $i$ -th subshell. A similar expression can also be written down for the ratio  $(\alpha_i/\alpha_j)_{\text{exp}}$  between the experimental ICC values for the  $i$ -th and  $j$ -th subshells,

$$\left(\frac{\alpha_i}{\alpha_j}\right)_{\text{exp}} = \frac{\alpha_i(M1)(1 + B_1^i\lambda + B_2^i\lambda^2) + \delta^2\alpha_i(E2)}{\alpha_j(M1)(1 + B_1^j\lambda + B_2^j\lambda^2) + \delta^2\alpha_j(E2)}. \quad (9)$$

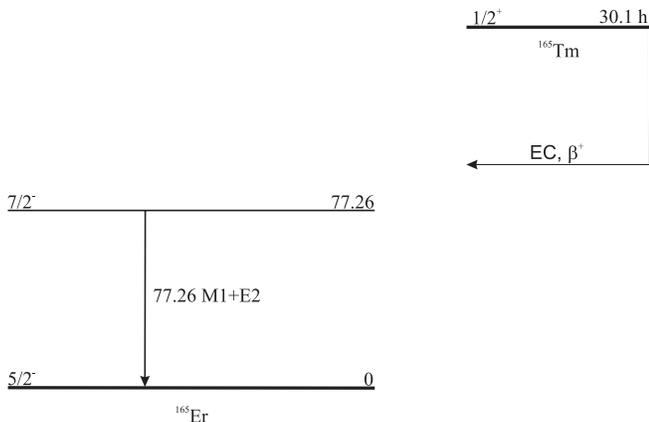


Fig. 1. Fragment of the schematic diagram of  $^{165}\text{Tm}$  decay

The system of equations (8) and (9) was solved by minimizing the following functional with the use of the least square technique,

$$\chi_{\min}^2 = \left( \frac{\alpha_{i,\text{exp}} - \alpha_i(\lambda, \delta)}{\Delta\alpha_{i,\text{exp}}} \right)^2 + \sum_{i,j} \left( \frac{(\alpha_i/\alpha_j)_{\text{exp}} - \alpha_i(\lambda, \delta)/\alpha_j(\lambda, \delta)}{\Delta(\alpha_i/\alpha_j)_{\text{exp}}} \right)^2, \quad (10)$$

where  $\Delta\alpha_{i,\text{exp}}$  and  $\Delta(\alpha_i/\alpha_j)_{\text{exp}}$  are the errors for the corresponding quantities;  $\alpha_i(\lambda, \delta)$ ,  $\alpha_j(\lambda, \delta)$ , and  $\alpha_i(\lambda, \delta)/\alpha_j(\lambda, \delta)$  are the theoretical ICC values and the ICC ratio for the  $i$ -th and  $j$ -th subshells, which depend on the parameters  $\lambda$  and  $\delta$ , which are the fitting ones in the  $\chi_{\min}^2$ -minimization method.

In order to avoid local minima, the initial values of  $\lambda$  and  $\delta$  were determined by solving the system of equations (8) and (9) graphically. Theoretical values for the ICC and electronic parameters were obtained by interpolating the values tabulated in works [24] and [23], respectively.

The standard errors were determined, by using the relation

$$\chi^2(\lambda^{\text{opt}} \pm \Delta\lambda) = \chi_{\min}^2 + 1, \quad (11)$$

where  $\lambda^{\text{opt}}$  is the optimum value of parameter  $\lambda$ , which minimizes the quantity  $\chi^2$ . All other parameters are fixed at that and correspond to their optimal values.

The errors for  $\delta$  were determined in a similar way.

#### 4. Experimental Part

The first experiments in this direction were executed by us at the beginning of the 1990s [25, 26]. Now, we continue this activity.

The spectra of internal-conversion electrons (ICEs) of  $\gamma$ -transitions were measured on a magnetic  $\beta$ -spectrometer of the  $\pi\sqrt{2}$ -type with an iron yoke and an equilibrium orbit radius of 50 cm [27]. The dependence of the electron counting rate on the voltage applied between the radiation source and the spectrometer chamber was measured. While measuring, the magnetic field was maintained constant, being stabilized at three points along the radius making use of the nuclear magnetic resonance technique. The resolving power of the spectrometer was 0.04% with respect to the pulse at a solid angle of 0.07% of  $4\pi$ . The spectrometer enabled the relative intensities of conversion lines to be determined with an error not worse than 1%.

For searching anomalies in the ICCs for mixed  $M1 + E2$ -transitions, it is very important to possess precision data concerning not only the relative, but also the absolute ICC values for various atomic subshells. Since it is very difficult to reach a required accuracy while determining ICCs by directly comparing the ICE and  $\gamma$ -ray intensities, we found them in the following manner. In the energy range of ICE and  $\gamma$ -ray spectra, which was close to the energy of the examined transition, a transition with well-known multipolarity was selected, and the theoretical values of its ICC were used as normalizing ones. Having measured the ratios between the ICE and  $\gamma$ -ray intensities for those two transitions, the absolute ICC values can be determined by the formula

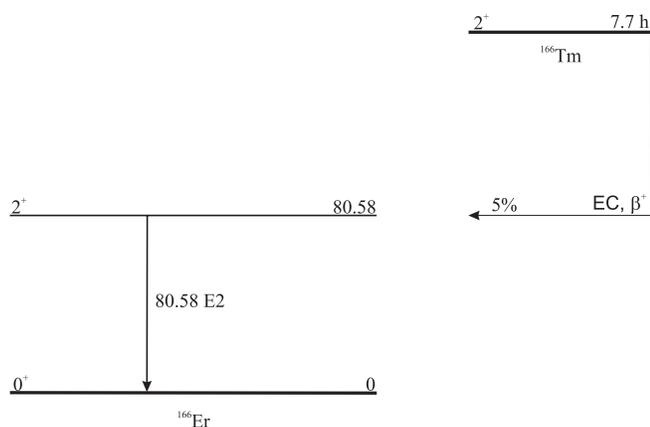
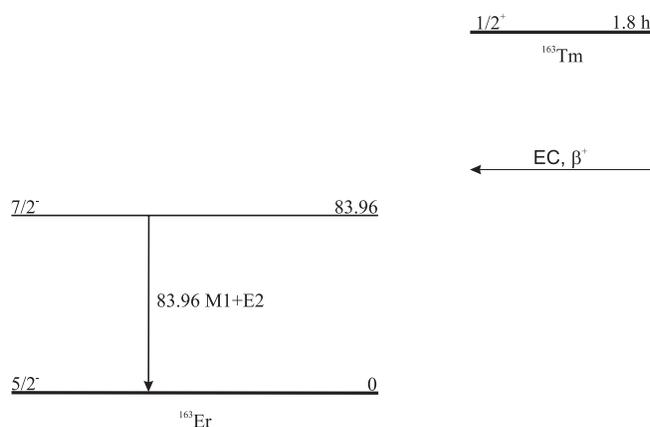
$$\alpha_i = \alpha_j \frac{I_e^i I_\gamma^j}{I_e^j I_\gamma^i}, \quad (12)$$

where  $I_e^{i,j}$  and  $I_\gamma^{i,j}$  are the experimental values of ICE and  $\gamma$ -ray, respectively, intensities for the corresponding transitions; and  $\alpha_{i,j}$  are the absolute values of their ICCs.

To measure the  $\gamma$ -spectra, we used hyper-pure germanium detectors 2 and 160 cm<sup>2</sup> in volume and with resolving powers of 490 and 800 eV, respectively, at the 122 keV  $\gamma$ -line of  $^{57}\text{Co}$ . The conversion and  $\gamma$ -spectra were treated by the software programs developed by us [27–29].

##### 4.1. $\gamma$ -transition with an energy of 77 keV in $^{165}\text{Er}$

The ratio between the ICCs for the  $K$ - and  $L$ -shells, as well as the absolute ICC for the  $K$ -shell, of  $^{165}\text{Er}$  for this  $\gamma$ -transition was obtained, while studying the conversion spectrum of  $^{165}\text{Tm}$  ( $T_{1/2} = 30.1$  hour) [30–33]. A fragment of the corresponding decay scheme is shown in Fig. 1. The accuracy of those measurements was low, so

Fig. 2. Fragment of the schematic diagram of  $^{166}\text{Tm}$  decayFig. 3. Fragment of the schematic diagram of  $^{163}\text{Tm}$  decay

that the results of different works are in bad agreement with one another. It was impossible to obtain the penetration,  $\lambda$ , and mixture,  $\delta(E2/M1)$ , parameters from those data, which explains their absence in Table 1. The results of correlation measurements [34] did not clarify the situation.

For the  $M1$ -conversion in the  $L_3$ -subshell, the ICC almost does not depend on the nuclear parameters  $\lambda$ , because the coefficients  $B_1$  and  $B_2$  in formula (8) are small or even equal zero. However, they contain information concerning the mixture of multiplicities  $\delta^2(E2/M1)$ . Therefore, together with the measurements of the ratios between ICE intensities for  $L$ -subshells, we also measured the absolute ICC for the  $L_3$ -subshell of  $^{165}\text{Er}$  for the  $\gamma 77$ -keV transition [26].

$^{165}\text{Tm}$  radiation sources were produced in the course of the  $(p, n)$  reaction, by bombarding targets of enriched  $^{166}\text{Er}$  with protons with the energy  $E_p = 23$  MeV on an U-240 isochronous cyclotron at the Institute for Nuclear Research (Kyiv) of the National Academy of Sciences of Ukraine. The targets were fabricated by sputtering erbium fluoride onto an aluminum substrate in vacuum. The thickness of a sputtered layer was about  $50 \mu\text{g}/\text{cm}^2$ .

To determine the absolute ICC for the  $L_3$ -subshell of  $^{165}\text{Er}$ , we used the fact that, when of an erbium target is irradiated, besides  $^{165}\text{Tm}$  nuclei,  $^{166}\text{Tm}$  ones ( $T_{1/2} = 7.7$  hour) are also created in the course of the

$(p, n)$  reaction. The decay of the latter nuclei includes the intense  $\gamma 81$ -keV transition of the  $E2$  multipolarity, which is close by energy to the  $\gamma 77$ -keV transition, and the theoretical ICC values for which can be used as normalizing ones. A fragment of the  $^{166}\text{Tm}$  decay scheme is shown in Fig. 2.

The intensity ratio  $I_\gamma(77)/I_\gamma(81)$  was measured with the help of an HPGe-detector  $160 \text{ cm}^3$  in volume. Since this ratio changes in time, the measurements of the  $\gamma$ -spectrum were carried out before the measurements of the ICE intensity ratio  $L_3(77)/L_3(81)$  taking the half-life periods for  $^{165}\text{Tm}$  and  $^{166}\text{Tm}$  into account. The results of measurements are quoted in Table 2. When determining the absolute ICC for the  $\gamma 77$ -keV transition in the  $L_3$ -subshell of  $^{165}\text{Er}$  by formula (12), the theoretical value  $\alpha_{L_3}(81) = 1.925$  [24] was used for the  $E2$ -transition  $\gamma 81$  keV.

The relative ICE intensities measured by us for  $L$ -subshells of  $^{165}\text{Er}$  agree with the data of works [30, 31, 33] within the experimental errors, but they are more accurate. The analysis of conversion data given in Table 2 allowed us to obtain the following value for the mixture parameter:  $|\delta(E2/M1)| = 6_{-3}^{+\infty}$  (at  $\lambda \cong 0$ ). It agrees well with the results of work [34]. Unfortunately, the accuracy achieved in the measurements of absolute and relative ICCs of the  $\gamma 77$ -keV transition in the  $L$ -subshells of  $^{165}\text{Er}$  turned out insufficient to simultaneously determine the magnitude of penetration parameter  $\lambda$ .

**Table 2.** Experimental values of the ICC for the  $\gamma 77$ -keV transition in the  $L_3$ -subshell and the relative ICE intensities for  $L$ -subshells of  $^{165}\text{Er}$

Subshell	$I_e$ , rel. units	ICC
$L_1$	$0.15 \pm 0.07$	–
$L_2$	$0.91 \pm 0.10$	–
$L_3$	1	$2.27 \pm 0.17$

#### 4.2. $\gamma$ -transition with an energy of 84 keV in $^{163}\text{Er}$

The situation with conversion data for this transition looks no better than that for the  $\gamma$ -transition with an energy of 77 keV in  $^{165}\text{Er}$ . At the same time, since the half-life period of  $^{163}\text{Tm}$  mother nucleus (see Fig. 3) is

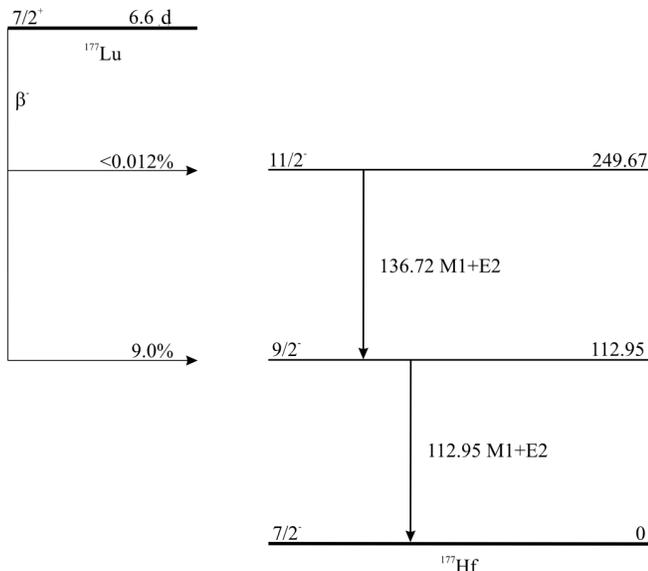


Fig. 4. Fragment of the schematic diagram of  $^{177}\text{Lu}$  decay

equal to only 1.8 h, it is much more difficult to obtain them. For the  $\gamma$ 84-keV transition in  $^{163}\text{Er}$ , the ICC ratio for the  $K$ - and  $L$ -shells and the absolute ICC for the  $K$ -shell were measured [35–37]. The accuracy of the data obtained turned out insufficient to determine the penetration,  $\lambda$ , and mixture,  $\delta(E2/M1)$ , parameters for this transition.

The corresponding analysis showed that this problem could be solved, provided that the absolute ICC for the transition in the  $L_3$ -subshell of  $^{163}\text{Er}$  would be measured. For this purpose, targets of enriched  $^{166}\text{Er}$  were bombarded, first, with protons with the energy  $E_p = 23$  MeV for 8 h to obtain  $^{166}\text{Tm}$  owing to the  $(p, n)$  reaction and, afterward, with protons with the energy  $E_p = 41$  MeV for 2 h to obtain  $^{163}\text{Tm}$  owing to the  $(p, 4n)$  reaction. Analogously to the previous case, the theoretical values of ICC for the  $E2$ -transition with an energy of 81 keV in the course of the decay of  $^{166}\text{Tm}$  were used as normalizing ones.

The intensity ratio  $I_\gamma(84)/I_\gamma(81)$  was measured on an HPGe-detector 160  $\text{cm}^3$  in volume. Since this ratio changed in time, the measurements of the  $\gamma$ -spectrum were carried out before the measurements of the ICE intensity ratio  $L_3(84)/L_3(81)$  taking the half-life periods for  $^{163}\text{Tm}$  and  $^{166}\text{Tm}$  into account. The absolute ICC for the  $\gamma$ -transition with an energy of 84 keV in the  $L_3$ -subshell of  $^{163}\text{Er}$  turned out equal to  $\alpha_{L_3}(84) = 0.82 \pm 0.16$ . The cumulative analysis of the most exact conversion data presented in work [37] and the absolute ICC value measured by us for the  $L_3$ -subshell al-

lowed us to determine the magnitudes of the mixture,  $|\delta(E2/M1)| = 1.5 \pm 0.1$ , and penetration,  $\lambda = 2.5 \pm 1.7$ , parameters for the  $M1$ -component of this transition.

### 4.3. $\gamma$ -transition with an energy of 113 keV in $^{177}\text{Hf}$

As is seen from Table 1, the  $M1$ -component of this transition has the largest prohibition factor among all known rotational transitions in deformed nuclei. This fact results in that the transition  $\gamma$ 113 keV in  $^{177}\text{Hf}$  is almost a pure  $E2$ -transition with a small admixture of  $M1$ -component. This circumstance also explains such a considerable number of discrepancies in the estimations of the mixture parameter  $\delta(E2/M1)$  obtained from both the conversion data and the results of correlation measurements.

For today, nine experimental values of this quantity are known, including our results, which were published by various authors at various times. All of them are presented in Table 3. Some of them are substantially different. The situation becomes even more complicated, because the possible anomalies in the ICC for the  $M1$ -component of this transition are induced by the penetration effect. This circumstance has to be taken into account while determining the mixture parameter.

To elucidate the situation, we carried out the precision measurements of the relative ICE intensities for this transition in the  $L$ -subshells of  $^{177}\text{Hf}$  [25]. Sources of  $^{177}\text{Lu}$  ( $T_{1/2} = 6.6$  day) were obtained in the  $(n, \gamma)$  reaction, while bombarding targets by a flux of thermal neutrons for two weeks to a dose of  $5 \times 10^{13}$  neutron/ $\text{cm}^2$  on a WWR-M reactor. The targets were fabricated by sputtering metallic lutetium onto an aluminum foil in vacuum. The measurements were carried out using sources with various thicknesses of a sputtered layer, ranging from 30 to 70  $\mu\text{g}/\text{cm}^2$ . A fragment of the decay scheme for  $^{177}\text{Lu}$  is depicted in Fig. 4.

Having averaged the results of several series of measurements, we obtained the following values for the relative ICE intensities of the  $\gamma$ 113-keV transition in

**Table 3.** Experimental values of the mixture parameter  $\delta(E2/M1)$  for the  $\gamma$ -transition with an energy of 113 keV in  $^{177}\text{Hf}$

$\delta(E2/M1)$	Source	$\delta(E2/M1)$	Source
$-(3.0 \pm 0.8)$	[38]	$-(3.7 \pm 0.3)$	[39]
$-(3.99 \pm 0.25)$	[22]	$-(4.0 \pm 0.2)$	[40]
$(4.20 \pm 0.11)^*$	[25]	$-(4.75 \pm 0.07)$	[21]
$-(4.5 \pm 0.3)$	[41]	$-(4.8 \pm 0.2)$	[42]
$-(4.85 \pm 0.05)$	[43]		

\* The  $|\delta(E2/M1)|$ -value is shown

the  $L$ -subshells of  $^{177}\text{Hf}$ :  $L_1/L_3 = 0.189 \pm 0.002$  and  $L_2/L_3 = 1.109 \pm 0.011$ . The indicated determination errors of the conversion line intensity include a possible systematic error, which, according to our estimations, did not exceed 1% [27].

Our results agree well with the data reported by Högberg *et al.* [41]; however, they cast doubt on the reliability of the results obtained by Novakov and Hollander [44]. Having appended our results to the available body of literature data concerning the absolute and relative ICC values for  $K$ -,  $L$ -, and  $M$ -shells, we managed to determine the magnitudes of the mixture,  $|\delta(E2/M1)| = 4.20 \pm 0.11$ , and penetration,  $\lambda = 1.8 \pm 1.4$ , parameters for the  $M1$ -component of this transition. The magnitude of penetration parameter is in good agreement with the available systematization for transitions of this type. In turn, this fact proves the correctness of the value determined for the mixture parameter. At the same time, if one takes, e.g., the value  $\delta(E2/M1) = -(4.85 \pm 0.05)$  from work [43] and uses it together with the data on the ICC to determine the penetration parameter, the value  $\lambda = -(4.7 \pm 0.5)$  will be obtained, which contradicts theoretical estimations. In the latter case,  $\chi_{\min}^2 = 6.1$  for the solution of the system of equations (9), (10) making use of the least square method, in contrast to  $\chi_{\min}^2 = 1.3$  obtained, if the  $\delta(E2/M1)$ -value is taken from work [25].

#### 4.4. $\gamma$ -transition with an energy of 137 keV in $^{177}\text{Hf}$

Table 1 also shows the data for one more hindered  $M1$ -transition in  $^{177}\text{Hf}$ , between the first and second excited levels of the ground-state rotational band (see Fig. 4). Its intensity at a decay of  $^{177}\text{Lu}$  is by two orders of magnitude lower than the intensity of the  $\gamma$ -transition with an energy of 113 keV. This circumstance complicates very much the determination of precision data for the ICC. A number of experimental works were fulfilled [22, 45], but the accuracy of those measurements was low. A high accuracy also cannot be achieved while studying the decay of  $^{177}\text{Ta}$  ( $T_{1/2} = 56$  h) [40]. We tried to obtain more accurate values for the relative and, when it was possible, absolute ICCs for this transition in the  $L$ -

**Table 4.** Experimental values of the ICC and the relative ICE intensities for the  $\gamma$ 137-keV transition in the  $L$ -subshells of  $^{177}\text{Hf}$

Subshell	$I_e$ , rel. units	ICC
$L_1$	$0.47 \pm 0.11$	–
$L_2$	$1.31 \pm 0.18$	$0.25 \pm 0.06$
$L_3$	1	$0.13 \pm 0.05$

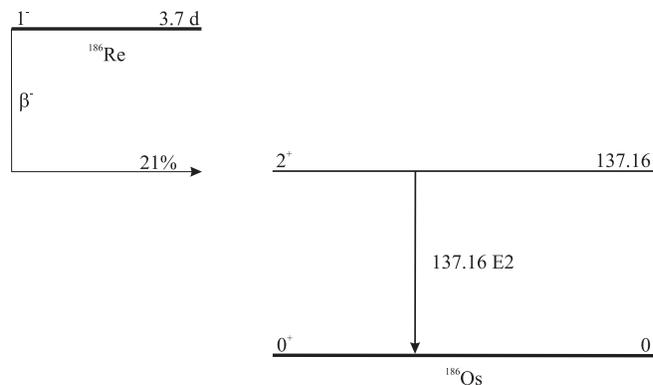


Fig. 5. Fragment of the schematic diagram of  $^{186}\text{Re}$  decay

subshells of  $^{177}\text{Hf}$ . For this purpose, to measure ICE spectra, we used a magnetic  $\beta$ -spectrometer, in which the electron registration took place with the help of microchannel plates [46].

The sources of radiation were targets fabricated of natural and enriched  $^{176}\text{Lu}$  with the thickness of a sputtered layer ranging from 30 to 100  $\mu\text{g}/\text{cm}^2$ . The targets were bombarded for a week on the reactor. For the measurements of absolute ICCs, we made targets consisting of a mixture of natural lutetium and enriched  $^{185}\text{Re}$ , with the total thickness of a sputtered layer not exceeding 70  $\mu\text{g}/\text{cm}^2$ .  $^{186}\text{Re}$  ( $T_{1/2} = 3.7$  day) created at the target bombardment with thermal neutrons has the  $\gamma$ 137-keV transition of the  $E2$ -multipolarity (see Fig. 5), which is close by energy to the examined transition, and the theoretical ICC values of which can be used as normalizing ones. The relative intensities of  $\gamma$ -transitions, which are of interest for us, were measured with the help of an HPGe-detector 2  $\text{cm}^3$  in volume and with a resolving power of 490 eV at the 122-keV  $\gamma$ -line of  $^{57}\text{Co}$ . The results of measurements are quoted in Table 4.

The cumulative analysis of the conversion data for the  $\gamma$ 137-keV transition in  $^{177}\text{Hf}$ , which are available in the literature, and those given in Table 4 allowed us to obtain the following value for the mixture parameter:  $|\delta(E2/M1)| = 2.3 \pm 0.7$  (at  $\lambda \cong 0$ ). As was in the case with the  $\gamma$ 77-keV transition in  $^{165}\text{Er}$ , the achieved measurement accuracy for the absolute and relative ICCs turned out insufficient to determine the magnitude of penetration parameter  $\lambda$ .

#### 4.5. $\gamma$ -transition with an energy of 129 keV in $^{191}\text{Ir}$

The ICCs for this  $\gamma$ -transition are convenient to be measured using the  $\beta$ -decay of  $^{191}\text{Os}$  ( $T_{1/2} = 15.4$  day). The corresponding fragment of the decay scheme is depicted

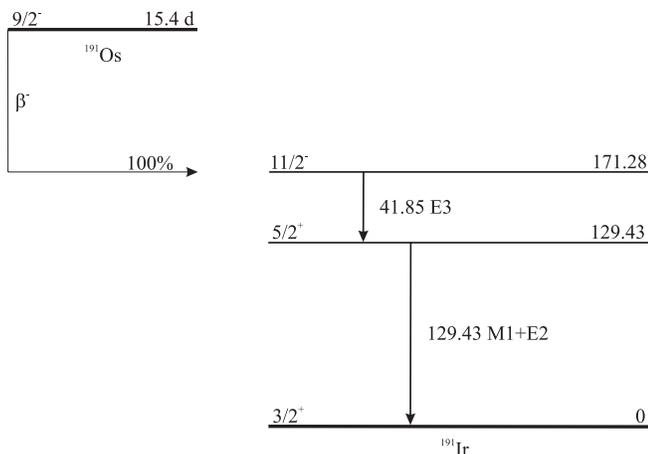


Fig. 6. Fragment of the schematic diagram of  $^{191}\text{Os}$  decay

in Fig. 6. The ratio between ICE intensities for the atomic  $L$ -subshells and the ICC for the  $K$ -shell were determined by various authors as early as in the 1960s [47–53]. The best accuracy achieved in those works was about 3%. However, it turned out insufficient for the magnitudes of penetration,  $\lambda$ , and mixture,  $\delta(E2/M1)$ , parameters to be determined reliably, although such an attempt has been made [54]. The accuracy of correlation measurements achieved for today [55] also does not allow this problem to be solved. A breakthrough in this direction was outlined after the publication of work [56], in which the authors managed to determine the ICC for the  $\gamma$ -transition with an energy of 129 keV in the  $K$ -shell of  $^{191}\text{Ir}$  with an accuracy better than 1%. We decided to try to measure, with the same accuracy, the ratio between ICE intensities for the  $L$ -subshells of iridium.

$^{191}\text{Os}$  radiation sources were obtained in the  $(n, \gamma)$  reaction, while bombarding osmium targets with thermal neutrons on the WWR-M reactor at the Institute for Nuclear Research of the NASU. The targets were fabricated by sputtering  $^{190}\text{Os}$  enriched to 91.2% onto an aluminum substrate in vacuum. The measured values of intensities for ICE lines were as follows:  $L_2/L_1 = 0.3076 \pm 0.0030$  and  $L_3/L_1 = 0.1653 \pm 0.0008$ . These values together with the data of work [56] on the ICC for this transition in the  $K$ -shell of  $^{191}\text{Ir}$ ,  $\alpha_K = 2.134 \pm 0.014$ , were used to find the mixture,  $\delta(E2/M1)$ , and penetration,  $\lambda$ , parameters. The value obtained for the mixture parameter,  $|\delta(E2/M1)| = 0.3907 \pm 0.0010$ , agrees well with the data of correlation measurements [55], but has a higher accuracy. The magnitude of penetration parameter for the  $M1$ -component of the  $\gamma$ -transition with an energy of 129 keV for  $^{191}\text{Ir}$ ,  $\lambda = 1.22 \pm 0.09$ , was determined for the first time.

### 5. Determination of Experimental Values of Penetration Matrix Elements. Comparison between the Theory and the Experiment

The quantity  $\lambda$  is defined by formula (5) as a ratio between the penetration and radiation emission matrix elements. This means that if the partial half-life period for a radiative transition has been measured, and the experimental value  $\lambda^{\text{exp}}$  has been found making use of the ICC, it is possible to determine the experimental value of nuclear penetration matrix element by the formula

$$\langle M_e \rangle^{\text{exp}} = \lambda^{\text{exp}} \times \langle U_\gamma \rangle^{\text{exp}}. \tag{13}$$

In turn, the experimental value of matrix element  $\langle U_\gamma \rangle^{\text{exp}}$  for the  $\gamma$ -emission with  $M1$ -multipolarity can be found from the reduced probability of  $\gamma$ -transition  $B(M1)$  as follows:

$$\langle U_\gamma \rangle^{\text{exp}} = [(2I_i + 1)B(M1)]^{1/2}. \tag{14}$$

Table 5 exposes experimental values of penetration matrix elements  $\langle M_e \rangle^{\text{exp}}$ , which were measured by us for rotational  $\gamma$ -transitions in  $^{163}\text{Er}$ ,  $^{177}\text{Hf}$ , and  $^{191}\text{Ir}$  nuclei. While carrying out calculations, the magnitudes for  $B(M1)$  from work [10] were used.

The sign of  $\lambda^{\text{exp}}$  can be determined from anomalous ICCs. The signs of penetration matrix elements  $\langle M_e \rangle^{\text{exp}}$  remain unknown, because only the absolute values of  $\langle U_\gamma \rangle^{\text{exp}}$  can be obtained from the given probabilities of  $\gamma$ -transitions. However, as was pointed out in work [3], the sign of  $\langle U_\gamma \rangle^{\text{exp}}$  can be determined from theoretical values  $\langle M_e \rangle^{\text{th}}$  as follows:

$$\text{sign} \langle U_\gamma \rangle^{\text{exp}} = \text{sign} \lambda^{\text{exp}} \times \text{sign} \langle M_e \rangle^{\text{th}}. \tag{15}$$

Since  $\text{sign} \lambda^{\text{exp}}$  is positive for all  $M1$ -transitions, which were considered by us, the sign of  $\langle U_\gamma \rangle^{\text{exp}}$  coincides with that of  $\langle M_e \rangle^{\text{th}}$ . It is negative for the one-quasi-particle neutron transitions  $7/2^-5/2[523] \rightarrow 5/2^-5/2[523]$  in  $^{163}\text{Er}$  and  $9/2^-7/2[514] \rightarrow 7/2^-7/2[514]$  in  $^{177}\text{Hf}$ , but positive for the one-quasi-particle proton transition  $5/2^+3/2[402] \rightarrow 3/2^+3/2[402]$  in  $^{191}\text{Ir}$ .

**Table 5.** Experimental values of penetration matrix elements  $\langle M_e \rangle^{\text{exp}}$  for rotational transitions in  $^{163}\text{Er}$ ,  $^{177}\text{Hf}$ , and  $^{191}\text{Ir}$

Nucleus	$E_\gamma$ keV	$\lambda^{\text{exp}}$	$\langle U_\gamma \rangle^{\text{exp}}$ , nucl. magn.	$\langle M_e \rangle^{\text{exp}}$ , nucl. magn.
$^{163}_{68}\text{Er}$	84.0	$2.5 \pm 1.7$	$-(0.141 \pm 0.010)$	$-(0.35 \pm 0.24)$
$^{177}_{72}\text{Hf}$	113.0	$1.8 \pm 1.4$	$-(0.088 \pm 0.010)$	$-(0.16 \pm 0.12)$
$^{191}_{77}\text{Ir}$	129.4	$1.2 \pm 0.1$	$0.528 \pm 0.006$	$0.64 \pm 0.05$

The theoretical expression for nuclear matrix elements of  $\gamma$ -emission and penetrations, as well as their peculiarities associated with their calculations in the framework of different nuclear models, are presented and discussed in works [3, 9, 57, 58].

The analysis of  $M1$ -transitions with  $\Delta K = 0, \pm 1$  in deformed nuclei is convenient to be carried out, by using the tables of radiation and conversion matrix elements calculated in the framework of the Nilsson model [59–62]. According to work [59], in the case  $K \neq 1/2$ , we have

$$\begin{aligned} \langle M_e \rangle^{\text{th}} = \langle P_1 \rangle \mu_n = 0, & 722A^{-1/3} C_{K, K' - K, K'}^{J1J'} \times \\ & \times [3(2J + 1)/16\pi]^{1/2} (N + 3/2) \times \\ & \times \{ (g_l - g_R) GML(K \rightarrow K') + \\ & + [(5/3)g_s - g_R] GMS(K \rightarrow K') + \\ & + g_s PME(K \rightarrow K') \}, \end{aligned} \quad (16)$$

where  $A$  is the mass number;  $J$  and  $K$  are the quantum numbers of the initial state, whereas  $J'$  and  $K'$  are those of the final state;  $C$  is the Clebsch–Gordan coefficient;  $N$  the principal quantum number;  $\mu_n$  is the nuclear magneton;  $g_l = 0$  for the neutron transitions and 1 for the proton ones;  $g_R$  is the gyromagnetic ratio for a rotational motion of the nucleus;  $GML(K \rightarrow K')$ ,  $GMS(K \rightarrow K')$ , and  $PME(K \rightarrow K')$  are the matrix elements tabulated in works [59, 60]; and  $g_s$  is the spin gyromagnetic ratio.

In the simple one-particle model, the gyromagnetic ratio  $g_s = 5.58$  for the one-proton transitions and  $-3.82$  for the one-neutron ones. However, the values of matrix elements calculated using those gyromagnetic ratios do not coincide accurately with experimental ones. In such a case, it is said about a renormalization of the  $g_s$ -factor in a real nucleus in comparison with the  $g_s$ -factor for a free nucleon. The ratio between the experimental and calculated matrix elements gives the renormalization factor or, simply, the renormalization.

To a certain extent, the renormalization can be explained by specifying the nuclear model used as a basis for the comparison between the theoretical and experimental nuclear matrix elements. However, if the matrix element is not hindered by the selection rules in the given model of the nucleus (just such are the penetration matrix elements  $\langle M_e \rangle$  in most cases), the specification of

the model, e.g., changing from the Nilsson wave functions to the Saxon–Woods ones while calculating  $\langle M_e \rangle$  in deformed nuclei or making allowance for superfluid corrections and Coriolis interaction, brings about minor alterations in the theoretical penetration matrix element in most cases [3] and does not explain the experimentally observed renormalization of  $g_s$ -factors.

As was marked several times by Listengarten *et al.* [11, 63], the study of intranuclear conversion gives a unique opportunity to experimentally determine the renormalization of  $g$ -factors at matrix elements with the operator  $\hat{\mathbf{S}}$  (the  $g_s$ -factors) in the Hamiltonian with residual interaction. The most important part of the Hamiltonian with residual interaction is the spin-multipole interaction, which contains tensor products of the spin operator  $\hat{\mathbf{S}}$  and the spherical vectors  $\mathbf{T}_{LM}^{(\nu)}$ . At each  $L$ , the operator of spin-multipole interaction includes three components with the superscripts  $\nu = 0, -1$ , and  $1$  in  $\mathbf{T}_{LM}^{(\nu)}$ , namely

$$\mathbf{ST}_{LM}^{(-1)}, \mathbf{ST}_{LM}^{(0)}, \mathbf{ST}_{LM}^{(+1)} \quad (17)$$

with corresponding constants  $\varkappa_L^\nu$ .

The matrix element of the operator  $\mathbf{ST}_{LM}^{(-1)}$  is included into the expressions for the magnetic multipole moments and the probability of magnetic multipole radiation emission; the matrix element of the operator  $\mathbf{ST}_{LM}^{(0)}$  into the penetration matrix element for the anomalous conversion of electric multipolarity, and the matrix element of operator  $\mathbf{ST}_{LM}^{(+1)}$  into the penetration matrix element for the anomalous magnetic conversion. Having determined the corresponding renormalizations  $g_s/g_s^{\text{free}}$  from the data on  $\gamma$ -emission and anomalous conversion, we thereby obtain experimental values for all matrix elements, which are included into the Hamiltonian with residual interaction. The renormalization value for the simple magnetic operator  $\mathbf{ST}_{LM}^{(-1)}$ , which is simply the operator  $\hat{\mathbf{S}}_Z$  in the dipole case ( $L = 1$ ), is well-known from the study of magnetic moments and magnetic  $\gamma$ -transitions: on the average, it equals about 0.6. At the same time, the only way for today to obtain a specific qualitative result and to find the renormalization values for two other operators in Eq. (17) consists in studying the anomalous conversion [3].

In formula (16), the quantity  $g_s$  is equal to the above-mentioned gyromagnetic ratio for the spin-multipole interaction in the form  $\mathbf{ST}_{LM}^{(+1)}$ , i.e. it is equal to the  $g_s$ -factor of anomalous magnetic conversion.

Table 6 exhibits the experimental values for the renormalization of the spin gyromagnetic ratio in the rotational  $M1$ -transitions in  $^{163}\text{Er}$ ,  $^{177}\text{Hf}$ , and  $^{191}\text{Ir}$  nu-

**Table 6.** Experimental values of renormalization for the spin gyromagnetic ratios  $g_s(M1)/g_s^{\text{free}}(p)$  and  $g_s(M1)/g_s^{\text{free}}(n)$  in  $M1$ -conversion transitions in  $^{163}\text{Er}$ ,  $^{177}\text{Hf}$ , and  $^{191}\text{Ir}$

Nucleus	$E_\gamma$ , keV	$g_s(M1)/g_s^{\text{free}}(p)$	$g_s(M1)/g_s^{\text{free}}(n)$
$^{163}_{68}\text{Er}$	84.0	–	$0.70 \pm 0.18$
$^{177}_{72}\text{Hf}$	113.0	–	$0.57 \pm 0.08$
$^{191}_{77}\text{Ir}$	129.4	$0.574 \pm 0.023$	–

clei, which were obtained by comparing the experimental penetration matrix elements from Table 5 with the theoretical ones calculated by formula (16), by using the Nilsson potential wave functions. The theoretical values of matrix elements  $GML(K \rightarrow K')$ ,  $GMS(K \rightarrow K')$ , and  $PME(K \rightarrow K')$  were calculated by interpolating the tabular data from works [59, 60]; the parameters of a quadrupole deformation were taken from works [64, 65].

The average renormalization value of  $g_s$ -factor for the neutron transitions in  $^{163}\text{Er}$  and  $^{177}\text{Hf}$  calculated according to the data of Table 6 is  $g_s(M1)/g_s^{\text{free}}(n) = 0.59 \pm 0.07$ . The renormalization value of  $g_s$ -factor calculated from the data on the anomalous  $M1$ -conversion is close to the renormalization of the  $g_s$ -factor for  $M1$ -transitions, which was obtained from the researches of nuclear magnetic moments (see, e.g., work [7]). It is difficult to say to what extent this coincidence is not accidental. However, it is important to continue the study of this problem.

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#### АНОМАЛІЇ В КОЕФІЦІЄНТАХ ВНУТРІШНЬОЇ КОНВЕРСІЇ ЗАГАЛЬМОВАНИХ РОТАЦІЙНИХ ГАММА-ПЕРЕХОДІВ

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## Резюме

Продовжено цикл робіт щодо дослідження ефекту проникнення в  $M1$ -компоненті загальмованих ротаційних  $\gamma$ -переходів. Ефект зумовлений різними правилами добору для матричних елементів  $\gamma$ -випромінювання та внутрішньоядерної конверсії. Вперше визначено величину параметра проникнення  $\lambda$  для  $M1$ -компоненти  $\gamma$ -переходу з енергією 129 кеВ в  $^{191}\text{Ir}$ . Наведено результати досліджень аналогічних переходів в  $^{163}\text{Er}$ ,  $^{165}\text{Er}$  та  $^{177}\text{Hf}$ . При порівнянні експериментальних матричних елементів проникнення з їх теоретичними значеннями, обчисленими в моделі Нільсона, знайдено перенормування гіромагнітних відношень для спин-мультипольної взаємодії в  $M1$ -конверсійних переходах:  $g_s(M1)/g_s^{\text{free}}(p) = 0,574 \pm 0,023$  та  $g_s(M1)/g_s^{\text{free}}(n) = 0,59 \pm 0,07$ .