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THE INFLUENCE OF MAGNETOELASTIC INTERACTION ON THE FIRST TRANSVERSE SOUND IN A FERROMAGNET OF CUBIC SYMMETRY IN A VICINITY OF THE MARTENSITIC TRANSFORMATION

The dispersion laws of coupled magnetoelastic waves have been calculated for all ground states of a ferromagnet with the cubic symmetry. It is shown that the magnetoelastic interaction with the first transverse sound takes place for all equilibrium directions of the magnetization vector. The obtained dispersion laws testify that the magnetoelastic interaction coefficient depends on the magnetization and wave vector directions. The quantitative calculations of the dispersion relations for the shape memory alloy Ni–Mn–Ga are made on the basis of the obtained results. The results of research demonstrate that a decrease in the elastic modulus gives rise to an appreciably stronger magnetoelastic interaction.

Keywords: magnetoelastic interaction, dispersion law, cubic ferromagnet, shape memory alloy, elastic modulus.

1. Introduction

Magnetoelastic waves have been studied for many years [1, 2], and the phenomenological model for their description is well developed [3, 4]. Nowadays, the study of the interaction between the magnetic and elastic subsystems gets a new impetus owing to numerous experiments [5–8] with magnetically ordered systems, in which such interaction can be rather strong. It is well known that the magnetoelastic interaction increases, as such systems approach spin-reorientation phase transitions [4, 9]. This fact induced the active researches of coupled magnetoelastic waves at phase transformations of different types.

Recently, the so-called structural phase transitions have been the objects of intense researches owing to their crucial role in such effects as superelasticity and shape memory. Of special interest are the so-called “martensitic transformations” and the structural phase transitions of the first kind from a highly symmetric structure into a low symmetric deformed one, which take place at low temperatures [5–8]. In materials with those phase transitions, the phenomenon of giant magnetostriction was discovered, which is governed by a drastic decrease of the elastic energy in vicinities of martensitic transformations [10]. It is the interaction between the magnetic subsystem and elastic waves that plays the key role in this phenomenon. In work [11], the influence of such interaction on one of the elastic moduli of a cubic ferromagnet with shape memory was calculated. However, the experimental data [5, 7] testify that the relevant theoretical calculations are extremely necessary for other elastic moduli as well, because they also appreciably change at martensitic phase transitions.

2. Dispersion Laws for Coupled Magnetoelastic Waves in a Cubic Ferromagnet

Consider a ferromagnet with cubic symmetry. For the description of the interaction between spin and elastic waves, the total energy density in a cubic crystal can be presented in the form

$$F = F_m + F_e + F_{me}. \tag{1}$$

The first term in expression (1) is the magnetic part of the energy density. In the case of cubic symmetry, it looks like [3]

$$F_m = \frac{\alpha}{2}\frac{\partial \boldsymbol{\mu}}{\partial x_i}\frac{\partial \boldsymbol{\mu}}{\partial x_k} + K_1\left(\mu_x^2\mu_y^2 + \mu_x^2\mu_z^2 + \mu_y^2\mu_z^2\right) + $$

$$+ K_2\mu_x^2\mu_y^2\mu_z^2 - \mathbf{MH}, \tag{2}$$

where α is the inhomogeneous exchange interaction constant; K_1 and K_2 are the constants of magnetic anisotropy in the cubic ferromagnet; $\mathbf{M}$ and $\mathbf{H}$ are the vectors of magnetization and external magnetic field, respectively; $\mu = \frac{\mathbf{M}}{M_0}$ is the normalized magnetization vector (since the constants in expression (2) have the dimensionality of energy), and M_0 is the saturation magnetization. The energy of demagnetizing fields is neglected in Eq. (2), because we do not consider the specific shape of a ferromagnetic specimen.

The energy density of elastic deformations looks like [12, 13]

$$F_e = \frac{3}{2}(C_{11} + 2C_{12})u_1^2 + \frac{1}{6}C'(u_2^2 + u_3^2) + $$

$$+ 2C_{44}(u_4^2 + u_5^2 + u_6^2). \tag{3}$$

Here, the quantities C_{11}, C_{12}, C_{44}, and $C' = (C_{11} - C_{12})/2$ are the elastic moduli of the second order of the crystal with cubic symmetry. The variables $u_1 = \frac{1}{3}(\mathrm{E}_{xx} + \mathrm{E}_{yy} + \mathrm{E}_{zz})$, $u_2 = \sqrt{3}\,(\mathrm{E}_{xx} - \mathrm{E}_{yy})$, $u_3 = (2\mathrm{E}_{zz} - \mathrm{E}_{xx} - \mathrm{E}_{yy})$, $u_4 = \frac{1}{2}(\mathrm{E}_{yz} + \mathrm{E}_{zy})$, $u_5 = \frac{1}{2}(\mathrm{E}_{xz} + \mathrm{E}_{zx})$, and $u_6 = \frac{1}{2}(\mathrm{E}_{xy} + \mathrm{E}_{yx})$ are the linear combinations of strain tensor components, which are transformed according to the one- (u_1), two- (u_2 and u_3), and three-dimensional (u_4, u_5, and u_6) irreducible representations of the crystal symmetry group.

The third term in Eq. (1) corresponds to the energy density for the interaction between the magnetic and elastic subsystems [13],

$$F_{me} = -\delta_0 u_1(\mu_x^2 + \mu_y^2 + \mu_z^2) -$$

$$- \delta_1\{\sqrt{3}u_2(\mu_x^2 - \mu_y^2) + u_3(2\mu_z^2 - \mu_x^2 - \mu_y^2)\} -$$

$$- \delta_2(u_4\mu_y\mu_z + u_5\mu_x\mu_z + u_6\mu_x\mu_y), \tag{4}$$

where the constants δ_0, δ_1, and δ_2 describe the magnetoelastic interaction.

From the minimization condition for the magnetic part of the energy, it is easy to show that there are three ground states for the magnetization vector in the cubic ferromagnet in the absence of an external magnetic field ($\mathbf{H} = 0$): along the fourth-order axis, $\mathbf{M} \parallel\mid \langle 001\rangle$, phase 1; along the diagonal of one of the cube edges, $\mathbf{M} \parallel\mid \langle 101\rangle$, phase 2; and along the spatial cube diagonal, $\mathbf{M} \parallel\mid \langle 111\rangle$, phase 3; every other possible direction of the magnetic moment is equivalent to one of those given above. In real experiments aimed at the study of the elastic and magnetic properties of materials [5–8], the external magnetic field direction coincides with one of the indicated directions of the magnetic moment, and the magnitude of $\mathbf{H}$ is rather large (∼1000 Oe). Hence, $\mathbf{M}$ in equilibrium can be considered as directed along one of those directions.

We consider small adiabatic oscillations of the magnetic moment density $\boldsymbol{\mu}$ in the ferromagnet [3]. Accordingly, we can write

$$\boldsymbol{\mu}(\mathbf{r}, t) = \boldsymbol{\mu}_0 + \mathbf{m}(\mathbf{r}, t), \tag{5}$$

where $\mathbf{m}(\mathbf{r}, t)$ stands for small deviations from the equilibrium $\boldsymbol{\mu}_0$-value as a result of fluctuations, and the equilibrium value of magnetization vector has the following components: $\boldsymbol{\mu}_0 = (0, 0, 1)$ in phase 1, $\boldsymbol{\mu}_0 = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ in phase 2, and $\boldsymbol{\mu}_0 = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ in phase 3. Using the condition $\partial F/\partial \mathrm{E}_{ik} = 0$, we can obtain the equilibrium values E_{ik}^0 of the strain tensor components for the ground states of a cubic ferromagnet (below, the corresponding expressions are given for each ground state of a cubic ferromagnet). Hence, every strain tensor component can also be written as the sum of a homogeneous part and a small deviation from it,

$$\mathrm{E}_{ik} = \mathrm{E}_{ik}^0 + \varepsilon_{ik}. \tag{6}$$

The inhomogeneous part of the elastic strain tensor can be expressed in terms of the vector of particle displacement $\mathbf{U}$, using the formula [4]

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right). \tag{7}$$

In order to obtain the dispersion laws for coupled magnetoelastic waves in all ground states of a cubic ferromagnet, let us use the dynamic equations for the magnetization vector $\boldsymbol{\mu}$ (the Landau–Lifshitz equations) and the particle displacement vector $\mathbf{U}$ [3, 4],

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\mu \times \mathbf{H}_{\text{eff}}, \tag{8}$$

$$\rho\ddot{\mathbf{U}} = -\frac{\delta F}{\delta \mathbf{U}}, \qquad (9)$$

where $\mathbf{H}_{\text{eff}} = -\frac{\delta F}{\delta \mathbf{m}}$ is an effective magnetic field, γ the gyromagnetic ratio, and ρ the density. We should expand the total energy density in a series in small deviations m_i and ε_{ik}, substitute it into Eqs. (8) and (9), and linearize them.

It is well known [14] that the following elastic waves can propagate in a crystal with cubic symmetry: longitudinal waves with $s_{l1}^2 = C_{11}/\rho$ and $s_{l2}^2 = (C_{11}+C_{12}+2C_{44})/2\rho$, where s_{l1} and s_{l2} are the velocities of the first and second longitudinal sounds, respectively; and transverse waves with $s_{t1}^2 = C_{44}/\rho$, $s_{t2}^2 = C'/\rho$, where s_{t1} and s_{t2} are the velocities of the first and second transverse sounds, respectively. The first transverse sound can be described in the cases of two directions of the wave vector of elastic vibrations [14]: along the fourth-order axis and along the cube face diagonal. Hence, for definiteness, let us consider the directions $\mathbf{k} \parallel \langle 100\rangle$ and $\mathbf{k} \parallel \langle 110\rangle$.

Let us change in Eqs. (8) and (9) to the Fourier components with respect to the time t and the coordinates $\mathbf{r}$ in the case of small deviations, $\mathbf{m} = \mathbf{m}_0 \exp\{i(\mathbf{kr} - \omega t)\}$ and $\mathbf{U} = \mathbf{U}_0 \exp\{i(\mathbf{kr} - \omega t)\}$, where ω is the frequency, and $\mathbf{k}$ the wave vector of collective waves. Then Eqs. (8) and (9) give rise to a system of six equations for the components of the vectors $\mathbf{m}_0$ and $\mathbf{U}_0$. From the condition that the determinant of this system equals zero, we obtain the dispersion laws for coupled magnetoelastic waves in the ground states of a cubic ferromagnet.

Below, we present the results obtained for each ground state.

Phase 1: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$.

The equilibrium values of strain tensor components are

$$E_{xx}^0 = E_{yy}^0 = \frac{\delta_0}{3(C_{11}+2C_{12})} - \frac{2\delta_1}{C_{11}-C_{12}},$$
$$E_{zz}^0 = \frac{\delta_0}{3(C_{11}+2C_{12})} + \frac{4\delta_1}{C_{11}-C_{12}},$$
$$E_{xz}^0 = E_{zx}^0 = E_{yz}^0 = E_{zy}^0 = E_{xy}^0 = E_{yx}^0 = 0.$$

Case $\mathbf{k} \parallel \langle 100\rangle$:

$$(\omega^2 - s_{t1}^2k^2)(\omega^2 - s_{l1}^2k^2)\Big[(\omega^2 - s_{t1}^2k^2)\times$$
$$\times(\omega^2 - \gamma^2 M_0^2\omega_{m1}^2) - \delta_3^2\left\{\frac{\omega_{m1}\gamma^2k^2}{4\rho}\right\}\Big] = 0. \qquad (10)$$

Case $\mathbf{k} \parallel \langle 110\rangle$:

$$(\omega^2 - s_{t2}^2k^2)(\omega^2 - s_{l2}^2k^2)\Big[(\omega^2 - s_{t1}^2k^2)\times$$
$$\times(\omega^2 - \gamma^2 M_0^2\omega_{m1}^2) - \delta_3^2\left\{\frac{\omega_{m1}\gamma^2k^2}{4\rho}\right\}\Big] = 0. \qquad (11)$$

In expressions (10) and (11), the notation $\omega_{m1} = \alpha k^2/M_0^2 + H/M_0 + 2K_1/M_0^2 + 72\delta_1^2/M_0^2(C_{11}-C_{12})$ was used.

Phase 2: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 101\rangle$.

The equilibrium values of strain tensor components are

$$E_{xx}^0 = E_{zz}^0 = \frac{\delta_0}{3(C_{11}+2C_{12})} + \frac{\delta_1}{C_{11}-C_{12}},$$
$$E_{yy}^0 = \frac{\delta_0}{3(C_{11}+2C_{12})} - \frac{2\delta_1}{C_{11}-C_{12}},$$
$$E_{xz}^0 = E_{zx}^0 = \frac{\delta_2}{8C_{44}},$$
$$E_{yz}^0 = E_{zy}^0 = E_{xy}^0 = E_{yx}^0 = 0.$$

Case $\mathbf{k} \parallel \langle 100\rangle$:

$$(\omega^2 - s_{t1}^2k^2)\Big[(\omega^2 - s_{t1}^2k^2)(\omega^2 - s_{l1}^2k^2)\times$$
$$\times(\omega^2 - \gamma^2 M_0^2\omega_{m2}\omega_{m3}) -$$
$$-\delta_2^2\left\{\frac{36\omega_{m2}\gamma^2k^2}{\rho}(\omega^2 - s_{t1}^2k^2)\right\} -$$
$$-\delta_3^2\left\{\frac{\omega_{m3}\gamma^2k^2}{8\rho}(\omega^2 - s_{l1}^2k^2)\right\}\Big] = 0. \qquad (12)$$

Case $\mathbf{k} \parallel \langle 110\rangle$:

$$(\omega^2 - s_{t1}^2k^2)(\omega^2 - s_{t2}^2k^2)(\omega^2 - s_{l2}^2k^2)\times$$
$$\times(\omega^2 - \gamma^2 M_0^2\omega_{m2}\omega_{m3}) - \delta_2^2\left\{\frac{18\omega_{m2}\gamma^2k^2}{\rho}(\omega^2 - s_{t1}^2k^2)\times\right.$$
$$\left.\times(\omega^2 - \frac{(s_{l2}^2+s_{t2}^2)}{2}k^2)\right\} - \delta_3^2\left\{\frac{3\omega_{m3}\gamma^2k^2}{16\rho}(\omega^2 - s_{t2}^2k^2)\times\right.$$
$$\left.\times(\omega^2 - \frac{(s_{l2}^2+2s_{t1}^2)}{3}k^2)\right\} = 0. \qquad (13)$$

In expressions (12) and (13), the notations

$$\omega_{m2} = \alpha k^2/M_0^2 + H/M_0 + K_1/M_0^2 +$$
$$+ K_2/2M_0^2 + 36\delta_1^2/M_0^2(C_{11}-C_{12}) + \delta_2^2/8M_0^2C_{44},$$
$$\omega_{m3} = \alpha k^2/M_0^2 + H/M_0 - 2K_1/M_0^2 + \delta_2^2/4M_0^2C_{44}.$$

were used.

Coefficient of magnetoelastic interaction with the first transverse sound in various ground states of a cubic ferromagnet

Wavevector direction	Phase 1: $\mathbf{H}\|\|\mathbf{M}\|\|\langle 001\rangle$ $\omega_{\rm sw}=\gamma M_0\omega_{m1}$	Phase 2: $\mathbf{H}\|\|\mathbf{M}\|\|\langle 101\rangle$ $\omega_{\rm sw}=\gamma M_0(\omega_{m2}\omega_{m3})^{1/2}$	Phase 3: $\mathbf{H}\|\|\mathbf{M}\|\|\langle 111\rangle$ $\omega_{\rm sw}=\gamma M_0\omega_{m4}$
$\mathbf{k}\|\|\langle 100\rangle$	$\xi=\frac{\omega_{m1}\gamma^2k^2}{4\rho}$	$\xi=\frac{\omega_{m3}\gamma^2k^2}{8\rho}$	$\xi=\frac{\omega_{m4}\gamma^2k^2}{9\rho}$
$\mathbf{k}\|\|\langle 110\rangle$	$\xi=\frac{\omega_{m1}\gamma^2k^2}{4\rho}$	$\xi=\frac{\omega_{m3}\gamma^2k^2}{16\rho}$	$\xi=\frac{\omega_{m4}\gamma^2k^2}{36\rho}$

Phase 3: $\mathbf{H}\parallel\mathbf{M}\parallel\langle 111\rangle$.

The equilibrium values of strain tensor components are

$$E^0_{xx}=E^0_{yy}=E^0_{zz}=\frac{\delta_0}{3(C_{11}+2C_{12})},$$
$$E^0_{xz}=E^0_{zx}=E^0_{yz}=E^0_{zy}=E^0_{xy}=E^0_{yx}=\frac{\delta_2}{12C_{44}}.$$

Case $\mathbf{k}\parallel\langle 100\rangle$:

$$(\omega^2-s_{t1}^2k^2)\Bigg[(\omega^2-s_{t1}^2k^2)(\omega^2-s_{l1}^2k^2)\times \times(\omega^2-\gamma^2M_0^2\omega_{m4}^2)-\delta_2^2\left\{\frac{32\omega_{m4}\gamma^2k^2}{\rho}(\omega^2-s_{t1}^2k^2)\right\}- -\delta_3^2\left\{\frac{\omega_{m4}\gamma^2k^2}{9\rho}(\omega^2-s_{l1}^2k^2)\right\}\Bigg]=0. \quad (14)$$

Case $\mathbf{k}\parallel\langle 110\rangle$:

$$(\omega^2-s_{t1}^2k^2)(\omega^2-s_{t2}^2k^2)(\omega^2-s_{l2}^2k^2)\times \times(\omega^2-\gamma^2M_0^2\omega_{m4}^2)-\delta_2^2\left\{\frac{32\omega_{m4}\gamma^2k^2}{\rho}\times \times(\omega^2-s_{t1}^2k^2)(\omega^2-\frac{(3s_{l2}^2+s_{t2}^2)}{4}k^2)\right\}- -\delta_3^2\left\{\frac{\omega_m\gamma^2k^2}{12\rho}(\omega^2-s_{t2}^2k^2)(\omega^2-\frac{(s_{l2}^2+2s_{t1}^2)}{3}k^2)\right\}- -\delta_2\delta_3\left\{\frac{4\omega_m\gamma^2k^2}{3\rho}(\omega^2-s_{t1}^2k^2)(\omega^2-s_{t2}^2k^2)\right\}=0. \quad (15)$$

In expressions (14) and (15), the notation $\omega_{m4}= =\alpha k^2/M_0^2+H/M_0-4K_1/3M_0^2-4K_2/9M_0^2+ +\delta_2^2/4M_0^2C_{44}$ was used.

Hence, expressions (10)–(15) are the dispersion laws for coupled magnetoelastic waves in a ferromagnet with cubic symmetry presented in the general form. Those dispersion equations are standard by their form [3, 4]. If the magnetoelastic interaction is neglected ($\delta_i\to 0$), they split into the classical dispersion laws for spin waves [3] and elastic waves in cubic crystals [14].

3. First Transverse Sound for a Shape Memory Alloy

The influence of the magnetic subsystem on the first transverse sound and, accordingly, on the elastic modulus C_{44} can be described, by considering the magnetoacoustic resonance at the frequency $\omega_{\rm ph}= =(C_{44}/\rho)^{1/2}k$. In this case, the dispersion laws (10)–(15) transform into a dispersion equation, which has the following general form for all ground states and wave vector directions:

$$(\omega^2-\omega_{\rm ph}^2)(\omega^2-\omega_{\rm sw}^2)-\delta_2^2\xi=0. \quad (16)$$

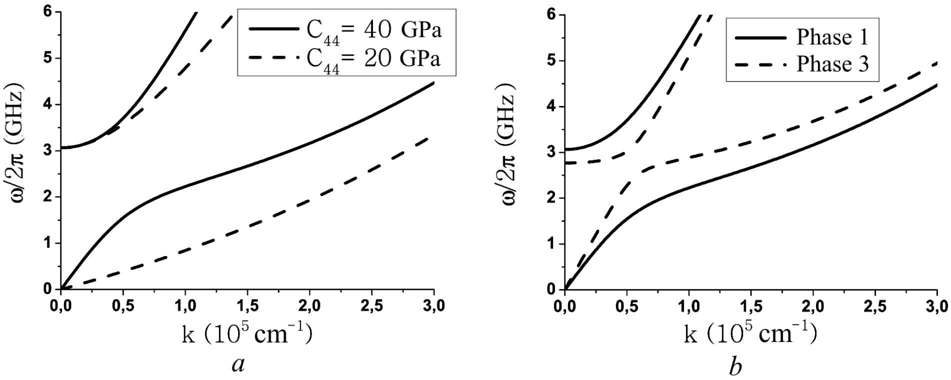
Here, ω_{sw} is the spin wave frequency, and ξ the coefficient of magnetoelastic interaction. Their specific values (see Table) depend on the direction of the ferromagnet magnetic moment and the direction of the wave vector of collective waves, and are presented in Table. The solution of Eq. (16) looks like

$$\omega_\pm^2=\frac{1}{2}\left\{\omega_{\rm ph}^2+\omega_{\rm sw}^2\pm[4\xi\delta_2^2+(\omega_{\rm ph}^2-\omega_{\rm sw}^2)^2]^{1/2}\right\}. \quad (17)$$

This dispersion law consists of two branches: quasimagnon and quasiphonon ones (see Fig. 1). From Eq. (17), it is easy to see that, as the system approaches the magnetoacoustic resonance ($\omega_{\rm sw}\to\omega_{\rm ph}$), it is the quantities ξ and δ_2 that determine the "repulsion" between the quasimagnon and quasiphonon branches.

For making some quantitative estimations, let us plot the obtained dispersion law (17) in various cases, by using a shape memory material as an example (Fig. 1). The magnitudes of constants entering Eq. (17) are taken in the case of Ni–Mn–Ga alloy, since this alloy is one of the most interesting representatives of shape memory materials for today. In a vicinity of room temperature, it undergoes a martensitic phase transformation: a transition from the cubic phase to the tetragonal one [15].

In specific calculations for Ni–Mn–Ga alloy, we selected the known experimental values of anisotropy



Dispersion laws for magnetoelastic waves: (a) in the ground state $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$ for two values of elastic modulus C_{44} and (b) in the ground state $\mathbf{H} \parallel \mathbf{M} \parallel \langle 101\rangle$ and $\mathbf{H} \parallel \mathbf{M} \parallel \langle 111\rangle$for $C_{44} = 40$ GPa

constants in the cubic phase (austenite) [16], which correspond to phase 1, namely, $K_1 = 2.7 \times 10^4$ erg/cm^3 and $K_2 = -6.1 \times 10^4$ erg/cm^3, and the magnitude of saturation magnetization $M_0 = 600$ Gs. The constant of inhomogeneous exchange interaction can be estimated from the expression [3] $\alpha \cong (k_\mathrm{B} T_c A^2 M_0)/\mu_\mathrm{B}$, where $T_c = 360$ K is the Curie temperature [16], $A = 0.41 \times 10^{-8}$ cm is the distance between magnetic atoms [16], μ_B is the Bohr magneton, and k_B the Boltzmann constant. The external magnetic field has to be strong enough in order to satisfy the existence conditions for the ground states ($\omega_{mi} \geqslant 0$, where $i = 1, 2, 3, 4$) and to correspond to the conditions of experimental researches, which are usually carried out with such materials. Therefore, we selected $H = 1000$ Gs. The elastic moduli were also taken as those for austenite: $C_{44} = 40$ GPa and $C' = 14$ GPa [17]. The constant of magnetoelastic interaction δ_2 was not evaluated earlier. Proceeding from the facts that it cannot be less that δ_1, and $\delta_1 \sim 10^7$ erg/cm^3 [13], we put $\delta_1 = 10^9$ erg/cm^3 for the sake of illustration.

4. Discussion and Conclusions

The dispersion laws (10)–(15) of coupled magnetoelastic waves calculated for a ferromagnet with cubic symmetry make it possible to estimate the influence of the magnetic subsystem on the elastic properties of the crystal, namely, on the corresponding elastic moduli. From the obtained laws of dispersion in a cubic ferromagnet, one can see that, contrary to other sound modes [11], the magnetoelastic interaction with the first transverse sound takes place for all equilibrium directions of the magnetic moment in a cubic ferromagnet.

The coefficient ξ of the magnetoelastic interaction between spin waves and the first transverse sound depends on the direction of the ferromagnet magnetic moment (see Table and Fig. 1, *b*). This interaction manifests itself most strongly in the ground state $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$. In addition, it turns out that, in the ground states $\mathbf{H} \parallel \mathbf{M} \parallel \langle 101\rangle$ and $\mathbf{H} \parallel \mathbf{M} \parallel \langle 111\rangle$, the coefficient of magnetoelastic interaction also depends on the direction of the wave vector of collective oscillations (Table).

Collective oscillations of spin waves and collective vibrations of the first transverse sound are described by the dispersion equation (17), which has identical character for each direction of the ferromagnet magnetic moment. From Eq. (17), it follows that if the elastic modulus C_{44} drastically decreases, the magnetoelastic interaction grows considerably. From Fig. 1, *a* plotted for Ni–Mn–Ga alloy as an example, one can see that even the two-fold reduction of the elastic modulus C_{44} brings about a considerable "repulsion" between the quasimagnon and quasiphonon branches in the dispersion law. Such a behavior of the quasiphonon mode can be responsible for even more undervalued C_{44}-magnitudes at resonance measurements.

It is also worth noting that the application of the expression for the magnetoelastic energy in the form (4) enables one to accurately determine the part of this energy (i.e. the constant δ_i) that is responsible for the interaction with a definite sound mode (unlike the classical form of the expression used, e.g., in work [4]). The analysis of the dispersion laws (10)–(15) demonstrates that the constant δ_0 does not enter them, so that the influence of the equilibrium part of the magnetoelastic energy is not taken into account. Really, while considering the dynamic phenomena, e.g., the magnetoelastic resonance, the influence of this term cannot be taken into consideration. A theoretical model that makes allowance for the influence of the equilibrium part of the magnetoelastic energy was proposed in work [18]. The constant δ_1 characterizes the influence of the magnetic subsystem on the second transverse sound and, accordingly, on the elastic modulus C' [11]. From Eq. (10), one can easily see that, as was shown earlier [11], the interaction with this sound mode cannot be described in phase 1. In turn, the constant δ_2 characterizes the influence of the magnetic subsystem on the first transverse sound and the elastic modulus C_{44}.

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ВПЛИВ МАГНІТОПРУЖНОЇ
ВЗАЄМОДІЇ НА ПЕРШИЙ ПОПЕРЕЧНИЙ ЗВУК
В ФЕРОМАГНЕТИКУ КУБІЧНОЇ СИМЕТРІЇ
В ОКОЛІ МАРТЕНСИТНОГО ПЕРЕТВОРЕННЯ

Р е з ю м е

Розраховано закони дисперсії зв'язаних магнітопружних хвиль для всіх основних станів феромагнетика кубічної симетрії. Показано, що магнітопружня взаємодія з першим поперечним звуком має місце для всіх рівноважних напрямків магнітного моменту. Отримані закони дисперсії показують, що коефіцієнт магнітопружної взаємодії залежить як від напрямку магнітного моменту феромагнетика, так і від напрямку хвильового вектора колективних коливань. На основі отриманих результатів зроблено кількісні розрахунки дисперсійних залежностей для сплаву NiMnGa з ефектом пам'яті форми. Отримані розрахунки показують, що зменшення пружного модуля приводить до помітного зростання магнітопружної взаємодії.