A.N. TARASOV

Akhiezer Institute for Theoretical Physics, National Science Center
"Kharkiv Institute of Physics and Technology", Nat. Acad. of Sci. of Ukraine
(1, Akademichna Str., Kharkiv 61108, Ukraine; e-mail: antarasov@kipt.kharkov.ua)

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MAGNETIC SUSCEPTIBILITIES OF DENSE SUPERFLUID NEUTRON MATTER WITH GENERALIZED SKYRME FORCES AND SPIN-TRIPLET PAIRING AT ZERO TEMPERATURE

Magnetic properties of a dense superfluid neutron matter (relevant to the physics of neutron star cores) at subnuclear and supranuclear densities (in the range $0.5 \lesssim n/n_0 \lesssim 3.0$, where $n_0 = 0.17$ (fm^{-3}) is the saturation nuclear density) with the so-called generalized Skyrme effective forces BSk18, BSk19, BSk20, BSk21 (containing additional unconventional density-dependent terms) and with spin-triplet p-wave pairing (with spin $S = 1$ and orbital moment $L = 1$) in the presence of a strong magnetic field are studied within the framework of the non-relativistic generalized Fermi-liquid theory at zero temperature. The upper limit for the density range of a neutron matter is restricted by the magnitude $3n_0$ in order to avoid the account of relativistic corrections growing with density. The general formula obtained in [1] (valid for any parametrization of the Skyrme forces) for the magnetic susceptibility of a superfluid neutron matter at zero temperature is specified here for the new BSk18-BSk21 parametrizations of the Skyrme interaction. As is known, all previous conventional Skyrme interactions predict spin instabilities in a normal (nonsuperfluid) neutron matter beyond the saturation nuclear density. It is obtained in the present work that, for the model of superfluid neutron matter with new generalized BSk18-BSk21 parametrizations, such phase transition to the ferromagnetic state occurs neither at subnuclear nor at supranuclear densities. Thus, the high-density ferromagnetic instability is removed in the neutron matter with new generalized Skyrme forces BSk18-BSk21 not only in normal, but also in superfluid states with anisotropic spin-triplet pairing.

K e y w o r d s: dense superfluid neutron matter, Skyrme forces, spin-triplet pairing.

1. Introduction

This work is a continuation and an improvement of our previous investigations [1] (see also [2] and references therein), which were devoted to the theoretical description of the dense superfluid neutron matter (SNM) with conventional Skyrme forces [3–5] (in [6, 7], someone can find reviews on different parametrizations of the Skyrme forces and neutron-star properties calculated with the Skyrme interaction) and with anisotropic spin-triplet pairing similar to ^{3}He − A [8–10] in strong magnetic fields, which can be realized in the liquid outer cores of pulsars and magnetars (strongly magnetized neutron stars) (see, e.g., [11–14]).

The outer core occupies the density range $0.5 \lesssim n/n_0 \lesssim 3.0$ (where $n_0 = 0.17$ fm^{-3} is the saturation nuclear density) and is several kilometers in thickness [15, 16]. Its matter consists of neutrons with several per cent admixture of protons p, electrons, and possibly muons μ (the so-called $npe\mu$ composition). The state of this matter is determined by the conditions of electric neutrality and beta equilibrium, supplemented by a model of many-body nucleon interaction. The beta equilibrium implies the equilibrium with respect to the beta (muon) decay of neutrons and inverse processes. All $npe\mu$-plasma components are strongly degenerate. The electrons and muons form almost ideal Fermi gases. The neutrons and protons, which interact via nuclear forces, constitute a strongly interacting Fermi liquid and can be in the superfluid state.

Proton superfluidity in the core is thought to be mainly produced by the singlet-state proton pairing, but neutrons in the core are in superfluid states with spin-triplet pairing. Critical temperatures T_c of various particle species have been calculated by many authors (see, e.g., reviews [17, 18] and references therein). The results are extremely sensitive to strong interaction models and many-body theories employed. In all the cases, the calculations give density-dependent critical temperatures $T_c(n) \lesssim 1$ MeV and lower. As a rule, superfluidities weaken and disappear at essentially supranuclear densities, where the attractive part of the strong interaction becomes inefficient.

Recently, we have published works [2] and [19] devoted to the theoretical description of superfluid phases in dense SNM with anisotropic spin-triplet p-wave type of pairing in strong magnetic fields with conventional parametrizations [3–5] and with generalized BSk18 parametrization [20] of the Skyrme forces, respectively. The expressions obtained in [19] for the phase transition temperatures of a dense neutron matter (NM) to spin-triplet superfluid states are realistic non-monotone functions of the density for BSk18 parametrization of the Skyrme forces (contrary to their monotone increase for all previous BSk and other conventional parametrizations [2]). An analogous investigation of a dense SNM with new generalized BSk19-Bsk21 parametrizations [21] of the Skyrme forces will be published elsewhere.

But here, the main aim of our study is the equilibrium magnetic properties of the superfluid phases of an infinite pure NM with the so-called generalized (improved) Skyrme effective forces BSk18 [20] and BSk19, BSk20, BSk21 [21] (containing additional unconventional density-dependent terms) and with anisotropic spin-triplet p-wave pairing (of the same type as that in [1, 2]) in the presence of a spatially uniform strong magnetic field. We carry out this study within the framework of the non-relativistic generalized Fermi-liquid theory [22] (see also [2]) valid at subnuclear and supranuclear densities of neutrons (in the range $0.5 \lesssim n/n_0 \lesssim 3.0$), which constitute the main component of the outer cores of neutron stars, as mentioned above.

Note that other authors have investigated previously the existence (or absence) of phase transitions of a highly degenerate NM from normal (nonsuperfluid) state to ferromagnetic state in the absence of a magnetic field and at zero temperature or with taking the temperature effect into account (see, e.g., [23–30] and references therein) and with the effects of a strong magnetic field (see, e.g., [31–33]) within other approaches and using different nucleon-nucleon effective and so-called realistic interactions in NM.

This paper is organized as follows. In the second section, we briefly outline the main steps and assumptions made for the derivation of general equations for the effective magnetic field (EMF) and for the order parameter (OP) of the SNM with generalized Skyrme forces [20, 21] and spin-triplet pairing of the $^3\mathrm{He}-\mathrm{A}_{1,2}$ type between neutrons. In the third section, we write down the general formula for the paramagnetic susceptibility in SNM (valid for any parametrizations of the Skyrme forces, i.e., for the conventional [6] and for the generalized ones) in a strong magnetic field at zero temperature. Then we specify this formula for the paramagnetic susceptibility in SNM at first for the conventional BSk17 parametrization [34] (the best among all previous parametrizations of the BSk type), which is qualitatively similar to the conventional parametrizations Sly2 [3], Gs [4], and RATP [5] (see [1]), and then for the generalized BSk18-BSk21 Skyrme forces. In Conclusion, the obtained results for SNM with triplet pairing in a high magnetic field are briefly discussed and compared with those of some other works.

2. General Equations for the EMF and the OP in SNM with Conventional and Generalized Skyrme Forces between Neutrons and with Triplet Pairing

For the theoretical description of a dense SNM with generalized effective Skyrme forces [20, 21] and with anisotropic spin-triplet p-wave pairing of neutrons (spin and orbital momentum of a Cooper pair are equal to 1) in the presence of a strong spatially homogeneous magnetic field $\mathbf{H}$, we have to write down the order parameter for SNM, which is similar to the OP for superfluid $^3\mathrm{He}-\mathrm{A}_{1,2}$ (see [10] and [1,2]):

$$\Delta^A_\alpha(\mathbf{p}) \equiv (\Delta_+\hat{\mathbf{d}}_\alpha + \imath\Delta_-\hat{\mathbf{e}}_\alpha)\psi(\hat{\mathbf{p}}), \tag{1}$$

$$\psi(\hat{\mathbf{p}}) \equiv (\hat{m}_j + \imath\hat{n}_j)\hat{p}_j, \quad \hat{\mathbf{p}} \equiv \frac{\mathbf{p}}{p}.$$

Here, $\Delta_\pm(T) \equiv (\Delta_\uparrow(T) \pm \Delta_\downarrow(T))/2$; $\hat{\mathbf{d}}$ and $\hat{\mathbf{e}}$ are mutually orthogonal real unit vectors in the spin space, $\hat{\mathbf{d}}\cdot\hat{\mathbf{e}} = 0$, $\hat{\mathbf{d}}^2 = \hat{\mathbf{e}}^2 = 1$; $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are mutually orthogonal real unit vectors in the orbital space, $\hat{\mathbf{m}}\cdot\hat{\mathbf{n}} = 0$, $\hat{\mathbf{m}}^2 = \hat{\mathbf{n}}^2 = 1$. Note also that a superfluid phase of the $^3\mathrm{He}-\mathrm{A}_1$ type can be realized under the condition, when $\Delta_\downarrow = 0$ and $\Delta_\uparrow \neq 0$.

As a result, using the general formulas (obtained by us previously in [35, 36]) for anomalous and normal distribution functions of quasiparticles (neutrons) for SNM in a magnetic field, we have derived (see more details in [1, 2]) a set of integral equations for $\xi(p)$, $\Delta^A_\uparrow$, and $\Delta^A_\downarrow$ in the framework of the generalized Fermi-liquid approach [22]. In this case for SNM, $\boldsymbol{\xi}(\mathbf{p}) = \xi(p)\mathbf{H}/H \equiv \equiv -\mu_n\mathbf{H}_{\text{eff}}(p)$ ($\mu_n \approx -0.60308\times 10^{-17}$ MeV/G is the magnetic dipole moment of a neutron, and $\mathbf{H}_{\text{eff}}(p)$ is the effective magnetic field renormalized inside SNM). For $\xi(p)$, we have the equation

$$\xi(p) = -\mu_n H + (r + sp^2)K_2(\xi) + sK_4(\xi). \tag{2}$$

Here, $r = t'_0 + (t'_3/6)n^\alpha$ and $s = (t'_1 - t'_2)/(4\hbar^2)$, $n \equiv \equiv yn_0$ is the neutron matter density;

$$t'_0 = t_0(1 - x_0), \quad t'_3 = t_3(1 - x_3), \tag{3}$$

$$t'_1(n) = t_1(1 - x_1) + t_4(1 - x_4)n^\beta, \tag{4}$$

$$t'_2(n) = t_2(1 + x_2) + t_5(1 + x_5)n^\gamma, \tag{5}$$

and $1/12 \le \alpha \le 1/3$ are the parameters of the generalized (improved) Skyrme interaction. It is worth to note here that, for the generalized Skyrme forces [20, 21], the new additional parameters t_4, t_5, x_4, and x_5 and the additional power exponents β and γ of the density dependence have originated in comparison with the conventional Skyrme forces.

The functionals $K_\sigma(\xi)$ ($\sigma = 2, 4$) in Eq. (2) have the same form as that in [1,2]:

$$K_\sigma(\xi) = \frac{1}{8\pi^2\hbar^3}\int\limits_{p_{\min}}^{p_{\max}} dq q^\sigma \int\limits_0^1 dx\kappa(q,x), \tag{6}$$

where

$$\kappa(q,x) = \frac{z(q)+\xi(q)}{E_+(q,x^2)}\tanh\left(\frac{E_+(q,x^2)}{2T}\right) - $$
$$-\frac{z(q)-\xi(q)}{E_-(q,x^2)}\tanh\left(\frac{E_-(q,x^2)}{2T}\right), \tag{7}$$

$$E^2_\pm = q^2\Delta^2_{\uparrow(\downarrow)}(1-x^2) + (z(q)\pm\xi(q))^2, \tag{8}$$

$z(q) = q^2/2m^*_n - \mu$ (m^*_n is the effective mass of a neutron, μ is the chemical potential). We have taken into account that, for SNM with pairing of the $^3\mathrm{He}-\mathrm{A}_{1,2}$ type, the OP can be written as $\Delta^A_{\uparrow(\downarrow)}(T,\xi,q) = = q\Delta_{\uparrow(\downarrow)}(T,\xi)$, where the functions $\Delta_{\uparrow(\downarrow)}(T,\xi)$ obey the following equations (whose structures are similar to those from [1, 2]):

$$\Delta_{\uparrow(\downarrow)}(T,\xi) = -\Delta_{\uparrow(\downarrow)}(T,\xi)\frac{c_3}{8\pi^2\hbar^3}\times$$

$$\times\int\limits_{p_{\min}}^{p_{\max}} dq q^4 \int\limits_0^1 dx(1-x^2)\frac{\tanh(E_\pm(q,x^2)/2T)}{E_\pm(q,x^2)}, \tag{9}$$

($p_{\max} \gtrsim p_{\mathrm{F}}$ and $(p_{\max} - p_{\min})/p_{\mathrm{F}} < 1$, where p_{F} is the Fermi momentum). It is significant that $c_3 \equiv \equiv t'_2(n)/\hbar^2 < 0$ is the coupling constant leading to the spin-triplet p-wave pairing of neutrons, which is expressed through the generalized parameters $t'_2(n)$ (see (5)) of the Skyrme interaction. Note that, in contrast with (9) for conventional Skyrme forces, $c_3 \equiv \equiv t_2(1 + x_2)/\hbar^2 < 0$ (see [1, 2]) does not depend on the density n of SNM. Here, we consider a model of neutron Cooper pairing in a shell symmetric with respect to the Fermi sphere, i.e., $p_{\max} - p_{\mathrm{F}} = = p_{\mathrm{F}} - p_{\min}$.

This system of nonlinear integral equations (2) and (9) for the EMF and OP gives us a possibility to describe the thermodynamics of superfluid non-unitary phases of the $^3\mathrm{He}-\mathrm{A}_{1,2}$ type in a dense SNM with generalized Skyrme forces [20, 21] and with spin-triplet p-wave pairing in a static uniform high magnetic field at arbitrary temperatures from the interval $0 \le T \le \le T_c(H)$. In the general case, these equations cannot be solved analytically, and it is necessary to use numerical methods for their solution. But we can solve Eqs. (2) and (9) with the use of analytical methods in the limiting case, at zero temperature ($T = 0$), and it is the theme of the next section.

3. Solutions of the Equations for EMF and OP in Dense SNM at $T = 0$ for Conventional and Generalized Skyrme Forces

Let us consider the SNM at $T = 0$. In this case, we have obtained the following solution of the integral equation (2) for the EMF on the Fermi surface in the first order in the small parameter $h_{\rm ext} \equiv |\mu_n| \times H/\varepsilon_{\rm F} \ll a < 1$:

$$\frac{|\mu_n| H_{\rm eff}(p_{\rm F}, H)}{\varepsilon_{\rm F}(y)} = \frac{h_{\rm ext}(H, y)}{1 - (r + 2sp_{\rm F}^2)\nu_{\rm F}/2}. \quad (10)$$

Here, r and s (see after (2)) are the density-dependent combinations of the Skyrme parameters (3)–(5), and the density of states $\nu_{\rm F} = (m_n^* p_{\rm F})/(\pi^2\hbar^3)$ at the Fermi surface is

$$\nu_{\rm F}(y) \approx 0.00419 \frac{m_n^*(y)}{m_n} y^{1/3}\ {\rm MeV}^{-1}{\rm fm}^{-3}. \quad (11)$$

It should be emphasized that the general approximate formula (10) for $H_{\rm eff}(p_{\rm F}, H)$ is valid for all parametrizations (conventional and generalized) of the Skyrme forces admissible for NM [6, 7], and $H_{\rm eff}$ is independent of the cutoff parameter $a < 1$

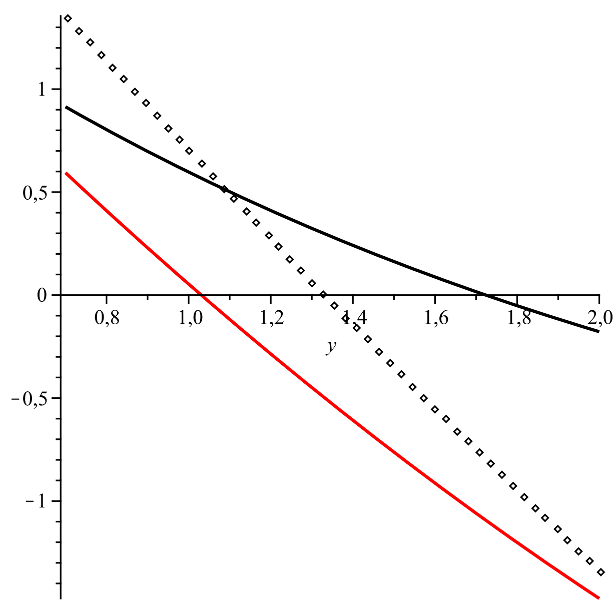

Fig. 1. Ratio $\chi_{\rm free}/\chi_{\rm Skyrme}(y)$ (see [1]) for SNM with RATP (lower line), Gs (points), and SLy2 (upper line) parametrizations of the Skyrme forces and a spin-triplet p-wave pairing of the ^{3}He – A type at $T = 0$ as a function of the reduced density $y = n/n_0$

($a \equiv \varepsilon_{\rm max}/\varepsilon_{\rm F} - 1$) and of the energy gap in the energy spectrum of neutrons in SNM (in the first order).

Formulas (7)–(11) contain the effective neutron mass m_n^*, which depends on the density of NM $n = yn_0$ according to the formula:

$$\frac{m}{m_n^*} = 1 + \frac{myn_0}{4\hbar^2}[t_1'(n) + 3t_2'(n)], \quad (12)$$

where $m \approx (m_p + m_n)/2 \approx 938.91897\ {\rm MeV/c^2}$ is the mean value of free nucleon mass [3]; the density-dependent parameters $t_1'(n)$ and $t_2'(n)$ (see (4) and (5)) have specific numerical values for each Skyrme parametrization. Note also that the Fermi energy $\varepsilon_{\rm F} \equiv p_{\rm F}^2/2m_n^*$ of the pure NM with density $n = yn_0$ is defined by the formula

$$\varepsilon_{\rm F} = (3\pi^2 yn_0)^{2/3}\frac{\hbar^2}{2m_n^*} \approx y^{2/3}\frac{m}{m_n^*} 60.902\ {\rm MeV}. \quad (13)$$

Now, let us consider the previous conventional SLy2, Gs, and RATP (see [1] and [3–5]) and recent BSk17 [34] parametrizations of the Skyrme forces for the sake of comparison. This concretization has given us a possibility to plot the figures for the ratio of the paramagnetic susceptibility of SNM with the Skyrme interaction $\chi_{\rm Skyrme}(y)$ and the Pauli susceptibility of a free neutron gas $\chi_{\rm free}$ (see (10))

$$\frac{\chi_{\rm Skyrme}(y)}{\chi_{\rm free}} = \frac{1}{1 - (r + 2sp_{\rm F}^2)\nu_{\rm F}/2}, \quad (14)$$

which describes the renormalization of a magnetic field inside SNM with a triplet p-wave pairing of the ^{3}He – A$_{1,2}$ type. See Fig. 1 (and details in [1]) for ratio with SLy2, Gs, and RATP variants of the conventional Skyrme interaction with power exponents $\alpha_{\rm RATP} = 0.20$, $\alpha_{\rm Gs} = 0.30$, and $\alpha_{\rm SLy2} = 1/6$ in their density dependence.

For the conventional BSk17 parametrization of the Skyrme forces [34] (with $\alpha_{\rm BSk17} = 0.30$), relation (14) yields

$$\frac{\chi_{\rm free}}{\chi_{\rm BSk17}(y)} \approx 1 - \frac{2y^{1/3}(0.407660y^{3/10} - 1.13208)}{(1 + 0.237909y)} - \frac{2.68438y}{(1 + 0.237909y)}. \quad (15)$$

See Figs. 1 and 2, where the points of intersection of lines with the abscissa axis correspond to

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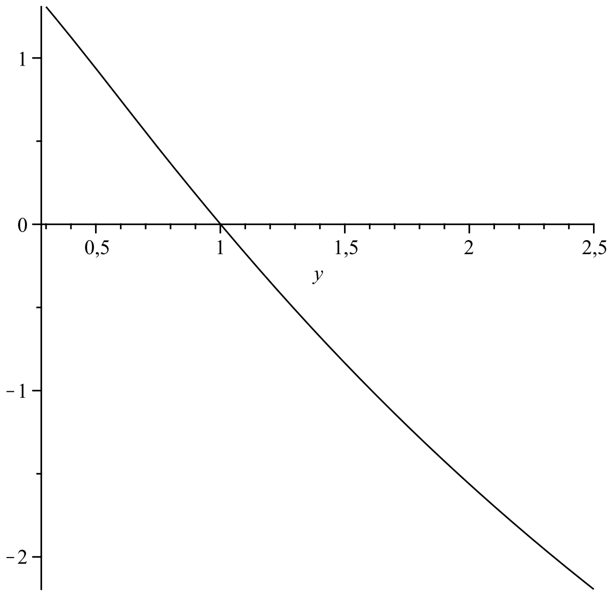

Fig. 2. Ratio $\chi_{\rm free}/\chi_{\rm BSk17}(y)$ (see (15)) for SNM with the conventional BSk17 parametrization [34] of Skyrme forces and a spin-triplet p-wave pairing of the ^{3}He $-$ A type at $T=0$ as a function of the reduced density $y=n/n_0$

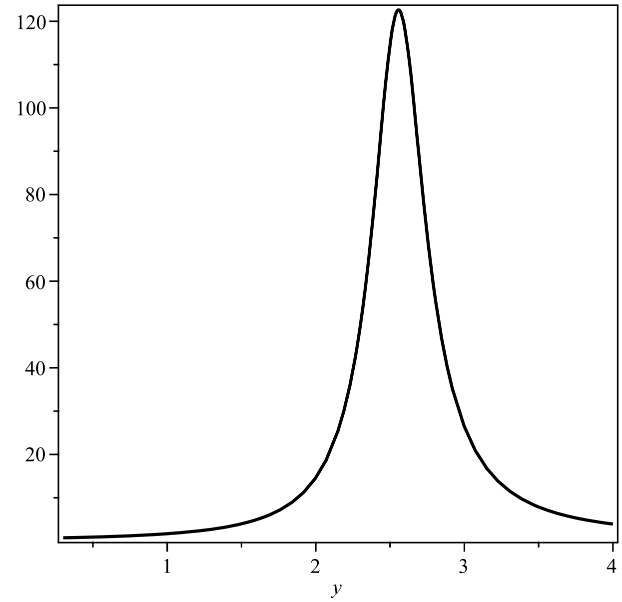

Fig. 3. Ratio $\chi_{\rm BSk19}(y)/\chi_{\rm free}$ (see (17)) for SNM with generalized BSk19 parametrization [21] of the Skyrme forces and a spin-triplet p-wave pairing of the ^{3}He $-$ A type at $T=0$ as a function of the reduced density $y=n/n_0$

the critical densities $n_C(\mathrm{SLy2}) \approx 1.72 n_0$, $n_C(\mathrm{Gs}) \approx$ $\approx 1.33 n_0$, $n_C(\mathrm{RATP}) \approx 1.03 n_0$, and $n_C(\mathrm{BSk17}) \approx$ $\approx 1.001 n_0$, which correspond to the phase transitions from the superfluid paramagnetic state of neutron matter with conventional Skyrme forces and the triplet pairing to the ferromagnetic state, which coexists with triplet superfluidity at the densities higher than n_C(Skyrme). Such phase transitions might occur in the liquid outer core of neutron stars.

By contrast, for SNM with generalized Skyrme forces BSk18-BSk21 [20, 21], the phase transition to the ferromagnetic state is removed. Really, for the generalized BSk18 parametrization of Skyrme forces (with additional dependences on the density, see (3)–(5)), relation (14) yields the expression

$$\frac{\chi_{\rm BSk18}(y)}{\chi_{\rm free}(y)} \approx \left[1-\frac{2y^{1/3}(0.373605y^{3/10}-1.11339)}{1+0.253920y}-\right.$$

$$\left.-\frac{4y(0.774775-0.209061y)}{1+0.253920y}\right]^{-1}. \qquad (16)$$

In a similar manner for the generalized BSk19, BSk20, and BSk21 parametrizations [21] of the Skyrme

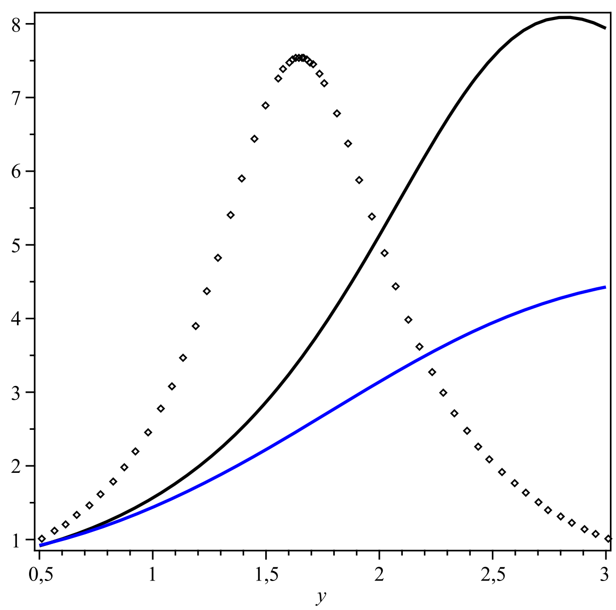

Fig. 4. Ratio $\chi_{\rm Skyrme}(y)/\chi_{\rm free}$ (see (16), (18), and (19)) for SNM with generalized parametrizations (BSk18 points, BSk20 upper line, and BSk21 lower line) of the Skyrme forces [20, 21] and a spin-triplet p-wave pairing of the ^{3}He $-$ A type at $T=0$ as a function of the reduced density $y=n/n_0$

forces, relation (14) yields

$$\frac{\chi_{\mathrm{BSk19}}(y)}{\chi_{\mathrm{free}}(y)} \approx$$
$$\approx \left[1 - \frac{2y^{1/3}(2.22596y^{1/12} - 2.58957)}{1 + y(2.86510y^{1/12} - 0.238482y^{1/3} - 2.77567)} - \right.$$
$$\left. - \frac{4y(1.16385 - 0.178829y^{1/3} - 0.716143y^{1/12})}{1 + y(2.86510y^{1/12} - 0.238482y^{1/3} - 2.77567)}\right]^{-1}, \tag{17}$$

$$\frac{\chi_{\mathrm{BSk20}}(y)}{\chi_{\mathrm{free}}(y)} \approx$$
$$\approx \left[1 - \frac{2y^{1/3}(1.35019y^{1/12} - 1.82731)}{1 + y(3.18344y^{1/12} - 0.305158y^{1/6} - 2.90372)} - \right.$$
$$\left. - \frac{4y(1.35095 - 0.228827y^{1/6} - 0.795714y^{1/12})}{1 + y(3.18344y^{1/12} - 0.305158y^{1/6} - 2.90372)}\right]^{-1}, \tag{18}$$

and

$$\frac{\chi_{\mathrm{BSk21}}(y)}{\chi_{\mathrm{free}}(y)} \approx$$
$$\approx \left[1 - \frac{2y^{1/3}(-0.133544y^{1/12} - 0.475909)}{1 + y(3.97930y^{1/12} + 0.0422618\sqrt{y} - 3.89571)} - \right.$$
$$\left. - \frac{4y(1.35338 + 0.0316905\sqrt{y} - 0.994643y^{1/12})}{1 + y(3.97930\, y^{1/12} + 0.0422618\sqrt{y} - 3.89571)}\right]^{-1}. \tag{19}$$

4. Conclusions

Thus, from our generalized integral equations (2) and (9), we have obtained that ferromagnetic instabilities take place not only for the normal NM (see, e.g., [23–33]), but also for SNM with the conventional Skyrme forces and the anisotropic spin-triplet pairing (see Fig. 1 and [1] for SLy2, Gs, and RATP and (15) and Fig. 2 for conventional BSk17 [34]).

But our main result is that the introduction of new additional Skyrme parameters dependent on the density (see (4) and (5)) for generalized Skyrme forces BSk18-BSk21 [20, 21] leads to the removal of the ferromagnetic instability not only in the normal NM but also in the SNM with BSk18-BSk21 forces and an anisotropic spin-triplet pairing of the $^3\mathrm{He} - \mathrm{A}$ type in a strong magnetic field. The density-dependent magnetic susceptibilities $\chi_{\mathrm{BSk18-21}}(n)$ have regular behavior on the interval $0.5 \lesssim n/n_0 \lesssim 3.0$ (see Figs. 3 and 4 and (16)–(19)). Note that the superfluid corrections to $\chi_{\mathrm{BSk18-21}}(n)$ have negligibly small values (which are of the order $(\Delta/\varepsilon_{\mathrm{F}})^2 \approx 10^{-6}$, where Δ is the maximal magnitude of anisotropic energy gap in spectrum (8) of quasiparticles (neutrons) in the SNM considered here). See, e.g., Fig. 1 in our recent paper [19] (for SNM with generalized BSk18 Skyrme force); whence it follows for BSk18 that the ratio $\Delta/\varepsilon_{\mathrm{F}} \lesssim 10^{-3}$ in the considered density range $0.5 \lesssim n/n_0 \lesssim 3.0$ (investigations for SNM with generalized BSk19-BSk21 Skyrme forces that are analogous to those in [19] will be published elsewhere). It is similar to the behavior of the magnetic susceptibility in the case of a phase transition between the real normal liquid $^3\mathrm{He}$ and the superfluid phases $^3\mathrm{He} - \mathrm{A}$ in a magnetic field, where their magnetic susceptibilities in the normal liquid $^3\mathrm{He}$ and in the superfluid $^3\mathrm{He} - \mathrm{A}$ almost coincide with each other (difference is $\sim 0.1\%$; see, e.g., [37]).

Note finally that the phenomenon of superfluidity in NM at very high densities $n > 3n_0$ (which can be realized inside the inner fluid cores of sufficiently massive neutron stars and magnetars) should be investigated in the framework of a relativistic approach and with a different interpretation of the hadron matter structure (including π-mesons, K-mesons, hyperons, quarks, and other possible constituents; see, e.g., [38–41] and references therein).

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О.М. Тарасов

МАГНІТНІ СПРИЙНЯТЛИВОСТІ
ГУСТОЇ НАДПЛИННОЇ НЕЙТРОННОЇ МАТЕРІЇ
З УЗАГАЛЬНЕНИМИ СИЛАМИ СКІРМА
ТА СПІН-ТРИПЛЕТНОЮ ВЗАЄМОДІЄЮ
ПРИ НУЛЬОВІЙ ТЕМПЕРАТУРІ

Р е з ю м е

Магнітні властивості густої надплинної нейтронної матерії (це стосується фізики ядер нейтронних зірок) при суб'ядерних та над'ядерних густинах (у діапазоні густин $0{,}5 \lesssim n/n_0 \lesssim 3{,}0$, де $n_0 = 0{,}17$ (фм$^{-3}$) – це ядерна густина насичення) з так званими узагальненими ефективними силами Скірма BSk18, BSk19, BSk20, BSk21 (з додатковими нетрадиційними членами, що залежать від густини) та зі спін-триплетними парами (спін та орбітальний момент яких $S = 1$ та $L = 1$) за наявності сильного магнітного поля і температури, що дорівнює нулю, вивчаються за допомогою нерелятивістської узагальненої теорії фермі-рідини. Верхня границя діапазону густин нейтронної матерії обмежена величиною $3n_0$, щоб запобігти урахування релятивістських поправок, які зростають з густиною. Отриманій у статті [A.N. Tarasov, Ukr. J. Phys. **55**, 644 (2010)] загальній формулі (яка справедлива для довільної параметризації сил Скірма) для магнітної сприйнятливості надплинної нейтронної матерії при температурі, що дорівнює нулю, надано конкретних виразів для нових BSk18-BSk21 параметризацій взаємодії Скірма. Як відомо, застосування усіх попередніх традиційних параметризацій взаємодії Скірма приводить до виникнення спінових нестійкостей у нормальній (ненадплинній) фазі нейтронної матерії при ядерних густинах та більших за ядерну густину насичення. В цій статті для моделі надплинної нейтронної матерії з новими узагальненими BSk18-BSk21 параметризаціями отримано, що такого фазового переходу до феромагнітного стану не відбувається ані при суб'ядерних, ані при над'ядерних значеннях густини. Таким чином, доведено, що у нейтронній матерії з новими узагальненими силами Скірма BSk18-BSk21 при великих густинах феромагнітна нестійкість усувається не тільки у нормальному, а й у надплинних станах зі спін-триплетними анізотропними парами.