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PACS 98.80.Cq

MAGNETIC FIELD OF COSMIC STRINGS IN THE EARLY UNIVERSE

Cosmic strings are topological defects which can be formed as a result of phase transitions with a spontaneous symmetry breaking in the early Universe. The possibility of the generation of a magnetic field around a cosmic string on the Grand Unification energy scale (GUT scale) in the early Universe immediately after the termination of the deconfinement-confinement phase transition has been analyzed. It is found that a circular current and a magnetic field directed along the string are induced around the string in the vacuum of a pseudoscalar matter consisting of charged pions. We also have studied the interaction between the magnetic flux tube surrounding the string (the string magnetosphere) and the cosmic plasma in the early Universe. A possibility of magnetization of the cosmic plasma surrounding the string owing to its interaction with the string magnetic field has been analyzed.

Keywords: cosmic string, phase transitions, vacuum polarization effect, ultrarelativistic plasma, vacuum of pseudoscalar matter consisting of charged pions, bow shock, magnetic tube.

1. Introduction

According to the standard cosmological model, the Universe is expanding and cooling down since the Big-Bang moment, but remains, as a whole, uniform and isotropic. There are the reasons to consider the Universe to pass through a chain of phase transitions in the course of its cooling [1, 2]. The phase transition associated with the separation of the strong interaction from the electroweak one – the end of Grand Unification Epoch – occurred in 10^{-35} s after the Big-Bang time moment, at a temperature of 2×10^{16} GeV. This phase transition was accompanied by the symmetry breaking: from a higher one characteristic of the unified interaction to symmetries inherent to plasma components at low temperatures [3]. The expansion of new phase regions, which initially had no causal links with various vacuum states stemming from the spontaneous symmetry breaking, can give rise to the emergence of topological defects at the interfaces between those regions. Cosmic strings are a type of topological defects, which can be formed owing to the phase transition with a spontaneous symmetry breakdown in the early Universe [4–6].

In 10^{-10} s after the Big Bang, the electroweak interaction separated. When the temperature fell down below 124 GeV, there emerged a phase with violated electroweak symmetry with the nonzero Higgs condensate and massive $W^{\pm}$ and Z bosons. The deconfinement–confinement transition, i.e. from a quark-gluon plasma to hadrons, took place in 10^{-5} s after the Big-Bang time moment, when the matter got cooled down to a temperature below 200 MeV [7, 8].

Linear defects – cosmic strings – are formed in the overwhelming majority of theoretical models dealing

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with the early Universe [2]. The topologically stable strings have no ends, i.e. they can be infinite or form closed loops. The specific mass per unit string length and the string tension (hereafter, we used the fundamental unit system, where $\hbar = c = k_{\rm B} = 1$) are of the order of $\mu \sim \eta^2$, where η is the energy scale of the symmetry breakdown. It is determined by the phase transition temperature in the Universe and, in turn, determines the Higgs field mass, $m_{\rm H} \sim \eta$.

For strings of the Grand Unification scale, the specific mass per unit length amounts to 10^{22} g/cm. The transverse string radius r_0 is determined from the relation $r_0 m_{\rm H} \sim 1$, and $r_0 \sim 10^{-30}$ cm for GUT-strings. As a result of the substantial tension, the string segments move at the velocities $V_s = \beta_{\rm s} c$ close to the velocity of light c. The average velocity within the correlation length approximately amounts to $\langle V \rangle \sim 0.15c$, and the root-mean-square velocity of a string in the radiation-dominating epoch is $V_{\rm rms} \sim 0.62c$ [5].

2. Magnetic Field around a GUT-String

Provided a charged field inside a string, the latter can behave as a superconductor [9]. Therefore, when a superconducting string moves, for example, in the intergalactic magnetic field, a current is generated that flows along the string. As a result, there arises the magnetosphere around the string; it is the own magnetic field of the string. If such a superconducting string moves through a cosmic plasma at relativistic velocities, the interaction of its magnetic field with the cosmic plasma will stimulate the generation of a shock wave around the string [10]. At the shock wave front, particles of the cosmic plasma will be accelerated to high energies. As a result, they will emit electromagnetic waves in a wide range of energies. In works [11, 12], the generation of non-thermal radiation at the interaction between a superconducting string and the cosmic plasma was described in detail.

In work [13] (see also works [14, 15]), it was shown that a magnetic field can be generated even near the surface of an ordinary non-superconducting string due to vacuum polarization effects in the quantized field of a charged matter around the string. Namely, the local cosmic string characterized by the tension $\mu \sim m_{\rm H}^2$ and the flux Φ of the gauge field in the string induces a current j that circulates in vacuum around the string and a magnetic field B directed along the string. The both quantities fall down exponentially at large distances from the string, being connected by the relation

$$B(r) = \int\limits_r^\infty dr \frac{\nu}{r} e j(r), \qquad (1)$$

where $\nu = (1 - 4G\mu)^{-1}$, and e is the electric charge of the quantized matter field. In the case of a field with mass m and zero spin, the total flux of the induced magnetic field equals

$$\Phi_{\rm B} = \frac{e}{6\pi}\left(F - \frac{1}{2}\right) F(1 - F)\nu^2 \ln \frac{m_{\rm H}}{m}, \qquad (2)$$

where $F = e\Phi\,(2\pi\hbar c)^{-1} - [\![e\Phi(2\pi\hbar c)^{-1}]\!]$, and $[\![u]\!]$ denotes the integer part of u [13].

Consider a cosmic string of the Grand Unification scale (a GUT-string) and its influence on the pseudoscalar matter vacuum consisting of charged pions. Such matter arises at an early stage of Universe's evolution, right after the phase transition "deconfinement–confinement" has terminated, as a result of the binding of the quarks u ($\bar{u}$) and $\bar{d}$ (d) into $\pi^{\pm}$-mesons [16]. A magnetic field is induced in vacuum around the string. According to the results of work [13], this field is given by the expression

$$B(r) \approx \frac{e\,[F \sin((1 - F)\pi) - (1 - F)\sin(F\pi)]}{2(4\pi)^2} \frac{\hbar c}{E_{\pi^\pm}} \times$$
$$\times \frac{e^{-2\frac{E_{\pi^\pm}}{\hbar c} r}}{r^3}, \qquad (3)$$

where $E_{\pi^\pm} = m_{\pi^\pm} c^2$, and $m_{\pi^\pm}$ is the mass of the charged pionic field. We also took into account that $\nu_{\rm GUT} \approx 1$. For F-values corresponding to the maximum value of $\Phi_{\rm B}$ (in particular, $F_1 \approx 0.8$ and $F_2 \approx 0.2$), we obtain

$$B(r) = B_0 \frac{e^{-2r/r_{\rm B}}}{(r/r_{\rm B})^3} \approx 2.7 \times 10^{13} \frac{e^{-2r/r_{\rm B}}}{(r/r_{\rm B})^3}\ ({\rm Gs}), \qquad (4)$$

where we took into consideration that a characteristic scaling factor $r_{\rm B} = \hbar c/E_{\pi^\pm} = 1.4 \times 10^{-13}$ cm can be introduced for the magnetic field.

3. Plasma Parameters around a GUT-String

At the examined early stage of Universe's evolution, the magnetic field of a string interacts with the surrounding cosmic plasma. At high temperatures typical of the early Universe, the rate of reactions between particles exceeds a characteristic time connected with the rate of Universe's expansion, so that the cosmic plasma is in thermal equilibrium with electromagnetic radiation. The main contribution to the Universe energy density is made by ultrarelativistic particles, for which, under the available conditions, $m_i c^2 \ll k_{\rm B}T(t)$ and $\mu_i = 0$, where $T(t)$ is the equilibrium temperature at the cosmological time moment t, m_i is the mass of particles of the i-th type, and μ_i is the corresponding chemical potentials [17]. Then, the energy density can be approximated as follows [16]:

$$e_{\rm th} = \left(\sum_{\rm b} N_{\rm b} + \frac{7}{8}\sum_{\rm f} N_{\rm f}\right)\frac{\pi^2}{30}\frac{k_{\rm B}^4 T^4}{\hbar^3 c^3} = \\ = \frac{\pi^2}{30}N(T)\frac{k_{\rm B}^4 T^4}{\hbar^3 c^3}, \qquad (5)$$

where N_b and N_f are the helicity numbers (the number of spin projections onto the momentum direction) for every boson and fermion, respectively; the summation is carried out over all bosonic and fermionic states; and $N(T)$ is the effective number of degrees of freedom. The number of degrees of freedom depends on the Universe composition and, therefore, depends on the temperature. In particular, $N\left(m_{\pi^\pm}c^2 < k_{\rm B}T < k_{\rm B}T_{\rm c}\right) = 69/4$, where $k_{\rm B}T_{\rm c} = 200$ MeV is the temperature of the phase transition "deconfinement–confinement". Notice that the energy density $e_{\rm th} = 1.9 \times 10^{36}$ erg/cm^3 at this temperature.

The relation between the temperature T in the Universe and the time t reckoned from the Big Bang moment during the radiation-dominating epoch is expressed by the simple formula $tT_{\rm MeV}^2 = 2.4\,[N(T)]^{-1/2}$, where t is measured in second units and $T_{\rm MeV}$ in megaelectronvolts [16]. The time corresponding to the deconfinement–confinement transition equals $t_{\rm c} = 1.4 \times 10^{-5}$ s.

From the expression for the concentration of ultrarelativistic particles

$$n = \left(\sum_{\rm b} N_{\rm b} + \frac{3}{4}\sum_{\rm f} N_{\rm f}\right)\frac{\zeta(3)}{\pi^2}\frac{k_{\rm B}^3 T^3}{\hbar^3 c^3}, \qquad (6)$$

where the value of zeta-function $\zeta(3) = 1.2$, it is possible to estimate the concentration of charged particles (at the temperature $m_{\pi^\pm}c^2 < k_{\rm B}T < k_{\rm B}T_{\rm c}$, these particles are $e^\pm$, $\mu^\pm$, and $\pi^\pm$), $n_{\rm ch} = 1.0 \times 10^{39}$ cm^{-3}, and the average distance between them $d_{\rm ch} = n_{\rm ch}^{-1/3}$ (at $T = T_{\rm c}$, we obtain $d_{\rm ch} = 9.9 \times 10^{-14}$ cm). The characteristic scale of a magnetic field, $r_{\rm B}$, turns out close to the average distance between charged particles, $r_{\rm B} \sim d_{\rm ch}$.

Let us also estimate the mean free path of a charged particle at this temperature. The energy of an ultrarelativistic particle is given by the relation $E \approx cp = 3k_{\rm B}T$, where p is particle's momentum. The mean free path is $\lambda \sim 1/(\sum_i \sigma_i n_i)$, where n_i is the concentration of particles of the i-th kind, and σ_i is the scattering cross-section for them. Taking into account that all scattering cross-sections for the electromagnetic interaction $\sigma_i \sim e^4/E^2$, where e is the electron charge, the mean free path is of the order $\lambda \sim 10^{-8}$ cm at $T = T_{\rm c}$.

At $T \sim 100$ MeV in the cosmic plasma, the concentration of non-relativistic particles is low in comparison with that of photons. Baryons are non-relativistic particles, and their total concentration $n_{\rm bar} \sim 10^{-9} n_\gamma \sim 10^{29}$ cm^{-3}, where n_γ is the concentration of photons [17]. This means that the order of magnitude for the ratio between the numbers of baryons and photons coincides with the value of baryon asymmetry in the Universe.

4. String Magnetosphere and Magnetic Field Transfer to Cosmic Plasma

Being surrounded with a shell-like magnetic field (a magnetic flux tube), the string moves at a typical relativistic velocity through the cosmic plasma of density ρ (in general, this velocity is higher than the sound speed in the plasma of ultrarelativistic particles, $a_{\rm s} = c/\sqrt{3}$). Therefore, in the framework of the hydrodynamic approximation, the flow of a relativistic (in the string reference frame) plasma around the string is similar to a non-relativistic case of the supersonic solar wind flowing around Earth's magnetosphere. As a result, there emerges a shock wave in the incident plasma flow, a contact discontinuity between the plasma behind the shock wave and the string magnetosphere [10, 11]. The shock wave radius along the string motion direction can be determined from the equality between the pressure of the incident plasma $P = P_{\rm th} + P_{\rm dyn}$ (here, $P_{\rm th}$ is the thermal pressure of ultrarelativistic gas, $P_{\rm dyn} = \gamma_{\rm s}^2 \rho c^2$ is the dynamical pressure, and $\gamma_{\rm s}$ is the Lorentz factor of the string and the magnetosphere with respect to the plasma, $\gamma_{\rm s}^2 \geq 1.5$) and the magnetic field pressure

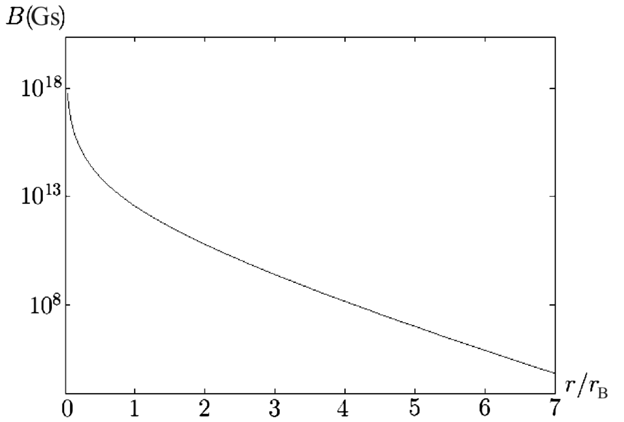
$P_\mathrm{B} = B^2\left(R_\mathrm{sh}\right)/8\pi \approx B_0^2 r_\mathrm{B}^6/(8\pi R_\mathrm{sh}^6)$ in the magnetosphere deformed by the plasma flux.

In this hydrodynamic scenario of the cosmic plasma flowing around the string magnetosphere, two channels of magnetic field appearance in the plasma are possible. One of them is associated with the emergence of instabilities at the contact discontinuity at the magnetosphere–downstream flux interface. As a result, some part of the plasma flux becomes magnetized with a subsequent enhancement of the field in the turbulent flux to values typical of relativistic fluxes $e_\mathrm{B} = \epsilon_\mathrm{B} e_\mathrm{th}$; i.e. the energy density of the turbulent magnetic field ($\epsilon_\mathrm{B} \sim 0.01 \div 0.1$) becomes comparable with the density of the thermal plasma energy e_th. The other channel is related to the emergence of a magnetic field in a vicinity of the shock wave in the non-magnetized plasma. This field will be removed into the region behind the shock wave, and its magnitude will depend on the relation between dissipative processes and processes that strengthen the field.

However, in the situation of the interaction between the string magnetic field and the cosmic plasma, which is considered here, the application of the hydrodynamic approximation is problematic. By equating the pressure of the incident plasma $P = P_\mathrm{th} + P_\mathrm{dyn}$ and the pressure of the magnetic field $P_\mathrm{B}\left(R_\mathrm{sh}\right)$, we can calculate the shock wave radius in the hydrodynamic approximation,

$$R_\mathrm{sh} \approx r_\mathrm{B}\left(\frac{B_0}{\sqrt{8\pi P}}\right)^{1/3} = 2.4 \times 10^{-15}\ \mathrm{cm}. \qquad (7)$$

For the hydrodynamic approximation to be valid, the sizes of a shock wave and a contact discontinuity must substantially exceed the mean free path of plasma particles. However, the condition of hydrodynamic approximation eligibility is not satisfied in our case, and the interaction between the string and plasma particles is reduced to separate scattering events of charged particles (the deflection of their trajectories) in the magnetic field of a string. In this case, the transfer of the magnetic field into the plasma should be considered separately. At the qualitative level, we may assume that, when the incident plasma flows around string's magnetic flux tube, it captures some part of the magnetic flux formed by field lines located at farther distances from the string than the average distance between particles d_ch. Since the Hubble volume (about $r_h^3 \sim (ct)^3$) includes about $r_h \sim ct$ of the string length, the plasma drifts a magnetic field with flux $d\Phi_\mathrm{cap}/dt \sim V_\mathrm{s} d_\mathrm{ch} B(d_\mathrm{ch})$ and energy $dW_\mathrm{cap}/dt = V_\mathrm{s} r_h d_\mathrm{ch}(B^2(d_\mathrm{ch})/8\pi)$ during a time unit. Owing to a rapid decrease of the magnetic field with increasing the distance from the string, the effective time of magnetic field energy transfer into the plasma equals $\Delta t \sim t_\mathrm{c}$, and the transferred energy in the Hubble volume, $\Delta W_\mathrm{cap} \sim (dW_\mathrm{cap}/dt)\,\Delta t \sim \beta_\mathrm{s} r_h^2(t_\mathrm{c}) d_\mathrm{ch} e_\mathrm{B}(d_\mathrm{ch})$, represents a very insignificant fraction of the thermal energy, $\sim(d_\mathrm{ch}/r_h(t_\mathrm{c}))\,(e_\mathrm{B}(d_\mathrm{ch})/e_\mathrm{th}(t_\mathrm{c})) \sim 10^{-18} \times 10^{-11} \sim 10^{-29}$. The further counteraction between the dissipation processes and the dynamo-processes that strengthen the magnetic field will be responsible for the final contribution of cosmic strings to the generation of the observed cosmological magnetic field [18].



Magnetic field B at various distances r from the string

5. Discussion and Conclusions

In this work, the generation of a magnetic field around a cosmic string of the Grand Unification energy scale in the early Universe at the times after the phase transition “deconfinement–confinement” is considered. The magnetic field is induced around a cosmic string in the vacuum of a pseudoscalar matter consisting of charged pions; this field is directed along the string. In Figure, the variation of the magnetic field with the distance from the string is depicted. The interaction between the magnetic field around the string and the ultrarelativistic cosmic plasma was studied. In particular, the parameters of the magnetic field and the thermodynamic characteristics of the plasma in the early Universe after the phase transition “deconfinement–confinement” are determined. The characteristic scale of the formed magnetic field flux tube is shown to be comparable with the average distance between charged particles in the plasma and smaller than their mean free path length. Therefore, at typical relativistic velocities of the string in the cosmic plasma, the latter will reduce the magnetic field only at rather long distances from the string; these are distances of the order of those between plasma particles, where the magnetic field is already suppressed substantially. As a result, only small fractions of the magnetic field flux and energy are transferred into the plasma. However, the resulting value of the field transferred into plasma will also depend on the subsequent field evolution governed by both the dissipative and enhancing (dynamo) processes.

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Received 05.11.12.
Translated from Ukrainian by O.I. Voitenko

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МАГНІТНЕ ПОЛЕ КОСМІЧНИХ СТРУН У РАННЬОМУ ВСЕСВІТІ

Резюме

Космічні струни – топологічні дефекти, що могли утворюватися під час фазових переходів зі спонтанно порушеною симетрією у ранньому Всесвіті. В роботі розглянута можливість утворення магнітного поля навколо струни енергетичного масштабу Великого Об'єднання в умовах раннього Всесвіту, одразу після фазового переходу деконфайнмент–конфайнмент. Навколо космічної струни у вакуумі псевдоскалярної матерії, що складається із заряджених піонів, індукуються коловий струм та магнітне поле, напрямлене вздовж струни. В роботі досліджено взаємодію магнітної силової трубки, що оточує струну – магнітосфери струни – з космічною плазмою в ранньому Всесвіті. Проаналізовано можливість замагнічення оточуючої струну плазми внаслідок її взаємодії з магнітним полем струни.