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LIGHT MODULATOR ON THE BASIS OF MAGNETO-OPTICAL CRYSTAL IN A BIMORPHIC STRUCTURE OPERATING IN THE MAGNETO-MECHANICAL VIBRATION MODE

A model of light modulator based on a bimorphic element consisting of a magneto-optical crystal layer on a magnetically passive substrate has been proposed. An equation describing magneto-mechanical vibrations in a rectangular bimorphic element with asymmetric thickness is derived, and an algorithm of its solution in the case of a specimen with free edges is proposed. The modulator is shown to be characterized by a two-dimensional distribution function for the rotation angle of the polarization plane in a light-beam cross-section. Calculations for bismuth-substituted yttrium ferrite garnet on a gadolinium-gallium substrate showed that the rotation angle of the light-beam polarization plane owing to the Faraday effect can reach 3° *for the fundamental mode of bimorphic element vibrations.*

Keywords: Faraday effect, magneto-optical crystal, bimorphic element.

1. Introduction

Embedding a magneto-optical crystal (MOC) into external dc and ac magnetic fields induces internal elastic deformations connected with the magnetostriction phenomenon. In the case of magneto-mechanic resonance, the mechanical stresses give rise to additional changes in the specimen magnetization [1]. By means of the Faraday and Cotton–Mouton effects, those changes affect the light polarization in MOC [2, 3]. In order to enhance the magnetization modulation and to reduce the resonance frequency of the specimen tension-compression vibrations, a composite structure was proposed in work [4]. Even a higher gain can be expected, if the MOC is a part of the bimorphic structure. At vibrations of bimorphic plates, the mechanical stress is known to be characterized by a distribution function in the plane of a bimorphic element. This circumstance is proposed to be used, while developing a magneto-optical modulator with a spatial distribution of polarization plane rotation angles over a light-beam transverse cross-section.

Note that the bimorphic structures are widely used in piezoelectric pressure sensors and acoustic radiators [5]. The combination of two piezoelements or a piezoelement and a metal plate into a bimorphic structure is known to result in the appearance of low-frequency bending vibrations, and the sensitivity to pressure becomes several tens of times higher [6].

Transverse bending vibrations of round bimorphic plates have been studied the most completely. For instance, in work [7], the equation of vibrations was derived, and its solution for symmetric piezoceramic round bimorphic plates was obtained. Magneto-mechanic vibrations of rectangular bimorphic plates were studied experimentally [8], and the corresponding sensitivity reached for them turned out by an order of magnitude higher than that for piezoceramic bimorphic transducers.

Vibrations of rectangular bimorphic plates were calculated in the cases of the clamped or hinged plate fastening, and the bimorphic element design was mainly supposed to have a symmetric arrangement of the neutral layer across the plate thickness [9]. However, the issues concerning the derivation of equations for the vibrations of asymmetric rectangular bimorphic plates, the determination of the neutral-layer coordinate for them, and the solutions of those equations under the condition of free plate edges remained unresolved and challenging.

2. Model of Magneto-Optical Modulator

In this work, we consider a magneto-optical modulator (Fig. 1) consisting of a rectangular bimorphic plate with linear dimensions $b \times l$ fabricated from

MOC *1* (ferrimagnet) of the thickness h_1, which was grown up on a magnetically passive substrate *2* of the thickness h_2. The plate edges are supposed to be free of contact (the soft suspension).

If the specimen is arranged in the dc (polarizing) magnetic field $H^0(x_k)$ and the ac magnetic field $H(x_k)\,e^{i\omega t}$ with the circular frequency ω, the ferrimagnet executes magneto-mechanic vibrations. Those vibrations are described by a two-dimensional stress distribution function in the plane $x_1 0 x_2$ and depend on the coordinate x_3. The mechanical stresses in MOC induce additional changes of the magnetization owing to the Villari effect [10]. The alternating component of the magnetization vector, which coincides with the axis $0x_3$ (the axis of light propagation), gives rise to a rotation of the light polarization plane by the angle ϕ owing to the Faraday effect. This angle depends on the stress at the given point in the plane $x_1 0 x_2$ of a bimorphic element and is characterized by the distribution function $\phi(x_1, x_2)$ over the transverse cross-section of the light beam.

In order to determine the function $\phi(x_1, x_2)$, we have to find firstly the function $w(x_1, x_2)$ describing the bending of a bimorphic element. In the general case, the thicknesses and the elastic properties of layers *1* and *2* are different. Therefore, we adopt in what follows that the neutral layer is located in the volume of layer *2*. Let us choose the right-handed Cartesian coordinate system, whose $x_1 0 x_2$ plane coincides with the neutral layer. Then the lower boundary of the ferrimagnet layer has the coordinate x_3^0. While describing the two-dimensional transverse bending, let us use the Kirchhoff hypotheses [9] for the plane transverse cross-sections $x_1 = \text{const}$ and $x_2 = \text{const}$ (Fig. 1).

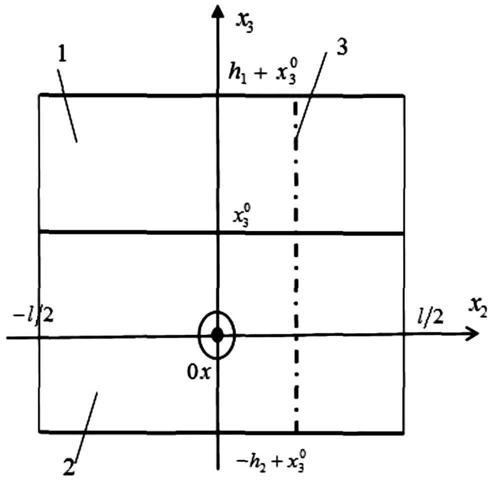

Fig. 1. Schematic diagram of a magneto-optical modulator: (*1*) layer *1* (MOC), (2) layer *2* (magnetically passive), (*3*) plane transverse cross-section ($x_2 = \text{const}$), x_3^0 is the coordinate of the neutral layer

The amplitudes of the elastic stresses harmonically changing in time in layers *1* and *2* will be denoted by $\sigma_\alpha^{(1)}(x_k)$ and $\sigma_\alpha^{(2)}(x_k)$, respectively, where ($\alpha = 1, 2, ..., 6$). Then

$$\sigma_\alpha^{(1)} = c_{\alpha\beta}^H \varepsilon_\beta^{(1)}(x_k) - m_{\rho k\alpha} H_\rho^0 H_k, \tag{1}$$

$$\sigma_\alpha^{(2)}(x_k) = c_{\alpha\beta} \varepsilon_\beta^{(2)}(x_k), \tag{2}$$

where $c_{\alpha\beta}^H$ are the elastic moduli of the elements, ε_β are the strains, and $m_{pq\alpha} \equiv m_{pkij}$ are the components of the matrix of magnetostriction constants for the demagnetized ferrimagnet. Using the relation between the strains ε_β and the bendings $w(x_1, x_2)$ from the theory of transverse bending [9] and taking formulas (1) and (2) into account, we obtain

$$\begin{aligned}
\sigma_1^{(1)} &= -x_3 \left(c_{11}^H \frac{\partial^2 w}{\partial x_1^2} + c_{12}^H \frac{\partial^2 w}{\partial x_2^2} \right) - m_{\rho q1} H_\rho^0 H_q;\\
\sigma_2^{(1)} &= -x_3 \left(c_{12}^H \frac{\partial^2 w}{\partial x_1^2} + c_{11}^H \frac{\partial^2 w}{\partial x_2^2} \right) - m_{\rho q2} H_\rho^0 H_q;\\
\sigma_1^{(2)} &= -x_3 \left(c_{11} \frac{\partial^2 w}{\partial x_1^2} + c_{12} \frac{\partial^2 w}{\partial x_2^2} \right);\\
\sigma_2^{(2)} &= -x_3 \left(c_{12} \frac{\partial^2 w}{\partial x_1^2} + c_{11} \frac{\partial^2 w}{\partial x_2^2} \right);\\
\sigma_6^{(1)} &= -x_3 \left(c_{11}^H - c_{12}^H \right) \frac{\partial^2 w}{\partial x_1 \partial x_2} - m_{\rho q6} H_\rho^0 H_q;\\
\sigma_6^{(2)} &= -x_3 (c_{11} - c_{12}) \frac{\partial^2 w}{\partial x_1 \partial x_2}.
\end{aligned} \tag{3}$$

Consider an elementary rectangular prism with the height $h_1 + h_2$ and the base sides dx_1 and dx_2. Proceeding from the dynamic equilibrium condition, we find that the lateral faces $(x_1, x_1 + dx_1)$ and $(x_2, x_2 + dx_2)$ undergo the action of forces with the linear densities

$$q_{13}(x_1, x_2) = -D_0 \frac{\partial}{\partial x_1}(\nabla^2 w), q_{23}(x_1, x_2) =$$
$$= -D_0 \frac{\partial}{\partial x_2}(\nabla^2 w), \tag{4}$$

respectively, where D_0 is the bending stiffness of the bimorphic element,

$$D_0 = \frac{1}{2}c_{11}^H\Big\{h_1(h_1+x_3^0)^2 - \frac{1}{3}[(h_1+x_3^0)^3-(x_3^0)^3]+ \\ + h_2[(h_1+x_3^0)^2-(x_3^0)^2]\Big\}+ \\ + \frac{1}{2}c_{11}\Big\{h_2(x_3^0)^3-\frac{1}{3}[(x_3^0)^2+(h_2-x_3^0)^3]\Big\}.$$

The force of inertia characterized by the surface density $\sigma_{\rm in} = -(h_1\rho_1+h_2\rho_2)\frac{\partial^2 w}{\partial t^2}$, where ρ_1 and ρ_2 are the densities of layers *1* and *2*, acts along the axis x_3. According to Newton's third law, in the absence of external forces,

$$\frac{\partial q_{13}(x_1,x_2)}{\partial x_1}+\frac{\partial q_{23}(x_1,x_2)}{\partial x_2}- \\ -(h_1\rho_1+h_2\rho_2)\frac{\partial^2 w}{\partial t^2}=0. \qquad (5)$$

From expression (5) with regard for Eqs. (4), we obtain the following equation for the vibrations of a bimorphic element in the case of uniform magnetic fields in its volume:

$$\nabla^2\nabla^2 w-\lambda^4 w=0, \qquad (6)$$

where $\lambda^4=\frac{\omega^2}{D_0}(h_1\rho_1+h_2\rho_2)$.

In order to solve Eq. (6), we have to know the neutral-layer coordinate x_3^0. For this purpose, let us use the fact that the resultants of normal stresses in the plane transverse cross-sections, which are parallel to the axis $0x_3$, are equal to zero. As a result, we obtain the expression

$$\frac{1}{2}\sigma_\alpha^{(1)}(x_3^0)x_3^0+\sigma_\alpha^{(1)}(x_3^0)h_1+\frac{1}{2}[\sigma_\alpha^{(1)}(h_1+x_3^0)- \\ -\sigma_\alpha^{(1)}(x_3^0)]h_1=\frac{1}{2}\sigma_\alpha^{(2)}(-h_2+x_3^0)h_2,\ \alpha=1,2,6 \qquad (7)$$

Taking into account that Newton's third law is obeyed at the interface between the layers, we find

$$-x_3^0(c_{11}^H-c_{12}^H)\frac{\partial^2 w}{\partial x_1\partial x_2}-m_{pq6}H_p^0H_q\Big|_{x_3^0}= \\ =-x_3^0(c_{11}-c_{12})\frac{\partial^2 w}{\partial x_1\partial x_2}. \qquad (8)$$

Applying expressions (3) and (7) to, e.g., the tangential stresses $\sigma_6^{(1)}$ and $\sigma_6^{(2)}$, we have

$$\Big\{\frac{1}{2}(c_{11}-c_{12})\Big[x_3^0-\Big(\frac{h_2}{x_3^0}-1\Big)(h_2-x_3^0)\Big]+ \\ +\Big(h_1+\frac{1}{2}\frac{h_2}{x_3^0}\Big)(c_{11}^H-c_{12}^H)\Big\}x_3^0\frac{\partial^2 w}{\partial x_1\partial x_2}= \\ =-h_1m_{pq6}(H_p^0H_q)|_{x_3^0}+ \\ +\frac{1}{2}h_1\Big[m_{pq6}(H_p^0H_q)\Big|_{h_1+x_3^0}-m_{pq6}(H_p^0H_q)\Big|_{x_3^0}\Big]. \qquad (9)$$

Using Eqs. (8) and (9) and adopting the magnetic fields to be uniform over the specimen volume, we obtain the sought quantity,

$$x_3^0=\frac{h_2^2(c_{11}-c_{12})-h_1^2(c_{11}^H-c_{12}^H)}{2(h_1+h_2)(c_{11}-c_{12})}. \qquad (10)$$

Let us consider the algorithm of solution for Eq. (6) under the condition of free bimorphic element sides. The normal stresses distributed over the transverse cross-sections $x_1=\mathrm{const}$ and $x_2=\mathrm{const}$ create the bending moments M_1 and M_2, which are determined by the linear densities m_1 and m_2, so that $dM_1=m_1dx_2$ and $dM_2=m_2dx_1$. In turn, the densities are determined by the equations

$$m_1=-D_1^*\left(\frac{\partial^2 w}{\partial x_1^2}+k\frac{\partial^2 w}{\partial x_2^2}\right)-m_1^H; \\ m_2=-D_2^*\left(k\frac{\partial^2 w}{\partial x_1^2}+\frac{\partial^2 w}{\partial x_2^2}\right)-m_2^H, \qquad (11)$$

where the notations

$$D_1^*=\frac{1}{3}\Big\{c_{11}\left[(x_3^0)^3+(h_2-x_3^0)^3\right]+ \\ +c_{11}^H\left[(h_1-x_3^0)^3-(x_3^0)^3\right]\Big\},$$

$$D_2^*=\frac{1}{3}\Big\{c_{12}\left[\left(x_3^0\right)^3+(h_2-x_3^0)^3\right]+ \\ +c_{12}^H\left[(h_1-x_3^0)^3-(x_3^0)^3\right]\Big\};\ k=D_2^*/D_1^*;$$

$$m_1^H=\int\limits_{x_3^0}^{h_1+x_3^0}x_3m_{\rho q1}H_\rho^0H_qdx_3;$$

$$m_2^H=\int\limits_{x_3^0}^{h_1+x_3^0}x_3m_{\rho q2}H_\rho^0H_qdx_3.$$

are used. If the magnetic field in the bimorphic element volume is uniform, then we have

$$m_\alpha^H = \frac{1}{2}\left[(h_1+x_3^0)^2-(x_3^0)^2\right] m_{pq\alpha} H_p^0 H_q,\ \alpha = 1,2.$$

The boundary conditions are reduced to the expressions

$$q_{13}^* = (\pm b/2,\ x_2) = 0, q_{23}^* = (x_1, \pm l/2) = 0, \tag{12}$$

$$m_1 = (\pm b/2,\ x_2) = 0, m_2 = (x_1, \pm l/2) = 0, \tag{13}$$

where $q_{13}^* = q_1 + \frac{\partial m_{12}}{\partial x_2}$, $q_{23}^* = q_2 + \frac{\partial m_{21}}{\partial x_1}$, $m_{12} = m_{21} = -D_{12}\frac{\partial^2 w}{\partial x_1 \partial x_2} - m_{12}^H$,

$$D_{12} = \frac{1}{3}\Big\{(c_{11}-c_{12})\left[(x_3^0)^3+(h_2-x_3^0)^3\right] + (c_{11}^H - c_{12}^H)\left[(h_1+x_3^0)^3-(x_3^0)^3\right]\Big\},$$

$$m_{12}^H = m_{21}^H = m_{pq6}\int\limits_{x_3^0}^{h_1+x_3^0} H_p^0 H_q dx_3.$$

The general solution of Eq. (6) is sought in the form

$$w = w_1 + w_2, \tag{14}$$

where

$$w_1(x_1,x_2) = \cos\alpha x_1[A\cos(x_2\sqrt{\alpha^2+\lambda^2}) + B\cos(x_2\sqrt{\alpha^2-\lambda^2})],$$
$$w_2(x_1,x_2) = \cos\beta x_2[C\cos(x_1\sqrt{\beta^2+\lambda^2}) + D\cos(x_1\sqrt{\beta^2-\lambda^2})].$$

Substituting expression (14) into the boundary conditions (13), we obtain

$$m_1^H N_m = D_1^*\frac{l}{2}\Big\{\left[\beta_m^2(1+k)+\lambda^2\right]C_m \times \cos\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right) + \left[\beta_m^2(1+k)-\lambda^2\right]D_m\cos\left(\frac{b}{2}\sqrt{\beta_m^2-\lambda^2}\right)\Big\}, \tag{15}$$

$$m_2^H M_n = D_1^*\frac{b}{2}\Big\{\left[\alpha_n^2(1+k)+\lambda^2\right]A_n \times \cos\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right) + \left[\alpha_n^2(1+k)-\lambda^2\right]B_n\cos\left(\frac{l}{2}\sqrt{\alpha_n^2-\lambda^2}\right)\Big\}, \tag{16}$$

where $N_m = \frac{2l}{\pi(1+2m)}(-1)^m$, $M_n = \frac{2b}{\pi(1+2n)}(-1)^n$, $\alpha_n = \frac{\pi}{b}(1+2n)$, $\beta_m = \frac{\pi}{l}(1+2m)$.

Then, from Eqs. (15) and (16), it follows that

$$C_m = C_m^0 - D_m f_m, \quad A_n = A_n^0 - B_n f_n, \tag{17}$$

where

$$A_n^0 = (-1)^n \times \frac{4m_2^H}{\pi(1+2n)\left[\alpha_n^2(1+k)+\lambda^2\right]D_1^*\cos\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right)},$$

$$C_m^0 = (-1)^m \times \frac{4m_1^H}{\pi(1+2m)\left[\beta_m^2(1+k)+\lambda^2\right]D_1^*\cos\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right)},$$

$$f_m = \frac{\left[\beta_m^2(1+k)-\lambda^2\right]\cos\left(\frac{b}{2}\sqrt{\beta_m^2-\lambda^2}\right)}{\left[\beta_m^2(1+k)+\lambda^2\right]\cos\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right)},$$

$$f_n = \frac{\left[\alpha_n^2(1+k)-\lambda^2\right]\cos\left(\frac{l}{2}\sqrt{\alpha_n^2-\lambda^2}\right)}{\left[\alpha_n^2(1+k)+\lambda^2\right]\cos\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right)}.$$

Hence, we arrive at the following formula for a bending of the bimorphic element $w(x_1,x_2)$:

$$w(x_1,x_2) = B_n\cos\alpha_n x_1\Big[\cos\left(x_2\sqrt{\alpha_n^2-\lambda^2}\right) - f_n(\alpha_n)\cos\left(x_2\sqrt{\alpha_n^2+\lambda^2}\right)\Big] + D_m\Big[\cos\left(x_1\sqrt{\beta_m^2-\lambda^2}\right) - f_m(\beta_m) \times \cos\left(x_1\sqrt{\beta_m^2+\lambda^2}\right)\Big]\cos\beta_m x_2 + w_0(x_1,x_2), \tag{18}$$

where

$$w_0(x_1,x_2) = A_n^0\cos(\alpha_n x_1)\cos\left(x_2\sqrt{\alpha_n^2+\lambda^2}\right) + C_m^0\cos\left(x_1\sqrt{\beta_m^2+\lambda^2}\right)\cos\beta_m x_2.$$

In order to find the coefficients B_n and D_m, let us rewrite the boundary conditions (12) in the form

$$\frac{\partial}{\partial x_1}\psi_2(x_1,x_2)\,|_{x_1=\pm b/2} = 0\,, \tag{19}$$

$$\frac{\partial}{\partial x_2}\psi_1(x_1,x_2)\,|_{x_2=\pm l/2} = 0\,, \tag{20}$$

where the notations $\psi_1(x_1,x_2) = (1+\xi)\frac{\partial^2 w}{\partial x_1^2} + + \frac{\partial^2 w}{\partial x_2^2}$, $\psi_2(x_1,x_2) = \frac{\partial^2 w}{\partial x_1^2} + (1+\xi)\frac{\partial^2 w}{\partial x_2^2}$, $\xi = D_{12}/D_0$ were introduced. One can easily see that condition (19) looks like

$$(-1)^n\alpha_n\beta_m\Big\{[\alpha_n^2 + (1+\xi)(\alpha_n^2-\lambda^2)] \times$$
$$\times\ \cos(x_2\sqrt{\alpha_n^2-\lambda^2} - f_n(\alpha_n)[\alpha_n^2+(1+\xi)(\alpha_n^2+\lambda^2)] \times$$
$$\times\ \cos(x_2\sqrt{\alpha_n^2+\lambda^2})\Big\} \pm D_m\Big\{\sqrt{\beta_m^2-\lambda^2}[\beta_m^2-\lambda^2 +$$
$$+ (1+\xi)\beta_m^2]\sin\left(\frac{b}{2}\sqrt{\beta_m^2-\lambda^2}\right) - f_m\sqrt{\beta_m^2+\lambda^2} \times$$
$$\times [\beta_m^2+\lambda^2+(1+\xi)\beta_m^2]\sin\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right)\Big\}\times$$
$$\times\ \cos(\beta_m x_2) = -\frac{\partial}{\partial x_1}\left[\frac{\partial^2 w_0}{\partial x_1^2} +\right.$$
$$\left.+ (1+\xi)\frac{\partial^2 w_0}{\partial x_2^2}\right]\Big|_{x_1=\pm b/2}. \tag{21}$$

Let us multiply both sides of equality (21) by $\cos(\beta_m x_2)$ and integrate over the variable x_2 from $-l/2$ to $l/2$. The obtained result can be expressed in the form

$$\mp\sum_{n=0}^{\infty} B_n F_{nm} \pm \frac{l}{2} D_m F_m =$$
$$= \pm\sum_{n=0}^{\infty} A_n^0 a_{nm} \mp \frac{l}{2} C_m^0 c_m, \tag{22}$$

where

$$a_{nm} = 2(-1)^n(-1)^m\alpha_n\beta_m\frac{\alpha_n^2+(1+\xi)(\alpha_n^2+\lambda^2)}{\alpha_n^2+\lambda^2-\beta_m^2}\times$$
$$\times\cos\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right),$$
$$c_m = \sqrt{\beta_m^2+\lambda^2}[\beta_m^2+\lambda^2+(1+\xi)\beta_m^2]\sin\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right),$$
$$F_{nm} = 2(-1)^n(-1)^m\beta_m\alpha_n\left\{\frac{\alpha_n^2+(1+\xi)(\alpha_n^2-\lambda^2)}{\alpha_n^2-\lambda^2-\beta_m^2}\times\right.$$
$$\times\cos\left(\frac{b}{2}\sqrt{\alpha_n^2-\lambda^2}\right) -$$
$$\left.- f_n\frac{\alpha_n^2+(1+\xi)(\alpha_n^2+\lambda^2)}{\alpha_n^2+\lambda^2-\beta_m^2}\ \cos\left(\frac{b}{2}\sqrt{\alpha_n^2+\lambda^2}\right)\right\},$$
$$F_m = \sqrt{\beta_m^2-\lambda^2}[\beta_m^2-\lambda^2+(1+\xi)\beta_m^2]\times$$
$$\times\sin\left(\frac{b}{2}\sqrt{\beta_m^2-\lambda^2}\right) -$$
$$- f_m\sqrt{\beta_m^2+\lambda^2}[\beta_m^2+\lambda^2+(1+\xi)\beta_m^2]\times$$
$$\times\ \sin\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right).$$

Analogous transformations of the boundary conditions (20) bring us to the equality

$$\mp\frac{b}{2}B_nF_n \pm \sum_{m=0}^{\infty} D_m F_{mn} =$$
$$= \pm\frac{b}{2}A_n^0 a_n \mp \sum_{m=0}^{\infty} C_m^0 c_{mn}, \tag{23}$$

where

$$a_n = \sqrt{\alpha_n^2+\lambda^2}[(1+\xi)\alpha_n^2+\alpha_n^2+\lambda^2]\sin\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right),$$
$$c_{mn} = 2(-1)^m(-1)^n\alpha_n\beta_m\frac{\beta_m^2+(1+\xi)(\beta_m^2+\lambda^2)}{\beta_m^2+\lambda^2-\alpha_n^2}\times$$
$$\times\ \cos\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right),$$
$$F_n = \sqrt{\alpha_n^2-\lambda^2}[(1+\xi)\alpha_n^2+\alpha_n^2-\lambda^2]\times$$
$$\times\ \sin\left(\frac{l}{2}\sqrt{\alpha_n^2-\lambda^2}\right) - f_n\sqrt{\alpha_n^2+\lambda^2}[(1+\xi)\alpha_n^2 +$$
$$+ \alpha_n^2+\lambda^2]\sin\left(\frac{l}{2}\sqrt{\alpha_n^2+\lambda^2}\right),$$
$$F_{mn} = 2(-1)^m(-1)^n\beta_m\alpha_n\left\{\frac{\beta_m^2+(1+\xi)(\beta_m^2+\lambda^2)}{\beta_m^2-\lambda^2-\alpha_n^2}\times\right.$$
$$\times\ \ \cos\left(\frac{b}{2}\sqrt{\beta_m^2-\lambda^2}\right) -$$
$$\left.- f_m\frac{\beta_m^2+(1+\xi)(\beta_m^2+\lambda^2)}{\beta_m^2+\lambda^2-\alpha_n^2}\ \cos\left(\frac{b}{2}\sqrt{\beta_m^2+\lambda^2}\right)\right\}.$$

The solution of the system of equations (22), (23) should be substituted into formula (18) to obtain an expression for the calculated bending $w(x_1,x_2)$. Note that the solutions of Eq. (18) contain the quantities proportional to the linear moments m_1^H and m_2^H, which are determined, in turn, by the parameters of an ac magnetic field.

Shear deformations in the ferrimagnetic layer give rise to an additional variation of the projection J_3 of the alternating magnetization vector component directed along the light beam, as well as to the rotation of the polarization plane, owing to the Faraday effect, by the angle $\phi = \int_{x_3^0}^{h_1+x_3^0}\sigma_6^{(1)}\Lambda\alpha_f dx_3$, where $\Lambda = \partial J_3/\partial\sigma_6^{(1)}$ [10], and $\alpha_{\rm F}$ is the specific angle of a polarization plane rotation. In view of expressions

(3) and (14), we obtain the distribution function of the polarization plane rotation $\phi(x_1, x_2)$ over the light beam position on the bimorphic element area,

$$\phi = \frac{\alpha_{\mathrm{F}}\Lambda\left[\left(h_1 + x_3^0\right)^2 - \left(x_3^0\right)^2\right]\left(c_{11}^H - c_{12}^H\right)}{2} \times$$
$$\times \left\{\alpha_n \sin \alpha_n x_1 \left[A_n \sqrt{\alpha_n^2 + \lambda^2} \sin(x_2 \sqrt{\alpha_n^2 + \lambda^2}) +\right.\right.$$
$$\left. + B_n \sqrt{\alpha_n^2 - \lambda^2} \sin(x_2 \sqrt{\alpha_n^2 - \lambda^2})\right] +$$
$$+ \beta_m \sin \beta_m x_2 \left[C_m \sqrt{\beta_m^2 + \lambda^2} \sin(x_1 \sqrt{\beta_m^2 + \lambda^2}) +\right.$$
$$\left.\left. + D_m \sqrt{\beta_m^2 - \lambda^2} \sin(x_1 \sqrt{\beta_m^2 - \lambda^2})\right]\right\}. \tag{24}$$

3. Main Results and Their Discussion

The system of equations (22), (23) has no exact solution. They will be solved approximately for the fundamental mode ($m = 0, n = 0$) of a bimorphic element $b \times l = 10 \times 10$ mm^2 in dimensions, in which layer *1* is made of bismuth-substituted yttrium ferrite garnet $Y_{3-x}Bi_xFe_5O_{12}$. The relevant parameters are $h_1 = 0.3$ mm, $c_{11}^H = 269$ GPa, $c_{12} = 108$ GPa, $\rho_1 = 5.7 \times 10^3$ kg/m^3, $m_{111} = m_{112} = 2$ N/A^2 (for $H_1^0 = 600$ A/m [1]), $\Lambda = 2 \times 10^{-5}$ T^{-1}, and $\alpha_{\mathrm{F}} = 3.2$ A^{-1} [11]. For layer *2* made of gadolinium-gallium garnet ($Gd_3Ga_5O_{12}$), $h_2 = 0.5$ mm, $c_{11} = 287$ GPa, $c_{12} = 163$ GPa, and $\rho_1 = 7.08 \times 10^3$ kg/m^3.

While searching the approximate solutions, we transform the system of equations (22), (23) into the form

$$\left(\mp 2 \sum_{n=0}^{\infty} B_n F_{nm}/lF_m\right) \pm D_m = \Delta_1 \tag{25}$$

$$\mp B_n \pm 2 \sum_{m=0}^{\infty} D_m F_{mn}/bF_n = \Delta_2, \tag{26}$$

where

$$\Delta_1 = 2\left(\pm \sum_{n=0}^{\infty} A_n^0 a_{nm} \mp \frac{l}{2} C_m^0 c_m\right) \Big/ lF_m,$$
$$\Delta_2 = 2\left(\pm \frac{b}{2} A_n^0 a_n \mp \sum_{m=0}^{\infty} C_m^0 c_{mn}\right) \Big/ bF_n.$$

The right-hand sides of expressions (25) and (26) are frequency-dependent constants. Note that, in the course of numerical calculations, the error obtained while keeping six first terms in the series for Δ_1 and Δ_2 did not exceed one percent of the exact result in vicinities of the magneto-mechanic resonance frequencies.

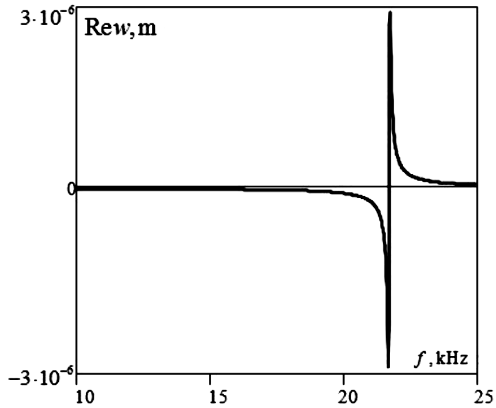

Fig. 2. Dependence of the real part of the bending function w at the center of a bimorphic plate on the linear resonance frequency for the fundamental mode of magneto-mechanical vibrations

At the linear resonance frequency $f_p = 21.641$ kHz for the fundamental mode, the values of Δ_1 and Δ_2 are substantially larger in comparison with the corresponding values for higher modes. For example, the ratio $\Delta_1|_{n=0,m=0}/\Delta_1|_{n=0,m=1} \approx 10^3$ at the frequency f_p. A similar relation is also observed for Δ_2. On the other hand, if the bimorphic plate shape approaches a square, then, at the equal m and n, it is possible to consider that the quantities B_n and D_m are of the same order of magnitude. Therefore, when the first term of the series, $2B_0F/lF_m|_{m=0}$, and the next one, $2B_1F_{1m}/lF_m|_{m=0}$, in Eq. (25) are numerically compared with each other at the frequency f_p, the latter and all the following terms can be neglected. As a result, Eq. (25) acquires a simplified form

$$\mp 2B_0F_{nm}/lF_m \pm D_0 = \Delta_1, \; n = 0, \; m = 0. \tag{27}$$

In a similar way, Eq. (26) can be transformed to the form

$$\pm 2D_0F_{mn}/bF_n \mp B_0 = \Delta_2, \; n = 0, \; m = 0 \tag{28}$$

The further solution of the system of equations (27), (28) allows one to easily determine the coefficients B_0 and D_0.

In Fig. 2, the dependence of the real part of the bending function w on the linear frequency f at the center of a bimorphic plate with the dimensions corresponding to the fundamental mode of

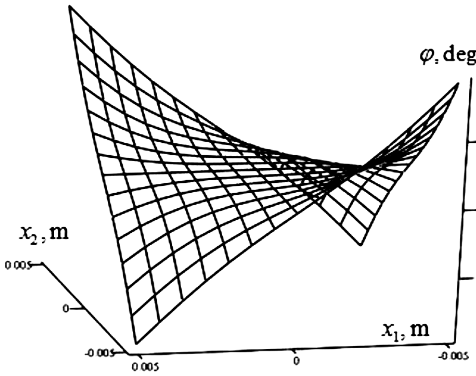

Fig. 3. Distribution function $\phi(x_1, x_2)$ of the light-beam polarization plane rotation angle over the position of a light beam on the surface of the bimorphic element at the resonance frequency for the fundamental vibration mode

magneto-mechanic vibrations is plotted using expression (18). The amplitude of the alternating component of the magnetic field strength vector was taken to equal $H_1 = 100$ A/m. The Q-factor of the vibrating system was taken into account, by substituting the wave number λ by $\lambda\,(1 + i/2Q)$, where $Q = 500$, in all expressions. In accordance with expression (10), the coordinate of the neutral layer was equal to $x_3^0 = 0.083$ mm.

In Fig. 3, the calculated dependence of the function $\phi(x_1, x_2)$ describing the rotation angle of the polarization plane on the light-beam coordinates on the bimorphic element area is shown for the linear resonance frequency $f_p = 21.641$ kHz. One can see that the magneto-optical light modulator on the basis of a magneto-optical crystal as a part of the bimorphic structure operating in the mode of magneto-mechanic vibrations and fabricating with the use of bismuth-substituted yttrium ferrite garnet on the gallium-gadolinium substrate can provide the amplitude of a polarization plane rotation angle up to 3° in the optical-beam cross-section.

The resonance frequency of the fundamental mode for a square bimorphic element 1×1 mm^2 in size amounts to 21.6 kHz. The obtained value of resonance frequency is an order of magnitude lower than the corresponding values obtained at tension-compression deformations of thin rods [1].

The Cotton–Mouton effect results in modulations of the optical radiation ellipticity. As a result of the square-law dependence of this effect on the transverse magnetization component and the relation $H_1 \ll H_1^0$, the alternating component of the ellipticity does not have a pronounced character and, therefore, was not taken into account.

4. Conclusions

On the basis of the model of magneto-mechanic vibrations of a bimorphic element asymmetric across the thickness and with free edges, the equation for specimen vibrations is obtained with the help of Kirchhoff hypotheses, and the position of the neutral layer in the asymmetric bimorphic element is found. To find the solution of this equation, an approximate technique is applied, which provides an error of about 10^{-3} for the bending amplitude within the interval of resonance frequencies for the fundamental mode. An expression was obtained for the two-dimensional distribution function for rotation angles of the light-beam polarization plane at the output of the optical modulator with the bimorphic structure consisting of a bismuth-substituted yttrium ferrite garnet crystal on the gallium-gadolinium substrate. Using this expression, it is shown that, owing to the Faraday effect, the amplitude of this angle can reach a value of 3° for the fundamental mode of vibrations of the bimorphic element. Modulators with the two-dimensional distribution function for polarization plane rotation angles can be used in rotation angle sensors.

1. I.V. Linchevskyi and O.N. Petrischev, Ukr. Fiz. Zh. **56**, 496 (2011).
2. I.V. Linchevskyi, T.I. Shevchenko, and V.D. Tron'ko, Opt. Spektrosk. **114**, 124 (2013).
3. I.V. Linchevskyi, Opt. Spektrosk. **115**, 785 (2013)
4. I.V. Linchevskyi, Radiofiz. Elektron. **2**, 43 (2014).
5. J.G. Smits, S I. Dalke, and T.K. Cooney, Sensors Actuat. A **28**, 41 (1991).
6. V. Sharapov, M. Musienko, and E. Sharapova, *Piezoelectric Sensors* (Tekhnosfera, Moscow, 2006) (in Russian).
7. F. Sammjura, K. Smyth, and S.-G. Kim, Ultrasonics **53**, 328 (2013).
8. J.K. Lee and M.A. Marcus, Ferroelectrics **32**, 93 (1981).
9. L.H. Donnell, *Beams, Plates, and Shells* (McGraw-Hill, New York, 1976).
10. R.M. Bozorth, *Ferromagnetism* (Van Nostrand, New York, 1959).
11. P. Hansen and J.-P. Krumme, Thin Solid Films **114**, 69 (1984).

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МОДУЛЯТОР СВІТЛА
НА МАГНІТООПТИЧНОМУ КРИСТАЛІ
В СКЛАДІ БІМОРФНОЇ СТРУКТУРИ
У РЕЖИМІ МАГНІТОМЕХАНІЧНИХ КОЛИВАНЬ

Р е з ю м е

Запропоновано модель модулятора світла на основі біморфного елемента, що складається з шару магнітооптичного кристала, вирощеного на магнітопасивній підкладці. Отримано рівняння магнітомеханічних коливань асиметричного по товщині біморфного елемента прямокутної форми і запропоновано алгоритм його розв'язку для вільного від закріплення гранями зразка. Показано, що такий модулятор характеризується двовимірною функцією розподілу кута повороту площини поляризації в площині перетину світлового пучка. На прикладі кристала вісмут ітрієвого ферит гранату на галій гадолінієвій основі показано, що амплітуда кута повороту площини поляризації світлового променя внаслідок ефекту Фарадея може досягати значень 3 град для основної моди коливань біморфного елемента.