STATES OF EVEN-EVEN NUCLEI IN NEUTRON CHAINS WITH N = 96, 98, 100

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On the basis of the Davydov–Chaban model, we study the evolution of changes in the spectrum of the levels of excited states for the ground-, β, and γ bands of even-even nuclei of neutron chains with N = 96, 98, and 100. The excited energy levels of these bands are considered for low and intermediate spins. It is shown that the model describes satisfactorily the energy levels of the above-mentioned neutron chains.

1. Introduction

A deformation of the nucleus surface occurs under the action of nucleons located outside closed shells: they attract nucleons of the core, by stretching their orbits. If the number of particles outside closed shells is sufficiently large, it is advantageous, by energy, for a nucleus to have a deformed equilibrium shape. As usual, it takes the form of an elongated ellipsoid of rotation [1]. Therefore, with regard for deviations of the nucleus shape due to a deformation of the quadrupole type, it is sufficient to approximate a nucleus by a three-axis ellipsoid. In the coordinate system connected with the nucleus at the given mean radius, the nucleus shape will depend on two parameters β and γ [1]. At their variation, the total energy of all nucleons of the nucleus changes adiabatically. This energy is the potential energy of surface oscillations of the nucleus, whose nucleons are present in the given one-particle state [2]. The possibility to describe the energy levels and the electromagnetic transitions between them on the basis of the Davydov–Chaban model for the exponential type of the potential energy of longitudinal surface oscillations was considered in [3, 4]. On the basis of the results obtained, it was clarified that the given model can describe the spectrum of excited levels of transient and nonspherical nuclei in a single way. But those works considered no specific cases, for example, the nuclei in isotopic, isobaric, and isotonic chains. The study of the energies of excited states of these chains allows one to analyze the evolution of variations in the spectrum of energy levels of excited states of the ground-, β-, and γ-bands from nucleus to nucleus both experimentally [5] and theoretically. The energy levels in atomic nuclei and the electromagnetic transitions between them vary with the number of protons and/or neutrons in the external shell, which is related to the transitions from one kind of collective behavior to another one [6]. In work [7], the Schrödinger equation with Bohr's Hamiltonian [8] was solved, by using the proper separation of the dynamical variables of longitudinal β-oscillations and transverse γ-oscillations. The authors of that work used the Davidson potential for longitudinal β-oscillations and the oscillatory potential for transverse γ-oscillations. We note that the potential energy $V(\beta,\gamma)$ is parametrized as $u = 2BV(\beta,\gamma)/\hbar^2$. In that work, the description of the energy levels of excited states included three parameters: the energy factor $\hbar\omega$ and the dimensionless parameters β_0 (it is called the Davidson parameter and characterizes the deformation of a nucleus in the ground state) and C (the stiffness of a nucleus relative to γ-oscillations). We now write the formula for energy levels taken from [7]:

$$E_{nn_\gamma I} = \hbar\omega\Bigg\{2n+1+$$

$$+\sqrt{\frac{I(I+1)-K^2}{3}+\frac{9}{4}+\beta_0^4+3C(n_\gamma+1)}\Bigg\}, \qquad (1)$$

Here, $n = 0,1,2,3,\ldots$ is the quantum number of β-oscillations; $n_\gamma = 0,1,2,\ldots$ is the quantum number of

γ-oscillations; I is the spin of an even-even nucleus, and K is the projection of a spin on the axis perpendicular to the symmetry axis of the nucleus. For K=0, $I = 0, 2, 4, 6, \ldots$, for $K \neq 0$, $I = K, K+1, K+2, \ldots$. The bands are characterized by the quantum numbers (n,n_γ,K): (0, 0, 0) for ground-bands; (1, 0, 0) for β-bands; and (0, 1, 2) for γ-bands. In that work, the ratios of the energies of excited levels of even-even nuclei from the rare-earth region and the region of actinides to the energy of the first excited level of the ground-band were calculated. These ratios in good agreement with experimental data [5]. As is known, there exist three regions of deformations of nuclei: at (Al, Mg), in the mass interval $150 < A < 190$ (lanthanides), and for $A > 200$ (actinides and heavy nuclei). Therefore, the new experimental data [5] on the collective states of deformable nuclei allow one

1. To determine the dependence of the spectrum of levels of excited states on the number of nucleons outside closed shells, which deform a nucleus and allow one to trace the evolution of their variation from nucleus to nucleus in neutron chains;
2. To find a region of deformable even-even nuclei, where their energy levels are sensitive to the dynamical or effective account for transverse γ-oscillations.

A good reserve for the solution of the above-mentioned problems was formed by work [9], where the energy levels with positive and negative parities of the ground band of axisymmetric nuclei ^{150}Nd, ^{152}Sm, ^{154}Gd, ^{156}Dy, and ^{158}Er in the neutron chain with N=90 were considered within the Sofia–Giessen model [9]. We note that this model is also based on the Bohr–Mottelson's Hamiltonian for axisymmetric nuclei. In the present work, we consider the evolution of variations of the spectrum of energy levels of excited states of the ground-, $\beta-$, and γ-bands of even-even nuclei in the neutron chain with N = 96, 98, and 100 within the Davydov–Chaban model. The obtained results will be compared with those in [7] and with the experimental data [5]. This presents a possibility to analyze a variation of the spectrum of energy levels of the nuclei under study with the dynamical effective account for transverse γ-oscillations.

2. Spectrum of Energy Levels of the Collective States of the Neutron Chains with N = 96, 98, and 100

In the present work, we consider the Davydov–Chaban model for collective excitations of the quadrupole type with regard for the coupling of the rotational motion with longitudinal and transverse oscillations of the nucleus surface (see [1, 4, 10]). This model explains a number of regularities in the excitation spectra of deformable nonaxial even-even nuclei observed in reactions of heavy ions with nuclei [5]. In the model [10], we should solve the Schrödinger equation

$$\left\{-\frac{\hbar^2}{2B}\frac{1}{\beta^3}\left(\beta^3\frac{d}{d\beta}\right)+V(\beta)+ \right.$$

$$\left. +\frac{\hbar^2}{4B\beta^2}\varepsilon_{I\tau}-E_{I\tau}\right\}F_{I\tau}(\beta)=0, \qquad (2)$$

where B is the mass parameter, $V(\beta)$ is the potential energy of β-oscillations, and the transverse γ-oscillations are taken into account by the introduction of the effective parameter $\gamma_{\rm eff}$ [10]. The eigenvalues of the equation for a rigid asymmetric rotator $\varepsilon_{I\tau}$ [10] in Eq. (2) depend on the parameter $\gamma_{\rm eff}$ and are determined for every value of this parameter. The index τ enumerates the eigenvalues referred to identical values of I. Each partial solution of Eq. (2) is related to the specific shape $V(\beta)$ of the potential. The solutions of the Schrödinger equation (2) for various types of the potential energy of β-oscillations were considered in [4, 10–15]. These works described satisfactorily the energies of excited levels, electric multipole transitions between them, and the mean values of electric multipole moments in these states. However, the mentioned works did not analyze the variation of the characteristics of excited states of nuclei from nucleus to nucleus in a wide interval of mass numbers. We now consider the solutions of the Schrödinger equation (2) with a potential energy of the exponential form,

$$V(\beta)=-\frac{C\beta_0^2}{2}\exp\left[-\frac{(\beta-\beta_0)^2}{\beta_0^2}\right], \qquad (3)$$

where C is the stiffness of a nucleus, β_0 is the parameter of a deformation of the nucleus in the ground state. At small values of the variable β, the potential energy $V(\beta)$ can be expanded in a series in a vicinity of the equilibrium state $\beta=\beta_0$:

$$V(\beta)=V(\beta_0)+\frac{(\beta-\beta_0)^2}{2}\frac{d^2V(\beta)}{d\beta^2}\mid_{\beta=\beta_0}+\ldots.$$

It is seen that, at $\frac{(\beta-\beta_0)^2}{\beta_0^2} \ll 1$, potential (3) coincides with the oscillatory potential. Hence, the oscillatory potential can be used in the case of small deformations. For large deformations, we choose a potential of the exponential form (3). The solution of Eq. (2) with oscillatory potential is given in [10] in detail. Here, we also use

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this method to solve the problem. Therefore, we give only the final formulas for the spectrum of energy levels and the wave functions of excited states. The energy spectrum in units $\hbar\omega$ takes the form

$$E_{\nu I\tau} = \left(\nu + \frac{1}{2}\right) \left\{2\mu^{-4} \left[2p_{I\tau} + 1 - p_{I\tau}^2 - \frac{3}{2p_{I\tau}}\right] \times \right.$$

$$\left. \times \exp[-(p_{I\tau} - 1)^2]\right\}^{1/2} + 0.5\mu^{-4}(p_{I\tau}^2 - p_{I\tau} - 1)\times$$

$$\times \exp[-(p_{I\tau} - 1)^2], \tag{4}$$

where $p_{I\tau} = \frac{\beta_{I\tau}}{\beta} > 1$ satisfies the condition

$$p_{I\tau}^3(p_{I\tau} - 1)\exp\{-(p_{I\tau} - 1)^2\} = \frac{\varepsilon_{I\tau} + 1.5}{2\mu^{-4}}, \tag{5}$$

which follows from the continuity of the potential energy of surface oscillations $V(\beta)$ [10], where $\mu = \left\{\frac{\hbar^2}{BC\beta_0^4}\right\}^{1/4}$ is the dimensionless parameter of the theory, and $\beta_{I\tau}$ is a new state of equilibrium corresponding to the states $I\tau$ [10]. The wave functions of Eq. (2) are as follows:

$$\phi(\xi) = N_\nu H_\nu(\xi)e^{-\xi^2/2}, \tag{6}$$

Here, N_ν is the coefficient of normalization, $H_\nu(\xi)$ is the first-order Hermite function, ν is a root of the transcendental equation

$$H_\nu\left(-\frac{p_{I\tau}}{\mu_{I\tau}}\right) = 0, \tag{7}$$

following from the boundedness of wave functions, and the variable

$$\xi = \frac{p_{I\tau}(\beta - \beta_{I\tau})}{\mu_{I\tau}\beta_{I\tau}},$$

changes in the interval

$$-\frac{p_{I\tau}}{\mu_{I\tau}} < \xi < \infty,$$

where

$$\mu_{I\tau} = \mu\left[2\left[2p_{I\tau} - p_{I\tau}^2 + 1 - \frac{3}{2p_{I\tau}}\right]\exp\{-(p_{I\tau} - 1)^2\}\right]^{-1/4}. \tag{8}$$

The energy levels of excited states are described by the quantum numbers $\nu I\tau$. The sequence of states in the energy bands can be denoted by $I_{\nu\tau}$.

The states with the quantum numbers ν=0, τ=1 are called the ground-band and are determined by the sequence of spins $I_{01}^+ = 0_{01}^+; 2_{01}^+; 4_{01}^+; 6_{01}^+; 8_{01}^+; \ldots$

The states with the quantum numbers ν=0, τ=2 are called the γ-band and are determined by the sequence of spins $I_{02}^+ = 2_{02}^+; 3_{01}^+; 4_{02}^+; 5_{01}^+; 6_{02}^+; \ldots$

The states with the quantum numbers ν=1, τ=1 are called the β-band and are determined by the sequence of spins $I_{11}^+ = 0_{11}^+; 2_{11}^+; 4_{11}^+; 6_{11}^+; 8_{11}^+; \ldots$

We note that the above-presented integer values of quantum number ν are conditional. In the general case, the values of this quantum number are not integers and are roots of the transcendental equations (7). Namely the roots of Eq. (7) are involved in the calculation of the energy levels of excited states (4).

Here, we use the following parameters: $\hbar\omega$ is the energy factor, and the effective amplitude $\gamma_{\rm eff}$ of transverse γ oscillations and the dimensionless parameter μ determine the "softness" of the nucleus with respect to surface deformations.

In the present work, we consider the collective states of the ground-, β-, and γ-bands of deformable even-even nuclei of neutron chains with N = 96, 98, and 100. We will consider the neutron chains for nuclei with N = 96 (^{164}Er, ^{166}Yb, ^{168}Hf, ^{170}W), with N = 98 (^{162}Gd, ^{166}Er, ^{168}Yb, ^{170}Hf), and with N = 100 (^{166}Dy, ^{168}Er, ^{170}Yb, ^{172}Hf).

The experimental data on the spectrum of energy levels for each band are considered up to the point of crossing of the bands. The crossing of the bands happens under the action of Coriolis forces on the pairs of nucleons in a rotating nucleus [16]. As a result of this effect, the nucleus passes at the point of crossing of the bands from the superfluid state to the normally fluid one [17]. Therefore, it is not expedient to describe the energy levels after the bandcrossing.

Columns 2–4 of Table 1 contain the parameters of the model: $\hbar\omega$, $\gamma_{\rm eff}$, and μ. Therefore, we can observe the evolution of variations of these parameters from nucleus to nucleus. It is seen that they vary quite smoothly. The second column presents the energy factor $\hbar\omega$, which takes close values for the neutron chains under study. In the third column, the parameter of "softness" μ takes values $\mu > 1/3$ for nuclei ^{168}Hf and ^{170}W, which corresponds to soft nuclei [10]. For the rest of nuclei of the neutron chains, this parameter takes values $\mu < 1/3$, which correspond to rigid nuclei [10]. In the fourth column, we give the parameter of "nonaxiality" $\gamma_{\rm eff}$, which also takes close values for the neutron chains under study. This allows us to conclude that the nuclei of the given neutron chain have a small "nonaxiality" [10].

Table 1. Values of parameters and significant characteristics used in the present work. The parameters $\hbar\omega$ and γ_{eff} are given in keV and degrees, respectively. The rest parameters μ, β_0, and C are dimensionless

Nuclei	$\hbar\omega$	μ	γ_{eff}	$\hbar\omega$	β_0	C	RMS	[7]	R_{041}^{exp}	E_{021}^{exp}
^{164}Er	93.4	0.2966	13.1°	572.2	1.5592	6.3	133.1	96.7	3.28	91.4
^{166}Yb	104	0.3327	12.4°	545	0.0261	6.9	182.4	150.1	3.23	102.4
^{168}Hf	120.2	0.3511	13°	549.8	0.0522	5.7	91.2	92.7	3.11	124.1
^{170}W	133.2	0.3473	14.8°	592.5	0.0486	4.8	98.8	80.2	2.95	156.7
^{162}Gd	69.5	0.2209	11.7°	714.5	2.8315	7.9	35	15.4	3.31	71.6
^{166}Er	72.2	0.2425	12.5°	648.8	2.5118	7	146.9	104.6	3.29	80.6
^{168}Yb	86.7	0.2760	11.9°	577.6	1.7827	7.7	54.4	32.1	3.27	87.7
^{170}Hf	100.9	0.3212	12.3°	523.5	0.0052	6.8	117.1	99.1	3.19	100.8
^{166}Dy	82.6	0.2652	12.8°	590.8	2.0887	6.7	77.4	56.6	3.31	76.6
^{168}Er	86.7	0.2777	13.5°	583	1.9742	6	112.8	73.4	3.31	79.8
^{170}Yb	92.4	0.2870	11.2°	565.7	1.0451	8.4	101.2	79.2	3.29	84.3
^{172}Hf	98.6	0.2890	11.8°	575	0.0041	7.4	186.2	155.2	3.25	95.2

Table 2. Comparison of experimental and theoretical energy levels of the neutron chain with $N = 96$

I	^{164}Er			^{166}Yb			I	^{168}Hf			^{170}W		
	Exp.	Theor.	[7]	Exp.	Theor.	[7]		Exp.	Theor.	[7]	Exp.	Theor.	[7]
2^+_{01}	91.3	112.6	106.9	102.4	124.3	111.5	2^+_{01}	124.1	147	121.7	156.7	165.2	140.9
4^+_{01}	299.4	354.5	343	330.5	383.8	355.6	4^+_{01}	385.9	445	385.1	462.3	499.3	442.9
6^+_{01}	614.3	691.8	683.7	667.9	734.8	702.9	6^+_{01}	757.3	835.5	755.8	875.5	935	861.9
8^+_{01}	1024.6	1093.9	1103.1	1098.3	1143.8	1124.7	8^+_{01}	1213.7	1277.5	1199.8	1363.4	1425.8	1359.0
10^+_{01}	1518.1	1539.3	1579.7	1605.9	1587.8	1598.8	10^+_{01}	1736.1	1747.4	1694.4	1901.5	1947.2	1907.5
12^+_{01}	2082.9	2014.7	2097.4	2176	2052.7	2109.3	12^+_{01}	2306.1	2232.7	2223.3	2464.3	2487.7	2490.2
14^+_{01}	2702.6	2512.8	2645	2779.5	2529.7	2645.7	14^+_{01}	2857.5	2726.1	2775.9	3118	3039.9	3096.2
16^+_{01}	3411.2	3028.6	3214.6	3490.1	3012.8	3200.7	16^+_{01}	3310.4	3221.1	3345.5	3815.9	3596.7	3718.7
2^+_{02}	860.2	891.6	922.2	932.4	1030.7	1015.5	2^+_{02}	875.9	958	933.5	937	977.1	920.3
3^+_{01}	946.4	977.4	1004.5	1039.1	1114	1096.6	3^+_{01}	1030.9	1056.8	1022.4	1073.6	1092.2	1023.8
4^+_{02}	1058.4	1092.8	1111.9	1162.7	1224.9	1202.1	4^+_{02}	1160.7	1189.4	1137.4	1220	1248.4	1157.1
5^+_{01}	1197.4	1224.8	1242.2	1327.9	1352.9	1329.9	5^+_{01}	1386.4	1336.4	1276.2		1417.3	1317.1
6^+_{02}	1358.6	1397.4	1393.5	1482.4	1515.1	1478	6^+_{02}	1551.3	1532.2	1436.2		1650.9	1500.5
7^+_{01}	1545.1	1558.6	1563.7	1704.5	1671.9	1644.3	7^+_{01}		1702.4	1614.8		1840.5	1704.1
8^+_{02}	1744.8	1793.6	1750.8	1812.5	1886.5	1826.7	8^+_{02}		1965.9	1809.7		2160	1925.2
9^+_{01}	1977.1	1961.4	1952.8	2150.3	2052.6	2023.4	9^+_{01}		2130.2	2018.8		2332.9	2161.1
10^+_{02}	2184.3	2267.2	2168.1	2143.1	2323.6	2232.5	10^+_{02}		2467.4	2240.1		2746.4	2409.8
11^+_{01}	2479.4	2417.6	2394.9	2646.7	2478.2	2452.4	11^+_{01}		2598.4	2472		2869.9	2669.3
12^+_{02}	2733.3	2802.4	2632.1	2609.6	2809.7	2682.2	12^+_{02}		3011.3	2713.1		3378.5	2938.9
13^+_{01}	3027.2	2913.8	2878.2	3196.6	2934.2	2920.2	13^+_{01}		3089.2	2962.1		3431.3	3214.9
0^+_{11}	1246	1071	1144.5	1043.1	954.2	1090.1	0^+_{11}	942	996.5	1099.7	952.5	1126.7	1185.0
2^+_{11}	1314.6	1206.8	1251.3	1144.3	1107.6	1201.6	2^+_{11}	1058.6	1178.8	1221.4	1202.2	1331.3	1325.9
4^+_{11}	1469.7	1489.7	1487.5	1342.5	1413	1445.6	4^+_{11}	1284.7	1528.8	1484.8	1578.3	1723.8	1627.9
6^+_{11}	1706.6	1869.1	1828.2	1608	1804.6	1792.9	6^+_{11}		1960.2	1855.1		2205.7	2046.9
8^+_{11}	2068.9	2305.5	2247.6	1852.9	2239.6	2214.8	8^+_{11}		2423.5	2299.5		2721.6	2544
10^+_{11}	2462.7	2774.2	2724.2	2319.6	2693.9	2688.8	10^+_{11}		2895.2	2974.1		3247.5	3092.5

Hence, these nuclei can be considered as axisymmetric [9].

In columns 5–7 of Table 1, we present the values of parameters from [7]: $\hbar\omega$, β_0, and C. These parameters also vary quite smoothly, except for the parameter β_0. In the fifth column, we give the values of energy factor $\hbar\omega$, which are close for the neutron chains under study. In the sixth column, the parameter β_0 shows a wide dis-

T a b l e 3. Comparison of experimental and theoretical energy levels of the neutron chain with $N = 98$

I	^{162}Gd			^{166}Er			I	^{168}Yb			^{170}Hf		
	Exp.	Theor.	[7]	Exp.	Theor.	[7]		Exp.	Theor.	[7]	Exp.	Theor.	[7]
2^+_{01}	71.6	80.2	74.8	80.6	84.9	81.0	2^+_{01}	87.7	102.1	95.7	100.8	120.6	107.9
4^+_{01}	236.4	261.8	246.2	264.9	274.8	265.3	4^+_{01}	286.6	326.6	309.5	322	375.1	343.7
6^+_{01}	490	533.2	507.4	545.5	553.4	542.9	6^+_{01}	585.3	648.3	622.9	642.9	723.6	679.1
8^+_{01}	826.2	880.1	849.2	911.2	902	900.9	8^+_{01}	970	1042.2	1014.8	1043.1	1133.6	1085.9
10^+_{01}	1237.9	1288.3	1261.4	1349.5	1304.2	1326.1	10^+_{01}	1425.5	1488.1	1466.4	1505.2	1582.4	1542.6
12^+_{01}	1718.6	1746.1	1733.7	1846.5	1748.2	1806.2	12^+_{01}	1936	1971.6	1962.9	2016.1	2055.7	2034.2
14^+_{01}	2260.2	2245.2	2256.8	2389.3	2226.3	2330.6	14^+_{01}	2488.5	2483.5	2492.9	2567	2544.6	2550.4
16^+_{01}	2857.1	2779.7	2822.3	2967.3	2733.6	2890.6	16^+_{01}	3073.2	3017.8	3048.2	3151.3	3043.3	3084.4
18^+_{01}		3345.9	3423.2	3577	3266.6	3479.4	18^+_{01}	3686.9	3570.3	3622.8	3766.5	3546.8	3631.5
2^+_{02}	864.7	864.1	863.5	785.9	793.3	822.8	2^+_{02}	984	993.3	1005.1	961.3	1028.2	967.1
3^+_{01}	930.7	935.1	929.9	859.4	866.1	892.8	3^+_{01}	1067.2	1073	1079.5	1087.6	1111.7	1045.6
4^+_{02}	1015.7	1030.8	1017.6	956.2	964.5	984.9	4^+_{02}	1171.4	1179.7	1177.0	1227.4	1223	1147.6
5^+_{01}		1144.5	1125.9	1075.3	1079.6	1098.3	5^+_{01}	1302.3	1305	1295.9		1351.8	1271.2
6^+_{02}		1289.4	1253.8	1215.9	1228.8	1231.3	6^+_{02}	1445.1	1463.8	1434.9		1515.2	1414.4
7^+_{01}		1437.3	1400.6	1376	1375	1383.2	7^+_{01}	1618.5	1622.5	1592.1		1674	1575.1
8^+_{02}		1636.2	1565.2	1555.7	1580.7	1552.7	8^+_{02}		1836.3	1766		1891.1	1751.2
9^+_{01}		1804.9	1746.5	1751.4	1741.5	1738.3	9^+_{01}		2011.8	1954.9		2060.6	1941
10^+_{02}		2066.1	1943.6	1964	2012.9	1938.9	10^+_{02}		2286.8	2157.3		2336	2142.8
11^+_{01}		2238.6	2155.4	2189.7	2168.3	2153.2	11^+_{01}		2459.8	2371.8		2495.5	2355
12^+_{02}		2572.1	2380.9	2428.8	2515.8	2380.1	12^+_{02}		2803.7	2597.2		2834.4	2576.4
13^+_{01}		2729.4	2619.1	2654.4	2645.4	2618.3	13^+_{01}		2954.7	2832.2		2964.8	2805.8
14^+_{02}		3145.8	2868.9	2880	3078.1	2867.1	14^+_{02}		3374.7	3075.7		3370	3042.2
0^+_{11}	1427.7	1427.5	1429.0	1460	1232.6	1297.6	0^+_{11}	1155.2	1144.9	1155.2	879.6	990.8	1046.9
2^+_{11}	1492.7	1518.2	1503.8	1528.4	1330.6	1378.6	2^+_{11}	1233.1	1266.3	1250.9	987	1138.7	1154.9
4^+_{11}		1722.1	1675.2	1678.8	1546.9	1562.9	4^+_{11}	1390.1	1526.7	1464.8	1156.7	1438.1	1390.7
6^+_{11}		2023	1936.4	1897.3	1858.4	1840.5	6^+_{11}		1888.7	1778.1		1828.4	1726
8^+_{11}		2402.3	2278.3	2194.6	2240.5	2198.5	8^+_{11}		2319.3	2170		2267.8	2132.8
10^+_{11}		2842.3	2690.5	2479.7	2673.2	2623.7	10^+_{11}		2793.9	2621.7		2731.7	2589.6
12^+_{11}		3329.4	3162.8	2656.9	3143	3103.8	12^+_{11}		3297.2	3118.1		3206.3	3081.2

persion from an even-even nucleus to the adjacent even-even nucleus. The seventh column contains the values of parameter C, which also takes close values for these neutron chains.

In columns 8–9, we give the root-mean-square deviations (RMSDs (in keV)) of experimental [5] and theoretical energy levels of the indicated bands for our work and work [7]. It is seen that, in both approaches, RMSD take satisfactory values, except for ^{166}Yb and ^{172}Hf. We note that RMSDs (at ≤ 100 keV) are good criterion of applicability of various models [9].

In the next column, we show the ratio of the second excited energy level to the first excited energy level of the ground-band $R^{\text{exp}}_{041} = E^{\text{exp}}_{041}/E^{\text{exp}}_{021}$. This ratio determines the collective rotation-vibrational behavior of excited levels [1]. At $2.7 < R^{\text{exp}}_{041} < 10/3$, the collective behavior of the spectrum of energy levels will be rotational or near-rotational. At $2 < R^{\text{exp}}_{041} < 2.4$, the spectrum is vibrational or near-vibrational [18]. Hence, the spectrum of energy levels of the neutron chains under consideration is rotational. The last column of Table 1 gives the energy of the first excited level of the ground band E^{exp}_{021}. In all neutron chains, this energy grows with the number of protons. The especially sharp increase is observed in chains with N=96 and 98. As is shown in [19], this energy gives information about deformations and the moment of inertia of the nucleus.

3. Comparison with Experimental Data

In Tables 2–4, we compare the experimental and theoretical (the results of our work and work [7]) values for the energy spectrum of excited levels of the ground-, β-, and γ-bands of the neutron chains under study.

T a b l e 4. Comparison of experimental and theoretical energy levels of the neutron chain with $N = 100$

I	^{166}Dy			^{168}Er			I	^{170}Yb			^{172}Hf		
	Exp.	Theor.	[7]	Exp.	Theor.	[7]		Exp.	Theor.	[7]	Exp.	Theor.	[7]
2^+_{01}	76.6	98.3	90.9	79.8	104.7	96.6	2^+_{01}	84.3	108.1	103.8	95.2	116.4	113.9
4^+_{01}	253.5	314.8	295.1	264	332.7	312.5	4^+_{01}	277.4	344.3	333.5	309.2	369.7	364.1
6^+_{01}	527	626.3	596.9	548.7	655.2	628.7	6^+_{01}	573.3	680.9	665.7	628.3	728.9	722
8^+_{01}	892	1008.4	977.8	928.3	1044.2	1024.3	8^+_{01}	963.3	1091.7	1075.7	1037.4	1164.7	1158.8
10^+_{01}	1341	1441.7	1420.5	1396.8	1479.5	1480.2	10^+_{01}	1437.5	1555.6	1542.6	1521.2	1654.2	1651.5
12^+_{01}	1868	1913.2	1910.6	1943.3	1949	1981.2	12^+_{01}	1983.4	2057.3	2050.7	2064.6	2181.3	2183.8
14^+_{01}	2467	2415	2436.9	2571.9	2446	2516.2	14^+_{01}	2580.4	2585.9	2588.9	2654.1	2735.7	2744.3
16^+_{01}	3119	2942.2	2991.1	3259.5	2965.9	3076.7	16^+_{01}	3195.7	3134.6	3149.3	3277.2	3310.5	3325.4
18^+_{01}		3490.8	3566.8		3504.7	3656.7	18^+_{01}	3806.8	3698.2	3726.3	3919.4	3901.1	3921.8
20^+_{01}		4057.4	4159.4		4058.6	4251.9	20^+_{01}	4436.6	4272.7	4316.2	4575.9	4503.1	4529.7
22^+_{01}		4638.6	4765.3		4624.1	4858.9	22^+_{01}	4854.4	5274.3	4916		5112.7	5146.5
24^+_{01}		5231.3	5381.9		5197.7	5475.4	24^+_{01}	5439.7	6032.3	5523.6		5725.7	5770.4
2^+_{02}	857.2	855.5	853.2	821.1	806.6	816.8	0^+_{11}	1069.4	1130.5	1131.4	871.3	1190.1	1150
3^+_{01}	928.7	935.9	927.7	895.7	891.2	895.4	2^+_{11}	1138.6	1160.1	1235.2	952.4	1329.7	1263.9
4^+_{02}	1023.4	1044.4	1025.3	994.7	1005.8	998.0	4^+_{11}	1292.5	1535.5	1464.9	1129.5	1625.4	1514.1
5^+_{01}	1141.3	1170.2	1144.5	1117.5	1136.3	1123.1	6^+_{11}		1914.8	1797.2		2029.9	1872
6^+_{02}		1333.9	1283.9	1263.9	1310	1268.8	2^+_{02}	1145.7	1163.8	1151.1	1075.3	1137.5	1112.6
7^+_{01}		1491.1	1441.8	1432.9	1468.8	1433.3	3^+_{01}	1225.4	1243.6	1226.9	1180.8	1224.6	1195.3
8^+_{02}		1715.6	1616.8	1624.5	1709.2	1614.8	4^+_{02}	1320.3	1349.8	1325.9	1304.7	1340.8	1303
0^+_{11}	1149	1180.4	1181.6	1217.1	1132.6	1165.9	5^+_{01}	1459.8	1475.7	1446.6	1462.8	1477.4	1433.8
2^+_{11}	1208	1296.1	1272.5	1276.2	1257.2	1262.5	6^+_{02}	1632.2	1621.5	1587.3		1649.3	1585.7
4^+_{11}		1546	1476.7	1411.1	1521.9	1478.4	7^+_{01}		1793.3	1746.3		1822	1756.5
6^+_{11}		1896.2	1778.6	1616.8	1884.7	1794.7	8^+_{02}		2000.8	1921.6		2051.6	1944.3

We conditionally divide the values of RMS and energies in Tables 2–4 into three parts. The first good part includes nuclei ^{162}Gd, ^{166}Dy, ^{168}Yb, ^{168}Hf, and ^{170}W. The second admissible one contains nuclei ^{164,166,168}Er, ^{170}Hf, and ^{170}Yb. The third worse part includes nuclei ^{166}Yb and ^{172}Hf. Hence, we may consider that the first part of nuclei is more sensitive to the exponential type of a potential of longitudinal surface deformations. It is seen from Tables 1–4 that the results obtained in [7] agree better with the experimental data than our ones. We note, however, that Table 1 indicates that the parameter μ varies more smoothly from nucleus to nucleus, than the parameter β_0. From the physical viewpoint, it is difficult to explain such a dispersion of values of the parameter β_0, because the nuclei of neutron chains differ only by two nucleons, except for nuclei ^{162}Gd and ^{166}Er of the neutron chain with N=98. The more consistent results can be obtained at a smooth variation of this parameter. We note that the solution of the Schrödinger equation for transverse oscillations is considered in [7] for small values of the γ-variable. The parameters μ and β_0 characterize the ground state of a nucleus.

4. Conclusion

We have considered the Davydov–Chaban model for the potential energy of surface β-oscillations of the exponential type. The energy levels for all bands (ground, β, and γ) depend on three parameters, $\hbar\omega$, γ_{eff}, and μ, which vary quite smoothly for the neutron chains under consideration. The nuclei are well deformable, i.e., $R_{041} \geq 2.95$.

As usual, the deformation of a nucleus (it can be calculated from the estimate of quadrupole moments) and its moment of inertia (it follows from the estimate of the first excited energy level of the ground-band with spin 2^+_{01}) increase for the nuclei, being far from closed neutron and proton shells [19]. For the nuclei with a half-filled shell, it is well known that the deformation attains a maximum and remains practically constant for a large region of nuclei. It is the region of nuclides, where the deformation is saturated and takes high values.

The nearest magic number for protons and neutrons for the neutron chains with N=82 is Z=82. We may consider that they are sufficiently remote from magic numbers. We note that the proton shell approaches the magic number, whereas the neutron one goes away from it.

We performed a comprehensive analysis of theoretical and experimental data [5] for lanthanides, actinides, and heavy and superheavy nuclei on the basis of the proposed model (the results of analysis of the spectrum of levels of excited states of actinides and heavy and superheavy nuclei are omitted here). We note that the properties of collective excited states of actinides and heavy and superheavy nuclei vary slowly. This can be observed by the first excited level of the ground-band $E_{021}^{\exp}$ [5], which varies slowly from nucleus to nucleus for even-even nuclei.

It is worth noting work [19], where the transition probabilities from the first excited level 2_{01}^{+} to the ground state of the nucleus are considered, and the growth factor $F \approx 70$ for the neutron chains under study. The shell effects affect significantly the low-lying excited states, which is supported by a quite sharp increase of $E_{021}^{\exp}$ and by the oscillation of RMSD at a change of the number of nucleons on the external shells of the neutron chains.

The parameters used in our work and work [7] agree with one another (see Table 1). The energy factor $\hbar\omega$ is identically defined in our work and work [7]. The parameters μ and β_0 are determined by β-oscillations, and the parameters γ_{eff} and C are related to γ-oscillations.

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ЗБУДЖЕНІ СТАНИ ПАРНО-ПАРНИХ ЯДЕР
У НЕЙТРОННИХ ЛАНЦЮЖКАХ З $N = 96, 98, 100$

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Р е з ю м е

На основі моделі Давидова–Чабана вивчено еволюцію зміни в спектрі енергій рівнів збуджених станів ground-, β- і γ-смуг парно-парних ядер нейтронних ланцюжків з $N = 96, 98, 100$. Розглянуто збуджені енергетичні рівні цих смуг для низьких і проміжних спінів. Показано, що дана модель задовільно описує рівні енергії розглянутих нейтронних ланцюжків.