G.A. MELKOV, D.V. SLOBODIANIUK
Taras Shevchenko National University of Kyiv, Faculty of Radiophysics
(64, Volodymyrs'ka Str., Kyiv 01601, Ukraine; e-mail: denslobod@ukr.net)

PACS 75.40.Gb, 73.30.Ds, 75.75.-c

A STRONGLY NONEQUILIBRIUM STATE IN MAGNETIC NANODOTS AT HIGH PUMPING LEVELS

A theoretical model describing a strongly excited magnon system in a magnetic nanodot has been developed. In this system, despite the discreteness of its spectrum, the parametric processes similar to those occurring in massive specimens take place, in particular, the processes of Suhl instability. Owing to a slight mismatch between the frequencies of modes that are engaged in the indicated processes, the threshold of the latter becomes somewhat higher and a non-resonant parametric interaction takes place. It is shown that, at certain power levels in the system, the processes similar to those of the so-called kinetic instability observed in massive specimens can emerge to excite the lowest-frequency mode of a nanoelement.

Keywords: spin waves, magnetic nanoelements, permalloy, parametric processes, nonlinear ferromagnetic resonance

1. Introduction

Interest in the nonlinear dynamics of magnetic nanostructures has considerably grown recently. These structures can be used for the creation of magnetic memory [1, 2], and they are rather promising. Very interesting is the issue concerning the behavior of such systems at large angles of the magnetic precession, which means a transition into a strongly nonlinear mode. The behavior observed at that has a considerably nonlinear character, so that the creation of corresponding theoretical models is needed.

First of all, such systems reveal the so-called foldover effect [3, 4], which is a nonlinear process of the lowest order. It consists in the variation of the magnetization owing to the growth of the uniform magnetization precession angle and results in a distortion of resonance curves in the system up to the onset of the bistability [5]. A theoretical explanation of those effects was made for the first time by Suhl [6], who took into account the interaction between the uniform precession and spin waves in the system. The next step consisted in that not only the interaction of excited spin waves with the uniform precession but also with one another was taken into consideration [7]. This stage finished the construction of the complete theory of nonlinear ferromagnetic resonance.

A characteristic feature of magnetic elements is a reconstruction of their spectrum induced by a reduction of their dimensions. First, the spectrum bottom frequency substantially grows in comparison with that for thick films owing to the increase of the role of exchange effects with a reduction of the element thickness down to the submicron scale. Second, in nanodots with linear dimension R, all spin-wave excitations with $0 < k < k_{cr} = 1/R$ disappear (see Fig. 1), and only the uniform mode with $k \approx 0$ survives.

The discretization of the spectrum is another feature of submicron-sized nanodots. Instead of continuous sets of frequencies and wave vectors, which are observed in the case of a continuous film, there emerges a discrete set of modes with discrete frequencies and corresponding wave numbers [8]. Therefore, it is reasonable to talk about the excitation of separate modes rather than spin waves in magnetic nanodots. To determine the frequencies of those modes is rather a complicated problem in the general case, because it is necessary to consider simultaneously the action of demagnetizing factors of the specimen, as well as the boundary and exchange effects. A model for the calculation of mode frequencies was developed in work [8]. The results of this work will be used below.

At present, the experiments aimed at studying the nonlinear dynamics of magnetic nanodots are mainly carried out with permalloy. A characteristic feature of this material is a substantial width of the ferromagnetic resonance line, $\Delta H = 50 \div 60$ Oe, in comparison with that for yttrium iron garnet (YIG). However, there is no theoretical model which would describe the nonlinear dynamics of magnetization in such systems. As a rule, the micromagnetic simulation with

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Fig. 1. Illustration of a spin-wave spectrum modification at changing from a continuous film (a) to a nanodot (b). The points correspond to discrete modes

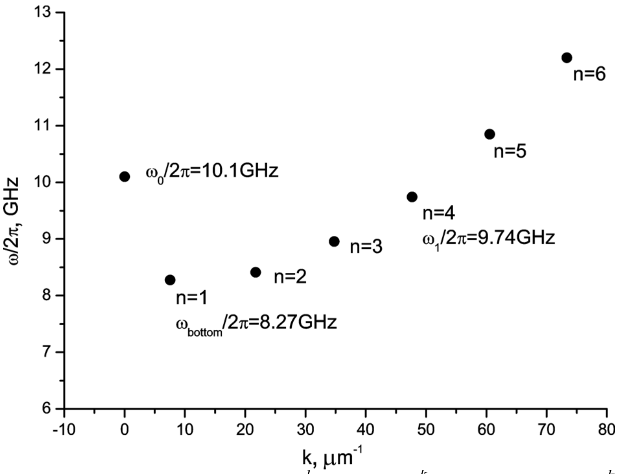

Fig. 2. Theoretically calculated spectrum of a circular nanodot with the radius $R = 250$ nm. The frequencies of uniform, ω_0, non-uniform, ω_1, and the lowest, $\omega_{\rm bottom}$, modes are indicated

the use of specialized software packages is usually applied.

The aim of this work is to develop a theoretical model on the basis of experimental results of work [9]. The model was intended to describe the nonlinear dynamics of magnetization in permalloy nanodots with regard for the following features: the nanodot spectrum becomes discrete, the frequency corresponding to the spectrum bottom grows as a consequence of exchange effects, and all long-wave excitations disappear but the uniform one [10].

2. Theoretical Model

2.1. Spin-wave spectrum of a nanodot

Before analyzing the processes that take place in the system, it is necessary to calculate the spectrum of a nanodot used in the experiment in work [9]. In the cited work, the experiments were carried out with permalloy nanodots of elliptic cross-section 500×250 nm^2 in dimensions and the thickness $t = 10$ nm. The uniform mode frequency in such nanodots can be calculated using the Kittel formula [11]

$$\omega_0^2 = \gamma^2 \left(H_{e0}+(N_x-N_z)M_0\right)\left(H_{e=0}+(N_y-N_z)M_0\right), \tag{1}$$

where γ is the gyromagnetic ratio, H_{e0} the magnitude of dc magnetic biasing field, M_0 the saturation magnetization, and N_x, N_y, and N_z are the demagnetizing factors. For an elliptic cylinder, those factors were found in work [12]. Using the results obtained in that work, we calculate that, in our case, $N_x/4\pi = 0.06$, $N_y/4\pi = 0.92$, and $N_z/4\pi = 0.02$. Substituting the values for the specimen magnetization $4\pi M_0 = 9500$ G and the external magnetic field $H_{e0} = 900$ Oe, we obtain the uniform mode frequency $\omega_0/(2\pi) = 10.1$ GHz. This value is only 4% larger than the experimental one, $\omega_{0\rm exp}/(2\pi) = 9.73$ GHz [9]. To calculate the higher modes of a magnetic nanodot is a more complicated problem. From the literature data, it is known that several types of oscillations can be distinguished in such a system in accordance with mechanisms that play a crucial role in that or another case [8]. We will confine the consideration to only one sort of modes, namely, quasibulk backward (BA) ones from work [8]. The calculations were carried out for the same parameters as those used while calculating the uniform mode. The results of calculations are depicted in Fig. 2.

The figure demonstrates that, in general, the obtained frequencies of characteristic modes in the system are in good agreement with those detected in the experiment [9]; in particular, the frequency of the lowest mode in the experiment was $\omega_{\rm bottom}^{\rm exp}/(2\pi) = 8.12$ GHz, and the frequency of uniform mode was $\omega_0^{\rm exp}/(2\pi) = 9.73$ GHz. It should be noted that the mismatches between the calculated and experimentally registered frequencies can be explained by the fact that the model of work [8] is suitable only to calculate the spectra of nanodots possessing the form of a circular cylinder. Hence, instead of the spectrum of elliptic cylinders that was studied experimentally, we calculated the spectrum for circular nanodots with the radius $R = 250$ nm.

2.2. Excitation of uniform and non-uniform modes

The first mode that can be excited in the system is the so-called uniform mode (or, more precisely, the quasiuniform one, because the matter concerns nanodots) if the arrangement geometry of an antenna and a specimen in the magnetic biasing field allows the processes of so-called perpendicular pumping [7, 11] to take place. Usually this geometry of the system is realized in experiments. When the power of the external electromagnetic pumping increases, the amplitude of the uniform mode grows. Next, there arise processes in the system that are analogous to the so-called Suhl instability in massive specimens [7, 11]. Provided that, besides the uniform mode with the frequency ω_0, there is also a non-uniform mode ω_n, which is degenerate with the former by frequency, the latter can be excited owing to the processes described by the formula

$$2\omega_0 = \omega_n + \omega_n. \qquad (2)$$

In the case of massive specimens, this process is called the second-order Suhl instability. It should be noted that, owing to the spectrum discreteness, the frequencies of the uniform and non-uniform modes in nanodots do not coincide in the general case, i.e. none of the frequencies of other modes equals to that of the uniform mode. Neverhless in this case, the indicated process does take place anyway, however with highest threshold: the mode, whose frequency is the nearest to that of the uniform mode, ω_0, is excited firstly. In this case, it is worth talking about the non-resonant parametric interaction described by the formula

$$2\omega_0 = \omega_n + \omega_n + 2\Omega, \qquad (3)$$

where $\omega_0 - \omega_n = \Omega$ is the frequency detuning between the uniform and non-uniform modes. Therefore, if the external pumping exceeds a certain level, two modes–the uniform, ω_0, and non-uniform, ω_n, ones–turn out to be excited in the system. In our case, the mode with $n = 4$ ($\omega_4 = 9.74$ GHz, see Fig. 2) is the nearest by frequency to the uniform mode. The interaction between the uniform and non-uniform modes manifests itself in a distortion of resonance curves, and the explanation to this effect was given in the framework of the so-called theory of nonlinear ferromagnetic resonance [7]. The governing equations are

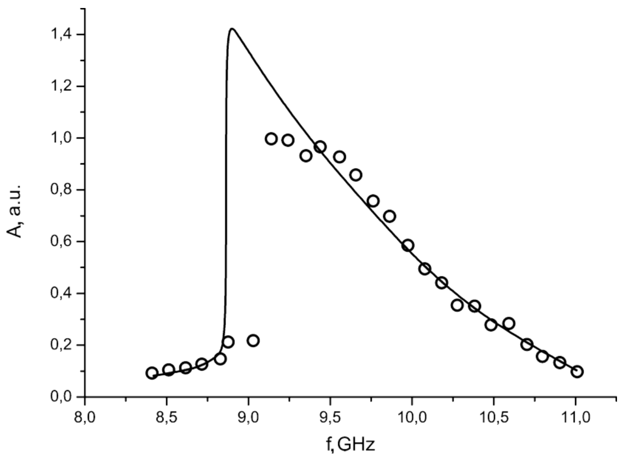

Fig. 3. Resonance curve in the nonlinear mode. The solid curve denotes the results of calculation in the framework of nonlinear resonance theory. The points exhibit the experimental data [9]. $S = 2 \times 10^{-12}$ cm^3/s and $h_{\rm ext}/h_{\rm thr} = 4.3$

$$\left[\frac{\partial}{\partial t}+\Gamma_0+i\left(\omega_0-\omega_p+2T_{00}|a_0|^2+2\sum_k T_{0k}|a_k|^2\right)\right]\times$$
$$\times a_0 + i\gamma h_{\rm ext} + i\left[\sum_k S_{0k}a_k^2\right]a_0^* = 0$$
$$\left[\frac{\partial}{\partial t}+\Gamma_k+i\left(\omega_k-\omega_p+2T_{0k}|a_0|^2+2\sum_{k'} T_{kk'}|a_{k'}|^2\right)\right]\times$$
$$\times a_k + i\left[S_{0k}a_0^2 + \sum_{k'} S_{kk'}a_{k'}a_{-k'}\right]a_{-k}^* = 0, \qquad (4)$$

where a_0 and $a_k (n = 4)$ are the amplitudes of the uniform and non-uniform modes, respectively; $T_{kk'}$ and $S_{kk'}$ are the nonlinearity parameters; and h is the amplitude of the external magnetic field. Using Eqs. (4), we calculated the resonance curve of the system, i.e. the dependence of the uniform mode amplitude a_0 on the frequency detuning of the external rf magnetic field, and compared it with that registered in the experiment [9]. The results of comparison are shown in Fig. 3. One can see a good correspondence between the theoretically calculated and experimentally measured resonance curves. The amplitude of the rf magnetic field is normalized by the excitation threshold for the non-uniform mode, $h_{\rm thr}$.

2.3. Behavior of the system in a strongly nonlinear regime

Hence, if a definite power level of the external electromagnetic pumping into the system is exceeded, the

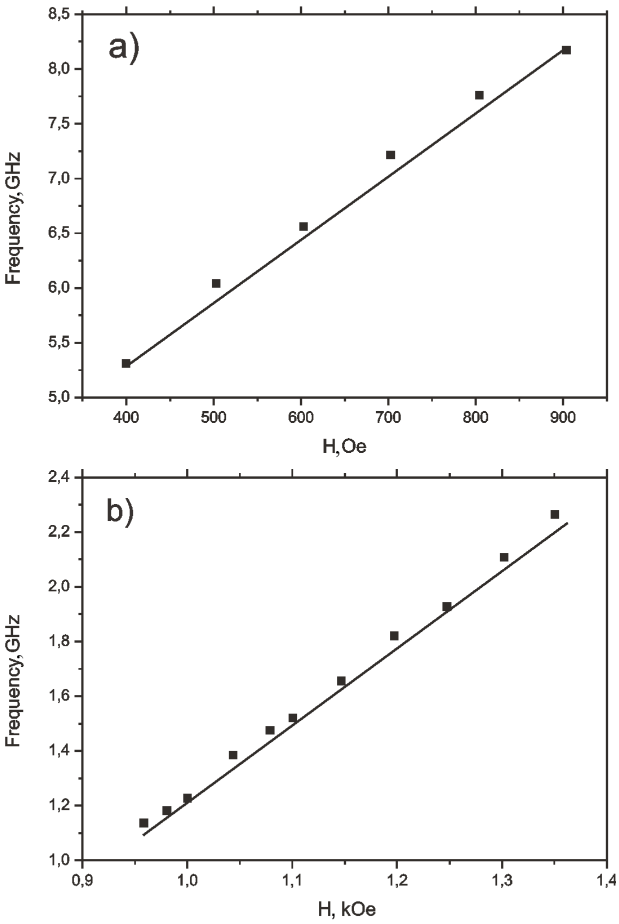

Fig. 4. Experimental data (points) and the results of corresponding theoretical calculations (curves) for (a) the lowest mode frequency of a nanodot [9] and (b) the bottom of the thin-film spin-wave spectrum [13]

uniform mode with the frequency ω_0 and the non-uniform one with the frequency ω_4 turn out excited. The further increase of the pumping power gives rise to that the system reveals nonlinear features of higher orders. The amplitude of the non-uniform mode with the frequency ω_4 becomes so large that this mode serves as a pump for other modes in the system. In particular, the processes described by the relation

$$\omega_0 + \omega_0 = \omega_n + \omega_m \quad (5)$$

are responsible for the energy redistribution in the system and the excitation of higher modes with the frequencies ω_n and ω_m. Here, we have to emphasize again that process (5) has an analog in massive specimens; it is called the kinetic instability [13]. The key feature of the kinetic instability processes is the fact that the mode excited owing to processes (5) corresponds to the mode with the lowest value of dissipation parameter in the system. According to T. Gilbert, owing to the frequency-dependent damping, it is the lowest mode in the spin-wave spectrum. In our case, it is the mode with the frequency ω_{bottom} ($n = 1$, see Fig. 2).

However, it is impossible to apply the results of work [13] directly in our case. First, the cited work was devoted to the research of the magnetic dynamics in YIG films, the latter being substantially different from magnetic nanodots possessing a discrete spectrum. Second, in work [13], the kinetic instability was studied in the so-called geometry of parallel pumping [11], which does not correspond to ours. Therefore, the extension of the available kinetic instability theory should consider the discrete character of modes of the system (it is the feature induced by nanodots) and the substantially different mechanism of non-uniform mode excitation; namely, as was mentioned above, it is excited through uniform mode not by external rf filed directly.

A convincing argument in favor of the kinetic instability processes is given by a comparison between the results of works [9] and [13] concerning the excited mode frequency (see Fig. 4). In both cases, the lowest mode of the system was excited. One can see that, in work [9], the excited mode frequency corresponds to the bottom of the spin-wave spectrum (Fig. 4,a). The experimental dependence of the frequency of electromagnetic radiation emitted from the specimen on the applied magnetic field, which was obtained in work [13], is exhibited in Fig. 4,b. The points in both panels correspond to the experimental data, and the solid curves to theoretically calculated dependences for the frequency of the spin-wave spectrum bottom taken from those works. Hence, irrespective of different materials used in experiments in both works, we may assert with confidence that the process of kinetic instability with the excitation of the lowest system mode takes place in both cases.

At last, another important difference between the processes in nanodots and films consists in that a definite group of spin waves is excited simultaneously in films and, as was shown earlier, only separate modes are excited in nanodots.

For the theoretical description of the excited magnetic system in a nanodot, we will use an analog of equations from the so-called S-theory [7], by extending the scope of its application onto the case of nanodots. As was already marked above, the main feature of this system is the absence of plane spin waves and the presence of the spectrum with a discrete collection of oscillations with corresponding frequencies and wave vectors.

Consider the evolution of the following three modes in this system. These are (i) the uniform mode with the frequency ω_0; it is excited by the external magnetic field, and its amplitude equals a_0; (ii) the nonuniform mode a_1 with the frequency ω_4; and (iii) the mode excited at high-power levels as a result of processes (5); it corresponds to the spectrum bottom a_1 with the frequency ω_{bottom} (see Fig. 2).

Now we can write down the governing system of equations for the evolution of the mode amplitudes a_i taking Eq. (4) as the basis and making allowance for the action of an external electromagnetic pumping and the kinetic processes in the system. The system of equations looks like

$$\begin{aligned} &\left[\frac{\partial}{\partial t} + \Gamma_0 + 2i\sum_i T_i|a_i|^2\right]\times \\ &\times a_0 + i\gamma h_{\text{ext}} + iS_{01}a_4^2 a_0^* = 0, \\ &\left[\frac{\partial}{\partial t} + \Gamma_4 - i\Omega + 2i\sum_i T_i|a_i|^2\right]\times \\ &\times a_4 + i\left(S_{01}a_0^2 + S_{11}a_4^2 + S_{12}a_1^2\right)a_4^* = 0, \\ &\left[\frac{\partial}{\partial t} + \Gamma_1 + 2i\sum_i T_i|a_i|^2\right]\times \\ &\times a_1 + i\left(S_{11}a_4^2 + S_{12}a_1^2\right)a_1^* = 0. \end{aligned} \tag{6}$$

Here, T and S are the nonlinearity parameters, and h_{ext} is the amplitude of the external electromagnetic field.

3. Discussion of the Results Obtained

The system of equations (6) was solved numerically for the following model parameters: the wave damping parameters $\Gamma_{0,4}/2\pi = 60$ MHz and $\Gamma_1/2\pi = 50$ MHz (the mode corresponding to the spectrum bottom has the lowest damping parameter), the nonlinearity parameter $S = 2\times10^{-12}$ cm^3/s [7], $T = -1.43S$, and the pump duration $\tau_p = 100$ ns [9].

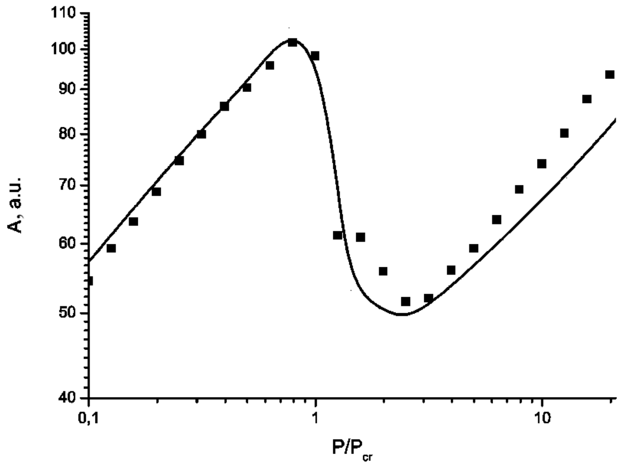

Fig. 5. Dependence of the uniform mode amplitude on the pump power P/P_{cr} in the nonlinear regime. The solid curve demonstrates the calculation results for the system of equations (6); the points correspond to experimental data of work [9]

We also selected the following initial conditions:

$$a_0(0) = a_1(0) = a_2(0) = \sqrt{\frac{2\gamma\hbar}{M_0}n_{t0}}, \tag{7}$$

with $n_{t0} = 3\times10^{13}$ cm^{-3}. In Fig. 5, the dependence of the uniform mode amplitude on the pump power is depicted. From the figure, one can see that, at the time moment when the mode corresponding to the spectrum bottom is excited, the amplitude of the uniform mode decreases in a step-like manner owing to processes (5).

Consider now the frequency shift of the lowest mode and its dependence on the pump power. If the level of an external electromagnetic pumping is high, a change in the magnetization of the system has to be taken into account. Then the lowest-mode frequency decreases, as the pump power grows, owing to the magnetization reduction. At the same time, the frequency of the excited uniform mode is rigidly linked to the frequency of the external exciting field and remains constant.

It is worth noticing that the ratio between the thresholds of the indicated nonlinearities, predicted theoretically and observed experimentally, is smaller approximately by 20%. This fact can be explained by the inaccuracy of the values used for mode frequencies while calculating the spectrum, as well as the values of dissipation parameters of various modes in a nanoelement.

4. Conclusions

To summarize, we have considered the processes that are running in the magnon system of a magnetic nanodot at high levels of the external electromagnetic pumping. The system concerned demonstrates the key features inherent to massive specimens. First of all, the external high-frequency ac magnetic field excites the uniform mode. Non-resonant processes of the form (2) are an analog of the Suhl instability; they give rise to the excitation of a non-uniform mode, the frequency of which can differ from that of the uniform mode. As a result, the threshold of the indicated process increases. When the pump power and, therefore, the amplitude of the non-uniform mode grow, the lowest mode can be excited by means of the processes of type (5), which are counterparts of the kinetic instability occurring in massive specimens [13]. The results of theoretical calculations were compared with the available experimental data obtained at studying the nonlinear high-frequency dynamics of magnetic nanodots. A good agreement between the theory and the experiment was obtained. The results of this work can be important for the analysis of the magnetic dynamics in submicron-sized nanodots, as well as devices on their basis.

This work was supported by the State Fund for Fundamental Researches of Ukraine (project No. UU34/008).

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Received 02.10.12.

Translated from Ukrainian by O.I. Voitenko

Г.А. Мелков, Д.В. Слободянюк

СИЛЬНО НЕРІВНОВАЖНИЙ СТАН В МАГНІТНИХ НАНОТОЧКАХ ПРИ ВИСОКИХ РІВНЯХ НАКАЧКИ

Р е з ю м е

Побудовано теоретичну модель, що описує сильно збуджену магнонну систему магнітної наноточки. В таких системах, незважаючи на дискретність спектра, мають місце параметричні процеси, аналогічні суцільним зразкам, зокрема процеси сулівської нестійкості. Внаслідок нечіткого збігання частот мод, що беруть участь у вказаних процесах поріг останніх дещо зростає і має місце нерезонансна параметрична взаємодія. Показано, що при певних рівнях потужності в системі можуть розвиватися процеси, аналогічні процесам так званої кінетичної нестійкості в суцільних зразках, які призводять до збудження найнижчої по частоті моди наноелемента.