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ON THE STRUCTURE OF $K^\pi = 3/2^+$ 362-keV LEVEL IN ^{165}Ho

High-precision measurements are carried out on a $\pi\sqrt{2}$ magnetic β-spectrometer for the relative intensities of the electron lines of internal conversion on the K- and L-shells of ^{165}Ho nucleus, with the 362-keV γ-transition being used. The penetration parameter λ for the M2-component of this transition and the multipole mixing ratio $\delta(E3/M2)$ are determined for the first time. The value of $E3$-multipolarity admixture corresponds to the probability of radiative transition $B(E3)$(362 keV) = (46 ± 6) W.u., which testifies to its collective nature and a probable octupole deformation of ^{165}Ho nucleus.

Keywords: radioactivity, ^{165}Dy, ^{165}Ho, magnetic spectrometer, internal conversion, penetration parameter, multipole mixing ratio.

1. Introduction

While studying the conversion spectrum of ^{165}Dy (the half-life $T_{1/2} = 2.34$ h) – see the fragment of the corresponding decay scheme depicted in Fig. 1 – we found that the γ-transition with an energy of 362 keV between the proton states $K^\pi = 3/2^+$ 3/2[411] and $K^\pi = 7/2^-$ 7/2[523] in ^{165}Ho is characterized by the mixed $(M2 + E3)$-multipolarity [1]. In comparison with theoretical estimations carried out in the framework of the Weisskopf model, the $M2$-component of this transition is hindered, $F_{\rm W}(M2) \approx 12$, and anomalies in the coefficients of internal conversion of γ-rays associated with the penetration effect are possible for it. This circumstance can substantially affect the results of calculations of the mixing parameter $\delta(E3/M2)$ carried out on the basis of conversion data; so it needs additional researches.

2. Experimental Technique

Sources of ^{165}Dy were obtained in the (n,γ) reaction by irradiating targets on a WWR-M reactor with a flux of thermal neutrons to a dose of 5×10^{13} neutron/cm^2. The targets were produced by the sputtering of enriched ^{165}Dy onto an aluminum substrate in vacuum. The thickness of a sputtered layer amounted to about 10 μg/cm^2.

The spectra of internal conversion electrons (ICEs) for the γ-transition with an energy of 362 keV were measured on a magnetic β-spectrometer of the $\pi\sqrt{2}$ type equipped with a positionally sensitive detector (PSD) [2]. In order to obtain more exact data concerning the intensities of ICE lines for this γ-transition on the K- and L-subshells of ^{165}Ho nucleus, we studied the dependence of the registration efficiency by the PSD composed of microchannel plates and located in the focal plane of a β-spectrometer at the detection place. This dependence was found to be nonlinear and varying within the limits of 10–15% along the focal plane of a spectrometer.

In Fig. 2, a section of the ^{165}Dy conversion spectrum located near the L-lines of the γ-transition with an energy of 362 keV is shown. The spectrum was corrected with regard for the registration efficiency of a PSD. The high resolution of a β-spectrometer (0.035% with respect to $B\rho$) made it possible to deter-

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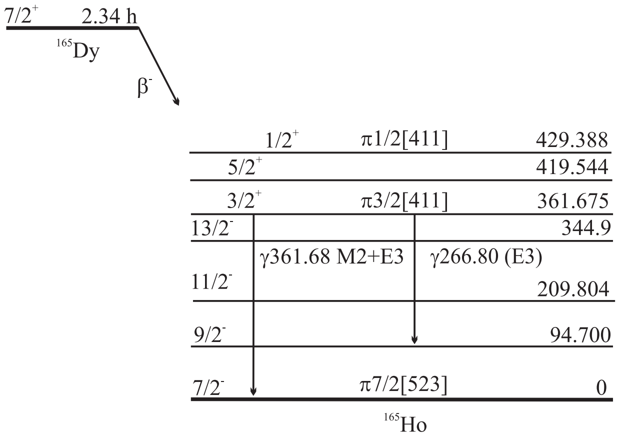

Fig. 1. Fragment of the ^{165}Dy decay scheme

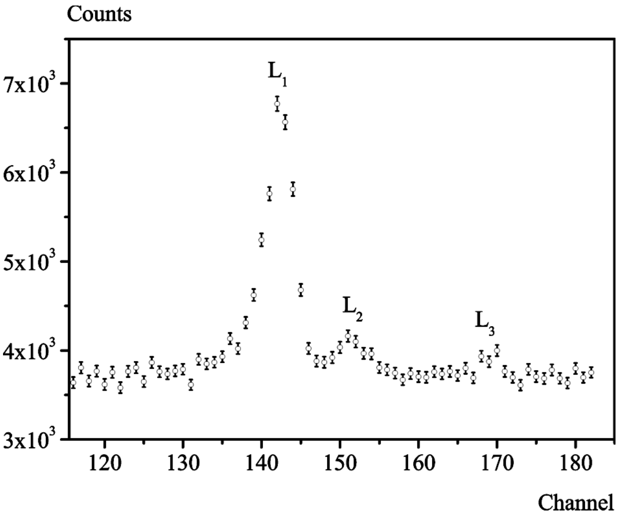

Fig. 2. Section of the conversion spectrum containing ICE lines of the γ-transition with an energy of 362 keV on the L-subshells of ^{165}Ho

Table 1. **Relative intensities of ICE lines of the γ-transition with an energy of 362 keV on the K- and L-shells of ^{165}Ho nucleus**

Shell	I_e, rel. units	Shell	I_e, rel. units
K L_1	643 ± 8 100	L_2 L_3	17.9 ± 0.8 6.9 ± 0.8

mine the relative intensities of conversion lines with an accuracy of 1%.

3. Results and Discussion

The conversion spectra were analyzed with the use of a specifically developed computer program. The corresponding values obtained for the intensities of ICE lines for the γ-transition with an energy of 362 keV on the K- and L-shells of ^{165}Ho nucleus are quoted in Table 1.

Concerning the results obtained by other authors, three experimental works in this domain are known for today. Persson and coauthors [3] determined the internal conversion coefficient (ICC) for the γ-transition with an energy of 362 keV on the K-shell of ^{165}Ho ($\alpha_K = 0.22 \pm 0.04$) and the ratio between the intensities of ICE lines on the K- and L-shells of ^{165}Ho (K/L $= 4.8 \pm 1.2$). In works [4] and [5], the ratios between the ICE line intensities for this transition – K/L $= 6.3 \pm 1.8$ and K/L$_1 = 4.8 \pm 0.5$, respectively – were measured. The result of our measurements, K/L $= 5.16 \pm 0.10$, agrees very well with the data of works [3, 4] and is characterized by a considerably higher accuracy; the ratio K/L$_1 = 6.43 \pm 0.08$ agrees with the result of work [5] to within the limits of three mean-square errors.

The new more accurate values of ICE line intensities were analyzed in view of the intranuclear conversion effect. The matter concerns the anomalies in the γ-ray internal conversion coefficients associated with the penetration effect. As the penetration effect or intranuclear conversion in the theory of internal conversion, we understand a correction to the ICC that arises, when the transient electromagnetic potentials calculated for a point-like nucleus (they arise at the moment, when a nucleus transits from one nuclear level onto another one) are substituted by the transient potentials calculated for a nucleus with finite dimension.

In the case of magnetic multipole transitions, the penetration effect can be described with the use of one nuclear parameter λ [6]. The technique used to analyze the anomalies in the ICCs for the transitions with mixed multipolarities was described in work [7] in detail. The penetration parameter λ and the mixing parameter δ are determined by solving the system of equations for the absolute or relative ICCs. For any i-th subshell of the mixed ($M2 + E3$)-transition, the experimental ICCs, which make allowance for the penetration effect in the $M2$-component, look like [8]

$$\alpha_{i,\,\mathrm{exp}} = \frac{\alpha_i(M2)(1 + B_1^i\lambda + B_2^i\lambda^2) + \delta^2\alpha_i(E3)}{1 + \delta^2}, \quad (1)$$

where B_1^i and B_2^i are parameters depending only on the electron wave functions and tabulated in work [8];

Table 2. **Energies and structures of the ground and first two excited states in ^{159,161,163,165}Ho isotopes**

Nucleus	K^π	Energy, keV		Structure
		experiment	theory	
^{159}Ho	$7/2^-$	0	0	523↑ 97%
	$3/2^+$	–	250	411↑ 94%
	$1/2^+$	206	380	411↓ 88% 411↑ + $Q_1(22)$ 9%
^{161}Ho	$7/2^-$	0	0	523↑ 97%
	$3/2^+$	299	260	411↑ 94% 411↓ + $Q_1(22)$ 3%
	$1/2^+$	211	380	411↓ 88% 411↑ + $Q_1(22)$ 9%
^{163}Ho	$7/2^-$	0	0	523↑ 98%
	$3/2^+$	360	240	411↑ 95% 411↓ + $Q_1(22)$ 2%
	$1/2^+$	298	390	411↓ 91% 411↑ + $Q_1(22)$ 8%
^{165}Ho	$7/2^-$	0	0	523↑ 99%
	$3/2^+$	362	160	411↑ 95%
	$1/2^+$	429	220	411↓ 95%

$\alpha_i(M2)$ and $\alpha_i(E3)$ are the theoretical values of ICCs on the i-th subshell for the $M2$- and $E3$-transitions, respectively; and $\alpha_{i,\,\exp}$ is the experimental ICC value for the i-th subshell. A similar expression can also be written down for the ICC ratios.

While searching the anomalies in the internal conversion coefficients for mixed transitions, it is very important to have precision data on not only relative but also absolute ICC values for various atomic subshells. For this reason, when carrying out the analysis, we used the data from Table 1 together with the value of α_K taken from work [3].

Analogously to what was done in work [9], the system of equations for the absolute and relative ICC values was solved by minimizing the following functional using the least-squares method:

$$\chi^2_{\min} = \left(\frac{\alpha_{i,\,\exp} - \alpha_i(\lambda,\delta)}{\Delta\alpha_{i,\,\exp}}\right)^2 + \\ + \sum_{i,j}\left(\frac{(\alpha_i/\alpha_j)_{\exp} - \alpha_i(\lambda,\delta)/\alpha_j(\lambda,\delta)}{\Delta(\alpha_i/\alpha_j)_{\exp}}\right)^2. \quad (2)$$

Here, $\alpha_{i,\,\exp}$, $\Delta\alpha_{i,\,\exp}$, $(\alpha_i/\alpha_j)_{\exp}$, and $\Delta(\alpha_i/\alpha_j)_{\exp}$ are the experimental ICC value for the i-th subshell and the ICC ratio between the i-th and j-th subshells, as well as their errors. The quantities $\alpha_i(\lambda,\delta)$ and $\alpha_j(\lambda,\delta)$ are the theoretical ICC values for the i-th and j-th subshells and $\alpha_i(\lambda,\delta)/\alpha_j(\lambda,\delta)$ is their ratio; they depend on λ and δ, which are the parameters of the fitting procedure using the $\chi^2_{\min}$-method.

In order to avoid local minima, the initial values of λ and δ were determined by solving the required system of equations graphically. The theoretical ICC values were obtained by interpolating the tabulated values from work [10]; the electron parameters were taken from work [8].

The standard errors were determined with the help of the relation

$$\chi^2(\lambda_{\text{opt}} \pm \Delta\lambda) = \chi^2_{\min} + 1, \quad (3)$$

where λ_{opt} is the optimum value of parameter λ, which minimizes the quantity χ^2. All other parameters are fixed at that and correspond to their optimum values. The errors for δ were determined analogously. The following results were obtained: $-2.7 \le \lambda_{\exp} \le$ ≤ 2.5 and $|\delta(E3/M2)| = 0.329 \pm 0.022$. The experimental value of penetration parameter does not contradict the value $\lambda_{\text{th}} \approx 1$ expected for this kind of transitions [7].

The half-life period of level $3/2^+$ 362 keV in ^{165}Ho is known. According to the compilation data presented in work [11], it amounts to $T_{1/2}(362.675\,\text{keV}) =$ $= (1.512 \pm 0.004) \times 10^{-6}$ s. The fraction of the $E3$-multipolarity admixture determined by us corresponds to the radiative transition probability $B(E3)(362$ keV$) = (46 \pm 6)$ W.u., which testifies to its collective nature and, probably, to the octupole deformation of ^{165}Ho nucleus. We note also that the probability of the γ267 keV radiative transition from level $3/2^+$ 362 keV onto the first rotational level $9/2^-$ 95 keV of the ground state of ^{165}Ho amounts also to a few one-particle units $B(E3)(267$ keV$) =$ $= (6.7 \pm 2.1)$ W.u. [11].

V.G. Solov'ev *et al.* [12, 13] calculated the structures of excited non-rotational states of ^{159,161,163,165}Ho isotopes in the framework of the superfluid nucleus model with regard for the interaction between quasiparticles and vibrational phonons. The results of their calculations for excitation energies lower that 500 keV are listed in Table 2. The fifth column of the table contains the contributions (in per cent) of the largest wave-function components obtained from the normalization condition of the wave function. For example, for the ^{165}Ho state with $K^\pi = 1/2^+$, the notation $411 \downarrow$ 88% means a contribution made by a one-quasiparticle state, whereas the notation $411 \uparrow + Q_1(22)$ 9% means the contribution of a component composed of a quasiparticle in the state $411 \uparrow$ and a phonon $Q_1(22)$.

According to the results of calculations [12, 13], the first excited non-rotational level in ^{159,161,163,165}Ho nuclei is the one-quasiparticle state with $K^\pi = 3/2^+$ 3/2[411], and the second one with $K^\pi = 1/2^+$ 1/2[411]. This sequence of levels agrees with experimental data only for ^{165}Ho. In the case of ^{161}Ho and ^{163}Ho nuclei, the levels are arranged in the inverse order. For ^{159}Ho, no level with $K^\pi = 3/2^+$ has been found yet.

The structural characteristics of excited states in ^{159,161,163,165}Ho isotopes also turned out similar. It was demonstrated that the excited states of those nuclei possess a complicated structure, so that only the lowest states and a small number of states with higher excitation energies were found to resemble one-quasiparticle ones.

According to the data quoted in Table 2, the contributions of one-quasiparticle state to the wave function structure for the levels with $K^\pi = 1/2^+$ and $K^\pi = 3/2^+$ are not very different. However, the experimental data demonstrate that it not so. In ^{159}Ho, ^{161}Ho, and ^{163}Ho nuclei, the probabilities of the radiative $E3$-transitions γ206, γ211, and γ298 keV from the level $1/2^+$ into the ground state are equal to $B(E3)(206\text{ keV}) = 0.025 \pm 0.004$, $B(E3)(211\text{ keV}) = 0.00281 \pm 0.00004$, and $B(E3)(298\text{ keV}) = 0.00264 \pm 0.00008$ (in terms of the single-particle Weisskopf units) [11]. An absolutely different picture is obtained for the discharge of level $3/2^+$ 362 keV in ^{165}Ho.

The authors of works [12, 13] drew conclusion that the deviation of the equilibrium deformation of a nucleus in the excited state from its equilibrium deformation in the ground state can substantially affects the energy and the structure of a state close to the one-quasiparticle one. Probably, just this situation takes place in the case of ^{165}Ho.

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ПРО СТРУКТУРУ РІВНЯ $K^\pi = 3/2^+$, 362 кеВ В ^{165}Но

Резюме

На магнітному β-спектрометрі типу $\pi\sqrt{2}$ з високою точністю поміряні відношення інтенсивностей електронів внутрішньої конверсії на K- та L-оболонках ^{165}Но для γ-переходу з енергією 362 кеВ. Вперше визначені параметр проникнення λ для $M2$-компонента цього γ-переходу та параметр змішування $\delta(E3/M2)$. Величина домішки $E3$-мультипольності відповідає ймовірності радіаційного переходу $B(E3)$(362 кеВ) = (46 ± 6) W.u., що свідчить про її колективну природу і, можливо, про октупольну деформацію ядра ^{165}Но.