METHOD FOR THE CALCULATION OF FREE ENERGY IN A THREE-DIMENSIONAL ISING-LIKE SYSTEM TAKING INTO ACCOUNT A CORRECTION FOR THE INTERACTION POTENTIAL AVERAGING

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The critical behavior of three-dimensional model systems has been studied theoretically. The partition function and the free energy for a one-component spin system have been calculated for a non-Gaussian distribution of order-parameter fluctuations. A specific feature of the proposed method of calculation consists in making allowance for the dependence of the Fourier transform of the interaction potential on the wave vector. Such an approach leads to a nonzero critical exponent η in the correlation function and the renormalization of the values of other critical exponents (for the correlation length, susceptibility, *etc.*). The calculation of those exponents was carried out with the use of the renormalization-group method and on the basis of obtained recurrence relations for the coefficients of fluctuation distributions in adjacent block structures.

1. Introduction

The main task for the physics of phase transitions is to describe relations between macroscopic experimentally measured quantities, by proceeding from the microscopic parameters of the system and the forces that act between particles composing the system. The fundamentals of the microscopic approach in the phase transition theory were established in works [1, 2], where ideas following from the scaling hypothesis [3, 4] were substantially used. A further development in the microscopic theory of phase transitions was obtained in the method of collective variables (CVs) generalized to the case of spin systems [5–7]. For a one-component spin model, not only the critical exponents for specific heat, average spin moment, susceptibility, and other characteristics were obtained, but also the explicit expressions for those characteristics in a vicinity of the phase transition temperature, T_c, were found and studied as the functions of the temperature, magnetic field, and microscopic parameters of the Hamiltonian. Anybody can come to know more about the CV method and a number of results obtained on its basis in works [8–12].

At a certain stage of calculations, the theoretical description of the critical behavior of real systems is reduced to a description of a phase transition in the framework of some model. The CV method, the basis of which with respect to the three-dimensional Ising model – the basic model for the consideration of phase transitions – was established in the 1970s–1980s, has been permanently improved. It is worth noting that the Ising model, despite its simplicity, has, on the one hand, a wide scope of realistic applications, and, on the other hand, it can be considered as a model, which serves as a standard for studying other models much more complicated by their construction. The development of the calculation method for main thermodynamic and structural characteristics of the basic phase transition model opens a way for the description of more complicated physical systems. For this reason, the solution for a three-dimensional Ising-like system, which would be as much complete as possible, is a key to the description of the critical behavior of many physical objects. Uniaxial magnets, simple liquids, binary alloys, micellar systems – that is not a complete list of objects, which the Ising-like behavior is inherent to. The critical behavior of the three-dimensional Ising universality class was discovered in systems with strong and electroweak interactions observed in high-energy physics.

In the CV method, when calculating the partition function of a three-dimensional Ising-like system, the

phase space of CVs $\rho_{\mathbf{k}}$ is divided into layers $\{\rho_{\mathbf{k}}\}_{B_{n+1},B_n}$ corresponding to certain values of the wave vector $\mathbf{k}$ ($B_{n+1} = B_n/s$, where $s > 1$ is the partition parameter). The partition function is calculated step-by-step, starting from the layers in the CV phase space that correspond to $\rho_{\mathbf{k}}$-variables with large wave vectors $\mathbf{k}$'s and finishing at $\rho_{\mathbf{k}}$ with $k \to 0$. In so doing, the Fourier transform of the interaction potential is replaced by its average value in every layer. This procedure gives rise to a zero value of critical exponent η, which characterizes the behavior of the pair correlation function at $T = T_c$.

The calculation of the so-called small critical exponent η is not a trivial procedure (see, e.g., researches in the framework of the nonperturbative renormalization group (RG) approach [13, 14], which is similar to the CV method). In this work, we calculated the thermodynamic characteristics in a vicinity of T_c with regard for a correction for the averaging of the Fourier transform of the potential. This correction results in a nonzero η-exponent value and was not considered in earlier researches (in particular, in [15–18]). The method proposed in this paper for the calculation of the thermodynamics of a system in the framework of the CV approach acquires a more general and complete character owing to the consideration of both the critical exponent of the correlation function and the changes (in comparison with the case $\eta = 0$) for the coordinates of a fixed point, the recurrence relations (RRs), and the very coefficients of thermodynamic characteristics.

2. Statement of the Problem

Let is consider a system composed of N interacting particles located at the sites of a simple cubic lattice with period c. The state of each particle is determined by the eigenvalue of the operator of its spin z-component, which can be equal to either $+1$ (the spin is oriented upward) or -1 (the spin is oriented downward). The Hamiltonian of the system is written down in the form

$$H = -\frac{1}{2}\sum_{\mathbf{i},\mathbf{j}} \Phi(r_{\mathbf{ij}})s_{\mathbf{i}}s_{\mathbf{j}} - h\sum_{\mathbf{i}} s_{\mathbf{i}}, \tag{1}$$

where $s_{\mathbf{i}}$ is the operator of spin z-component at the $\mathbf{i}$-th site, $\Phi(r_{\mathbf{ij}})$ is the interaction potential, which is expressed by the exponentially decreasing function

$$\Phi(r_{\mathbf{ij}}) = A\exp(-r_{\mathbf{ij}}/b), \tag{2}$$

A is a constant, $r_{\mathbf{ij}}$ the distance between the $\mathbf{i}$-th and $\mathbf{j}$-th lattice sites, and b the radius of the effective interaction. The partition function of the model in the representation

of CVs $\rho_{\mathbf{k}}$ in the absence of an external magnetic field H (or at $h = \mu_{\mathrm{B}}\mathcal{H} = 0$, where μ_{B} is the Bohr magneton) looks like

$$Z = 2^N 2^{(N'-1)/2} e^{a_0' N'} \int \exp\left\{-\frac{1}{2}\sum_{k\leq B'} d'(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \right.$$
$$\left. -\frac{1}{4!}(N')^{-1}\sum_{k_1,\ldots,k_4\leq B'} a_4'\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\right\}(d\rho)^{N'}. \tag{3}$$

Here, the approximation of quartic basis measure density [8, 19] was used, $B' = (b\sqrt{2})^{-1}$, $N' = Ns_0^{-3}$, $s_0 = B/B'$, $B = \pi/c$ is the boundary of the Brillouin half-zone,

$$d'(k) = a_2' - \beta\tilde{\Phi}(k), \tag{4}$$

and $\beta = 1/(kT)$ is the reciprocal temperature. For the Fourier transform of the interaction potential (2), we use the parabolic approximation

$$\tilde{\Phi}(k) = \begin{cases} \tilde{\Phi}(0)(1 - 2b^2k^2), & k \leq B', \\ 0, & B' < k \leq B, \end{cases} \tag{5}$$

which does not affect, in essence, the description of events in a vicinity of the phase transition point. The major contribution to the thermodynamic and structural functions is obtained from the wave-vector range with $k \to 0$. The interval with large values of $\mathbf{k}$ forms only initial values of the coefficients a_0', a_2', and a_4', which do not influence the values of critical exponents. In approximation (5), we have

$$a_0' = \ln\left[\frac{1}{\sqrt{2\pi}}(y')^{1/2}e^{(y')^2/4}U(0,y')\right],$$
$$a_2' = y'U(y'), \qquad a_4' = (y')^2\varphi(y'), \tag{6}$$

where $y' = 2^{1/4}\sqrt{3}\pi^{3/2}(b/c)^{3/2}$, and the functions $U(y')$ and $\varphi(y')$ are expressed in terms of parabolic cylinder functions (the Weber functions) $U(a,y')$ as follows [19]:

$$U(y') = U(1,y')/U(0,y'),$$
$$\varphi(y') = 3U^2(y') + 2y'U(y') - 2. \tag{7}$$

The coefficients a_{2l}' depend on the ratio between the potential action radius b and the lattice constant c. In the case $b = c$, we obtain

$$a_0' = -0.921747, \quad a_2' = 0.988929, \quad a_4' = 0.021120.$$

The expression for the partition function (3) is calculated by carrying out the "layer-by-layer" integration (see, e.g., works [7, 8, 11]). In every n-th layer of CVs $\rho_{\mathbf{k}}$, which corresponds to the wave-vector interval

$B_{n+1} < k \leq B_n$, the Fourier transform $\tilde{\Phi}(k)$ is substituted by its average value

$$\tilde{\Phi}(B_{n+1}, B_n) = \tilde{\Phi}(0) - 2\tilde{\Phi}(0)b^2 B_n^2 \bar{q}. \qquad (8)$$

Here, $B_n = B's^{-n}$, and the quantity $\bar{q}$ corresponds to the average value of k^2 in the interval $(1/s, 1]$. Following the geometrical way of averaging, which is used in our present calculations, we obtain $\bar{q} = 3(1 - s^{-5})/[5(1 - s^{-3})]$. The RRs arising in the course of integration, read [7, 19]

$$d_{n+1}(B_{n+2}, B_{n+1}) = d_n(B_{n+1}, B_n)N(x_n) - \Delta_n,$$
$$a_4^{(n+1)} = a_4^{(n)}s^{-3}E(x_n), \qquad (9)$$

where $d_n(B_{n+1}, B_n) = a_2^{(n)} - \beta\tilde{\Phi}(B_{n+1}, B_n)$, $\Delta_n = \beta\tilde{\Phi}(0)s^{-2n}(1 - s^{-2})\bar{q}$. As for the functions $N(x_n)$ and $E(x_n)$, we have

$$N(x_n) = \frac{y_n U(y_n)}{x_n U(x_n)}, \qquad E(x_n) = s^6 \frac{\varphi(y_n)}{\varphi(x_n)}. \qquad (10)$$

The arguments x_n and y_n in Eqs. (10) are defined by the expressions

$$x_n = \sqrt{3}d_n(B_{n+1}, B_n)\left(a_4^{(n)}\right)^{-1/2},$$
$$y_n = s^{3/2}U(x_n)\left(3/\varphi(x_n)\right)^{1/2}. \qquad (11)$$

In the specific case $x_n \gg 1$, the general RRs (9) are reduced to Wilson's RRs [20,21]. The application of RRs (9) enables the properties of three-dimensional systems to be studied in the critical region.

Now, while calculating the partition function of the system, we take the following quantity into account:

$$\Delta\tilde{\Phi}(k) = q - 2b^2\beta\tilde{\Phi}(0)k^2. \qquad (12)$$

It corresponds to a deviation of the quantity $\beta\tilde{\Phi}(k)$ from its average value $\beta\tilde{\Phi}(B_1, B')$, i.e. we make allowance for a correction for the $\tilde{\Phi}(k)$ averaging. In Eq. (12), $q = \bar{q}\beta\tilde{\Phi}(0)$. Below, assuming the quantity $\Delta\tilde{\Phi}(k)$ to be a small correction for the corresponding average value, we study its influence on the critical properties of the system and develop a method for the calculation of the free energy of the system with regard for the dependence of the Fourier transform of the potential, $\tilde{\Phi}(k)$, on the wave vector.

3. Calculation of the Partition Function of the System

The result of the integration of the partition function over those CVs $\rho_{\mathbf{k}}$, the indices $\mathbf{k}$ of which fall within the interval $B_1 < k \leq B'$ ($B_1 = B'/s$), looks like [8, 10, 22]

$$Z = 2^N 2^{(N_1 - 1)/2} Q_0[Q(P)]^{N_1} \times$$

$$\times \int \exp\left\{-\frac{1}{2}\sum_{k \leq B_1}[d'(k) - d'(B_1, B')]\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right\} \times$$

$$\times (1 + \hat{\Delta}_g + \cdots)\exp\left[-\frac{1}{2}R_2\sum_{k \leq B_1}\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{1}{4!}(N_1)^{-1}\times\right.$$

$$\left.\times R_4\sum_{k_1,...,k_4 \leq B_1}\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\right](d\rho)^{N_1}, \qquad (13)$$

where $N_1 = N's^{-3}$, $Q_0 = [e^{a_0'}Q(d)]^{N'}$ is the partial partition function of the zeroth layer in the CV phase space [19],

$$Q(d) = (2\pi)^{1/2}\left(3/a_4'\right)^{1/4}e^{x^2/4}U(0, x),$$
$$Q(P) = (2\pi)^{-1/2}s^{3/4}\left(a_4'/\varphi(x)\right)^{1/4}e^{y^2/4}U(0, y),$$
$$x = x_{n=0} = \sqrt{3}d'(B_1, B')(a_4')^{-1/2},$$
$$y = s^{3/2}U(x)\left(3/\varphi(x)\right)^{1/2}. \qquad (14)$$

For R_2 and R_4, we have [22]

$$R_2 = d'(B_1, B')N(x), \qquad R_4 = a_4's^{-3}E(x). \qquad (15)$$

The functions $N(x)$ and $E(x)$ are defined in Eqs. (10). The operator $\hat{\Delta}_g$ is given by the expression [11, 22]

$$\hat{\Delta}_g = \frac{1}{2(2\pi i)^2}\sum_{\mathbf{l}_1,\mathbf{l}_2}(N')^{-1}\sum_{B_1 < k \leq B'}\Delta g(k)e^{-i\mathbf{k}(\mathbf{l}_1 - \mathbf{l}_2)} \times$$

$$\times\left\{\left(\frac{S_4}{3!}\right)^2\frac{1}{(2\pi i)^6}(N')^{-3}\sum_{k_1,...,k_6 \leq B_1}\frac{\partial^6}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_6}}\times\right.$$

$$\times\exp[-i(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\mathbf{l}_1 - i(\mathbf{k}_4 + \mathbf{k}_5 + \mathbf{k}_6)\mathbf{l}_2] +$$

$$+2\frac{S_4 S_6}{3!5!}\frac{1}{(2\pi i)^8}(N')^{-4}\sum_{k_1,...,k_8 \leq B_1}\frac{\partial^8}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_8}}\times$$

$$\times\exp[-i(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\mathbf{l}_1 - i(\mathbf{k}_4 + \cdots + \mathbf{k}_8)\mathbf{l}_2] +$$

$$+\frac{1}{(2\pi i)^{10}}(N')^{-5}\sum_{k_1,...,k_{10} \leq B_1}\frac{\partial^{10}}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_{10}}}\left[2\frac{S_4 S_8}{3!7!}\times\right.$$

$$\times\exp[-i(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\mathbf{l}_1 - i(\mathbf{k}_4 + \cdots + \mathbf{k}_{10})\mathbf{l}_2] +$$

$$+\left(\frac{S_6}{5!}\right)^2\exp[-i(\mathbf{k}_1 + \cdots + \mathbf{k}_5)\mathbf{l}_1 -$$

$$-i(\mathbf{k}_6 + \cdots + \mathbf{k}_{10})\mathbf{l}_2]\Big] + 2\frac{S_6 S_8}{5!7!}\frac{1}{(2\pi i)^{12}}(N')^{-6}\times$$

$$\times\sum_{k_1,...,k_{12} \leq B_1}\frac{\partial^{12}}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_{12}}}\exp[-i(\mathbf{k}_1 + \cdots + \mathbf{k}_5)\mathbf{l}_1 -$$

$$-i(\mathbf{k}_6 + \cdots + \mathbf{k}_{12})\mathbf{l}_2] + \left(\frac{S_8}{7!}\right)^2\frac{1}{(2\pi i)^{14}}(N')^{-7}\times$$

$$\times \sum_{k_1,\ldots,k_{14}\leq B_1} \frac{\partial^{14}}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_{14}}} \exp\left[-i(\mathbf{k}_1+\cdots+\mathbf{k}_7)\mathbf{l}_1-\right.$$

$$\left.-i(\mathbf{k}_8+\cdots+\mathbf{k}_{14})\mathbf{l}_2]+\cdots\right\}. \tag{16}$$

Here,

$$S_2 = (2\pi)^2\left(\frac{3}{a_4'}\right)^{1/2}U(x), \quad S_4 = (2\pi)^4\frac{3}{a_4'}\varphi(x), \tag{17}$$

$$S_6 = (2\pi)^6\left(\frac{3}{a_4'}\right)^{3/2}\varphi_1(x), \quad S_8 = (2\pi)^8\left(\frac{3}{a_4'}\right)^2\varphi_2(x),$$

and the functions $\varphi_l(x)$ are defined by the relations

$\varphi(x) = 3U^2(x) + 2xU(x) - 2,$

$\varphi_1(x) = 30U^3(x) + 30xU^2(x) + 4(x^2-6)U(x) - 4x,$

$\varphi_2(x) = 630U^4(x) + 840xU^3(x) + 84(3x^2-8)U^2(x) +$

$+8x(x^2-45)U(x) - 8x^2 + 120.$

The summation over the sites $\mathbf{l}_1$ and $\mathbf{l}_2$ in formula (16) is carried out for a lattice with the period $c' = \pi b\sqrt{2}$. The wave-vector magnitudes change within the interval $[0, B']$. The quantity $\Delta g(k)$ is defined as

$$\Delta g(k) = \frac{\Delta\tilde\Phi(k)}{1 - S_2(2\pi)^{-2}\Delta\tilde\Phi(k)}. \tag{18}$$

The operator $\hat\Delta_g$ in expression (13) acts on the function

$$I(R_2, R_4) = \exp\left[-\frac{1}{2}R_2\sum_{k\leq B_1}\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{1}{4!}(N_1)^{-1}\times\right.$$

$$\left.\times R_4\sum_{k_1,\ldots,k_4\leq B_1}\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\right], \tag{19}$$

in which we will confine the consideration to the first term in the exponent. This circumstance is associated with a small contribution from R_4 in comparison with that from R_2 (the numerical estimates of the ratio $R_4/(6R_2^2)$ for various values of RG parameter s are quoted in work [22], being of the order of 10^{-4}).

The correction inserted by the operator $\hat\Delta_g$ is considered in the linear approximation in $\Delta\tilde\Phi(k)$. In so doing, we confine expression (16) to the first term, which is proportional to $\frac{\partial^6}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_6}}$. In this approximation,

$$\hat\Delta_g^{(1)} = \frac{1}{2}\sum_{k_1,\ldots,k_6\leq B_1}\left(\frac{\varphi(x)}{2a_4'}\right)^2\frac{\partial^6}{\partial\rho_{\mathbf{k}_1}\cdots\partial\rho_{\mathbf{k}_6}}(N')^{-4}\times$$

$$\times\sum_{B_1<k\leq B'}\Delta g(k)\sum_{\mathbf{l}_1,\mathbf{l}_2}\exp\left[-i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k})\mathbf{l}_1-\right.$$

$$\left.-i(\mathbf{k}_4+\mathbf{k}_5+\mathbf{k}_6-\mathbf{k})\mathbf{l}_2\right]. \tag{20}$$

A short remark concerning the consideration of other terms in expression (16) will be given below.

Operator (20) being applied to function (19) produces the following result:

$$\Delta_g^{(1)} = -\frac{3}{4}\left(\frac{\varphi(x)}{a_4'}\right)^2 R_2^3\sum_{\mathbf{l}}\mathcal{I}_1(\bar x)\mathcal{I}_2(\bar x)\times$$

$$\times\left[N'\mathcal{I}_1^2(\bar x) - 3R_2\mathcal{I}_1(\bar x)\sum_{k\leq B_1}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}e^{-i\mathbf{k}\mathbf{l}} + \frac{3}{2}R_2^2 s^{-3}\times\right.$$

$$\left.\times(N_1)^{-1}\sum_{k_1,\ldots,k_4\leq B_1}\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}e^{-i(\mathbf{k}_1+\mathbf{k}_3)\mathbf{l}}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\right]. \tag{21}$$

Here, $\mathbf{l} = \mathbf{l}_1 - \mathbf{l}_2$ and $\bar x = l/c'$. The quantities $\mathcal{I}_1(\bar x)$ and $\mathcal{I}_2(\bar x)$ are calculated with the help of a transition to the spherical Brillouin zone and integrating over $k \in [0, B_1]$:

$$\mathcal{I}_1(\bar x) = \frac{1}{N'}\sum_{k\leq B_1}e^{-i\mathbf{k}\mathbf{l}} = \delta_{\mathbf{l}} - Y(\bar x),$$

$$\mathcal{I}_2(\bar x) = \frac{1}{N'}\sum_{B_1<k\leq B'}\Delta g(k)e^{-i\mathbf{k}\mathbf{l}} = qt(Y(\bar x) - \mathcal{Z}(\bar x)). \tag{22}$$

For the functions $Y(\bar x)$ and $\mathcal{Z}(\bar x)$, which are present in formulas (22), we obtain

$$Y(\bar x) = \frac{1}{N'}\sum_{B_1<k\leq B'}e^{-i\mathbf{k}\mathbf{l}} = \frac{3}{(\pi\bar x)^3}\left[\sin(\pi\bar x)-\right.$$

$$\left.-\pi\bar x\cos(\pi\bar x) - \sin\left(\frac{\pi\bar x}{s}\right) + \frac{\pi\bar x}{s}\cos\left(\frac{\pi\bar x}{s}\right)\right],$$

$$\mathcal{Z}(\bar x) = \frac{2\beta\tilde\Phi(0)b^2}{q}\frac{1}{N'}\sum_{B_1<k\leq B'}k^2 e^{-i\mathbf{k}\mathbf{l}} =$$

$$= \frac{3}{\pi^2\bar q\bar x^2}\left\{3\frac{\sin(\pi\bar x)}{\pi\bar x} - \cos(\pi\bar x) + 6\left(\frac{\cos(\pi\bar x)}{(\pi\bar x)^2}-\right.\right.$$

$$\left.-\frac{\sin(\pi\bar x)}{(\pi\bar x)^3}\right) - s^{-3}\left[3\frac{\sin(\pi\bar x/s)}{\pi\bar x/s} - \cos(\pi\bar x/s)+\right.$$

$$\left.\left.+6\left(\frac{\cos(\pi\bar x/s)}{(\pi\bar x/s)^2} - \frac{\sin(\pi\bar x/s)}{(\pi\bar x/s)^3}\right)\right]\right\}. \tag{23}$$

The quantity

$$t = \left\langle\frac{1}{1 - \sqrt{3/a_4'}U(x)\Delta\tilde\Phi(k)}\right\rangle_{B_1,B'},$$

where the symbol $\langle\cdots\rangle_{B_1,B'}$ means the geometrical averaging over the interval $(B_1, B']$, is determined by the

formulas

$$t = \sqrt{\frac{\bar{a}_4'}{3}} \frac{1}{U(x)} t_0, \quad t_0 = \frac{3}{1 + s^{-1} + s^{-2}} + \frac{3a}{1 - s^{-3}} I_0,$$

$$\bar{a}_4' = a_4'(\beta\tilde{\Phi}(0))^{-2}, \quad a = \bar{q} - \sqrt{\frac{\bar{a}_4'}{3}} \frac{1}{U(x)},$$

$$I_0 = \begin{cases} \frac{1}{2\sqrt{a}} \ln \left| \frac{(1-\sqrt{a})(s^{-1}+\sqrt{a})}{(1+\sqrt{a})(s^{-1}-\sqrt{a})} \right|, & a > 0, \\ \frac{1}{\sqrt{|a|}} \left(\arctan \frac{1}{\sqrt{|a|}} - \arctan \frac{1}{\sqrt{|a|s}} \right), & a < 0. \end{cases} \quad (24)$$

Note that the separation of the Kronecker symbol $\delta_{\mathbf{l}}$ in the expression for $\mathcal{I}_1(\bar{x})$ (see Eqs. (22)) was proposed in work [8], being an important element of calculations. This procedure enables one to select the interval $k \in (B_1, B']$ from the range of wave vectors $k \in [0, B']$, in which the correction for the potential averaging is taken into account. The absence of this procedure in work [23] brought about the first unsuccessful attempt to calculate the critical exponent η in the framework of the CV method. In the case $s \to 1$, we have $Y(\bar{x}) \to 0$ and $B_1 \to B'$, so that the quantity $\mathcal{I}_1(\bar{x})$ transforms into the Kronecker symbol $\delta_{\mathbf{l}}$.

Returning back to Eq. (21), we find

$$\Delta_g^{(1)} = -\frac{3}{4} qtR_2^3 \left(\frac{\varphi(x)}{a_4'}\right)^2 \left\{ N'F_0 - 6R_2 A_0 \times \right.$$

$$\times \frac{1}{2} \sum_{k \leq B_1} \rho_{\mathbf{k}} \rho_{-\mathbf{k}} + 3R_2 B_0 \frac{1}{2} \sum_{k \leq B_1} (c')^2 k^2 \rho_{\mathbf{k}} \rho_{-\mathbf{k}} + 3s^{-3} \times$$

$$\times R_2^2 C_0 \frac{1}{2} (N_1)^{-1} \sum_{k_1,\ldots,k_4 \leq B_1} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_4} \delta_{\mathbf{k}_1 + \cdots + \mathbf{k}_4} \left. \right\}. \quad (25)$$

The quantities F_0, A_0, B_0, and C_0 are given by the relations

$$F_0 = -\sum_{\bar{x} \neq 0} Y^3(\bar{x})(Y(\bar{x}) - \mathcal{Z}(\bar{x}))\rho(\bar{x}),$$

$$A_0 = \sum_{\bar{x} \neq 0} Y^2(\bar{x})(Y(\bar{x}) - \mathcal{Z}(\bar{x}))\rho(\bar{x}),$$

$$B_0 = \sum_{\bar{x} \neq 0} Y^2(\bar{x})(Y(\bar{x}) - \mathcal{Z}(\bar{x}))\bar{x}^2 \rho_1(\bar{x}),$$

$$C_0 = -\sum_{\bar{x} \neq 0} Y(\bar{x})(Y(\bar{x}) - \mathcal{Z}(\bar{x}))\rho(\bar{x}). \quad (26)$$

While summing up in Eqs. (26), we take into account that $\bar{x}$ is the distance to particles: $\bar{x} = 1$, $\sqrt{2}$, $\sqrt{3}$, and so on. Besides the functions $Y(\bar{x})$ and $\mathcal{Z}(\bar{x})$, Eqs. (26) also include the number of particles, $\rho(\bar{x})$, which are located at the sites of a simple cubic lattice at the distance $\bar{x}$ from the coordinate origin, as well as the number of the

same particles, $\rho_1(\bar{x}) = \sum_\Theta \rho_\Theta(\bar{x}) \cos^2 \Theta$, which takes their angular distribution into account. The notation $\rho_\Theta(\bar{x})$ is used for the number of particles, for which the values of $\cos^2 \Theta$ are identical (Θ is the angle between the axis Oz and the vector directed to the particle). Note that $\rho(\bar{x}) = \sum_\Theta \rho_\Theta(\bar{x})$. The values of $\rho(\bar{x})$, $\rho_\Theta(\bar{x})$, and $\rho_1(\bar{x})$ can be found in works [11, 22]. The quantities F_0, A_0, B_0, and C_0 are calculated with regard for the averaging correction and give rise to the emergence of a nonzero critical exponent of the correlation function. The numerical estimates for those quantities are presented in work [11] for intermediate values of the RG parameter s, close to the value $s = s^* = 3.5862$. The latter, at $\Delta\tilde{\Phi}(k) = 0$, corresponds to the average value of the coefficient in the squared term in the effective measure density, which is equal to zero at the fixed point. Just such s-values are optimum for the presented calculation procedure. This fact is associated with two circumstances. First, at small s-values, the presence of a unit element in the system must be taken into consideration. For its separation, the RRs should be presented in the form of series of perturbation theory with respect to the Gaussian distribution [21, 24]. Second, at large s-values, there emerge the large intervals of wave vectors, in which $\tilde{\Phi}(k)$ is averaged. In this case, the correction $\beta\tilde{\Phi}(k) - \beta\tilde{\Phi}(B_{n+1}, B_n)$ is substantial, so that its account in the linear approximation is incorrect.

It should be emphasized that the consideration of the next terms in Eq. (16), which are proportional to higher orders of operators $\partial/\partial\rho_{\mathbf{k}}$, does not change the functional form of expression (25) for $\Delta_g^{(1)}$ (see works [11, 22]). Only the coefficients F_0, A_0, B_0, and C_0 become different. In our following calculations, we will neglect a similar renormalization of those coefficients.

After the successive integration over the zeroth, the first, ..., the n-th layers in the CV phase space, we arrive at the following expression for the partition function of the system in the linear approximation in $\Delta\tilde{\Phi}(k)$:

$$Z = 2^N 2^{(N_{n+1}-1)/2} \tilde{Q}_0 \tilde{Q}_1 \cdots \tilde{Q}_n [Q(P_n)]^{N_{n+1}} \times$$

$$\times \int \exp \left\{ -\frac{1}{2} \sum_{k \leq B_{n+1}} \tilde{d}_{n+1}(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} - \frac{1}{4!} (N_{n+1})^{-1} \times \right.$$

$$\times \sum_{k_1,\ldots,k_4 \leq B_{n+1}} \tilde{a}_4^{(n+1)} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_4} \delta_{\mathbf{k}_1 + \cdots + \mathbf{k}_4} \left. \right\} (d\rho)^{N_{n+1}}. \quad (27)$$

In comparison with the results obtained earlier without taking the correction for the potential averaging into account (see, e.g., works [8,9,11]), expression (27) includes

new quantities. In particular,

$$f(x_n) = -\frac{3\sqrt{3}}{4} s^6 U(x_n) \frac{U^3(y_n)}{y_n} \frac{q_n t_n}{\sqrt{\tilde{a}_4^{(n)}}} F_0 \qquad (28)$$

characterizes a correction for partial partition functions, which now look like

$$\tilde{Q}_0 = \left[Q_0 e^{f(x)} \right]^{N'}, \quad \tilde{Q}_1 = \left[(3/\varphi(y))^{1/4} e^{(x_1^2+y^2)/4} \times \right.$$

$$\times U(0, x_1) U(0, y) e^{f(x_1)} \right]^{N_1}, \ldots,$$

$$\tilde{Q}_n = \left[(3/\varphi(y_{n-1}))^{1/4} e^{(x_n^2+y_{n-1}^2)/4} U(0, x_n) \times \right.$$

$$\times U(0, y_{n-1}) e^{f(x_n)} \right]^{N_n}. \qquad (29)$$

The arguments of the functions are

$$x_n = \sqrt{3} \tilde{d}_n(B_{n+1}, B_n) \left(\tilde{a}_4^{(n)} \right)^{-1/2},$$

$$y_{n-1} = s^{3/2} U(x_{n-1}) (3/\varphi(x_{n-1}))^{1/2}. \qquad (30)$$

Expression (28) for $f(x_n)$, in addition to $q_n = q\frac{1+\alpha_0}{s^2}\frac{1+\alpha_1}{s^2}\cdots\cdots\frac{1+\alpha_{n-1}}{s^2}$, contains the quantity

$$t_n = \sqrt{\frac{\tilde{a}_4^{(n)}}{3}} \frac{1}{U(x_n)} \frac{s^2}{1+\alpha_0} \frac{s^2}{1+\alpha_1} \cdots \frac{s^2}{1+\alpha_{n-1}} t_0^{(n)} \times$$

$$\times \frac{1}{\beta\tilde{\Phi}(0)}, \qquad (31)$$

where

$$t_0^{(n)} = \frac{3}{1+s^{-1}+s^{-2}} + \frac{3a_n}{1-s^{-3}} I_0.$$

The quantity I_0 is described by formula (24), with an accuracy of a substituted by

$$a_n = \bar{q} - \sqrt{\frac{\tilde{a}_4^{(n)}}{3}} \frac{1}{U(x_n)} \frac{s^2}{1+\alpha_0} \frac{s^2}{1+\alpha_1} \cdots \frac{s^2}{1+\alpha_{n-1}} \times$$

$$\times \frac{1}{\beta\tilde{\Phi}(0)}. \qquad (32)$$

The quantity

$$\alpha_n = \frac{9\pi^2}{4} s^6 U^4(y_n) \bar{q} t_n B_0 \qquad (33)$$

determines the correction for potential averaging in the n-th layer of the CV phase space. Note that, in this work, we use the notations $x_0 \equiv x$, $y_0 \equiv y$, $q_0 \equiv q$, $t_0 \equiv t$, and $a_0 \equiv a$.

The coefficients $\tilde{d}_{n+1}$ and $\tilde{a}_4^{(n+1)}$ satisfy the following RRs:

$$\tilde{d}_{n+1}(B_{n+2}, B_{n+1}) = \tilde{d}_n(B_{n+1}, B_n) \tilde{N}(x_n) -$$

$$- q\frac{1+\alpha_0}{s^2}\frac{1+\alpha_1}{s^2}\cdots\frac{1+\alpha_{n-1}}{s^2} \left(1 - \frac{1+\alpha_n}{s^2}\right),$$

$$\tilde{a}_4^{(n+1)} = \tilde{a}_4^{(n)} s^{-3} \tilde{E}(x_n). \qquad (34)$$

The corrections in the expressions

$$\tilde{N}(x_n) = N(x_n) (1 - G(x_n) A_0),$$

$$\tilde{E}(x_n) = E(x_n) (1 + \mathcal{K}(x_n) C_0) \qquad (35)$$

for the contribution made by the potential averaging are given by the terms $G(x_n) A_0$ and $\mathcal{K}(x_n) C_0$ Here,

$$G(x_n) = \frac{9\sqrt{3}}{2} s^6 \frac{U^3(y_n)}{y_n} U(x_n) \frac{q t_n}{\sqrt{\tilde{u}_n}},$$

$$\mathcal{K}(x_n) = 27\sqrt{3} s^3 \frac{U^5(y_n)}{y_n \varphi(y_n)} U(x_n) \frac{q t_n}{\sqrt{\tilde{u}_n}}. \qquad (36)$$

In terms of the variables

$$\tilde{r}_n = \frac{s^2}{1+\alpha_0} \frac{s^2}{1+\alpha_1} \cdots \frac{s^2}{1+\alpha_{n-1}} \tilde{d}_n(0),$$

$$\tilde{u}_n = \frac{s^4}{(1+\alpha_0)^2} \frac{s^4}{(1+\alpha_1)^2} \cdots \frac{s^4}{(1+\alpha_{n-1})^2} \tilde{a}_4^{(n)}, \qquad (37)$$

RRs (34) read

$$\tilde{r}_{n+1} = \frac{s^2}{1+\alpha_n} \left[(\tilde{r}_n + q) \tilde{N}(x_n) - q \right],$$

$$\tilde{u}_{n+1} = \frac{s}{(1+\alpha_n)^2} \tilde{u}_n \tilde{E}(x_n). \qquad (38)$$

For t_n (Eq. (31)), we have

$$t_n = \sqrt{\frac{\tilde{u}_n}{3}} \frac{1}{U(x_n)} t_0^{(n)} \frac{1}{\beta\tilde{\Phi}(0)},$$

$$t_0^{(n)} = \frac{3}{1+s^{-1}+s^{-2}} + \frac{3a_n}{1-s^{-3}} \int_{1/s}^{1} \frac{dk}{k^2 - a_n}, \qquad (39)$$

where

$$a_n = \bar{q} - \sqrt{\frac{\tilde{u}_n}{3}} \frac{1}{U(x_n)} \frac{1}{\beta\tilde{\Phi}(0)}. \qquad (40)$$

There are two essential differences between RRs (38) and those obtained in works [11, 25] without regard for the correction for $\Delta\tilde{\Phi}(k)$. The first of them consists in a specific substitution of variables (37), which includes the multipliers $(1+\alpha_0)(1+\alpha_1)\cdots(1+\alpha_{n-1})$, hence being different from the corresponding substitution without the

account for the correction. The second difference concerns the transformation of the special functions $N(x_n)$ and $E(x_n)$ (10) into the functions $\tilde{N}(x_n)$ and $\tilde{E}(x_n)$ (35). This difference, as we will see further, results in a shift of the fixed-point coordinates, being associated with corrections to the critical exponents of thermodynamic functions.

4. Analysis of Recurrence Relations

As a partial solution, RRs (38) have a new fixed point $(\tilde{r}, \tilde{u})$, which, at $\Delta\tilde{\Phi}(k) \neq 0$, differs from the fixed point $(r^{(0)}, u^{(0)})$ for the case $\Delta\tilde{\Phi}(k) = 0$ [11, 25, 26]. Really, supposing that

$$\tilde{r}_{n+1} = \tilde{r}_n = \tilde{r}, \qquad \tilde{u}_{n+1} = \tilde{u}_n = \tilde{u}, \tag{41}$$

we obtain, from the second equation in (38), the equality

$$\tilde{E}(\tilde{x}) = \frac{(1 + \alpha^{(0)})^2}{s}, \tag{42}$$

and use the latter to find the quantity

$$\tilde{x} = \sqrt{3}(\tilde{r} + q)(\tilde{u})^{-1/2}. \tag{43}$$

The calculation accuracy for $\tilde{x}$ must correspond to the approximation used for the derivation of RRs (38). Therefore, in the linear approximation in $\Delta\tilde{\Phi}(k)$, we arrive at the formulas

$$\tilde{x} = x^{(0)} - \frac{\mathcal{K}(x^{(0)})C_0 - 2\alpha^{(0)}}{\psi(x^{(0)})}, \quad \psi(x^{(0)}) = \frac{E'(x^{(0)})}{E(x^{(0)})}. \tag{44}$$

Here, the quantity $x^{(0)}$ is the solution of the equation $E(x^{(0)}) = s^{-1}$ and characterizes the argument x_n at the fixed point found without regard for the dependence of the Fourier transform of the potential on the wave vector. For $\alpha^{(0)}$ and $\mathcal{K}(x^{(0)})$, we obtain

$$\alpha^{(0)} = \frac{9\pi^2}{4}s^6 U^4(y^{(0)})\bar{q}t^{(0)}B_0,$$

$$\mathcal{K}(x^{(0)}) = 27\sqrt{3}s^3 \frac{U^5(y^{(0)})}{y^{(0)}\varphi(y^{(0)})}U(x^{(0)})\frac{qt^{(0)}}{\sqrt{u^{(0)}}}, \tag{45}$$

where $t^{(0)} = t_n(u^{(0)}, x^{(0)})$, and $y^{(0)}$ is defined by the expression

$$y^{(0)} = s^{3/2}U(x^{(0)})\left(3/\varphi(x^{(0)})\right)^{1/2}.$$

Let the coordinates of the fixed point be presented in the form

$$\tilde{r} = -\tilde{f}\beta\tilde{\Phi}(0), \qquad \tilde{u} = \tilde{\varphi}(\beta\Phi(0))^2, \tag{46}$$

where

$$\tilde{f} = \bar{q}\frac{\tilde{N}(\tilde{x}) - 1}{\tilde{N}(\tilde{x}) - (1 + \alpha^{(0)})/s^2},$$

$$\tilde{\varphi} = 3\bar{q}^2\left[\frac{s^2 - \alpha^{(0)} - 1}{s^2\tilde{n}(\tilde{x}) - \tilde{x}(1 + \alpha^{(0)})}\right]^2. \tag{47}$$

Here, the following notations were introduced:

$$\tilde{n}(\tilde{x}) = n(\tilde{x})(1 - G(\tilde{x})A_0),$$

$$n(\tilde{x}) = \frac{\tilde{y}U(\tilde{y})}{U(\tilde{x})},$$

$$G(\tilde{x}) = \frac{9\sqrt{3}}{2}s^6\frac{U^3(\tilde{y})}{\tilde{y}}U(\tilde{x})\frac{\bar{q}\tilde{t}}{\sqrt{\tilde{\varphi}}}, \tag{48}$$

as well as $\tilde{t} = t_n(\tilde{\varphi}, \tilde{x})$. In the linear approximation in $\Delta\tilde{\Phi}(k)$, we obtain

$$\tilde{f} = f_0\left\{1 + \frac{\alpha^{(0)}x^{(0)}/s^2}{n(x^{(0)}) - x^{(0)}/s^2} + \right.$$
$$+ \frac{[x^{(0)}n'(x^{(0)}) - n(x^{(0)})]\Delta x - n(x^{(0)})G(x^{(0)})A_0x^{(0)}}{[n(x^{(0)}) - x^{(0)}][n(x^{(0)}) - x^{(0)}/s^2]} \times$$
$$\left. \times (1 - s^{-2})\right\}, \tag{49}$$

$$\tilde{\varphi} = \varphi_0\left\{1 - \frac{2\alpha^{(0)}/s^2}{1 - s^{-2}} + \right.$$
$$\left. + 2\frac{[-n'(x^{(0)}) + s^{-2}]\Delta x + n(x^{(0)})G(x^{(0)})A_0 + x^{(0)}\alpha^{(0)}/s^2}{n(x^{(0)}) - x^{(0)}/s^2}\right\}.$$

The quantities f_0 and φ_0 characterize the coordinates of the fixed point without taking the dependence $\tilde{\Phi}(k)$ on the wave vector into account [11, 19], and $\Delta x = \tilde{x} - x^{(0)}$. We have

$$n'(x^{(0)}) = n(x)\left[\frac{y'}{y} + y'\frac{U'(y)}{U(y)} - \frac{U'(x)}{U(x)}\right]\Bigg|_{\substack{x=x^{(0)} \\ y=y^{(0)}}},$$

$$G(x^{(0)}) = \frac{9\sqrt{3}}{2}s^6\frac{U^3(y^{(0)})}{y^{(0)}}U(x^{(0)})\frac{qt^{(0)}}{\sqrt{u^{(0)}}}. \tag{50}$$

Table 1 exhibits the values of the quantities $\alpha^{(0)}$, Δx, $\tilde{f}$, and $\tilde{\varphi}$ for some intermediate values of the RG parameter s.

Let us linearize RRs (38) in a vicinity of the fixed point,

$$\begin{pmatrix} \tilde{r}_{n+1} - \tilde{r} \\ \tilde{u}_{n+1} - \tilde{u} \end{pmatrix} = \tilde{\mathcal{R}}\begin{pmatrix} \tilde{r}_n - \tilde{r} \\ \tilde{u}_n - \tilde{u} \end{pmatrix}, \tag{51}$$

and determine the matrix elements for this transformation. The general expressions for the elements $\tilde{R}_{ij}$ of the

matrix $\tilde{\mathcal{R}}$ look like

$$\tilde{R}_{11} = \frac{s^2}{1+\tilde{\alpha}} n(\tilde{x}) \left[T(\tilde{x}) - G(\tilde{x})A_0(\Pi(\tilde{x}) + T(\tilde{x})) \right],$$

$$\tilde{R}_{12} = \frac{s^2}{1+\tilde{\alpha}} \frac{n(\tilde{x})(\tilde{u})^{-1/2}}{2\sqrt{3}} \left[1 - \tilde{x}T(\tilde{x}) + G(\tilde{x})\tilde{x}A_0 \times \right.$$
$$\times (\Pi(\tilde{x}) + T(\tilde{x})) \big],$$

$$\tilde{R}_{21} = \frac{sE(\tilde{x})}{(1+\tilde{\alpha})^2} (3\tilde{u})^{1/2} \left[\psi(\tilde{x}) + \mathcal{K}(\tilde{x})C_0 \times \right.$$
$$\times (\psi(\tilde{x}) + \Pi_1(\tilde{x})) \big],$$

$$\tilde{R}_{22} = \frac{sE(\tilde{x})}{(1+\tilde{\alpha})^2} \left\{ 1 - \frac{1}{2}\tilde{x}\psi(\tilde{x}) + \frac{1}{2}\mathcal{K}(\tilde{x})C_0 \times \right.$$
$$\times \left[1 - \tilde{x}(\psi(\tilde{x}) + \Pi_1(\tilde{x})) \right] \Big\}, \tag{52}$$

where $\tilde{\alpha} = \alpha_n(\tilde{\varphi}, \tilde{x})$ and the following notations are used:

$$T(\tilde{x}) = \frac{\tilde{y}'}{\tilde{y}} + \tilde{y}'\frac{U'(\tilde{y})}{U(\tilde{y})} - \frac{U'(\tilde{x})}{U(\tilde{x})} = \frac{1}{n(\tilde{x})} \left[\frac{\partial n(x_n)}{\partial x_n} \right] \Big|_{\substack{x_n=\tilde{x} \\ y_n=\tilde{y}}},$$

$$\Pi(\tilde{x}) = -\frac{\tilde{y}'}{\tilde{y}} + 3\tilde{y}'\frac{U'(\tilde{y})}{U(\tilde{y})} + \frac{U'(\tilde{x})}{U(\tilde{x})} = \frac{1}{G(\tilde{x})} \left[\frac{\partial G(x_n)}{\partial x_n} \right] \Big|_{\substack{x_n=\tilde{x} \\ y_n=\tilde{y}}},$$

$$\psi(\tilde{x}) = \tilde{y}'\frac{\varphi'(\tilde{y})}{\varphi(\tilde{y})} - \frac{\varphi'(\tilde{x})}{\varphi(\tilde{x})} = \frac{1}{E(\tilde{x})} \left[\frac{\partial E(x_n)}{\partial x_n} \right] \Big|_{\substack{x_n=\tilde{x} \\ y_n=\tilde{y}}},$$

$$\Pi_1(\tilde{x}) = -\frac{\tilde{y}'}{\tilde{y}} + \tilde{y}'\left[5\frac{U'(\tilde{y})}{U(\tilde{y})} - \frac{\varphi'(\tilde{y})}{\varphi(\tilde{y})} \right] + \frac{U'(\tilde{x})}{U(\tilde{x})} =$$
$$= \frac{1}{\mathcal{K}(\tilde{x})} \left[\frac{\partial \mathcal{K}(x_n)}{\partial x_n} \right] \Big|_{\substack{x_n=\tilde{x} \\ y_n=\tilde{y}}},$$

$$\mathcal{K}(\tilde{x}) = 27\sqrt{3}s^3 \frac{U^5(\tilde{y})}{\tilde{y}\varphi(\tilde{y})} U(\tilde{x}) \frac{\bar{q}\tilde{t}}{\sqrt{\tilde{\varphi}}}. \tag{53}$$

As was done before, all calculations are carried out in the approximation of a linear contribution from the correction $\Delta\tilde{\Phi}(k)$. In this approximation, from Eqs. (52), we obtain the matrix element

$$\tilde{R}_{11} = R_{11}(1 - \alpha^{(0)}) + R_{11}^{(1)}\Delta x + R_{11}^{(2)}A_0. \tag{54}$$

T a b l e   1.   Calculated values for the quantity $\alpha^{(0)}$ from Eqs. (45), the shift $\Delta x = \tilde{x} - x^{(0)}$ of the basic argument x_n at the fixed point, the quantities $\tilde{f}$ and $\tilde{\varphi}$ characterizing the fixed-point coordinates (46) with regard for the correction for the averaging of the potential, and the eigenvalues $\tilde{E}_1$ and $\tilde{E}_2$ of the transformation matrix $\tilde{\mathcal{R}}$ from Eq. (51)

s	$\alpha^{(0)}$	Δx	$\tilde{f}$	$\tilde{\varphi}$	$\tilde{E}_1$	$\tilde{E}_2$
3	0.0113	0.2551	0.3635	0.4975	6.8596	0.4109
s^*	0.0235	0.3332	0.4308	0.6353	9.2168	0.3801
4	0.0330	0.3782	0.4709	0.7261	11.0407	0.3649
5	0.0595	0.5047	0.5143	0.8630	16.1822	0.3381

where

$$R_{11} = s^2 n(x^{(0)})T(x^{(0)}),$$

$$R_{11}^{(1)} = s^2 [n(x^{(0)})T'(x^{(0)}) + n'(x^{(0)})T(x^{(0)})],$$

$$R_{11}^{(2)} = -s^2 n(x^{(0)})G(x^{(0)})[\Pi(x^{(0)}) + T(x^{(0)})]. \tag{55}$$

Here, the derivative $T'(x^{(0)})$ is calculated using the formula

$$T'(x^{(0)}) = \left\{ y''\left(\frac{1}{y} + \frac{U'(y)}{U(y)} \right) + (y')^2 \left[\frac{U''(y)}{U(y)} - \right.\right.$$
$$- \left.\left(\frac{U'(y)}{U(y)} \right)^2 - \frac{1}{y^2} \right] - \frac{U''(x)}{U(x)} + \left(\frac{U'(x)}{U(x)} \right)^2 \Bigg\} \Bigg|_{\substack{x=x^{(0)} \\ y=y^{(0)}}}, \tag{56}$$

in which

$$y'' = y'\left[\frac{U'(x)}{U(x)} - \frac{1}{2}\frac{\varphi'(x)}{\varphi(x)} \right] + y\left[\frac{U''(x)}{U(x)} - \left(\frac{U'(x)}{U(x)} \right)^2 - \right.$$
$$- \left.\frac{1}{2}\frac{\varphi''(x)}{\varphi(x)} + \frac{1}{2}\left(\frac{\varphi'(x)}{\varphi(x)} \right)^2 \right]. \tag{57}$$

Let us write down the matrix elements $\tilde{R}_{12}$ and $\tilde{R}_{21}$ in the form

$$\tilde{R}_{12} = \tilde{R}_{12}^{(0)}(\tilde{u})^{-1/2}, \qquad \tilde{R}_{21} = \tilde{R}_{21}^{(0)}(\tilde{u})^{1/2}. \tag{58}$$

Then, for $\tilde{R}_{12}^{(0)}$, we obtain

$$\tilde{R}_{12}^{(0)} = R_{12}^{(0)}(1 - \alpha^{(0)}) + R_{12}^{(1)}\Delta x + R_{12}^{(2)}A_0, \tag{59}$$

where

$$R_{12}^{(0)} = \frac{s^2}{2\sqrt{3}} n(x^{(0)})[1 - x^{(0)}T(x^{(0)})],$$

$$R_{12}^{(1)} = \frac{s^2}{2\sqrt{3}} \left\{ n'(x^{(0)})[1 - x^{(0)}T(x^{(0)})] - n(x^{(0)}) \times \right.$$
$$\times [T(x^{(0)}) + x^{(0)}T'(x^{(0)})] \Big\},$$

$$R_{12}^{(2)} = \frac{s^2}{2\sqrt{3}} n(x^{(0)})G(x^{(0)})x^{(0)}[\Pi(x^{(0)}) + T(x^{(0)})], \tag{60}$$

and, for $\tilde{R}_{21}^{(0)}$, we have

$$\tilde{R}_{21}^{(0)} = R_{21}^{(0)}(1 - 2\alpha^{(0)}) + R_{21}^{(1)}\Delta x + R_{21}^{(2)}C_0, \tag{61}$$

where

$$R_{21}^{(0)} = s\sqrt{3}E(x^{(0)})\psi(x^{(0)}),$$

$$R_{21}^{(1)} = s\sqrt{3}E(x^{(0)})[\psi'(x^{(0)}) + \psi^2(x^{(0)})],$$

$$R_{21}^{(2)} = s\sqrt{3}E(x^{(0)})\mathcal{K}(x^{(0)})[\psi(x^{(0)}) + \Pi_1(x^{(0)})]. \tag{62}$$

Here,

$$
\psi'(x^{(0)}) = \left\{ y'' \frac{\varphi'(y)}{\varphi(y)} + (y')^2 \left[\frac{\varphi''(y)}{\varphi(y)} - \left(\frac{\varphi'(y)}{\varphi(y)} \right)^2 \right] + \right.
$$
$$
\left. + \left(\frac{\varphi'(x)}{\varphi(x)} \right)^2 - \frac{\varphi''(x)}{\varphi(x)} \right\} \Bigg|_{\substack{x=x^{(0)} \\ y=y^{(0)}}}. \tag{63}
$$

The matrix element $\tilde{R}_{22}$ satisfies the relation

$$
\tilde{R}_{22} = R_{22}(1 - 2\alpha^{(0)}) + R_{22}^{(1)} \Delta x + R_{22}^{(2)} C_0, \tag{64}
$$

where

$$
R_{22} = sE(x^{(0)}) \left[1 - \frac{1}{2} x^{(0)} \psi(x^{(0)}) \right],
$$
$$
R_{22}^{(1)} = sE(x^{(0)}) \left\{ \psi(x^{(0)}) \left[1 - \frac{1}{2} x^{(0)} \Psi(x^{(0)}) \right] - \right.
$$
$$
\left. - \frac{1}{2} \left[\psi(x^{(0)}) + x^{(0)} \psi'(x^{(0)}) \right] \right\},
$$
$$
R_{22}^{(2)} = \frac{sE(x^{(0)})}{2} \mathcal{K}(x^{(0)}) \left\{ 1 - x^{(0)} [\psi(x^{(0)}) + \Pi_1(x^{(0)})] \right\}. \tag{65}
$$

Note that the quantities R_{11}, $R_{12}^{(0)}$, $R_{21}^{(0)}$, and R_{22} in Eqs. (54)–(65) coincide with the expressions for the matrix elements obtained in works [11, 19] without regard for the correction for the averaging of the potential. The contributions to matrix elements $\tilde{R}_{ij}$, which are proportional to $R_{ij}^{(1)} \Delta x$, correspond to a shift of the fixed point owing to the account for the dependence of the Fourier transform of the interaction potential on the wave vector. The terms like $\tilde{R}_{ij}^{(2)} A_0$ and $\tilde{R}_{ij}^{(2)} C_0$ describe a direct contribution to $\tilde{R}_{ij}$ made by the correction for the averaging. The fixed point remains to be the saddle one, because the following inequalities are satisfied for the eigenvalues of the matrix $\tilde{\mathcal{R}}$: $\tilde{E}_1 > 1$ and $\tilde{E}_2 < 1$ (see Table 1).

Having calculated the eigenvectors and the eigenvalues of the transformation matrix $\tilde{\mathcal{R}}$ from Eq. (51), we obtain the explicit solutions of RRs (38):

$$
\tilde{r}_n = \tilde{r} + \tilde{c}_1 \tilde{E}_1^n + \tilde{c}_2 \tilde{R} \tilde{E}_2^n,
$$
$$
\tilde{u}_n = \tilde{u} + \tilde{c}_1 \tilde{R}_1 \tilde{E}_1^n + \tilde{c}_2 \tilde{E}_2^n. \tag{66}
$$

Here,

$$
\tilde{c}_1 = [r' - \tilde{r} - (a_4' - \tilde{u})\tilde{R}]/\tilde{D}, \quad \tilde{R} = \tilde{R}_{12}/(\tilde{E}_2 - \tilde{R}_{11}),
$$
$$
\tilde{c}_2 = [a_4' - \tilde{u} - (r' - \tilde{r})\tilde{R}_1]/\tilde{D}, \quad \tilde{R}_1 = (\tilde{E}_1 - \tilde{R}_{11})/\tilde{R}_{12},
$$
$$
\tilde{D} = (\tilde{E}_1 - \tilde{E}_2)/(\tilde{R}_{11} - \tilde{E}_2), \quad r' = a_2' - \beta\tilde{\Phi}(0), \tag{67}
$$

the initial coefficients a_2' and a_4' are given in Eq. (6), and the fixed-point coordinates $\tilde{r}$ and $\tilde{u}$ in Eqs. (46) and

(49). In view of notations (37), we obtain the relations

$$
\tilde{d}_n(B_{n+1}, B_n) = s^{-2n} \left[\prod_{m=0}^{n-1} (1 + \alpha_m) \right] \times
$$
$$
\times \left[\tilde{r} + q + \tilde{c}_1 \tilde{E}_1^n + \tilde{c}_2 \tilde{R} \tilde{E}_2^n \right],
$$
$$
\tilde{a}_4^{(n)} = s^{-4n} \left[\prod_{m=0}^{n-1} (1 + \alpha_m)^2 \right] \left[\tilde{u} + \tilde{c}_1 \tilde{R}_1 \tilde{E}_1^n + \tilde{c}_2 \tilde{E}_2^n \right]. \tag{68}
$$

The major difference between solutions (68) and solutions obtained earlier in works [8, 9, 19], in which no correction for the potential averaging was made, consists in the presence of multipliers of the type $(1 + \alpha_m)$. In the general case, they depend on the temperature and the ordinal number of a block structure m. However, in the case $T = T_c$, the coefficients $\tilde{d}_n$ and $\tilde{a}_4^{(n)}$ in Eqs. (68) behave themselves specifically. Since, at $T = T_c$,

$$
\lim_{m \to \infty} \alpha_m(T_c) = \alpha^{(0)},
$$

we obtain the following asymptotics for those quantities:

$$
\tilde{d}_n(B_{n+1}, B_n) = (\tilde{r} + q)s^{-n(2-\eta)},
$$
$$
\tilde{a}_4^{(n)} = \tilde{u} s^{-2n(2-\eta)}. \tag{69}
$$

The quantity η in the exponent powers is given by the formula

$$
\eta = \frac{\alpha^{(0)}}{\ln s} \tag{70}
$$

and corresponds to the critical exponent of the correlation function.

Hence, the account for the correction associated with the averaging of the Fourier transform of the potential $\tilde{\Phi}(k)$ in the course of integration of the partition function of the system gives rise, for some block structures, to a change of the asymptotics for the coefficients $\tilde{d}_n$ and $\tilde{a}_4^{(n)}$ at $T = T_c$ (in contrast to the case $\Delta\tilde{\Phi}(k) = 0$, the exponent powers in Eqs. (69), except for n, also contain η).

5. Free Energy of the System

The general scheme developed in works [8, 11, 19, 25] to calculate the free energy was applied to the case where the correction for the potential averaging was taken into account. The free energy of the system will be calculated, by separating the contributions made by the short- and long-wave modes of spin density oscillations. Despite the changes in the solutions of RRs (68), it can be easily shown that

$$
\tilde{c}_1 = \tilde{c}_{10}\tau, \tag{71}
$$

where $\tau = (T - T_c)/T_c$, and $\tilde{c}_{10}$ is a constant different from zero at $T = T_c$. This allows us to determine the point m_τ, where the system goes out from the critical regime (CR). Analogously to the results of works [11, 19, 25], we use the condition

$$\tilde{d}_{m_{\tau+1}}(0) = 0 \tag{72}$$

to find the exit point from the CR fluctuations at $T > T_c$. We obtain the formula

$$m_\tau = -\frac{\ln \tau}{\ln \tilde{E}_1} + m_0 - 1. \tag{73}$$

Here, $\tilde{E}_1$ is the larger eigenvalue of the matrix of transformation (51) and

$$m_0 = \frac{\ln |\tilde{r}/\tilde{c}_{10}|}{\ln \tilde{E}_1}. \tag{74}$$

We illustrate the calculation procedure for the free energy of the system in the case of temperatures higher than T_c. The expression for the free energy at $T > T_c$ is written down in the form

$$F = F_0 + F_{\text{CR}} + F_{\text{LGR}}, \tag{75}$$

where the term $F_0 = -kTN \ln 2$ corresponds to the free energy of noninteracting spins, F_{CR} to the contribution to the free energy made by the short-wave oscillation modes of the spin-moment density (the CR region), and F_{LGR} to the contribution from the long-wave oscillation modes (the region of the limiting Gaussian regime, LGR). For F_{CR}, we obtain

$$F_{\text{CR}} = -kT \sum_{n=0}^{m_\tau} \ln \tilde{Q}_n. \tag{76}$$

The quantity $\ln \tilde{Q}_n$ can be expressed in the form

$$\ln \tilde{Q}_n = \ln Q_n + N_n f(x_n). \tag{77}$$

Here, Q_n is the partial partition function for the n-th block structure calculated without regard for the dependence of the Fourier transform of the potential $\tilde{\Phi}(k)$ on the wave vector, and $f(x_n)$ describes an extra contribution, which arises, if $\Delta\tilde{\Phi}(k)$ is taken into account. The contribution is convenient to be written down in the form

$$f(x_n) = -\frac{3\sqrt{3}}{4} s^6 F_0 q \alpha_n' t_n, \tag{78}$$

where the expression for the quantity α_n' looks like

$$\alpha_n' = \frac{1}{\sqrt{\tilde{u}_n}} U(x_n) U^3(y_{n-1}) y_{n-1}^{-1}. \tag{79}$$

In a vicinity of the fixed point, the quantity α_n' satisfies the relation

$$\alpha_n' = \frac{1}{\sqrt{\tilde{u}_n}} \frac{U(x^{(0)})}{(y^{(0)})^4} \left(1 - \frac{9}{2}\frac{1}{(y^{(0)})^2}\right) \times$$
$$\times \left[1 + A_1''(x_{n-1} - x^{(0)}) + A_2''(x_{n-1} - x^{(0)})^2 + \right.$$
$$+ A_3''(x_n - x^{(0)}) + A_4''(x_n - x^{(0)})^2 +$$
$$\left. + A_5''(x_{n-1} - x^{(0)})(x_n - x^{(0)})\right]. \tag{80}$$

Here, the following notations were used:

$$A_l'' = \omega_0 A_l',$$
$$\omega_0 = -\frac{3\sqrt{3}}{4} s^6 F_0 t^{(0)} \bar{q} \frac{1}{\sqrt{\varphi_0}} \frac{U(x^{(0)})}{(y^{(0)})^4} \left(1 - \frac{9}{2}(y^{(0)})^{-2}\right),$$
$$A_1' = -4r_1 + 18r_1/[2(y^{(0)})^2 - 9],$$
$$A_2' = 10r_1^2 - 4r_2 + 9(2r_2 - 11r_1^2)/[2(y^{(0)})^2 - 9],$$
$$A_3' = U'(x^{(0)})/U(x^{(0)}), \quad A_4' = \frac{1}{2}U''(x^{(0)})/U(x^{(0)}),$$
$$A_5' = U'(x^{(0)})/U(x^{(0)})\left\{-4r_1 + 18r_1/[2(y^{(0)})^2 - 9]\right\}, \tag{81}$$

as well as $t^{(0)} = t_n(\varphi_0, x^{(0)})$. The quantities r_1 and r_2, which are defined through the functions $U(x^{(0)})$, $\varphi(x^{(0)})$, and their derivatives are presented in works [11, 22]. For the contribution to the free energy of the system made by the CR region, we obtain

$$F_{\text{CR}} = -kTN'\left[a_0' + \ln Q(d) + \tilde{\gamma}_{01} + \tilde{\gamma}_{02}\tau + \tilde{\gamma}_{03}\tau^2 - \right.$$
$$\left. - \tilde{\gamma}' s^{-3(m_\tau+1)}\right]. \tag{82}$$

The coefficients a_0' and $Q(d)$ are defined in Eqs. (6) and (14), respectively. For other coefficients in Eq. (82), we obtain

$$\tilde{\gamma}_{01} = s^{-3}\left[\frac{f_{\text{CR}}^{(0)} + \omega_0}{1 - s^{-3}} + \frac{d_2\tilde{c}_{2k}\tilde{E}_2}{1 - \tilde{E}_2 s^{-3}} + \frac{\tilde{d}_4\tilde{b}_0\tilde{E}_2^2}{1 - \tilde{E}_2^2 s^{-3}}\right],$$

$$\tilde{\gamma}_{02} = s^{-3}\left[\frac{d_2\tilde{c}_{2k1}\tilde{E}_2}{1 - \tilde{E}_2 s^{-3}} + \frac{\tilde{d}_4\tilde{b}_1\tilde{E}_2^2}{1 - \tilde{E}_2^2 s^{-3}} + \frac{d_1\tilde{c}_{1k}\tilde{E}_1}{1 - \tilde{E}_1 s^{-3}} + \right.$$
$$\left. + \frac{\tilde{d}_5\tilde{c}_{1k}\tilde{c}_{2k}\tilde{E}_1\tilde{E}_2}{1 - \tilde{E}_1\tilde{E}_2 s^{-3}} + \frac{\tilde{d}_7\tilde{b}_0\tilde{c}_{1k}\tilde{E}_1\tilde{E}_2^2}{1 - \tilde{E}_1\tilde{E}_2 s^{-3}}\right],$$

$$\tilde{\gamma}_{03} = s^{-3}\left[\frac{d_2\tilde{c}_{2k2}\tilde{E}_2}{1 - \tilde{E}_2 s^{-3}} + \frac{\tilde{d}_5(\tilde{c}_{1k1}\tilde{c}_{2k} + \tilde{c}_{1k}\tilde{c}_{2k1})\tilde{E}_1\tilde{E}_2}{1 - \tilde{E}_1\tilde{E}_2 s^{-3}} + \right.$$
$$\left. + \frac{\tilde{d}_4\tilde{b}_2\tilde{E}_2^2}{1 - \tilde{E}_2^2 s^{-3}} + \frac{d_1\tilde{c}_{1k1}\tilde{E}_1}{1 - \tilde{E}_1 s^{-3}} + \frac{\tilde{d}_3\tilde{c}_{1k}^2\tilde{E}_1^2}{1 - \tilde{E}_1^2 s^{-3}} + \right.$$

$$+\frac{\tilde{d}_6 \tilde{c}_{2k} \tilde{c}_{1k}^2 \tilde{E}_1^2 \tilde{E}_2}{1 - \tilde{E}_1^2 \tilde{E}_2 s^{-3}} + \frac{\tilde{d}_7 (\tilde{b}_0 \tilde{c}_{1k1} + \tilde{b}_1 \tilde{c}_{1k}) \tilde{E}_1 \tilde{E}_2^2}{1 - \tilde{E}_1 \tilde{E}_2^2 s^{-3}} +$$

$$+\frac{\tilde{d}_8 \tilde{b}_0 \tilde{c}_{1k}^2 \tilde{E}_1^2 \tilde{E}_2^2}{1 - \tilde{E}_1^2 \tilde{E}_2^2 s^{-3}} \Bigg],$$

$$\tilde{\gamma}' = \frac{f_{\mathrm{CR}}^{(0)} + \omega_0}{1 - s^{-3}} + \frac{d_1 \tilde{f}}{1 - \tilde{E}_1 s^{-3}} + \frac{\tilde{d}_3 \tilde{f}^2}{1 - \tilde{E}_1^2 s^{-3}}. \qquad (83)$$

Formulas (83) differ from analogous expressions obtained without regard for the dependence of the Fourier transform of the potential on the wave vector by the renormalized eigenvalues of $\tilde{E}_1$ and $\tilde{E}_2$, and the coefficients $\tilde{d}_l$ (with $l \geq 3$) which, in the case where the aforementioned dependence is made allowance for, look like

$$\tilde{d}_3 = d_3 + B_3^2 A_5'' \tilde{E}_1^{-1}, \qquad \tilde{d}_4 = d_4 + B_1^2 A_5'' \tilde{E}_2^{-1},$$

$$\tilde{d}_5 = d_5 + B_1 B_3 A_5'' \left(\tilde{E}_1^{-1} + \tilde{E}_2^{-1} \right),$$

$$\tilde{d}_6 = d_6 + B_3 B_4 A_5'' \left(\tilde{E}_1^{-1} \tilde{E}_2^{-1} + \tilde{E}_1^{-1} \right) +$$

$$+ B_1 B_6 A_5'' \left(\tilde{E}_2^{-1} + \tilde{E}_1^{-2} \right),$$

$$\tilde{d}_7 = d_7 + B_1 B_4 A_5'' \left(\tilde{E}_1^{-1} \tilde{E}_2^{-1} + \tilde{E}_2^{-1} \right) +$$

$$+ B_2 B_3 A_5'' \left(\tilde{E}_2^{-2} + \tilde{E}_1^{-1} \right),$$

$$\tilde{d}_8 = d_8 + B_2 B_6 A_5'' \left(\tilde{E}_1^{-2} + \tilde{E}_2^{-2} \right) +$$

$$+ B_3 B_5 A_5'' \left(\tilde{E}_1^{-1} + \tilde{E}_1^{-1} \tilde{E}_2^{-2} \right) + B_4^2 A_5'' \tilde{E}_1^{-1} \tilde{E}_2^{-1} +$$

$$+ B_1 B_7 A_5'' \left(\tilde{E}_2^{-1} + \tilde{E}_1^{-2} \tilde{E}_2^{-1} \right). \qquad (84)$$

Note that the expressions obtained for the quantities $f_{\mathrm{CR}}^{(0)}$, d_l, and B_l are similar to those presented in works [11, 19]. One should substitute the quantity A_l in them by the quantity $\tilde{A}_l = A_l + A_l''$; the quantities f_0 and φ_0, which characterize the coordinates of the fixed point, by new quantities $\tilde{f}$ and $\tilde{\varphi}$, respectively; and the quantities E_l and R_{ij} by the renormalized eigenvalues $\tilde{E}_l$ and matrix elements $\tilde{R}_{ij}$, respectively. The same is true for the quantities $\tilde{c}_{lkm}$ and $\tilde{b}_l$, which differ from their counterparts c_{lkm} and b_l.

Separating the temperature dependence in Eq. (82), we arrive at the formula

$$F_{\mathrm{CR}} = -kTN' \left[\tilde{\gamma}_{01}' + \tilde{\gamma}_1 \tau + \tilde{\gamma}_2 \tau^2 - \tilde{\gamma}_{10} \tau^{3\nu} \right], \qquad (85)$$

where

$$\tilde{\gamma}_{01}' = \bar{a}_0 + \tilde{\gamma}_{01}, \quad \tilde{\gamma}_1 = \bar{a}_1 + \tilde{\gamma}_{02},$$
$$\tilde{\gamma}_2 = \bar{a}_2 + \tilde{\gamma}_{03}, \quad \tilde{\gamma}_{10} = \tilde{\gamma}' s^{-3m_0}. \qquad (86)$$

The expressions for $\bar{a}_l$ are the same as in works [11, 19], in which, however [22],

$$x_{0c} = \sqrt{3} [\bar{q} - \tilde{f} + \tilde{c}_{2k} \tilde{R}^{(0)} \tilde{\varphi}^{-1/2}] / \sqrt{\tilde{\varphi} + \tilde{c}_{2k}},$$
$$\tilde{R}^{(0)} = \tilde{R}_{12}^{(0)} / (\tilde{E}_2 - \tilde{R}_{11}).$$

We emphasize that the nonanalytical part of the free energy in the CR in Eq. (85) is determined by the last term from the renormalized critical exponent of the correlation length,

$$\nu = \frac{\ln s}{\ln \tilde{E}_1}. \qquad (87)$$

The renormalization of this exponent influences the critical behavior of the specific heat and other characteristics of the system in a vicinity of the temperature T_c.

While calculating the partition function of the system with regard for the correction for the potential averaging, the functional form of quantity $\tilde{d}_{n+1}(k)$ changes. For the initial $d'(k)$-coefficient, we have the expression

$$a_2' - \beta \tilde{\Phi}(k) = a_2' - \beta \tilde{\Phi}(0) + 2b^2 \beta \tilde{\Phi}(0) k^2.$$

After averaging k^2 over the interval (B_1, B') and integrating over the zeroth layer in the CV phase space, the expression for the coefficient $d'(k)$ transforms into the following one:

$$\tilde{d}_1(k) = \tilde{a}_2^{(1)} - \beta \tilde{\Phi}(0) + 2b^2 \beta \tilde{\Phi}(0)(1 + \alpha_0) k^2.$$

The average value of $\tilde{d}_1(k)$ in the interval of the wave vectors $(B_2, B_1]$ can be written down in the form

$$\tilde{d}_1(B_2, B_1) = \tilde{d}_1(0) + \bar{q} \beta \tilde{\Phi}(0) s^{-2(1 - \eta_0/2)},$$

where the notation

$$\eta_0 = \frac{\alpha_0}{\ln s}$$

was introduced. In terms of the average values calculated taking the correction $\Delta \tilde{\Phi}(k)$ into account, the interaction potential becomes renormalized,

$$(1 + \alpha_0) \langle k^2 \rangle_{B_2, B_1} = \langle k^{2 - \eta_0} \rangle_{B_2, B_1}. \qquad (88)$$

For the n-th block structure, we obtain, in a similar way,

$$\tilde{d}_n(B_{n+1}, B_n) = \tilde{d}_n(0) + 2\beta \tilde{\Phi}(0) b^2 \times$$

$$\times \langle k^2 \rangle_{B_{n+1}, B_n} \prod_{m=0}^{n-1} (1 + \alpha_m), \qquad (89)$$

which corresponds to the equality

$$\tilde{d}_n(B_{n+1}, B_n) = \tilde{d}_n(0) + \bar{q} \beta \tilde{\Phi}(0) \prod_{m=0}^{n-1} s^{-2(1 - \eta_m/2)}. \qquad (90)$$

Here,

$$\eta_m = \frac{\alpha_m}{\ln s},$$

and the quantity α_m is defined by Eq. (33). In the CR region, the following equalities are satisfied:

$$\alpha_0 = \alpha_1 = \cdots = \alpha_m = \alpha^{(0)}. \tag{91}$$

Therefore, the quantity $\tilde{d}_n(B_{n+1}, B_n)$ can be presented in the form

$$\tilde{d}_n(B_{n+1}, B_n) = \tilde{d}_n(0) + \bar{q}\beta\tilde{\Phi}(0)s^{-2n(1-\eta/2)}, \tag{92}$$

where η is given in Eq. (70). As a result, we obtain

$$(1 + \alpha^{(0)})^n \langle k^2 \rangle_{B_{n+1}, B_n} = \langle k^{2-\eta} \rangle_{B_{n+1}, B_n}. \tag{93}$$

If the temperature is equal to the critical one, then

$$\lim_{n \to \infty} \tilde{d}_n(0) = 0 \tag{94}$$

and the following limiting relation can be written down for $\tilde{d}_n(k)$:

$$\lim_{n \to \infty} \tilde{d}_n(k) = \lim_{k \to 0} 2\beta\tilde{\Phi}(0)b^2 k^{2-\eta}. \tag{95}$$

Let us examine a contribution to the free energy of the system made by long-wave fluctuations. The wavelength, which corresponds to these fluctuations, exceeds the correlation length. At $T > T_c$, besides the CR, there exists the LGR in the system. It corresponds to the wave-vector interval

$$0 \le k < B_{m_\tau}, \tag{96}$$

where $B_{m_\tau} = B' s^{-m_\tau}$. In the region of LGR (or long-wave fluctuations), the quantity x_n (and, respectively, the quantity y_n) increases with n. This results in a drastic reduction of the coefficients α_n and the functions $f(x_n)$, $G(x_n)$, and $\mathcal{K}(x_n)$. While calculating the contribution to the free energy F_{LGR} from the LGR region, it is convenient, as was done in works [11, 19], to distinguish two regions of wave-vector values. The first one is the transition region (TR), which corresponds to $\mathbf{k}$-values close to B_{m_τ}, i.e. to the layers in the CV phase space located immediately beyond the exit point from the CR. The second one, the Gaussian region, includes small wave-vector values ($k \to 0$), in which the phase distribution of spin-moment density fluctuations has a Gaussian character.

The part of the free energy of the system, which corresponds to the TR, looks like

$$F_{\text{TR}} = -kTN'\tilde{f}_{\text{TR}}\tau^{3\nu}. \tag{97}$$

Here,

$$\tilde{f}_{\text{TR}} = s^{-3m_0} \sum_{m=0}^{m''} s^{-3m} f_{m_\tau + m + 1},$$

$$f_{m_\tau + m + 1} = \frac{1}{2} \ln y_{m_\tau + m} + \frac{9}{4}(y_{m_\tau + m})^{-2} +$$

$$+ \frac{x_{m_\tau + m + 1}^2}{4} + \ln U(0, x_{m_\tau + m + 1}), \tag{98}$$

and the values for $x_{m_\tau + m + 1}$ and $y_{m_\tau + m}$ are determined from the formulas

$$x_{m_\tau + m + 1} = \sqrt{3}\frac{\bar{q} + \tilde{f}(\tilde{E}_1^m - 1)}{(\tilde{\varphi} + \tilde{f}\tilde{\varphi}^{1/2}\tilde{R}_1^{(0)}\tilde{E}_1^m)^{1/2}},$$

$$y_{m_\tau + m} = s^{3/2}U(x_{m_\tau + m})(3/\varphi(x_{m_\tau + m}))^{1/2}. \tag{99}$$

The quantity m'', which characterizes the TR size $[m_\tau + 1, m_\tau + m'' + 1]$, can be determined, by following the technique described in works [11, 19]. The corresponding value does not depend on the temperature and, for intermediate values of RG parameter s, corresponds to 2–3 layers in the CV phase space.

The Gaussian fluctuation regime (the Gaussian region) is described by the expression

$$Z' = 2^{-1/2} \left(\pi P_2^{(m_\tau' - 1)}\right)^{-\frac{N_{m_\tau'}}{2}} \times$$

$$\times \int \exp\left[-\frac{1}{2} \sum_{k \le B_{m_\tau'}} \tilde{d}_{m_\tau'}(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right] (d\rho)^{N_{m_\tau'}}, \tag{100}$$

where $m_\tau' = m_\tau + m'' + 2$,

$$\tilde{d}_{m_\tau'}(k) = \left(P_2^{(m_\tau' - 1)}\right)^{-1} + \beta\left[\tilde{\Phi}(B_{m_\tau'}, B_{m_\tau' - 1}) - \tilde{\Phi}(k)\right],$$

$$P_2^{(m_\tau' - 1)} = \left(3/\tilde{a}_4^{(m_\tau' - 1)}\right)^{1/2} U(x_{m_\tau' - 1}). \tag{101}$$

In view of the condition $\tilde{c}_1\tilde{E}_1^{m_\tau + 1} = \beta\tilde{\Phi}(0)\tilde{f}$, we obtain

$$\tilde{a}_4^{(m_\tau' - 1)} = s^{-2(2-\eta)\left(-\ln\tau/\ln\tilde{E}_1 + m_0\right)}s^{-4m''}\tilde{u}_{m_\tau' - 1}$$

or

$$\tilde{a}_4^{(m_\tau' - 1)} = \tau^{2(2-\eta)\nu}s^{-2(2-\eta)m_0 - 4m''}\tilde{u}_{m_\tau' - 1}. \tag{102}$$

Here,

$$\tilde{u}_{m_\tau' - 1} = \tilde{u} + \beta\tilde{\Phi}(0)\tilde{f}\tilde{R}_1\tilde{E}_1^{m''}. \tag{103}$$

Then, in accordance with Eqs. (101), the quantity $\tilde{d}_{m_\tau'}(k)$ satisfies the relation

$$\tilde{d}_{m_\tau'}(k) = \tau^{2(2-\eta)\nu}s^{-2(m_0 + m'') + m_0\eta}\tilde{G} +$$

$$+ 2\beta\tilde{\Phi}(0)b^2(1 + \alpha^{(0)})^{m_\tau + 1}k^2, \tag{104}$$

in which

$$\tilde{G} = \beta\tilde{\Phi}(0)\tilde{g},$$

$$\tilde{g} = \left(\frac{\tilde{u}_{m'_\tau-1}}{3}\right)^{1/2}\frac{1}{\beta\tilde{\Phi}(0)U(x_{m'_\tau-1})} - \bar{q}. \quad (105)$$

Integrating in Eq. (100) over every variable $\rho_\mathbf{k}$ with $k \neq 0$ and introducing an infinitesimally low external magnetic field into consideration, we obtain the following expression for the component Z' of the partition function:

$$Z' = \left(P_2^{(m'_\tau-1)}\right)^{-\frac{N_{m'_\tau}}{2}}(2\pi)^{-1/2}\prod_{k\neq0}^{B_{m'_\tau}}\left(\tilde{d}_{m'_\tau}(k)\right)^{-1/2}\times$$

$$\times\int\exp\left[-\frac{1}{2}\tilde{d}_{m'_\tau}(0)\rho_0^2 + \beta\sqrt{N}h\rho_0\right]d\rho_0. \quad (106)$$

The part of the free energy of the system, which corresponds to Z', looks like

$$F' = \frac{1}{2}kT\left[N_{m'_\tau}\ln P_2^{(m'_\tau-1)} + \sum_{k=0}^{B_{m'_\tau}}\ln\tilde{d}_{m'_\tau}(k)\right] -$$

$$-N\frac{s^{2(m_0+m'')s^{-m_0\eta}}}{2\tilde{\Phi}(0)\tilde{g}}h^2\tau^{-(2-\eta)\nu}. \quad (107)$$

At last, taking Eqs. (101) and (104) into account and summing up in Eq. (107) over the wave vectors, we arrive at the formula

$$F' = -kTN'\tilde{f}''\tau^{3\nu} - \beta N\tilde{\gamma}_4^+h^2\tau^{-(2-\eta)\nu}, \quad (108)$$

where

$$\tilde{f}'' = \tilde{f}'\tilde{c}_\nu^3s^{-3(m''+1)},$$

$$\tilde{\gamma}_4^+ = \tilde{c}_\nu^{-(2-\eta)}\bar{\gamma}_4^+/(\beta\tilde{\Phi}(0)), \qquad \bar{\gamma}_4^+ = s^{2m''}/(2\tilde{g}),$$

$$\tilde{f}' = -\frac{1}{4}\ln 3 + \frac{1}{3} + \frac{1}{4}\ln\left(\tilde{\varphi} + \tilde{f}\tilde{\varphi}^{1/2}\tilde{E}_1^{m''}\tilde{R}_1^{(0)}\right) -$$

$$-\frac{1}{2}\ln(s^{-2} + \tilde{g}) - \tilde{g}'\left(1 - \sqrt{\tilde{g}'}\arctan\frac{1}{\sqrt{\tilde{g}'}}\right) -$$

$$-\frac{1}{2}\ln U(x_{m'_\tau-1}), \quad (109)$$

$$\tilde{c}_\nu = (\tilde{c}_{1k}/\tilde{f})^\nu, \quad \tilde{R}_1^{(0)} = (\tilde{E}_1 - \tilde{R}_{11})/\tilde{R}_{12}^{(0)}, \quad \tilde{g}' = \tilde{g}s^2.$$

Note that $s^{-m_0} = \tilde{c}_\nu$.

The calculated contributions to the free energy of the system – F_{CR} (see Eq. (85)) and $F_{\text{LGR}} = F_{\text{TR}} + F'$ (see Eqs. (97) and (108)) – enable, according to Eq. (75), the corresponding complete expression to be written down. As a result, we have

$$F = -kTN'\left[\tilde{\gamma}_0 + \tilde{\gamma}_1\tau + \tilde{\gamma}_2\tau^2 + \tilde{\gamma}_3\tau^{3\nu}\right] -$$

$$-\beta N\tilde{\gamma}_4^+h^2\tau^{-(2-\eta)\nu}. \quad (110)$$

Here,

$$\tilde{\gamma}_0 = \tilde{\gamma}'_{01} + s_0^3\ln 2,$$

$$\tilde{\gamma}_3 = -\tilde{\gamma}_{10} + \tilde{f}_{\text{TR}} + \tilde{f}'', \quad (111)$$

the coefficients $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ are defined in Eqs. (86), and the coefficient $\tilde{\gamma}_4^+$ in Eqs. (109). The entropy and the specific heat of the system can be found from expression (110) for the free energy by differentiating it with respect to the temperature at $h = 0$.

The susceptibility per one particle of the system can be obtained by calculating the second derivative of the free energy with respect to the external field $\mathcal{H}$,

$$\chi = \tilde{\Gamma}^+\tau^{-\gamma}\frac{\mu_{\text{B}}^2}{\tilde{\Phi}(0)}. \quad (112)$$

The critical exponent of the susceptibility is given by the expression

$$\gamma = (2 - \eta)\nu. \quad (113)$$

The quantity

$$\tilde{\Gamma}^+ = 2\tilde{c}_\nu^{-(2-\eta)}\bar{\gamma}_4^+ \quad (114)$$

is the critical amplitude.

The critical exponents of the correlation function, η, correlation length, ν, susceptibility, γ, and specific heat, $\alpha = 2 - 3\nu$, were calculated in the approximation of the quartic basis measure density for the RG parameter $s = 4$ at $\Delta\tilde{\Phi}(k) = 0$ and in the linear approximation in $\Delta\tilde{\Phi}(k)$. The obtained values are quoted in Table 2.

Note that the found critical exponents, in particular, for the correlation function, $\eta = 0.024$, agree with the data obtained by other authors. For instance, the estimations $\eta = 0.0335(25)$, $\eta = 0.0362(8)$, and $\eta = 0.033$ were obtained in the framework of the field-theory approach (seven-loop calculations) [27], Monte-Carlo simulations [28], and nonperturbative RG approach (an expansion into a series up to the ∂^4-order of derivatives) [14], respectively.

T a b l e 2. Critical exponents calculated with regard for the correction for the averaging of the Fourier transform of the potential ($\Delta\tilde{\Phi}(k) \neq 0$) and without it ($\Delta\tilde{\Phi}(k) = 0$)

Condition	η	ν	γ	α
$\Delta\tilde{\Phi}(k) = 0$	0	0.612	1.225	0.163
$\Delta\tilde{\Phi}(k) \neq 0$	0.024	0.577	1.141	0.268

6. Conclusions

The known theoretical approaches aimed at solving the problems in the domain of phase transitions and critical phenomena (see, e.g., [20, 29–35]) are mainly based on using the Gaussian measure density as a basics one. In the framework of such an approach, the calculation of the free energy and other thermodynamic functions of the system is carried out with regard for the dependence of the Fourier transform of the potential $\tilde{\Phi}(k)$ on the wave vector. However, the application of the Gaussian measure density as the basics one implies the application of perturbation theory. Using the methods of standard perturbation theory with the Gaussian distribution in the critical region gives rise to the emergence of an infinite set of diagrams for the free energy, which are divergent at $T \to T_c$. As a result, while calculating the partition function, there emerges an alternative: either to use the perturbation theory and to consider the dependence of the Fourier transform of the interaction potential on the wave vector $\mathbf{k}$ or, by averaging the Fourier transform of the interaction potential over certain k-intervals, to use a non-Gaussian distribution of fluctuations (generally speaking, with an arbitrary complexity) as a basics one. The latter way gives an opportunity to avoid the problems dealing with the emergence of unphysical divergences at the calculation of the free energy in a vicinity of T_c and to obtain not only the correct qualitative consequences, but also quantitative results. However, in a number of works (see, e.g., works [15–18]), this approach was used together with an approximation which led to the zero value for the critical exponent η of the correlation function ($\eta = 0$). The technique of calculations reported in this work allows one to obtain not only a critical exponent different from zero, but also the more general temperature dependences of thermodynamic characteristics for a three-dimensional system near the phase transition point.

In the present work, a method is developed to calculate the free energy of a three-dimensional Ising-like system in the case where the quartic measure density is used as a basic one (the ρ^4 model), and the dependence of the Fourier transform of the interaction potential on the wave vector is taken into consideration. The correction for the potential $\tilde{\Phi}(k)$ averaging is taken into account in the linear approximation. In this case, the RRs for the coefficients of effective measure densities are derived and studied. It is shown that the account for the correction for the potential averaging gives rise to a change of the asymptotics for the RR solutions at $T = T_c$. An ana-

lytical procedure for the calculation of the small critical exponent of the correlation function that arises owing to the correction is developed, and the value of the critical exponent is found.

Considering the correction for potential averaging brings about a renormalization of critical exponents for the correlation length, susceptibility, and specific heat of the system. The critical amplitudes are changed as well. The renormalization of the critical exponent for the correlation length, ν, if comparing with the case $\eta = 0$, is associated with a change of the larger eigenvalue for the matrix of the RG linear transformation. In contrast to ν, the critical exponent of the susceptibility, γ, explicitly depends on η and is determined according to formula (113). The specific heat of the system is characterized by the exponent α, the expression for which contains the renormalized critical exponent of the correlation length, ν.

As we see from the data given in Table 2, the account for the nonzero exponent η in the framework of the CV method leads to a reduction of the value of the critical exponent for the correlation length, ν (as it took place in the nonperturbative RG approach [14]). For better quantitative estimations to be obtained for this and other critical exponents, it is necessary that other distributions, more complicated than the quartic one (e.g., the sextic one), should be take into account, in the framework of which the exponent ν, calculated without the correction for the potential averaging ($\eta = 0$), acquires larger values in comparison with that obtained in the ρ^4 model [11].

The developed technique of researches and the results obtained can be used at studying the properties of various systems, in particular, ferromagnets, antiferromagnets, ferroelectrics, liquid–gas system, binary mixtures, lattice liquid model, *etc.*

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МЕТОД РОЗРАХУНКУ ВІЛЬНОЇ
ЕНЕРГІЇ ТРИВИМІРНОЇ ІЗИНГОПОДІБНОЇ
СИСТЕМИ З ВРАХУВАННЯМ ПОПРАВКИ
НА УСЕРЕДНЕННЯ ПОТЕНЦІАЛУ ВЗАЄМОДІЇ

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Р е з ю м е

Роботу присвячено теоретичному вивченню критичної поведінки тривимірних модельних систем. Виконано розрахунок статистичної суми та вільної енергії однокомпонентної спінової системи з використанням негаусового розподілу флуктуацій параметра порядку. Особливістю запропонованого методу розрахунку є врахування залежності фур'є-образу потенціалу взаємодії від хвильового вектора. Це приводить до відмінного від нуля критичного показника кореляційної функції η та перенормування значень інших критичних показників (кореляційної довжини, сприйнятливості тощо). Розрахунок останніх здійснено з використанням методу ренормалізаційної групи на основі отриманих у роботі рекурентних співвідношень між коефіцієнтами розподілів флуктуацій у суміжних блочних структурах.