
**THE INFLUENCE OF SCREENING EFFECTS
ON THE GRAIN CHARGE IN A THERMAL
DUSTY PLASMA**

D.YU. MISHAGLI

I.I. Mechnikov Odesa National University, Department of Theoretical Physics
(2, Dvoryans'ka Str., Odesa 65026, Ukraine; e-mail: *mishagli@onu.edu.ua*)

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The influence of an inhomogeneous screened electric field on the charging of dust grains in a thermal dusty plasma is studied. The electric field of charged grains is considered within the cell approach, where the problem is reduced to a one-particle one. Within the model of quasichemical equilibrium, which is generalized to the case of an inhomogeneous screened electric field, we obtain the value of mean charge of dust grains, the distribution function of grains over charges, and the variance of this distribution. We also give the criterion of an inhomogeneity $g(z)$ of the electric field and show that the influence of screening effects can be neglected in the case of the rarefied subsystem of dust grains (for $r_c/r_p \gg 1$) and in the case of small-radius grains ($r_p \sim 10^{-5}$ cm).

determination of the charge of identical dust grains of radius r_p . Within this method, the analog of the ionization energy I in the Saha equation is the work function of electrons leaving the surface of grains, which have charge z : $I \Rightarrow W_0 + ze^2/r_p$, where W_0 is the work function for the surface of neutral grains, and the term ze^2/r_p (z is the charge in units of the electron charge e) has meaning of the work needed to move an electron from the surface of a grain to infinity. It was shown that, under the made assumptions, the mean charge Z of dust grains for $Z \gg 1$ is given by the expression

$$Z = \frac{kTr_p}{e^2} \ln \frac{n_{es}}{n_e} + \frac{1}{2}, \quad (1)$$

where $n_{es} = 2(2\pi m_e kT/h^2)^{3/2} \exp(-W_0/kT)$ is the equilibrium density of electrons near the surface of an emitting grain, and n_e is the mean density of electrons in a plasma.

1. Introduction

The thermal dusty plasma characterized by the equality of the temperatures of all components occurs mainly under the terrestrial conditions in the combustion products of various fuels [1]. One of the important problems of the theory of a dusty plasma is the problem of determination of the charge of dust grains [2,3]. This problem is solved with the use of various methods considering its specificity with that or other degree of adequacy. Below, we discuss the most typical works.

Within the approach developed in [4], which will be called quasichemical, works [5–7] considered the possibility to form a negative charge and constructed the distribution function over charges of grains. It was shown that the coefficient kTr_p/e^2 is the distribution variance. We note that a small number of dust grains can be negatively charged, but, on the average, the dust grains in a plasma of combustion products have a positive charge.

In work [4], the existence of an analogy between the process of thermoionization of dust grains and the process of ionization of atoms was used for the determination of the charge of dust grains. The charge of atoms at the ionization varies by at most several values of electron charge. Therefore, it is possible to neglect the screening effect and to consider that the electrons pass a sufficiently large (infinite) path, by leaving the ions emitting them. The same approach was applied in work [4] to the

In work [8], the charge of dust grains of the same size r_p is determined from the condition of balance of currents: the thermoemission current is balanced by the current of electrons, which recombine with charged dust grains. The thermoemission current is determined, like that in [4], by the Richardson–Dushman equation. It is assumed that all electrons colliding with a dust grain recombine. The recombination rate is estimated in the

approximation of binary collisions as $-\alpha n_e n_p$, where $\alpha = \sigma v$, σ is the cross-section of a dust grain ($\sigma \approx \pi r_p^2$), v is the mean thermal velocity of relative motion, and n_p is the mean density of dust grains in a plasma. The balance equation for flows allows one to establish the connection between the mean charge Z and the mean density of electrons n_e : $Z = kTr_p/e^2 \ln n_{es}/n_e$. In fact, this result coincides with (1). In the same work, the value of n_e was taken from experimental data (in principle, it can be calculated theoretically), which gave the estimate of the value of charge. For example, for $n_e = 3.47 \times 10^{10} \text{ cm}^{-3}$, $n_p = 2 \times 10^7 \text{ cm}^{-3}$, $T = 3285 \text{ K}$, and for the grains with the radius $r_p = 10^{-4} \text{ cm}$, the charge Z was calculated to be 1700.

There is no basic difference between the above-mentioned approaches, which was indicated in [9, 10], where the formula of equilibrium constants was obtained by the kinetic way. This was further used in [4–7].

Works [11–15] present the attempts to consider the inhomogeneous distribution of electrons near a dust grain and the influence of the appropriate electric field on the recombination. In [11], the screening effects were considered in the linear approximation. Work [15] dealt with a separate dust grain, and the distribution of the electric field in its neighborhood was found as a solution of the Poisson equation in the Debye–Hückel approximation.

A certain contradiction is inherent in the above approaches: the introduction of the notion “the mean density” is not proper, because it was assumed that the thermoemission electrons go to infinity.

Therefore, some different approaches were naturally developed. There, it was considered that the thermoemission electrons remain in a bounded region including the dust grain [16–20]. This region (we will call it the cell) is electrically neutral. The electrons can pass from one cell to another one, but the mean number of electrons in each cell under the given conditions remains fixed. Such an approach allows one to consider only one single dust grain in a cell, i.e., the problem becomes one-particle. We note that, in the frame of cell approaches, the introduction of the mean density of emitted electrons is quite proper.

The system composed from identical spherical dust grains, electrons emitted by them, and a buffer gas, was considered in [16]. Each dust grain is located inside an electrically neutral spherically symmetric cell of radius r_c . The size of a cell is determined from the geometric reasoning as half the mean distance between dust grains:

$$r_c = \frac{1}{2} \left(\frac{3}{4\pi n_p} \right)^{1/3}. \quad (2)$$

The dust grains have the same charge Ze . The electric field inside a cell is determined by the numerical solution of the Poisson equation under relevant boundary conditions. In [16], it was accepted that, on the boundary of a cell, the potential and the strength of the electric field are zero. For the cases of weak and strong screenings of a grain by the electron cloud, the approximation formulas for the electric field potential were obtained. In work [17], it was shown that the solution of the electrostatic problem in the case of the weak screening ($r_D \gg r_p$) transfers in (1), if the subsystem of dust grains is rarified.

In [18, 19], the electric field inside a cell was determined by the solution of a linearized Poisson equation supplemented by two boundary conditions, which establish: (i) the equality of the electric field strength on the boundary of a cell to zero and (ii) the connection of the electric field strength on the surface of a grain with its charge.

We note that the boundary condition (i) is trivial (it follows from the Ostrogradskii–Gauss theorem) and gives no new results. Moreover, the influence of the electric field on the charging of dust grains was not considered in the proper way.

Work [20], which was also based on the cell approach to the description of properties of a dusty plasma, used the notion “the plasma potential” (“bulk plasma potential”) introduced earlier in [21, 22]. It was proposed to reckon the electric field potential ϕ from this “plasma potential”. The authors commented on such a choice of the reference point for the potential that, only in this case, one can set $\phi(r)|_{r \rightarrow \infty} \rightarrow 0$. In the previous works, “the plasma potential” was defined so that the Boltzmann distribution was consistent with the condition of the electric neutrality of a plasma. This proposition is not proper, since the process of thermoelectronic emission from an isolated grain is nonstationary, and the use of the Boltzmann distribution is impossible.

The consideration of the processes of charging and the influence of the screening effects on them has become especially actual after the discovery of ordered structures in the thermal dusty plasma [23]. This problem was analyzed, in particular, in works [24–26], where the influence of screening effects on the charging of a separate grain was studied. In work [24], the behavior of the screened field of a dust grain, which was described with a linearized Poisson equation, was studied by numerical methods. It was shown that the essential deviation from a linear theory of screening should be expected for grains, whose radius is of the order of the Debye one. The dynamics of the charging of a dust grain in the pres-

ence of external sources of ionization with regard for the photoemission from its surface was numerically studied in [25]. The value of charge was determined from the condition of equality of flows. The densities of electrons and ions in the vicinity of a dust grain and the charge of a grain as a function of the time were calculated. In work [26], the influence of boundaries on the screening of a point-like dust grain was studied.

In addition, the Saha equation became again used as a tool to describe the properties of a dusty plasma in the recent works [27, 28] (see also [21, 22]). This is explained by the simplicity and the physical clearness of this approach. Therefore, the necessity to comprehensively study the possibility of the use of the quasichemical approach seems obvious.

In the present work, we study the value and the variance of the mean charge of dust grains in the thermal plasma. To this end, we use: (i) the cell approximation and (ii) an approach based on the quasichemical model of the charging of dust grains. No buffer gas is considered.

2. Definition of a Cell and the Electrostatic Field Energy

Here, we will formulate the basic positions of the cell model and will determine the energy of the electrostatic field created by a charged dust grain. The limiting case of a weakly charged grain will be analyzed as well.

2.1. The cell model of a dusty plasma

Let us consider the identical dust grains of radius r_p with the mean charge Ze , which are in equilibrium with electrons emitted by them. We assume that it is possible to separate an electrically neutral cell around each grain. The cell radius r_c is given by relation (2). The condition of electric neutrality of a cell takes the form

$$Ze + 4\pi \int_{r_p}^{r_c} \rho(\mathbf{r}) r^2 dr = 0, \quad (3)$$

where the distance r is reckoned from the center of a grain.

We assume that the distributions of the bulk charge $\rho(\mathbf{r})$ and the potential $\phi(\mathbf{r})$ of the electrostatic field inside a cell have spherical symmetry: $\rho(\mathbf{r}) \Rightarrow \rho(r)$ and $\phi(\mathbf{r}) \Rightarrow \phi(r)$. The distribution of the potential is described in the approximation of self-consistent field: the potential $\phi(r)$ satisfies the Poisson equation, in which

the charge density $\rho(r)$ is determined by the Boltzmann distribution.

We note that the charge density in the vicinity of a charged grain is not always described by the Boltzmann distribution. For example, it is not proper if the nonstationary problems, involve a change of the charge of a dust grain in the course of time. In this case, it is expedient to use the methods of nonequilibrium thermodynamics, as it was proposed in [29, 30].

In order to solve the problem posed in Introduction, we turn to the linearized Poisson equation. This is justified by the following reasoning. The potentials, which are the solutions of the linearized and nonlinear Poisson equations, are close for $r_p < r < r_p + r_D$ and $r_p + r_D < r < r_c$ (r_D is the Debye radius of dust grains). In the second region, the linearized Poisson equation is generally quite adequate. The considerable difference between the potentials is significant only for $r \sim r_D$. This circumstance is of importance in the problems, in which the detailed behavior of the potential is defining. To estimate the mean charge Z within the method used by us, it is necessary to calculate the energy of the electrostatic field, which is a functional of the potential. Therefore, the "fine" details of the behavior of the potential are insignificant.

In the dimensionless form, the linearized Poisson equation reads

$$\Delta_{\tilde{r}} \psi(\tilde{r}) - \frac{1}{\lambda^2} \psi(\tilde{r}) = 0, \quad (4)$$

where $\Delta_{\tilde{r}}$ is the dimensionless radial part of the Laplace operator. The dimensionless parameters are as follows:

$$\tilde{r} = \frac{r}{r_p}, \quad \varsigma = \frac{r_c}{r_p}, \quad \psi(\tilde{r}) = 1 + \frac{e\phi(\tilde{r})}{kT}, \quad (5)$$

where ς and $\psi(\tilde{r})$ are, respectively, the dimensionless radius of a cell and the dimensionless potential of the electrostatic field inside a cell, e is the elementary charge, and k is the Boltzmann constant.

The quantity λ is the dimensionless Debye radius equal to

$$\lambda = \sqrt{\frac{1}{\tilde{n}_{e0}}}, \quad \tilde{n}_{e0} = \frac{n_{e0}}{n_*}, \quad n_* = \frac{kT}{4\pi e^2 r_p^2}, \quad (6)$$

where n_{e0} is the mean density of electrons for $\phi(r) = 0$, i.e., on the boundary of a cell. We consider that the screening of a grain is realized only by electrons emitted from its surface.

Equation (4) is supplemented by boundary conditions on the boundary of a cell and on the surface of a dust

grain:

$$\begin{cases} \psi(\varsigma) = 1, \\ \left. \frac{\partial \psi(\tilde{r})}{\partial \tilde{r}} \right|_{\tilde{r}=1} = -\frac{Z}{Z_0}, \end{cases} \quad (7)$$

where $Z_0 = kTr_p/e^2$ (for grains of a radius of 10^{-4} cm at the temperature $T = 3000$ K, $Z_0 \sim 10^2$ by the order of magnitude). The solution of Eq. (4) satisfying the boundary conditions (7) takes the form

$$\psi(\tilde{r}) = \frac{1}{\tilde{r}} \frac{(Z/Z_0)\lambda \operatorname{sh} \frac{\varsigma - \tilde{r}}{\lambda} + \varsigma (\lambda \operatorname{sh} \frac{\tilde{r}-1}{\lambda} + \operatorname{ch} \frac{\tilde{r}-1}{\lambda})}{\lambda \operatorname{sh} \frac{\varsigma-1}{\lambda} + \operatorname{ch} \frac{\varsigma-1}{\lambda}}. \quad (8)$$

It is possible to show that the electric field strength

$$\begin{aligned} \tilde{E}(\tilde{r}) = & \frac{1}{\tilde{r}} \frac{(Z/Z_0) \operatorname{ch} \frac{\varsigma - \tilde{r}}{\lambda} - \varsigma (\frac{1}{\lambda} \operatorname{sh} \frac{\tilde{r}-1}{\lambda} + \operatorname{ch} \frac{\tilde{r}-1}{\lambda})}{\lambda \operatorname{sh} \frac{\varsigma-1}{\lambda} + \operatorname{ch} \frac{\varsigma-1}{\lambda}} + \\ & + \frac{1}{\tilde{r}^2} \frac{(Z/Z_0)\lambda \operatorname{sh} \frac{\varsigma - \tilde{r}}{\lambda} + \varsigma (\lambda \operatorname{sh} \frac{\tilde{r}-1}{\lambda} + \operatorname{ch} \frac{\tilde{r}-1}{\lambda})}{\lambda \operatorname{sh} \frac{\varsigma-1}{\lambda} + \operatorname{ch} \frac{\varsigma-1}{\lambda}} \end{aligned} \quad (9)$$

(here, $\tilde{E}(\tilde{r}) = eE(r)r_p/kT$) on the boundary of an electrically neutral cell is equal to zero. This is a natural consequence of the Ostrogradskii–Gauss theorem. Hence, the chosen boundary conditions (7) are quite suitable for the complete solution of the electrostatic problem, and the boundary condition $E(r_c) = 0$ should not be considered instead of $\phi(r_c) = 0$. We note that $\tilde{E}(\varsigma) = 0$ with regard for the dependence of the Debye radius λ on the grain charge Z (see the following subsection).

The energy of the electrostatic field W_{el} can be calculated in the standard way:

$$W_{el} = \frac{1}{8\pi} \int_V \mathbf{E}^2 dV. \quad (10)$$

Performing all necessary calculations, we obtain that the dimensionless energy of the electrostatic field $\tilde{W}_{el}(Z) = W_{el}(Z)/kT$ reads

$$\tilde{W}_{el}(Z) = \alpha Z^2/Z_0 - \beta Z + \gamma Z_0, \quad (11)$$

where the coefficients α , β , and γ are as follows:

$$\alpha = \frac{\lambda \operatorname{sh} 2\frac{\varsigma-1}{\lambda} + 2\lambda^2 \operatorname{ch} 2\frac{\varsigma-1}{\lambda} + 2(\varsigma - \lambda^2 - 1)}{8D},$$

$$\beta = \frac{\varsigma [(\varsigma - \lambda^2 - 1) \operatorname{sh} \frac{\varsigma-1}{\lambda} + \lambda(\varsigma - 1) \operatorname{ch} \frac{\varsigma-1}{\lambda}]}{2\lambda D},$$

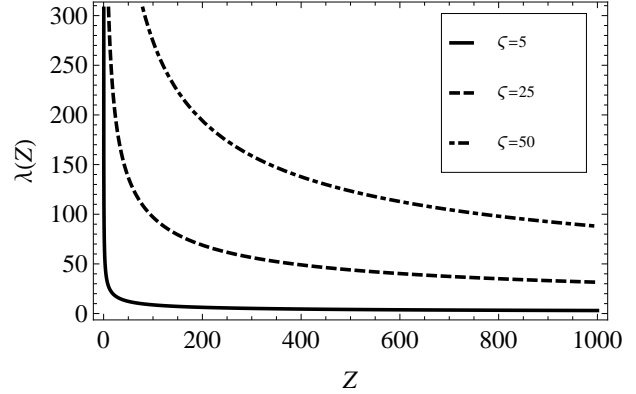


Fig. 1. Debye dimensionless radius $\lambda(Z)$ for various radii of a cell ς

$$\begin{aligned} \gamma = & \frac{\varsigma}{8\lambda^2 D} \left[\lambda [(\varsigma - 4)\lambda^2 + \varsigma] \operatorname{sh} 2\frac{\varsigma-1}{\lambda} + \right. \\ & + 2\lambda^2(\varsigma - 1 - \lambda^2) \operatorname{ch} 2\frac{\varsigma-1}{\lambda} + \\ & \left. + 2 [(\varsigma^2 + \lambda^2)(\lambda^2 - 1) + \varsigma] \right], \\ D = & \left(\lambda \operatorname{sh} \frac{\varsigma-1}{\lambda} + \operatorname{ch} \frac{\varsigma-1}{\lambda} \right)^2. \end{aligned} \quad (12)$$

These coefficients depend implicitly on the charge of a dust grain Z . To take this fact into account, we determine the dependence $\lambda(Z)$.

2.2. Dependence of the Debye radius on the charge of a dust grain

It is easy to verify that the equation of electric neutrality (3) can be written in the form

$$\frac{Z}{Z_0} - \frac{1}{\lambda^2} \int_1^\varsigma \psi(\tilde{r}) \tilde{r}^2 d\tilde{r} = 0. \quad (13)$$

By substituting solution (8) in (13), we obtain

$$\frac{Z}{Z_0} = \frac{1}{\lambda} \left[(\varsigma - \lambda^2) \operatorname{sh} \frac{\varsigma-1}{\lambda} + \lambda(\varsigma - 1) \operatorname{ch} \frac{\varsigma-1}{\lambda} \right]. \quad (14)$$

This equation establishes the interrelation between the dimensionless Debye radius λ and the mean charge Z of a grain. The corresponding dependence for various sizes of a cell ς is presented in Fig. 1. It is seen that, as the size of a cell increases (i.e., as the mean distance between

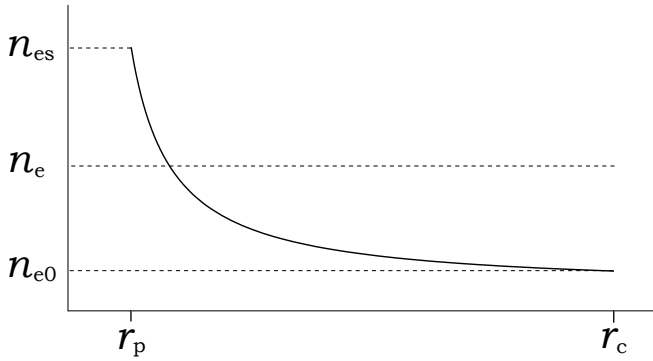


Fig. 2. Schematic distribution of the density of thermoemission electrons in an electrically neutral cell. Here, n_{es} is the equilibrium density of electrons near the surface of a grain ($r = r_p$), n_{e0} is the density of electrons for $\phi = 0$ ($r = r_c$), and n_e is the mean density of electrons in a cell

dust grains increases), the dimensionless Debye radius λ becomes much more than 1.

Since $\tilde{n}_{e0} = \lambda^{-2}$, Eq. (14) gives also the dependence of the density n_{e0} of thermoemission electrons on the boundary of a cell on the mean charge of a grain Z . The knowledge of the value of n_{e0} allows us to determine the mean density n_e of thermoemission electrons in a cell:

$$n_e = \frac{4\pi}{V_c} \int_{r_p}^{r_c} n_e(r) r^2 dr, \quad (15)$$

where $V_c = 4\pi(r_c^3 - r_p^3)/3$ is the cell volume free of a dust grain. In view of $n_e(r) \simeq n_{e0} \left(1 + \frac{e\phi(r)}{kT}\right)$, we obtain

$$n_e(Z) = \frac{3}{\zeta^3 - 1} \frac{Z}{Z_0} n_* = \frac{3}{4\pi} \frac{Z}{r_c^3 - r_p^3}. \quad (16)$$

It is clear that the calculated mean density of electrons n_e in a cell in the limiting case $\zeta \rightarrow \infty$ should coincide with n_{e0} (see Fig. 2), which has meaning, in this case, of the mean density of electrons in the system.

Equation (14) is significantly simplified in the limiting case of small charges of a dust grain ($Z \rightarrow 0$), which corresponds to the condition $\lambda \rightarrow \infty$. In this case, we have a small parameter $\sqrt{\tilde{n}_{e0}}$ and can expand the right-hand side of Eq. (14) in the series in it. To within $(Z/Z_0)^{3/2}$, we obtain

$$\tilde{n}_{e0}(Z) \simeq \left(\frac{Z}{a_2 Z_0}\right) \left(1 - \frac{a_4}{a_2^2} \frac{Z}{Z_0}\right), \quad (17)$$

where $a_2 = (\zeta^3 - 1)/3$ and $a_4 = (\zeta^5 - 5\zeta^3 + 5\zeta^2 - 1)/30$ are the coefficients of the relevant degrees of $\sqrt{\tilde{n}_{e0}}$. The

domain of applicability of the asymptotic formula (17) is determined by the inequality $\tilde{n}_{e0} \ll a_2/a_4$.

Thus, as $\lambda \rightarrow \infty$, we have the relation

$$\lambda(Z) \simeq \left(a_2 \frac{Z_0}{Z}\right)^{1/2} \left(1 + \frac{a_4}{2a_2^2} \frac{Z}{Z_0}\right), \quad (18)$$

which is valid for $\lambda \gg \sqrt{a_2/a_4}$.

2.3. Behavior of the potential and the energy of the electrostatic field for large Debye radii ($\lambda \gg 1$)

In this limiting case, the obtained formulas for the potential (8) and the energy (11) of the electrostatic field are considerably simplified. We now show that, as $\lambda \rightarrow \infty$, we arrive at the case considered in Appendix.

Indeed, the condition $\lambda \rightarrow \infty$ is equivalent to $Z \rightarrow 0$. Therefore, taking (18) into account and expanding (8) in the series in Z , we obtain (A3). Such a behavior of the cell potential was indicated in work [17].

If we restrict ourselves by the first term in the expansion, then the formulas for α , β , and γ become

$$\alpha \simeq \frac{\zeta - 1}{2\zeta} + \dots, \quad \beta \simeq \frac{\zeta^3 - 3\zeta + 2}{2\zeta(\zeta^3 - 1)} \frac{Z}{Z_0} + \dots, \quad (19)$$

$$\gamma \simeq \frac{\zeta^6 - 5\zeta^3 + 9\zeta - 5}{10\zeta(\zeta^3 - 1)^2} \left(\frac{Z}{Z_0}\right)^2 + \dots$$

Hence, the electrostatic energy (11) in the approximation $\lambda \rightarrow \infty$ ($Z \rightarrow 0$) is equal to (A5). In the limiting case $\zeta \rightarrow \infty$, the energy W_{el} passes into the energy of a nonscreened grain: $W_{el} \Rightarrow (Ze)^2/2r_p$.

3. The Model of Quasichemical Equilibrium

In this section, we recall the main results of works [4–7] and generalize the approach developed in them. For this purpose, we use the results obtained in the previous section. We will construct the distribution function of grains over charges, like that in [7] but with regard for the performed corrections, and will determine the variance Z_D of this distribution.

3.1. Generalization to the case of a screened field

It is known [31] that, from the thermodynamic viewpoint, the ionization equilibrium is a particular case

of the chemical equilibrium corresponding to simultaneously running “reactions of ionization.” These reactions can be written as follows:

$$A_0 = A_1 + e^-, \quad A_1 = A_2 + e^-, \quad \dots, \quad (20)$$

where the symbol A_0 means a neutral atom, A_1, A_2, \dots are atoms ionized one, two, *etc.* times, and e^- is an electron.

Under the assumption that such a model of charging holds also for dust grains, the authors of works [5–7] obtained the equilibrium constants

$$K_z = n_{es} \exp\left(-\frac{ze^2}{r_p kT}\right). \quad (21)$$

As was mentioned above (see Introduction), the factor ze^2/r_p in relation (21) represents the additional work made by an electron, by moving from the surface of a grain with charge z to infinity.

The consideration of the influence of the electric field becomes essential in a number of cases. For example, for grains of the radius $r_p = 10^{-5}$ cm with $Z = 100$, the ratio of the factor Ze^2/r_p to the work function $W_0 = 2.75$ eV is equal to 0.52. We obtain the same value also for grains of the radius $r_p = 10^{-4}$ cm with $Z = 1000$. Thus, the influence of the electric field becomes essential (for grains of the radius $r_p = 10^{-5}$ – 10^{-4} cm) for $Z \sim 100$ – 1000 . Such charges are typical of a dusty plasma.

The Saha equation $n_p^{(z)} n_e/n_p^{(z-1)} = K_z$ allows one to represent the mean density of dust grains with charge z as $n_p^{(z)} = \prod_{i=0}^{z-1} K_z/n_e$ and the mean densities of dust grains n_p and thermoemission electrons n_e as

$$n_p = \sum_{z=-\infty}^{\infty} n_p^{(z)}, \quad n_e = \sum_{z=-\infty}^{\infty} zn_p^{(z)}. \quad (22)$$

In the above-mentioned works, the condition of electric neutrality $n_e = Zn_p$ yields the formula for the mean charge Z of dust grains:

$$Z = \frac{\sum_{z=-\infty}^{\infty} z \exp\left(-\frac{z(z-1)e^2}{2r_p kT} + z \ln \frac{n_{es}}{n_e}\right)}{\sum_{z=-\infty}^{\infty} \exp\left(-\frac{z(z-1)e^2}{2r_p kT} + z \ln \frac{n_{es}}{n_e}\right)}. \quad (23)$$

Equation (23) for Z is not closed, since the quantity n_e remains undetermined. In works [5–7], it was taken from experimental data. In addition, the probability for a grain to obtain a negative charge is determined, obviously, only by the energy of the electrostatic interaction.

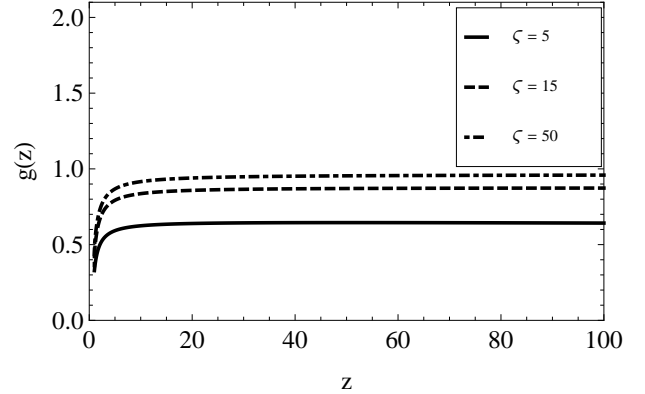


Fig. 3. Function $g(z)$ for various values of the dimensionless radius of a cell ζ

Therefore, for $z \leq 0$, we should set $n_{es} = n_e$, and we obtain $\ln n_{es}/n_e = 0$ in (23).

We now generalize the result in (23), by considering (i) the existence of a finite domain (electrically neutral cell); (ii) the influence of screening effects. Then the equilibrium constants (21) take the form

$$K_z = n_{es} \exp\left(-\frac{W_{el}(z) - W_{el}(z-1)}{kT}\right), \quad (24)$$

where the difference $W_{el}(z) - W_{el}(z-1)$ represents the additional work made by an electron, by moving from the surface of a grain into the cloud of electrons surrounding a grain.

In Fig. 3, we present the function

$$\frac{W_{el}(z) - W_{el}(z-1)}{ze^2/r_p} \equiv g(z) \quad (25)$$

for various sizes of a cell ζ . It is seen from the figure that, for $\zeta \gg 1$, i.e., for distances between dust grains to be much more than their radii, the function $g(z) \rightarrow 1$. Hence, we may consider that the thermoemission electrons move from grains to infinity. Therefore, for $r_c \gg r_p$, we may neglect the screening and use the equilibrium constants (21).

The equilibrium constants (24) lead to the result

$$Z = \frac{\sum_{z=-\infty}^{\infty} z \exp[f(z, Z)]}{\sum_{z=-\infty}^{\infty} \exp[f(z, Z)]}, \quad (26)$$

where

$$f(z, Z) = \begin{cases} -\tilde{W}_{el}(z), & z \leq 0; \\ -\tilde{W}_{el}(z) + z \ln \frac{n_{es}}{n_e}, & z > 0. \end{cases} \quad (27)$$

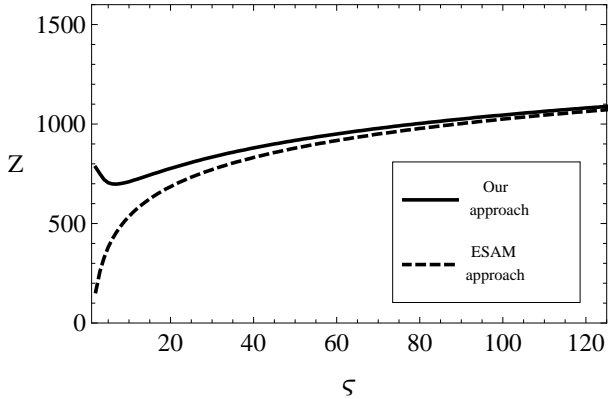


Fig. 4. Mean charge Z of dust grains CeO_2 of the radius $r_p = 5 \times 10^{-5}$ cm vs the dimensionless radius of a cell ς at $T = 2500$ K

Here, the mean density of thermoemission electrons n_e in a cell is a function of the mean charge of dust grains Z (see Eq. (16)).

The mean charges of dust grains CeO_2 ($W_0 = 2.75$ eV) for various temperatures and sizes of a cell (or, what is the same, for various mean distances between dust grains) are given in Tables 1–3.

First, we calculated the charge Z for given ς , T , and r_p by formula (26) (column Z_{cell}). In this case, we determined self-consistently the values of mean density of electrons n_e in a cell by formula (16). Then we calculated the values of charge by the “old” formula (23) (column Z_∞), where we used the calculated value of n_e . The computations were performed with the help of the developed algorithm in language `Java` with the use of class `BigDecimal` from packet `Math`.

In Fig. 4, we present the mean charge Z of dust grains CeO_2 of the radius $r_p = 5 \times 10^{-5}$ cm versus the dimensionless radius of a cell ς at $T = 2500$ K. It is seen that the role of the inhomogeneity is significant for small cells. As the size of a cell increases, formula (26) passes in (23). In addition, the value of Z decreases for $\varsigma \leq 5$ and starts to grow for $\varsigma > 5$. We explain such a result by that the quasichemical approach considered in the present work is not applicable for very dense systems ($\varsigma \sim 1$).

3.2. Distribution of grains over charges

The undoubtful advantage of the approach under consideration is the possibility to construct the distribution function of dust grains over charges like that in work [7], where the equilibrium constants (21) were used. Thus, by assuming that the charge of grains varies continu-

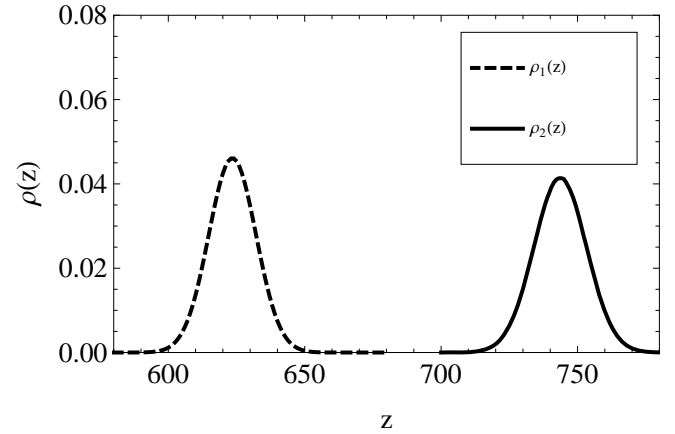


Fig. 5. Distribution functions $\rho_1(z)$ and $\rho_2(z)$ at $T = 2500$ K, $r_p = 5 \times 10^{-5}$ cm, and $\varsigma = 15$

ously, the following relation was obtained:

$$\frac{n_p^{(z)}}{n_p} = \frac{1}{\sqrt{2\pi Z_0}} \exp\left(-\frac{z(z-1)}{2Z_0} + z \ln \frac{n_{\text{es}}}{n_e}\right) \equiv \rho_1(z). \quad (28)$$

The variance of this distribution is $Z_0 = kTr_p/e^2$.

In the cell approximation, the distribution law for the charge z of dust grains reads

$$\frac{n_p^{(z)}}{n_p} = \frac{\exp[f(z, Z)]}{\sum_{z=-\infty}^{\infty} \exp[f(z, Z)]} \equiv \rho_2(z). \quad (29)$$

In Fig. 5, we present the distribution functions $\rho_1(z)$ and $\rho_2(z)$ at $T = 2500$ K, $r_p = 5 \times 10^{-5}$ cm, and $\varsigma = 15$. It is seen that the insignificantly small part of grains is negatively charged.

The variance Z_D of distribution (29) can be determined in the standard way as

$$Z_D = \mu_2 - \mu_1^2, \quad (30)$$

where μ_i is the i -th moment of the distribution, which is

$$\mu_i = \sum_{z=-\infty}^{\infty} z^i \rho_2(z). \quad (31)$$

For the same parameters as in Fig. 4, we give the dependence $Z_D(\varsigma)$. It is seen that the variance Z_D is somewhat different from Z_0 for small radii of a cell, and $Z_D(\varsigma) \rightarrow Z_0$ as $\varsigma \rightarrow \infty$.

T a b l e 1. Mean charges of dust grains calculated by the generalized formula (26) and by formula (23) for various radii of a cell ζ and grains r_p at the temperature $T = 2000$ K

ζ	$T = 2000$ K								
	$r_p = 10^{-4}$ cm			$r_p = 5 \times 10^{-5}$ cm			$r_p = 10^{-5}$ cm		
	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}
5	757	1.46×10^{12}	427	261	4.02×10^{12}	153	13	2.55×10^{13}	9
10	863	2.06×10^{11}	662	338	6.46×10^{11}	263	30	7.10×10^{12}	24
15	942	6.67×10^{10}	797	385	2.18×10^{11}	328	40	2.84×10^{12}	35
25	1061	1.62×10^{10}	966	449	5.49×10^{10}	410	54	8.24×10^{11}	50
35	1149	6.40×10^9	1078	495	2.20×10^{10}	465	63	3.53×10^{11}	60
50	1248	2.38×10^9	1196	545	8.33×10^9	523	74	1.41×10^{11}	71
150	1582	1.12×10^8	1562	713	4.04×10^8	705	107	7.57×10^9	106

T a b l e 2. Mean charges of dust grains calculated by the generalized formula (26) and by formula (23) for various radii of a cell ζ and grains r_p at the temperature $T = 2250$ K

ζ	$T = 2250$ K								
	$r_p = 10^{-4}$ cm			$r_p = 5 \times 10^{-5}$ cm			$r_p = 10^{-5}$ cm		
	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}
5	1250	2.41×10^{12}	676	472	7.27×10^{12}	264	36	7.01×10^{13}	23
10	1265	3.02×10^{11}	955	520	9.95×10^{11}	398	56	1.35×10^{12}	45
15	1326	9.38×10^{10}	1113	562	3.18×10^{11}	474	68	4.81×10^{12}	59
25	1441	2.20×10^{10}	1308	627	7.66×10^{10}	570	83	1.27×10^{11}	77
35	1533	8.54×10^9	1436	675	3.01×10^{10}	633	94	5.28×10^{11}	89
50	1641	3.14×10^9	1571	731	1.12×10^{10}	700	105	2.01×10^{11}	102
150	2013	1.42×10^8	1988	918	5.20×10^8	907	143	1.01×10^{10}	142

T a b l e 3. Mean charges of dust grains calculated by the generalized formula (26) and by formula (23) for various radii of a cell ζ and grains r_p at the temperature $T = 2500$ K

ζ	$T = 2500$ K								
	$r_p = 10^{-4}$ cm			$r_p = 5 \times 10^{-5}$ cm			$r_p = 10^{-5}$ cm		
	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}	Z_{cell}	n_{e0}, cm^{-3}	Z_{∞}
5	1782	3.43×10^{12}	933	705	1.09×10^{13}	381	76	9.27×10^{13}	44
10	1681	4.02×10^{11}	1255	710	1.36×10^{12}	536	97	1.21×10^{13}	75
15	1719	1.22×10^{11}	1434	744	4.21×10^{11}	624	109	3.71×10^{12}	93
25	1827	2.79×10^{10}	1654	807	9.87×10^{10}	733	125	8.48×10^{11}	115
35	1922	1.07×10^{10}	1797	858	3.82×10^{10}	804	137	3.21×10^{11}	129
50	2038	3.89×10^9	1949	918	1.40×10^{10}	879	151	1.14×10^{11}	145
150	2447	1.73×10^8	2415	1125	6.36×10^8	1110	180	1.27×10^{10}	178

4. Discussion of Results

Here, we have presented the results of theoretical studies of the influence of the screened electric field on the charging of dust grains in a thermal plasma. We have applied the following approximations: (i) the cell model of dusty plasma for the description of the distribution of the electrostatic field; (ii) the quasichemical approach for the determination of the mean charge of dust grains, distribution of grains over charges, and variance of this distribution.

The introduction of an electrically neutral cell containing a grain allowed us to: (i) consider the circumstance that the emitted electrons remain in the vicinity of a dust grain, rather than move to infinity; (ii) describe satisfactorily the distribution of the potential of the electrostatic field of a charged grain. Due to the introduction of a cell, we obtained a closed system of equations (14), (16), and (26) for the determination of the mean charge Z .

The influence of the inhomogeneity of the distribution of the electrostatic field is significant for $g(z) < 1$. In Fig. 6, we demonstrate the line corresponding to $g(z) =$

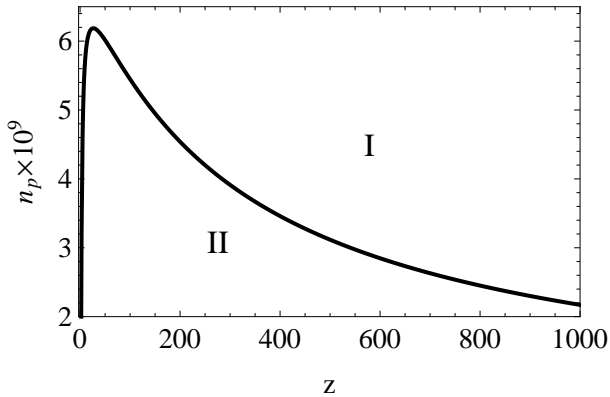


Fig. 6. Line corresponding to the values of n_p and z (at $T = 2500$ K and $r_p = 5 \times 10^{-5}$ cm), for which the condition $g(z) = 0.9$ is satisfied. Regions I and II correspond to $g(z) < 0.9$ and $g(z) > 0.9$, respectively

0.9 at the temperature $T = 3000$ K and the radius of grains $r_p = 10^{-4}$ cm. Above this line (region I), we deal with densities n_p and charges z , for which it is necessary to use the corrected formulas for the determination of the mean charge of dust grains. Below this line (region II), the influence of the screening effect can be neglected.

We have shown that the screening causes an increase of the mean charge of dust grains as compared with the prediction of the theory omitting the screening effects. As the size of a cell increases, i.e., as the mean distance between dust grains increases, result (26) is transformed into the well-known Einbinder–Smith–Arshinov–Musin formula (23).

For the grains of the radius $r_p = 10^{-5}$ cm, the differences in the predictions of the two approaches are slight. This is explained by the fact that the grains of such sizes have very small charges (~ 1), and the screening plays no significant role.

In the experiments [32] with grains CeO_2 ($W_0 = 2.75$ eV), the mean density of grains n_p varied in the limits $(0.2\text{--}5.0) \times 10^7$ cm^{-3} , and the temperature T was changed in the interval (1700–2200) K. The measured mean density of electrons n_e was in the limits $(2.5\text{--}7.2) \times 10^{10}$ cm^{-3} , the mean radius of grains $r_p = 4 \times 10^{-5}$ cm, and the lower experimental bound of the mean charge $Z \approx 500$.

For the grains with the indicated radius and the density $n_p = 2 \times 10^6$ cm^{-3} , the dimensionless radius of a cell $\zeta \approx 62$. At the temperature $T = 2000$ K, the mean charge and the mean density of thermoemission electrons are, respectively, $Z \approx 442$ and $n_e \approx 6.91 \times 10^9$ cm^{-3} according to (26) and (14); the charge distribution variance is $Z_D \approx 49$ according to (30). At the temper-

ature $T = 2200$ K, we have determined $Z \approx 560$, $n_e \approx 8.76 \times 10^9$ cm^{-3} , and $Z_D \approx 55$.

The calculated mean charge is close to the experimental result ($Z \approx 500$). However, the calculated densities of electrons are less than experimental ones by one order. This fact can be apparently explained by the neglect of the influence of an ionized buffer gas in the calculations. Therefore, the comparison of the theoretical and experimental results is not quite proper.

The study of the influence of the screening of a buffer gas will be performed separately.

We note also that the essential excess of the mean charge Z over Z_0 is accompanied by the violation of the condition of applicability of the linearized Poisson equation. The error of the calculated mean charge Z increases with Z , because, in this case, the contribution of the region, where the nonlinear effects are of importance, increases as well. This will be studied in further work.

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APPENDIX

Electric Field of a Weakly Charged Spherical Grain in a Cell

Small charges Z of a dust grain correspond to the condition $r_D \gg r_c - r_p$. In this case, the bulk density ρ of thermoemission electrons in a cell has a constant value $\rho = -en_e$, where n_e is the mean density of electrons in a cell, which is determined by the obvious relation

$$n_e = \frac{3}{4\pi} \frac{Z}{r_c^3 - r_p^3}. \quad (\text{A1})$$

The potential ϕ satisfies the Poisson equation

$$\Delta_r \phi = 4\pi en_e. \quad (\text{A2})$$

In the dimensionless variables (5), Eq. (A2) supplemented by the boundary conditions (7) has the solution

$$\psi(\tilde{r}) = 1 + \frac{Z/Z_0}{\zeta^3 - 1} \left[\frac{\zeta^3}{\tilde{r}} - \frac{3}{2}\zeta^2 + \frac{1}{2}\tilde{r}^2 \right]. \quad (\text{A3})$$

To within the designations, this formula coincides with that obtained in [16] in the case of the weak screening.

The strength and the energy of the electrostatic field of a weakly charged grain in a cell are given, respectively, by the relations

$$\tilde{E}(\tilde{r}) = \frac{Z/Z_0}{\zeta^3 - 1} \left(\frac{\zeta^3}{\tilde{r}^2} - \tilde{r} \right), \quad (\text{A4})$$

$$\tilde{W}_{\text{el}} = \frac{1}{2} \frac{Z^2/Z_0}{(\zeta^3 - 1)^2} \left[\zeta^6 - \frac{9}{5}\zeta^5 + \zeta^3 - \frac{1}{5} \right]. \quad (\text{A5})$$

We note that, in the limiting case of small charges (without the screening), we have strictly $E(\zeta) = 0$.

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ВПЛИВ ЕФЕКТІВ ЕКРАНУВАННЯ НА ЗАРЯД ПИЛИНКИ В ТЕРМІЧНІЙ ЗАПИЛЕНІЙ ПЛАЗМІ

Д.Ю. Миуазі

Резюме

В роботі досліджено вплив неоднорідного екранованого електричного поля на процес зарядження пилових частинок в термічній запыленій плазмі. Електричне поле заряджених частинок враховано в межах коміркового підходу, за допомогою якого задачу зведено до одночастинкової. В межах моделі квазіхімічної рівноваги, яку узагальнено на випадок неоднорідного екранованого електричного поля, отримано середній заряд пилових частинок, функцію розподілу частинок за зарядами та дисперсію цього розподілу. Також отримано критерій неоднорідності $g(z)$ електричного поля та показано, що впливом ефектів екранування можна знехтувати у випадку розрідженої підсистеми пилових частинок (при $r_c/r_p \gg 1$) та у випадку частинок малих радіусів ($r_p \sim 10^{-5}$ см).